Nonlinear effects in the propagation of shortwave transverse sound in pure superconductors

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Various mechanisms are analyzed which lead to nonlinear phenomena (e.g., the dependence of the absorption coefficient and of the velocity of sound on its intensity) in the propagation of transverse shortwave sound in pure superconductors (the wavelength of the sound being much less than the mean free path of the quasiparticles). It is shown that the basic mechanism, over a wide range of superconductor parameters and of the sound intensity, is the so-called momentum nonlinearity. The latter is due to the distortion (induced by the sound wave) of the quasimomentum distribution of resonant electrons interacting with the wave. The dependences of the absorption coefficient and of the sound velocity on its intensity and on the temperature are analyzed in the vicinity of the superconducting transition point. The feasibility of an experimental study of nonlinear acoustic phenomena in the case of transverse sound is considered.

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As is well known (see, for example, Ref. 1, p. 80), the absorption of transverse sound in superconductors can differ significantly from the absorption of longitudinal sound. Thus, in the transition to the superconducting state, the absorption coefficient of transverse sound decreases sharply in a number of cases, and upon a further lowering of the temperature T it falls off more smoothly, in proportion to $\exp[-\Delta(T)/T]$, where $\Delta(T)$ is the width of the superconducting gap at the temperature T. The qualitative explanation of this phenomenon, given in Ref. 1, is that the electromagnetic fields which accompany the propagation of transverse sound in a superconductor are screened as a consequence of the Meissner effect. Therefore, whereas in a normal metal the electromagnetic fields make a significant contribution to the sound absorption, on going to the superconducting state they are screened and the absorption decreases. Correspondingly, a singularity develops in the temperature dependence of the transverse sound velocity near the transition temperature T_c (Ref. 2).

Electromagnetic sound absorption has been studied theoretically and experimentally in a number of works (Refs. 3–8 and others). It was shown in these researches that the absorption coefficient of shortwave sound (with a wavelength $2\pi/q$ much shorter than the free path length l of the electrons) in the vicinity of T_c is directly connected with the response function of the superconductor to electromagnetic excitation. This circumstance offers a unique opportunity of determining the time and space dispersion of the complex electrical conductivity of the superconductor (we note that in the expression for the surface impedance, from which the response function is usually assessed, contains an integral of this function over all the wave vectors $\mathbf{q}$). Similar information can be obtained by using the temperature and frequency dependences of the velocity of transverse sound in the vicinity of T_c.[2] In this connection, a number of works have recently appeared on the study of the electromagnetic properties of superconductors by acoustical methods.

At the present time, experimenters have succeeded in introducing sound waves of sufficient intensity (both longitudinal and transverse) into a superconducting crystal, so that nonlinear phenomena are clearly observed in the absorption and change in velocity of the sound.[9] Along with this, in the overwhelming majority of theoretical researches on the electromagnetic effects in transverse-sound propagation, the analysis has been carried out within the framework of a theory linear in the sound wave amplitude. The only exception has been the works of Bulyzhenkov and Ivlev,[10,11] where the effect on the sound absorption of the heating of the particles by the field of the superconductor, created by the sound wave, was considered in the case in which the sound wavelength is much smaller than the coherence length of the superconductor ξ_0, and also the work of Ivlev and Kopnin,[12] where this phenomenon was considered for a very high frequency transverse sound [$\hbar\omega > 2\Delta(T)$ where ω is the sound frequency].

The purpose of the present work was the consideration of different mechanisms of nonlinearity of absorption and change in the velocity of transverse shortwave sound, and a comparison of their contributions. We limit ourselves to the case of shortwave sound of sufficiently low frequency, in which, along with the condition $ql \gg 1$, the conditions

$$\hbar\omega \ll \Delta, \quad q\xi_0 \ll 1 \tag{1}$$

are also satisfied. These inequalities are compatible with the condition $ql \gg 1$ only in very pure superconductors, for which $l \gg \xi_0$. We shall consider just such superconductors.

In the situation of interest to us, the interaction of sound with the quasiparticles of a superconductor is conveniently studied with the help of the kinetic equation for the distribution function $n_\mathbf{p}$ of the quasiparticles of the superconductor:[13,14]

$$\frac{\partial n_\mathbf{p}}{\partial t} + \frac{\partial \tilde{\varepsilon}}{\partial \mathbf{p}}\frac{\partial n_\mathbf{p}}{\partial \mathbf{r}} - \frac{\partial \tilde{\varepsilon}}{\partial \mathbf{r}}\frac{\partial n_\mathbf{p}}{\partial \mathbf{p}} + \hat{I}(n_\mathbf{p}) = 0, \tag{2}$$

where $\tilde{\varepsilon}(\mathbf{p}, \mathbf{r})$ is the Hamiltonian function of the quasiparticle, while $\hat{I}(n_\mathbf{p})$ is the collision operator. The Hamiltonian function $\tilde{\varepsilon}(\mathbf{p}, \mathbf{r})$ which describes the interaction of the electrons with the sound wave can be rep-

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resented in a set of coordinates comoving with the deformed lattice in the form[15]

$$\tilde{\varepsilon}(\mathbf{p},\mathbf{r})=[\tilde{\xi}^2(\mathbf{p},\mathbf{r})+\Delta^2]^{1/2}-e\mathbf{vA}/c-m_0\mathbf{v}\dot{\mathbf{u}}, \tag{3}$$

$$\tilde{\xi}(\mathbf{p},\mathbf{r})=\varepsilon_0(\mathbf{p})-\varepsilon_F+m_0\Lambda(v_x\partial/\partial x)\mathbf{v}\dot{\mathbf{u}}, \tag{4}$$

where m_0 is the mass of the free electron, $\varepsilon_0(\mathrm{p})$ is the bare energy spectrum, $\mathbf{v}=\partial\varepsilon_0/\partial\mathbf{p}$ is the electron velocity, $\mathbf{A}$ is the vector potential of the electromagnetic field (in our gauge, $\operatorname{div}\mathbf{A}=0$), $\mathbf{u}$ is the displacement vector in the sound field ($\mathbf{u}\perp\mathbf{q}\parallel x$), and ε_F is the Fermi energy. The quantity Λ can be assumed to be a function only of the components of the quasimomentum that are transverse relative to $\mathbf{q}$; it is even and the characteristic scale of its change is of the order of the Fermi momentum p_F. The Hamiltonian function (3) is obtained with the help of the Bogolyubov-De Gennes equations[16] by substituting in them of the Hamiltonian function of the normal state

$$\tilde{\xi}(\mathbf{p},\mathbf{r})-e\mathbf{vA}/c-m_0\mathbf{v}\dot{\mathbf{u}}.$$

The collision operator in (2) has the standard form.[13,14]

We now proceed to a comparison of the various mechanisms of nonlinearity. One can imagine a competition between three mechanisms: 1) heating of the quasiparticles by the field of the sound wave, considered in Refs. 10–12 for the case $q\xi_0\gg 1$; 2) change in the modulus of the order parameter Δ due to pair breaking by the vortex currents flowing in the sample; 3) "momentum nonlinearity," due to significant distortion of the quasimomentum distribution of the quasiparticles responsible for the interaction with shortwave sound. For the case of a normal conductor, this mechanism is considered in Ref. 15.

We begin with the first mechanism. According to Ref. 10, in the presence of an intense sound wave, the distribution function n_ε averaged over the surface of constant energy ε can deviate significantly from equilibrium in the region of small energies, determined by the estimate

$$\varepsilon-\Delta\leqslant\varepsilon_0=\Delta(\varepsilon_F q u_0/\Delta)^{4/5}(w\omega\tau_\varepsilon/v_F)^{2/5}, \tag{5}$$

where u_0 is the displacement amplitude in the sound wave, w is the speed of sound, v_F is the Fermi velocity, τ_ε is the relaxation time of the energy of the quasiparticles. We shall show that this estimate is also valid in the region of low frequencies (1). Integrating the kinetic equation (2) up to second order in the amplitudes of the displacement and of the electric field, and then averaging over the constant-energy surface and over the period of the sound wave, we obtain the following equation for n_ε in the region $\varepsilon-\Delta\ll\Delta$:

$$\alpha\frac{(eE_0^e v_F)^2}{qv_F}\left(\frac{\varepsilon-\Delta}{\Delta}\right)^{1/2}\frac{d}{d\varepsilon}\frac{\Delta}{\varepsilon-\Delta}\frac{dn_\varepsilon}{d\varepsilon}+\hat{I}_\varepsilon(n_\varepsilon)=0, \tag{6}$$

where α is a number of the order of unity, E_0^e is the amplitude of the effective electric field

$$\mathbf{E}^e=-\dot{\mathbf{A}}^e/c,\quad \mathbf{A}^e=\mathbf{A}+m_0(1+\Lambda)\dot{\mathbf{u}}/e,$$

$\hat{I}_\varepsilon$ is the operator of collision with phonons, averaged over the constant energy surface. If we set $\hat{I}_\varepsilon(n_\varepsilon)=(n_\varepsilon-n_0)/\tau_\varepsilon$ (n_0 is the equilibrium distribution function), we find that the nonequilibrium part of the distribution function is not small in a low-energy region, of width

$$\varepsilon-\Delta\approx\varepsilon_0\approx\Delta(eE_0^e v_F/\Delta)^{4/5}(\tau_\varepsilon/qv_F)^{2/5}. \tag{7}$$

We shall be interested below in the region of comparatively low frequencies, in which the electromagnetic effects play the dominant role in the normal state. This region is defined by the inequality

$$\omega\ll\omega_1=(\omega_p w/c)(3\pi w/4v_F)^{1/2}, \tag{8}$$

where ω_p is the plasma frequency. In a typical metal, $\omega_1\approx 10^9\ \mathrm{sec}^{-1}$. In the region (8)[8] $eE_0^e\approx m\omega^2 v_F u_0/w$. Substituting this estimate in (7), we again obtain the inequality (5). We note that the right hand side of (5) contains the product of Δ and a quantity which is small at any reasonable sound intensity, $\varepsilon_0\ll\Delta$. Therefore, the heating cannot change the order of magnitude of the absorption coefficient—only the logarithmic contribution changes, which is small at[1)] $T_c-T\ll T_0$ (Ref. 8). This contribution is determined by the energies $\varepsilon-\Delta\approx\hbar\omega$. Therefore the nonlinearity connected with the heating can appear at $\varepsilon_0\gtrsim\hbar\omega$. For a comparison of the heating with the momentum nonlinearity, we take it into account that for the latter the characteristic parameter, as will be seen, is the quantity

$$a=q\tau(eE_0^e v_F/m\omega)^{1/2}\approx q\tau(qu_0\varepsilon_F/m)^{1/2},\quad \tau\approx l/v_F \tag{9}$$

(the momentum nonlinearity can appear at $a\gtrsim 1$). It follows from (5) that

$$\varepsilon_0/\hbar\omega=a^{8/5}(\Delta/\hbar\omega)(\varepsilon_F/\Delta q^2l^2)^{4/5}(w\omega\tau_\varepsilon/v_F)^{2/5}. \tag{9a}$$

Therefore, in order that the heating appear before the momentum nonlinearity, it is necessary that the coefficient of $a^{8/5}$ in (9a) exceed unity, or

$$(ql)^2<(\varepsilon_F/\Delta)(\Delta/\hbar\omega)^{5/4}(w\omega\tau_\varepsilon/v_F)^{1/2}. \tag{10}$$

At $\Delta=1$ K, $\omega=2\cdot 10^8\ \mathrm{sec}^{-1}$, $\tau_\varepsilon=10^{-9}$ sec the right side of (10) is of the order of 10^5. We note that, even upon satisfaction of the condition (10), when the heating begins to appear earlier, allowance for the momentum nonlinearity is significant. Actually, the heating cannot change the order of magnitude of the absorption coefficient. At the same time, the momentum non-linearity significantly changes the order of the magnitude of the absorption coefficient. Furthermore, in the regime of developed momentum nonlinearity, the region of low energies no longer determines the absorption.

We now discuss the second mechanism. We compare the density of the vortical currents with the density of the pair-breaking current $p_s v_F\sim\Delta$. Using the linear-theory expression for the current density[8] we show that the dimensionless parameter that determines the role of pair-breaking has the form

$$(a/ql)^2(p_F w/T_c)[T_c/(T_c-T)]^{3/2}. \tag{11}$$

If the ratio (11) is small, then the corrections are determined by its square. If the ratio is of the order of unity, then the pair breaking effects can lead to a significant nonlinearity in the absorption of the sound. At typical values of the parameters of the metal and at $T_c\approx 10$ K, the coefficient of a^2 is $10(ql)^{-2}[T_c/(T_c-T)]^{3/2}$. We shall assume it to be small (by virtue of the large value of ql) and neglect the pair breaking effects.

We now proceed to a more detailed analysis of the momentum nonlinearity. The first part of the problem consists in the calculation of the nonlinear response of the flow to the acoustic perturbation, for which we must solve the kinetic equation (2) in the approximation linear in the sound amplitude. For this, as also in the case of a normal metal,[15] it is convenient to transform to new variables $\mathbf{p}'$ and $\mathbf{r}'$, so that the kinetic equation takes on a much simpler form. We carry out this transformation with the help of the function

$$\Phi(\mathbf{r}, \mathbf{p}', t) = \mathbf{p}'\mathbf{r} - m_0\Lambda(\mathbf{v}'\mathbf{u}). \qquad (12)$$

With account of the condition $|\partial u_i/\partial x_k| \ll 1$ that the deformation in the sound wave be small, this transformation has the form

$$p_i = p_i' - m_0\Lambda\nabla_i(\mathbf{v}'\mathbf{u}), \quad r_i' = r_i - m_0\nabla_{p_i'}\Lambda(\mathbf{v}'\mathbf{u}). \qquad (13)$$

The Hamiltonian function for the normal state in the new system of variables is $\xi(\mathbf{p}') - e\mathbf{A}^e\mathbf{v}'/c$, where $\xi(\mathbf{p}) = \varepsilon_0(\mathbf{p}) - \varepsilon_F$. Its substitution in the Bogolyubov-De Gennes equation gives

$$\tilde{\varepsilon}'(\mathbf{p}', \mathbf{r}') = \varepsilon(\mathbf{p}') - e\mathbf{A}^e\mathbf{v}'/c, \quad \varepsilon(\mathbf{p}') = [\xi^2(\mathbf{p}') + \Delta^2]^{1/2}. \qquad (14)$$

We note that the energies of the quasiparticles in the coordinate frame comoving with the deformed lattice, as expressed in the new coordinates $\tilde{\varepsilon}'(\mathbf{p}', \mathbf{r}') - m_0\Lambda(\mathbf{v}'\dot{\mathbf{u}})$, should appear in the arguments of the δ-functions in the collision operator of the kinetic equation. With account of this, representing the solution of the kinetic equation in the form $n_0[\tilde{\varepsilon}(\mathbf{p}', \mathbf{r}')] + n^{(1)}$, we can obtain the following formal solution of the kinetic equation (2) (for brevity, we shall omit the primes on the variables in what follows), cf. Ref. 15:

$$n^{(1)} = \{m_0\Lambda(\mathbf{v}\dot{\mathbf{u}}) + \hat{B}^{-1}[e\mathbf{E}_0^e\mathbf{v} - m_0\hat{I}\Lambda(\mathbf{v}\dot{\mathbf{u}})]\}(-\partial n_0/\partial\varepsilon(\mathbf{p})), \qquad (15)$$

where

$$\hat{B} = \partial/\partial t - \mathbf{v}(\xi(\mathbf{p})/\varepsilon(\mathbf{p}))(\partial/\partial\mathbf{r}) - (\partial/\partial\mathbf{r})(e\mathbf{v}\mathbf{A}^e/c)(\partial/\partial\mathbf{p}) \qquad (16)$$

is the operator of the kinetic equation. The second term in the square brackets in (15) describes the so-called mechanism of incomplete dragging of the electrons. At $ql \gg 1$ this mechanism makes a negligibly small contribution to the sound absorption in the range of frequencies of interest to us. Moreover, this mechanism is determined by the nonresonant electrons, and therefore the indicated contribution to the absorption does not depend on the sound intensity S for reasonable values of S. By virtue of this fact, we shall not take into account below the second term in the square brackets in (15), and for the determination of all the quantities of interest to us we need only construct the operator $\hat{B}^{-1}$ that is the inverse of the kinetic-equation operator $\hat{B}$. Actually, knowing $n^{(1)}$, we can calculate the mean absorbed power P, which determines the sound absorption coefficient Γ:

$$P = \langle E_i^e \hat{\sigma}_{ik}^e E_k^e \rangle, \qquad (17)$$

where the angle brackets denote averaging over the period of the sound wave, and the operator

$$\hat{\sigma}_{ik}^e = e^2 \int \frac{2d^3p}{(2\pi\hbar)^3} v_i \hat{B}^{-1} v_k (-\partial n_0/\partial\varepsilon) \qquad (18)$$

determines the effective conductivity. The field $\mathbf{E}^e$ in (17) should be determined with the help of Maxwell's equations with the bare current density

$$j_i = -\frac{c}{4\pi} K_{ik} A_k^e + \hat{\sigma}_{ik}^e E_k^e + C_{ik} e N_0 \dot{u}_k, \qquad (19)$$

where N_0 is the concentration of the electrons. The expressions for the tensors C_{ik} and K_{ik}, which describe the nondissipative part of the electromagnetic response, are easily obtained from the kinetic equation. For brevity, we shall only write them down for an isotropic quadratic spectrum, when these tensors degenerate into scalars:

$$C = \Lambda m_0/m, \quad K = \delta_L^{-2} N_s/N_0. \qquad (20)$$

Here m is the effective mass of the electron, $\delta_L = (4\pi N_0 e^2/mc^2)^{-1/2}$ is the London penetration depth,

$$N_s = N_0 \left(1 - \int_0^\infty dx\, \mathrm{ch}^{-2}[x^2 + (\Delta/2T)^2]^{1/2}\right)$$

is the so-called concentration of superconducting electrons, equal to $2N_0(T_c - T)/T_c$ at $T_c - T \ll T_c$. Thus, the first part of the problem is the construction of the operator $\hat{B}^{-1}$ and the determination of the effective conductivity operator. A similar procedure for the case of the normal metal was carried out in Ref. 15. We shall now show that over a rather wide range of sound intensities, we can use the results obtained for the normal state.

As in the normal state, the principal contribution to the absorption and the electronic correction to the sound velocity are made by quasiparticles whose group-velocity x component is close to w. Since $w \ll v_F$ and, as will be seen, typical values of the ratio ξ/ε are of the order of unity, the quasiparticles with small v_x are important. Therefore, only the "departure" part in the collision operator is important and we can represent it in the form $\hat{I}(n^{(1)}) = n^{(1)}|\xi|/\varepsilon\tau$. Setting

$$n^{(1)}(x, t) = \chi(x - wt)(-\partial n_0/\partial\varepsilon),$$

we have the following equation for χ:

$$\hat{B}\chi = \left[\left(v_x\frac{\xi}{\varepsilon} - w\right)\frac{\partial}{\partial x} + \frac{e}{c}\frac{\partial}{\partial x}(\mathbf{A}^e\mathbf{v})\frac{\partial}{\partial p_x} + \frac{|\xi|}{\varepsilon\tau}\right]\chi = e\mathbf{E}^e\mathbf{v}. \qquad (21)$$

The equations of the characteristics for (21) have the first integral

$$\varepsilon(\mathbf{p}) - p_x w - e\mathbf{A}^e\mathbf{v}/c = \text{const}. \qquad (22)$$

As in the linear regime, the principal role is played by resonant quasiparticles, for which the difference $U = v_x\xi/\varepsilon - w$ is small. It can be seen from Eq. (22) that the trajectories of these quasiparticles are generally more complicated in the superconducting state than in the normal conductor. Actually, in the case of motion along the trajectory, both v_x and the ratio ξ/ε change. To sum up, two types of processes of reflection of the quasiparticles from the profile of the sound wave are possible: 1) ordinary reflection, when v_x vanishes at a certain point, and 2) reflection of the Andreev type, when ξ vanishes at some point and the group velocity changes sign without change in the sign of v_x—the quasielectron transforms into a quasihole after reflection. The role of Andreev reflection for the case of longitudinal sound has been studied by Galaĭko, Shumeĭko and Krzyszton'.[17] We restrict ourselves here to the case in which the Andreev reflection is unimportant; this case occurs over a wide range of sound intensities. We shall prove this. Let the difference U vanish at some value $p_x = p_0$. Expanding U in powers of $(p_x - p_0)$ near this point, we get[2)]

$$\delta\varepsilon = \frac{\xi_0}{\varepsilon_0}\left(1+\frac{mw^2\Delta^2}{\xi_0^3}\right)\frac{(p_x-p_0)^2}{2m}.$$

It then follows that if $mw^2\Delta^2/\xi_0^3 \ll 1$, account of the term $-p_x w$ in (22) is unimportant. Since we are interested in the case of the immediate vicinity of T_c, this ratio is of the order of $mw^2(T_c - T)/T_c^2$, which is always a small quantity under the conditions of observation of the electromagnetic contribution to the absorption. If we assume that the ratio ξ/ε changes little along all the entire trajectory, we get from (22)

$$U=\pm\left(\frac{2e}{mc}|\mathbf{v}\mathbf{A}_0^e|\frac{|\xi_0|}{\varepsilon_0}\right)^{1/2}[\mathscr{E}-\beta_1\beta_2 b(q-wt)]^{1/2}, \qquad (23)$$

where $\mathscr{E} \approx 1$ is the dimensionless energy, $A^e(x,t) = A_0^e\, b(x - wt)$, $\beta_1 = \text{sign}\,(\mathbf{v}\mathbf{A}_0^e)$, $\beta_2 = \text{sign}\,\xi_0$; b plays the role of the dimensionless "profile of the wave." Thus the characteristic value of U on the trajectory is max $(\bar{v}, w)$, where $\bar{v} = (eE_0^e v_F/mw)^{1/2}$. By comparing the amplitude of the change in U (which is of the order of v) with the characteristic width of the region making a contribution to the linear absorption $\sim(q\tau)^{-1}$,[8] we show that the momentum nonlinearity is important under the condition $q\bar{v}\tau \gtrsim 1$. (It is seen that the parameter a introduced above is equal to $q\bar{v}\tau$.)

We now find the extent to which the ratio ξ/ε changes in motion of the quasiparticle along the trajectory. The relative change of this quantity is

$$\delta \ln(\xi/\varepsilon) \approx (p_x v_x \Delta^2/\varepsilon_0^2\xi_0) \approx p_x v_x (T_c - T)/T_c^2.$$

At $\bar{v} \ll w$ this ratio is identical with what we estimated above, and is always small. If $\bar{v} \gg w$, we have the following condition for applicability of our theory: $m\bar{v}^2(T_c - T)/T_c^2 \ll 1$. Combining these two criteria and expressing $\bar{v}$ in terms of the dimensionless parameter a, we have

$$a \ll \max\,(\omega\tau,\ qlT_c/[\varepsilon_F(T_c-T)]^{1/2}). \qquad (24)$$

As a rule, the second quantity is the larger; the ratio $T_c/[(T_c - T)\varepsilon_F]^{1/2}$ under the conditions of observation of the electromagnetic absorption in a superconductor amounts to 0.1 – 1 (at $T_c = 5$ K, $\varepsilon_F = 1$ eV; $T_c - T = 0.1$ K, this ratio is equal to 0.16). Thus, there exist a broad region of sound intensities in which the momentum nonlinearity is important ($a > 1$) but the Andreev localization still does not appear. We shall consider precisely this case.

The fact that we can assume $\xi(\mathbf{p})$ to be constant on the trajectory simplifies the problem considerably. Essentially, it reduces the problem to that already solved in Ref. 15 for the case of a normal metal, and we shall give here only the corresponding results. In sum, the operator of the effective conductivity is connected with the corresponding operator for the normal metal, constructed in Ref. 15, by the relation

$$\frac{\hat{\sigma}^*}{\hat{\sigma}_n^*} = 2\int_\Delta^\infty \frac{\varepsilon^{3/2}\,d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}\left(-\frac{\partial n_\varepsilon}{\partial\varepsilon}\right). \qquad (25)$$

In the case $T_c - T \ll T_c$ of interest to us, this integral is determined by energies of the order of T_c. Therefore the nonequilibrium of the distribution function n_ε in the region of low energies is unimportant in its calculation—the heating of the excitations by the field of the sound wave, which is important in a regime that is close to linear, has practically no effect in the regime of developed momentum nonlinearity. Therefore the function n_ε in (25) can be replaced by the equilibrium value n_0. The right side of (25) can then be represented in the form $1 + 1.80(\Delta/T)$. In the immediate vicinity of T_c, where the electromagnetic contribution to the absorption is observed, the deviation of this quantity from unity is small. Thus, in the nonlinear regime, the effective conductivity (at $T_c - T \ll T_c$) is close to the corresponding conductivity of the normal metal and is smaller than the conductivity in the linear regime by a factor a.

The subsequent analysis is similar to that given in Ref. 15 for the case of a normal metal (the difference is essentially connected with the presence of a nondissipative current response which, however, is determined by the nonresonant electrons and does not depend on the sound intensity). With the help of such an analysis, we can analyze the dependence of the absorption coefficient Γ on the sound intensity S for all the cases of interest.

We pause to consider the most interesting case, in which the electromagnetic absorption dominates in the normal state and Γ increases with increase in S. This is the range of parameters

$$\omega \ll \omega_1, \quad 1 \ll s \ll (\omega_1/\omega)^4, \qquad (26)$$

where $s = S/S_1$, $S_1 = \rho w^3(ql)^{-4}$, and ρ is the density of the crystal. Numerically, $\rho w^3 \approx 4 \times 10^9$ W/cm². Setting $\omega = 2\cdot 10^8\,\text{c}^{-1}$, $l = 0.3$ cm, we get $S_1 \approx 0.5$ W/cm² (here $\omega_1/\omega_2 \approx 5$). In the indicated region in the normal state, $\Gamma_n/\Gamma_{0n} \sim \sqrt{s}$, where Γ_{0n} is the linear absorption coefficient. The electromagnetic contribution to the linear absorption coefficient in the superconducting state is (cf. Ref. 8)

$$\Gamma_{0s} = \Gamma_{0n}[1+\Theta^2]^{-1}, \quad \Theta = \frac{8v_F}{3\pi w}\frac{T_c-T}{T_c}. \qquad (27)$$

It dominates over the contribution of the mechanism of incomplete dragging at $\Theta < \sqrt{ql}$, which is satisfied in all the regions of interest. We note that the dissipative part of the response of the superconductor in the nonlinear regime decreases by a factor a compared with the linear response, and the parameter a, just as in the normal state, should be determined self-consistently from Maxwell's equations—it depends on S and Θ. A calculation similar to that given in Ref. 15, for the case of the normal metal gives

$$a = (\Theta\sqrt{2})^{-1}[(1+4\Theta^2 s)^{1/2}-1]^{1/2}. \qquad (28)$$

Thus, at $\Theta s \ll 1$ we have $a \approx \sqrt{s}$, as in the normal metal; at $\Theta s \gg 1$ we have $a \approx \Theta^{-1/2}s^{1/4}$. The electromagnetic contribution to the nonlinear coefficient of absorption is then

$$\Gamma_s = \Gamma_{0n}\frac{\sqrt{2}}{\Theta}\frac{[(1+4\Theta^2 s)^{1/2}-1]^{1/2}}{(1+4\Theta^2 s)^{1/2}+1}. \qquad (29)$$

This expression describes the $\Gamma(S)$ dependence in the vicinity of T_c. At sufficiently small values of the difference $T_c - T$ (when $\Theta s \ll 1$) $\Gamma \sim \sqrt{s}\Gamma_{0n} \approx \Gamma_n$ as in the normal metal. If $\Theta s \gg 1$, then $\Gamma \sim \Gamma_0 \Theta^{-1/2} s^{-1/4}$. We note that the linear absorption coefficient in the regime of rapid falloff of the absorption is proportional to Θ^{-2}. Thus, the ratio $\Gamma_s/\Gamma_{0s} \approx \Theta^{1/2} s^{-1/4}$. This ratio can be either smaller or greater than unity. The $\Gamma(S)$ dependence is nonomotonic at a given temperature: with increase in the sound intensity, Γ initially increases like $\sqrt{s}$ and then begins to fall off like $S^{-1/4}$. This falloff takes place up to the point where the contribution of incomplete dragging begins to dominate, after which the absorption ceases to depend on the sound intensity. The temperature dependence of the sound absorption in the nonlinear regime differs from the corresponding dependence in linear regime. In the linear regime, Γ depends weakly on the temperature at $\Theta < 1$ and then falls off like Θ^{-2}. In the nonlinear regime, Γ depends weakly on the temperature at $\Theta < s^{-1} \ll 1$, and then falls off like $\Theta^{-3/2}$.

We now proceed to the consideration of the sound velocity in the vicinity of T_c. As a rule, the nonlinear corrections to the sound velocity are small (of the order of w^2/v_F^2) and are beyond the limits of accuracy of contemporary experiment. An exception is the case of the propagation of transverse sound, in a superconductor at $T_c - T \ll T_c$. In this situation, the corrections to the sound velocity, as shown by Ozaki and Mikoshiba,[2] can depend strongly on the temperature and can be of the order of w/v_F at the maximum. The reason for such a dependence can easily be understood qualitatively: in the normal state a significant contribution to the rigidity of the lattice is made by the electromagnetic fields, diminish rapidly in the superconducting state with decrease in temperature, as a consequence of the Meissner effect. Therefore, the sharp decrease in the absorption should be accompanied by a sharp peak in the sound velocity. The electronic corrections to the sound velocity are calculated by the standard method: the nonequilibrium distribution function of the quasiparticles is calculated from the kinetic equation and is then substituted in the expression for the force exerted on the lattice by the electrons. The dispersion equation thus obtained for sound waves gives the renormalized sound velocity. Such a procedure gives

$$\frac{|w_s - w_n|}{w_n} \equiv \left|\frac{\delta w}{w}\right| = D(N_s/N_0)\left[(N_s/N_0)^2 + (3\pi w/4v_F)^2\right]^{-1} \qquad (30)$$

in the linear approximation.[2] The coefficient $D \approx (w/v_F)^2$, therefore, as $T \to T_c$ we have $|\delta w/w| \approx (w/v_F)^2$. However, at $N_s = 3\pi w N_0/4v_F$ the sound velocity has a sharp peak, while the maximum change in the sound velocity is of the order of w/v_F. The two terms in the denominator of (30) are proportional to the squares of the nondissipative and dissipative responses of the current to the vector potential.

How does this picture change in the nonlinear regime? We have already seen that the nondissipative part does not change. As for the dynamic conductivity, both heating and momentum nonlinearity can lead to its change. In the first case, the logarithmic contribution can change by an amount of the order its own magnitude. This should lead to an increase in the change of the sound velocity and to a shift of the maximum in the direction of T_c.[11] In the case of developed momentum nonlinearity, we have

$$\left|\frac{\delta w}{w}\right| \approx \frac{w}{v_F}\frac{\Theta a^2}{a^2\Theta^2+1} = -\frac{w}{v_F}\sqrt{s}\,F(2\Theta\sqrt{s}), \qquad (31)$$
$$F(x) = x^3\left[(1+x^2)^{1/2}-1\right]^2.$$

The function $F(x)$ reaches at $x = x_m = \sqrt{3}$ a maximum equal to 0.2. At $x \ll x_m$ it is proportional to x, at $x \gg x_m$, it tends to x^{-1}. Since under our conditions $s \gg 1$, the maximum change in the sound velocity can be much greater than in the linear regime; the absorption maximum shifts toward T_c, since it corresponds to the condition $\Theta \approx s^{-1/2}$, while in the linear regime the maximum is achieved at $\Theta \approx 1$. Thus, both heating and the momentum nonlinearity lead to the same tendency of the change in the sound velocity—a shift of the maximum change of the sound velocity toward T_c and an increase in its absolute value. In the experimental work of Ref. 9, just such a behavior of the sound velocity as a function of its intensity was observed. However, it is difficult to identify the mechanism of the observed change in the sound velocity, since the condition for strong momentum nonlinearity has not been satisfied. Therefore the contributions of both mechanisms can generally be of the same order. It is also impossible to identify the mechanism of nonlinear absorption observed in this research. The point is that the absorption of a weak signal was studied in this experiment against the background of a powerful pump, with the frequencies of the signal and the pump significantly different. The sound intensity was then such that the conditions of strong momentum nonlinearity were not satisfied. In order to compare the experimental data with the theory, further experiments are necessary—a careful study of the $\Gamma(S)$ dependence over a wider drange of sound intensities, using a single powerful signal.

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