Localized resistive regions in superconducting thin films

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A phenomenological model for resistive domains produced in semiconducting thin films on passage of a transport current through them is presented. The resistivity is pronouncedly nonequilibrium and is due to a magnetic flux through the specimen. The domains appear at sites of edge defects or inhomogeneities whose role reduces to lowering of the potential barrier to the entrance of the vortices. The kinetics of the flux in the specimen and the dissipation caused by it are considered. The heat-balance equation for a film with a domain is solved and the current-voltage characteristic (CVC) is calculated. Some quantitative features of the CVC are predicted, viz., absence of hysteresis at thermostat temperature T_0 close to the superconductor critical temperature T_c, the presence of a voltage discontinuity under given-current conditions, passage of the differential conductivity $\sigma(T_0)$ of the initial resistive part of the CVC through a maximum, the presence of an excess current in the resistive part on the forward CVC after the temperature instability sets in, and others. Results are presented of an experimental verification of the model by measuring the CVC of thin indium films at thermostat temperatures $\zeta_0 = 1 - T_0/T_c$ between 10^{-4} and 10^{-1}. The experimental and theoretical results are compared qualitatively and quantitatively.

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INTRODUCTION

The destruction of the superconducting state by the transport current in thin films of Al, In, Sn and other soft superconductors 100 to 3000 Å thick is local in character.[1] It is the result of penetration of the magnetic flux at the location of edge defects or inhomogeneities, and causes local overheating of the sample, while in the given-current regime it is accompanied by development of temperature instabilities.[2] Under certain conditions the resistive (in particular cases, normal) regions produced where the magnetic flux penetrates do not extend over the entire sample, and either remain localized[3] or can move as a unit, as predicted by Gurevich and Mintz.[4]

It should be noted that the question of coexistence and stability of finite regions of normal and superconducting phases was investigated quite some time ago in connection with the onset of an intermediate state in a magnetic field. The intermediate state, however, is isothermally and thermodynamically in equilibrium, whereas the superconducting state considered by Mintz[5] and by Gurevich and Mintz[3,4] is essentially nonequilibrium (albeit stationary) and is not isothermal. The onset of nonisothermal local dissipative regions was investigated experimentally in Ref. 2. Their appearance on the current-voltage characteristics (CVC) differed substantially from that predicted by Gurevich and Mintz. This, however, is not surprising, since the analysis in Refs. 3–5 does not take into consideration the specifics of the processes that take place in thin films, and is applicable only to the situation realized in the case of thin combined superconductors.

We propose in this paper a phenomenological model that describes the onset of resistive domains in superconducting films. The main predictions of this model are compared with the results of experiments aimed at its verification.

DESCRIPTION OF MODEL

As shown in Refs. 1 and 2, the appearance of resistivity in thin superconducting films with weak pinning is due to the penetration of the vortices or flux tubes into regions whose energy barrier for the penetration is lowered for some reason. The vortices that penetrate in the film move perpendicular to the current direction. The energy dissipation along their trajectory, which takes place in the superconductor, leads to a local heating of the sample. Since the thermal length λ_T greatly exceeds all other characteristic lengths of the superconductor, one can use the one-dimensional heat-balance equation with a δ-function term whose form and derivation are given in Refs. 2, 3, and 5. The solution of this equation is written in the form

$$|x| = \int_T^{T_m} \frac{dT'\,K(T')}{[2S(T')]^{1/2}}, \quad S(T) = \int_{T_0}^{T} dT\,K(T)\,[W(T) - Q(T)], \tag{1}$$

where $K(T)$ is the thermal-conductivity coefficient, $W(T) = h(T)(T - T_0)/d$ is the heat transferred to the external medium, $Q(T)$ is the specific heat release, $h(T)$ is the heat-transfer coefficient, d is the sample thickness, and T_0 is the thermostat temperature.

The minimum temperature T_m at the center of the domain is determined from the equation

$$\frac{1}{8} F^2(T) = S(T), \quad F(T) = \int_{-l}^{l} dx[Q(T,x) - Q(T) - W(T,x) + W(T)], \tag{2}$$

where $Q(T,x) = jE(T,x)$ and $W(T,x)$ are the values of $Q(T)$ and $W(T)$ due to the inhomogeneity, $2l$ is the region of localization of the inhomogeneity, $E(T,x)$ is the electric field, x is the coordinate in the current direction, and j is the current density.

It is reasonable to assume that, just as in the case of thin channels,[6] the change of the longitudinal electric field takes place over a characteristic length[7,8] $l_E = l_D(4T/\pi\Delta)^{1/2}$ and is determined by an equation of the

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diffusion type, so that $E(x)$ can be expressed in the form

$$E(x)=[E(0)-E_0]\exp(-x/l_E)+E_0, \quad x>0, \tag{3}$$

where l_0 is the diffusion length of the quasiparticles, Δ is the modulus of the order parameter, $E(0)$ is the value of E at $x=0$, and E_0 is a constant field connected with the flux produced when the current exceeds the critical value $j_c(T)$ in the homogeneous part of the sample.

The quantities $E(0)$ and E_0 are determined by the character of the flow of the magnetic flux in the sample and depend therefore on T and j. To determine the distribution of the temperature $T(x)$ of the domain and the voltage on it at different values of the current it is necessary to obtain expressions for $E(0, j, T)$ and $E_0(j, T)$ in explicit form. The gist of the proposed model is in fact the determination of these quantities.

One of the assumptions of the model is the following. Since the inhomogeneity on which the domain is produced is localized in a region with a linear dimension λ_e of the order of the size of the entering vortex, and this is the smallest length ($\lambda_e \ll \lambda_T \ll L$), where L is the width of the film, we can assume with good accuracy ($\sim\lambda_e/L$) that the heat-release conditions along the entire film are the same. Therefore only the local change of the heat release $[Q(T,x)-Q(T)]$ contributes to $F(T)$.

For the same reason, in contrast to Gurevich and Mintz,[3] we neglect the local change of the electric and thermal characteristics of the sample in the location of the domain. The defects or inhomogeneities that give rise to the domain perform a single function—they decrease the critical current at which flux from $j_c(T)$ to $j_1(T)$ begins.

When studying the d.c. current-voltage characteristics, the quantity $E(x)$ in (3) should be regarded as the time-averaged longitudinal field. Therefore $E(x)$ is connected with the mean value of the gauge-invariant scalar potential Φ in the following manner:

$$E(x)=-\nabla\overline{\Phi}, \tag{4a}$$

$$\Phi=\varphi+\frac{\hbar}{2e}\frac{\partial\chi}{\partial t}, \tag{4b}$$

where the bar denotes time averaging, φ is the scalar potential of the electric field, and χ is the phase of the order parameter.

Recognizing that the line $x=0$ is the trajectory of the vortices, we integrate Eq. (4a) in a direction perpendicular to the trajectory, and make the distance between the initial (1) and final (2) points equal to the coherence length $\xi'(T)$ (the dimension of the vortex core); these points are on opposite sides of the line $x=0$. Since the field and the electric potential are constant at a distance on the order of $\xi'(T)$, the results of this operation lead to the relation

$$E(0)=\frac{\overline{\Phi}_2-\overline{\Phi}_1}{\xi'(T)}=\frac{\hbar}{2e\xi'(T)}\overline{\frac{\partial\delta}{\partial t}}. \tag{5}$$

Here $\delta=\chi_2-\chi_1$ is the phase difference between points 1 and 2. Since passage of one flux quantum Φ_0 through the line joining points 1 and 2 changes the phase difference by 2π, the average time derivative of the phase difference is $2\pi/\tau$, where τ is the time required for the vortex to negotiate the distance $a(j,T)$ between two neighboring vortices in a direction perpendicular to the x axis.

In the case of viscous motion of the vortices

$$v=[j-j_p(T)]\Phi_0/c\eta, \tag{6}$$

where $j_p(T)$ is the pinning current, $\Phi_0=ch/2e$ is the magnetic-flux quantum, c is the speed of light, $\eta=\pi\hbar^2/e^2[\xi'(T)]^2\rho$ is the viscosity coefficient, and ρ is the resistivity of the film in the normal state. Consequently,

$$E(0)=\frac{\pi}{2}\rho\frac{\xi'(T)[j-j_p(T)]}{a(j,T)}. \tag{7}$$

When determining $a(j,T)$ we shall make use of the following assumptions which are based on the experimentally observed characteristics.[1] In the vicinity of the critical current $j_1(T)$, when the barrier preventing the entry of the vortices into the inhomogeneity region first vanishes and the flux sets in, the characteristic is almost linear. We therefore have in this current interval [when $j_p(T)$ can be neglected] $a(j,T)\sim[j-j_1(T)]^{-1}$. At larger currents $j>j_c(T)$ the vortex density becomes sufficiently large and can therefore be assumed to depend little on the current in a wide range of currents, owing to the appreciable repulsion forces. The latter causes the characteristic to become linear again, but with a large differential resistance. The simplest function that satisfies these conditions can be written in the form

$$a(j,T)=\frac{\pi}{2}\xi'(T)\frac{j-j_1(T)+A(j,T)}{j-j_1(T)}, \tag{8}$$

where $A(j,T)$ is a certain function that depends little on the current and takes effectively into account the influence of the repulsion that saturates the vortex density. The coefficient in this formula was chosen such as to obtain the usual ohmic characteristic at $T=T_c$. The dependence of A on the temperature can be obtained by assuming that $\Phi_0 A(T)/c$ is of the same order of magnitude as the repulsion force between two vortices separated by a distance $\xi'(T)$, for if such a vortex density can be reached as a result of the current, any further increase of the current will not increase the density, and can only increase the vortex velocity. Naturally, this reasoning can only overestimate A, but the character of the temperature dependence is expected to be correctly represented.

Using the expression given by Pearl[9] for the vortex-interaction force, we obtain for $A(T)$ the expression

$$A(T)=\alpha j_c[(T_c-T)/T_c]^{1/2}, \tag{9}$$

where α is a dimensionless constant and serves as the parameter of our model.

Relations (7)–(9) determine $E(0)$. As for E_0, its main difference from $E(0)$ is that the barrier for the entry of the vortices vanishes at a different value of the current. We shall therefore use for E_0 the same expression as for $E(0)$, but with $j_c(T)$ in place of $j_1(T)$.

PRINCIPAL RESULTS

From the model described above, recognizing that near the critical temperature of the superconductor (T_c) the functions $j_p(T)$, $j_1(T)$, and $j_c(T)$ are linear[1,2,10] $[j_i(T)=j_i(1-T/T_c), i=(1,p,c)]$, we easily obtain the CVC of a film with a domain:

$$\frac{U}{E_p}=\frac{2\lambda_E(\zeta_p-\zeta_m)(\zeta_1-\zeta_m)}{\zeta_m^{1/4}(\zeta_1-\zeta_m+\alpha_1\zeta_m^{3/2})}\theta(\zeta_1-\zeta_m)$$
$$+2^{1/2}\int_{\zeta_m}^{\zeta_c}\frac{d\zeta\, K(\zeta)(\zeta_c-\zeta)(\zeta_p-\zeta)\theta(\zeta_c-\zeta_m)}{S^{1/2}(\zeta)(\zeta_c-\zeta+\alpha\zeta^{3/2})}, \qquad (10)$$

where U is the sample voltage, $E_p=\rho j_p$, $\lambda_E=l_E\zeta^{1/4}$, $\zeta=1-T/T_c$, $\zeta_m=1-T_m/T_c$, $\zeta_i=j/j_i$, $i=(1,p,c)$, $\alpha_1=\alpha j_c/j_1$, and $\theta(\zeta)$ is the Heaviside step function.

As seen from (10), to calculate the CVC we must solve Eq. (2), to determine the explicit dependence of ζ_m on the current and on ζ_0. In the general case it is impossible to obtain the corresponding analytic relation. We shall therefore write down certain analytic results obtained for the case $j_i(T)\ll j_c(T)$ with the aid of known mathematical procedures. Before presenting the corresponding data, we describe the behavior of ζ_m as a function of the current in different ranges of ζ_0, obtained as a result of a qualitative analysis of the solutions of Eq. (2).

1. $0\leq\zeta_0\leq\zeta_{0t}$, $\zeta_{0t}=32\gamma^{1.33}$, where

$$\gamma=j_\lambda j_p/j_c^2,\quad j_\lambda=(T_c/E_p\lambda_E)[kh(T)/d]^{1/2}.$$

The dimensionless coefficient γ characterizes the degree of nonisothermy of the sample in the expression for the CVC ($\gamma\to\infty$ and $\zeta=\zeta_0$ in the entire film).[1]

In this interval of ζ_0, the relative temperature ζ_m remains equal to ζ_0 when j increase from 0 to $j_1\zeta_0$, after which it decreases monotonically and irreversibly into the region of negative values.

2. $\zeta_{0t}<\zeta_0<\bar{\zeta}_0=(j_c/j_1)^{2.33}\gamma^{1.33}$. Just as in the preceding region, up to $j=\zeta_0 j_1$ the relative temperature is $\zeta_m=\zeta_0$. This is followed by an interval of reversible change of ζ_m when j increases from $\zeta_0 j_1$ to $j_s=0.73 j_c\gamma^{1/2}\zeta_0^{0.625}$, at which ζ_m jumps down to values lower than ζ_c. Further increase of j leads to a decrease of ζ_m to negative values.

When j decreases from values larger than j_s, ζ_m first increases continuously, and then jumpwise[2] at $\bar{j}=\varkappa j_c(\gamma\zeta_0)^{0.57}$ $[j_1(T)<\bar{j}<j_s]$. After the jump, ζ_m remains smaller than ζ_0, which it reaches at $j=j_1\zeta_m$.

3. $\bar{\zeta}_0<\zeta_0<\zeta_{0\min}=[\alpha j_c j_p j_\lambda/j_1^2(j_1-j_p)]^4$. The character of the variation of ζ_m remains the same as in region 2, but ζ_m immediately becomes equal to ζ_0 after the jump that occurs with decreasing j.

4. $\zeta_0>\zeta_{0\min}$. Here ζ_m varies with the current in the same manner as in region 3, the only difference being that ζ_m remains equal to ζ_0 with increasing current up to $j=j_s$.

In regions 2 and 3, as j increases within the interval $\bar{j}<j<j_s$, the state with $\zeta_m<\zeta_0$ becomes metastable and a perturbation of sufficient strength can lead to a jump-like decrease of ζ_m to a value lower than ζ_c.

The character of variation of ζ_m makes it possible to reconstruct the CVC of a film with a domain in various regions of ζ_0. The value $\zeta_m=\zeta_0$ corresponds to the nondissipative segment of the characteristic with $U=0$. The segment of reversible variation of ζ_m with increasing current up to a value $j=j_s(T)$ in regions 1–3 corresponds to the almost linear segment of the CVC, whose conductivity σ goes through a maximum at $U=0$ when ζ_0 changes from 0 to $\zeta_{0\min}$, in accord with the formula[3]

$$\sigma=\sigma_0(j_1/j_c)^2\frac{\zeta_{0\min}}{\gamma}y^{3/4}(1-y^{1/4}), \qquad (11)$$

where $\sigma_0^{-1}=2\lambda_E\rho$ and $y=\zeta_0/\zeta_{0\min}$. The interval of metastability of ζ_m corresponds to the interval of metastability of the CVC, the jump of T_m corresponds to the jump of U, etc.

As for the CVC segment characterized by $\zeta_m<\zeta_c$, in the case when $\zeta_0\gg\zeta_c\gg\zeta_k=(\gamma\zeta_0\alpha)^{0.444}$ the $U(j)$ dependence takes the form

$$U=\frac{2E_p\lambda_E}{\zeta_k^{1/4}}(\zeta_p-\zeta_k)+\frac{2E_p\lambda_T\zeta_p\zeta_c}{(\zeta_0^2-\zeta_T\zeta_p\zeta_c)^{1/2}}, \qquad (12)$$

where $\zeta_T=j/j_T$ and $j_T=T_c h(T)/2dE_p=\lambda_E j_\lambda/4\lambda_T$.

If, in addition, the following inequality holds

$$1\gg\frac{\lambda_T}{\lambda_E}\zeta_k^{1/4}\frac{\zeta_c}{\zeta_0}, \qquad (13)$$

the characteristic is linear:

$$U=R(\zeta_0)[J-J_0(\zeta_0)], \qquad (14)$$

where $J=jdL$ is the current while $R(\zeta_0)$ and $J(\zeta_0)$ are the resistance and excess current, given by

$$R(\zeta_0)=2\lambda_E\rho/dL(\zeta_0\gamma/\alpha)^{0.111}, \qquad (15)$$
$$J_0(\zeta_0)=j_p dL(\zeta_0\gamma/\alpha)^{0.444}. \qquad (16)$$

EXPERIMENTAL PROCEDURE

To check on the model that describes the behavior of the localized resistive regions in superconducting films, we plotted experimentally the CVC of thin films of indium (In) in a wide range of thermostat temperatures, $10^{-4}<\zeta_0<5\cdot10^{-1}$.

Since the main formulas obtained from the theoretical analysis are valid in a narrow temperature interval, where $\zeta_{0t}<\zeta_0\leq\gamma^{0.57}\sim10^{-2}$, particular importance attaches to CVC plotted with ζ_0 in the immediate vicinity of 0 or T_0 is in the immediate vicinity of T_c. The temperature was measured with a carbon resistor of the Allen-Bradley type, calibrated against saturated He^4 vapor pressure after each heating of the system to room temperature. The thermostat temperature T_0 could be stabilized in the experiment in the range from 2 to 4 K accurate to 10^{-4} K. This was also the accuracy of the temperature measurements.

The investigated samples were prepared by sputtering In on a cover-glass substrate in vacuum of $\sim10^{-5}$ Torr, and ranged from 100 to 3000 Å in thickness, from 1 to 2 cm in length, and from 0.1 to 0.5 cm in width. They were granulated systems with granule sizes from several hundred angstroms to a micron. Altogether, more than 30 samples were investigated. Their CVC are in full agreement with the proposed model.

The CVC of the films were recorded by a four-probe method that made it possible to exclude the resistance of the supply leads, in both the given-current and given-voltage regimes. It was ascertained beforehand that, apart from several details that are inessential for our present purposes and will not be considered, the CVC recorded in the different regimes are equivalent in the region of positive differential resistances.

COMPARISON OF THEORY WITH EXPERIMENT

Figure 1 shows the J–U characteristics of a thin indium film with the largest value of ζ_{0t}(In1), recorded at different values of the thermostat temperature in the given-voltage regime. In this regime, the CVC at $\zeta_0 = \zeta_{0t} = 1.46 \cdot 10^{-2}$ is particularly pronounced. At $\zeta_0 < \zeta_{0t}$ the characteristics are fully reversible, and at $\zeta_0 > \zeta_{0t}$ there appears a CVC segment with negative differential resistance. In the given-current regime, voltage jumps, represented by dashed lines, appear on the CVC.

In accordance with the classification of the theoretical part of the paper, curves 1–8 pertain to the first range of variation of ζ_0, curves 10–12 to the second, and 13–14 to the third. The second region is characterized by the presence of a resistive segment at voltages close to zero in both directions of the CVC, while in the third region the resistive segment is observed only in the forward direction.

The lines and points in Fig. 2 show the experimental $J_s(T)$ and $\bar{J}(T)$ dependences for a sample with small ζ_{0t}(In2) in the thermostat temperature interval $6.86 \cdot 10^{-4} = \zeta_{0t} \ll \zeta_0 \leq \gamma^{0.57} = 10^{-2}$ ($\gamma = 3.15 \cdot 10^{-4}$). The straight line corresponds to $J_t(T)$.

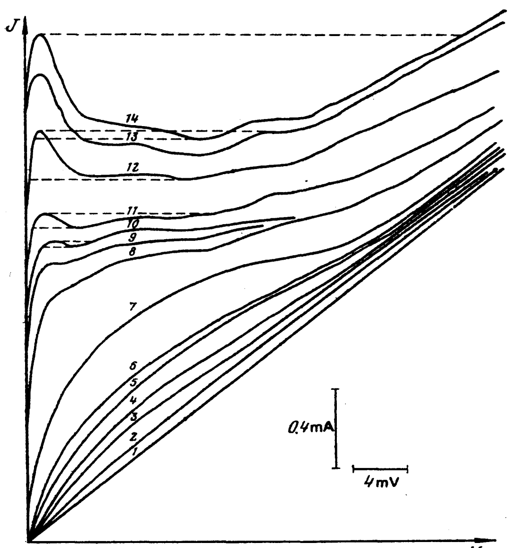

FIG. 1. Set of current-voltage characteristics (CVC) of the sample in 1, plotted for different thermostat temperatures $\zeta_0 = 1 - T_0/T_c$. Illustration of the onset of hysteresis in the CVC. ζ_0 takes on the following value-: 1) 0; 2) $8 \cdot 10^{-4}$; 3) $4.7 \cdot 10^{-3}$; 4) $6.1 \cdot 10^{-2}$; 5) $7.5 \cdot 10^{-3}$; 6) $9.1 \cdot 10^{-3}$; 7) $1.19 \cdot 10^{-2}$; 8) 1.4×10^{-2}; 9) $1.46 \cdot 10^{-2}$; 10) $1.52 \cdot 10^{-2}$; 11) $1.63 \cdot 10^{-2}$; 12) 1.79×10^{-2}; 13) $1.9 \cdot 10^{-2}$; 14) $2.07 \cdot 10^{-2}$.

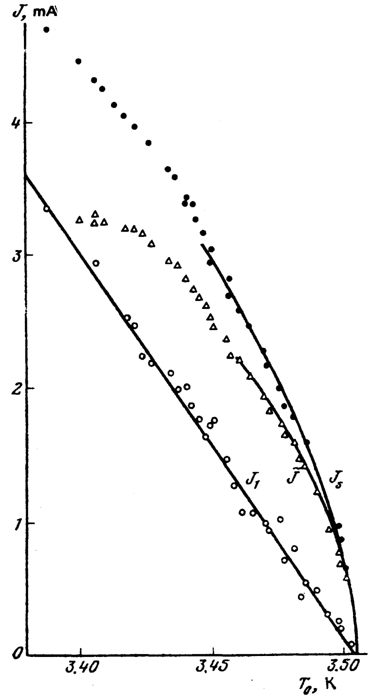

FIG. 2. Theoretical (lines) and experimental (points) dependences of the following quantities: of the current $J_1(T_0)$ at which the vortices enter the inhomogeneity, and of the currents $J_s(T_0)$ and $\bar{J}(T_0)$ at which temperature instability develops in the forward and backward directions of the CVC.

Figure 3a shows a series of CVC plotted at different thermostat temperatures for the sample In3($\zeta_{0\min} = 0.0196$). The slopes of the thin straight lines reflect the differential conductivity of the initial resistive segment. Figure 3b shows the experimental $\sigma(\zeta_0)$ dependences for three samples: In2, In3, and In4 ($\zeta_{0\min} = 0.0687$; 0.0196, 0.121). The solid line is a plot of Eq. (11) and reflects the theoretical $\sigma(\zeta_0)$ dependence.

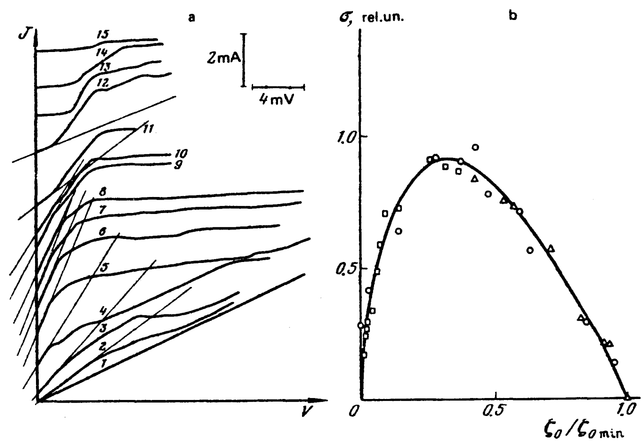

FIG. 3. a) Set of CVC of the sample In3, plotted at different temperatures of the thermostat ζ_0: 1) 0, 2) $1 \cdot 10^{-4}$; 3) $6 \cdot 10^{-4}$; 4) $2.6 \cdot 10^{-3}$; 5) $4.9 \cdot 10^{-3}$; 6) $6.7 \cdot 10^{-3}$: 7) $7.8 \cdot 10^{-3}$: 8) 8.7×10^{-3}; 9) $1.06 \cdot 10^{-2}$; 10) $1.15 \cdot 10^{-2}$; 11) $1.35 \cdot 10^{-2}$; 12) 1.73×10^{-2}; 13) $1.96 \cdot 10^{-2}$; 14) $2.16 \cdot 10^{-2}$; 15) $2.4 \cdot 10^{-2}$. Illustration of the passage of the differential conductivity $\sigma(\zeta_0)$ of the initial resistive segment of the CVC through a maximum. b) Theoretical curve and experimental points of the function $\sigma(\zeta_0/\zeta_{0\min})$ for the samples In2 (□), In3 (○), and In4 (△).

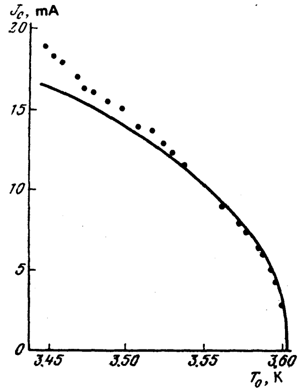

FIG. 4. Theoretical curve and experimental points of the temperature dependence of the excess current J_0 of the resistive segment of the CVC after the onset of the temperature instability.

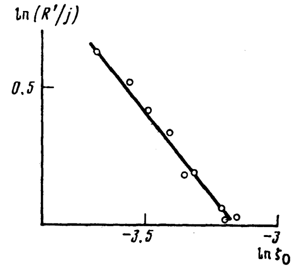

FIG. 5. Theoretical (line) and experimental (points) dependences of the reduced differential resistance R'/j (j is a certain fixed value of the current) of the resistive CVC segment that describes a domain with the maximum at temperature at the center $T > T_c(j)$ [$T_c(j)$ is the critical temperature of the start of the homogeneous flux at a given j] in the case $\zeta_0 > 2.2 \times 10^{-2}$. The coordinate scales are logarithmic.

Curves 13–15, shown in Fig. 3a, pertain to the fourth range of ζ_0. They show no initial resistive segment of the CVC, and the voltage jump in the forward direction of the CVC follows directly the nondissipative part of the characteristic.

To conclude this section, we compare the experimental data with the theoretical calculations in the temperature region $\zeta_m \leq \zeta_c$. On the CVC plots, ζ_m becomes less than ζ_c after the voltage jump in the forward direction, and remains less all the way to the voltage jump in the backward direction. It is convenient to carry out the comparison in the region of not too small values of ζ_0, when Eq. (13) is valid and the CVC of a sample with localized resistive domain is linear [see Eq. (14)]. The dependence of the excess current on the thermostat temperature $J_0(\zeta_0)$ should be determined in this case by Eq. (6), and the dependence of the resistance of the resistive section on ζ_0 should be determined by Eq. (15).

Figure 4 shows the experimental points of the function $J_0(T_0)$. The line is the theoretical plot of $J_0 \sim (1 - T_0/T_c)^{0.444}$ [see (16)]. It follows from the figure that at $T_0 > 3.53$ K or $\zeta_0 < 2.2 \cdot 10^{-2}$ the experimental points fit this curve well.

In the region $\zeta_0 > 2.2 \cdot 10^{-2}$ the discrepancy is substantial. This, however, is natural, since in this region the resistive-section currents are so large that Eq. (13) ceases to be valid because of the large ζ_c ($\zeta_c = j/j_c$). The CVC is then described by Eq. (12) rather than (14). In the case when the second term of the right-hand side of (12) exceeds the first [$\xi_k > (\xi_0 \lambda_B / \lambda_T \xi_c)^4$] and the inequality $\zeta_0^2 \gg \zeta_T \zeta_p \zeta_c$ is satisfied, in this region of ζ_0 the sample voltage is a quadratic function of the current, and the differential resistance is

$$R' = dU/dj = \frac{4E_p \lambda_T}{\zeta_0 j_p j_c} j. \tag{17}$$

Since E_p, j_p, and j_c are film parameters independent of ζ_0, and λ_T depends weakly on ζ_0, it follows from (17) that $R'/j \sim \zeta_0^{-1}$.

Figure 5 shows the experimental values of R'_j as functions of ζ_0 in a logarithmic scale. The tangent of the angle between the abscissa axis and the line drawn through this point is -1.2, in rough agreement with Eq. (17). The small discrepancy can be attributed to the $\lambda_T(\zeta_0)$ dependence, or to the fact that the term $\zeta_p \zeta_T \zeta_c$ cannot be neglected compared with ζ_0^2, or to both causes.

As for the temperature and current region where the condition (13) is satisfied ($7.4 \cdot 10^{-4} < \zeta_0 < 2.2 \cdot 10^{-2}$) and the function $R(\zeta_0)$ is given by Eq. (15), the agreement between theory and experiment is even better there.

It can be thus concluded from the experimental data reported in this section that the proposed phenomenological model describes correctly the localized resistive states in superconducting thin films.

CONCLUSION

Peculiarities of the CVC of superconducting thin films containing localized resistive domains were predicted theoretically and investigated in experiment. With respect to questions not touched upon in the present paper, it should be noted that, past the voltage jump in the forward CVC direction, the temperature $T = T_c(1 - \zeta_m)$ in the center of the domain differs insignificantly from T_c in a wide range of currents and of T_0, being located in the interval $T_c(0) < T_m < T_c(j)$ or $0 < \zeta_m < \zeta_c$. Under typical experimental conditions $\zeta_c \sim 10^{-2}$, so that the difference between T and T_c does not exceed 1% in most cases. This explains the linearity of the corresponding resistive segment of the characteristic.

In the case of large currents and large $\zeta_0 \sim 10^{-1}$, the resistive domain may become heated past the voltage jump on the CVC to a temperature much higher than T_c, and an appreciable normal region can be produced in its center. This situation was investigated in detail experimentally in Ref. 2. It can be shown that the main results of Ref. 2, particularly the treatment of the kink in the backward direction of the CVC, and the explanation that the hysteresis is due to overheating of the center of the domain (see also Ref. 11) the thermostat temperature and current ranges in which the investigation was made, do not contradict the phenomenological model described in the present paper and are only particular results for the case of films with poor heat dissipation to the ambient.

We note also the following fact. The critical temperature of a superconductor was determined in this paper by extrapolating to zero the linear $j_1(T)$ dependence, or the power-law dependences of the currents at which the forward and backward CVC are different (see Fig. 2). Frequently, nonetheless, it turns out that at the value of T_c determined in this manner, or even at T somewhat higher than T_c (they sometimes differ by as much as 0.5%) the CVC of the sample is not ohmic. This disparity can be ascribed either to fluctuation effects or to inhomogeneity of T_c along the sample.

It should be added also that the singularities on the current-voltage characteristics of superconducting thin films, similar to those considered in the present paper, were noted also by I. M. Dmitrenko *et al.* (see, e.g., Refs. 12–14). In particular, we call attention to the fact that the hysteresis vanishes at temperatures not too far from T_c.[13]

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[1)] It is possible to obtain for γ an estimate in the form $\gamma = Ll_i^2 d/\lambda_E\lambda_T\lambda_L(0)$, where l_i is the elastic effective free path of the electrons, $\lambda_L(0)$ is the London depth of penetration of the magnetic field at $T=0$. In typical experimental situations $\gamma \sim 10^{-4}-10^{-5}$.

[2)] The coefficient $\varkappa$ varies closely and monotonically with changing ζ_0, from a value $\varkappa = 0.88$ at $\zeta_0 = \zeta_{0i}$ to $\varkappa = 1$ at $\zeta_0 \gg \zeta_{0i}$.

[3)] Formally, the relation (11) yields $\sigma = 0$ at $\zeta_0 = 0$. However, the use, in Sec. 1 of the assumption that in dissipation region is local restricts the minimum of the possible ζ_0 to the value $(\lambda_E/\lambda_T)^4$.

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