Resonant absorption, due to laser radiation, of neutrons by nuclei in a crystal

A. Ya. Dzyublik

Nuclear Research Institute, Ukrainian Academy of Sciences
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The cross section is calculated for resonant capture of neutrons to p-levels of a compound nucleus under the influence of laser radiation. It is shown that the dependence of this cross section of induced absorption on the crystal phonons is determined by the same factor as in the case of ordinary capture to s-levels. Detailed estimates are presented of the magnitude of the effect. The influence of classical nuclear oscillations in an ionic crystal under the influence of an electromagnetic wave on the neutron capture is considered.

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1. INTRODUCTION

The capture of slow neutrons to the p-level of a compound nucleus under the influence of laser radiation was recently predicted.[1-3] It was shown that this shifts the p-resonance energy by an amount equal to the laser-photon energy. It was proposed in Refs. 1–3 that the nuclei form an ideal gas with a Maxwellian velocity distribution. In the present paper we construct the theory of induced scattering (absorption) of neutrons by nuclei bound in a crystal. We refine also the estimates of the induced width of the p-level, which do not depend on the medium in which the nuclei are located. A quantum theory of the influence of polaritons on the capture of neutrons by nuclei in an ionic crystal was developed in Ref. 4. It was assumed there that the forced oscillations of the nucleus are small. We consider this effect without assuming the oscillations to be small.

2. SCATTERING MATRIX

We consider a system consisting of a nucleus in a crystal, a neutron, and an electromagnetic field. We write the Hamiltonian of the system in the form $\hat{\mathscr{H}}=\hat{\mathscr{H}}_0+\hat{V}$. Let the zeroth-approximation Hamiltonian be

$$\hat{\mathscr{H}}_0=\hat{H}_{cr}+\hat{H}_{nucl}+\hat{H}_{rad}-\frac{\hbar^2}{2m}\hat{\nabla}_r^2+U(\rho), \tag{1}$$

where $\hat{H}_{cr}$, $\hat{H}_{nucl}$, and $\hat{H}_{rad}$ are respectively the Hamiltonians of the crystal, of the nucleus, and of the field, m is the mass of the neutron, $\mathbf{r}$ is its radius vector, $\rho=\mathbf{r}-\mathbf{R}$, $\mathbf{R}$ is the radius vector of the nucleus, and $U(\rho)$ is a central potential that approximates the interaction $\hat{V}_n$ of the neutron with the nucleus. The Hamiltonian of the field is of the form

$$\hat{H}_{rad}=\sum_{\mathbf{k},\nu}\hbar\omega_{\mathbf{k}\nu}\hat{a}_{\mathbf{k}\nu}^+\hat{a}_{\mathbf{k}\nu}, \tag{2}$$

where $\hat{a}_{k\nu}^+$ and $\hat{a}_{k\nu}$ are the operators for the creation and annihilation of a photon with a wave vector $\mathbf{k}$ and a polarization unit vector $\mathbf{e}_\nu(\mathbf{k})$.

The perturbation operator is then

$$\hat{V}=(\hat{V}_n-U(\rho))+\hat{V}_r, \tag{3}$$

where $\hat{V}_r$ is the interaction of the nucleus with the field.

Since we are interested only in $E1$ transitions, it suffices to choose

$$\hat{V}_r=-\frac{e}{mc}\sum_{i=1}^{Z}\hat{\mathbf{p}}_i\hat{\mathbf{A}}(\mathbf{r}_i), \tag{4}$$

where e is the proton charge, $\mathbf{r}_i$ and $\mathbf{p}_i$ are the radius vector and momentum of the i-th proton of the nucleus, and $\hat{\mathbf{A}}(\mathbf{r})$ is the operator of the vector potential of the field:

$$\hat{\mathbf{A}}(\mathbf{r})=\sum_{\mathbf{k}}\left(\frac{2\pi\hbar c}{kL^3}\right)^{1/2}\sum_{\nu=1,2}\{\mathbf{e}_\nu(\mathbf{k})\hat{a}_{\mathbf{k}\nu}e^{i\mathbf{k}\mathbf{r}}+\mathbf{e}_\nu^*(\mathbf{k})\hat{a}_{\mathbf{k}\nu}^+e^{-i\mathbf{k}\mathbf{r}}\}, \tag{5}$$

where L^3 is the volume occupied by the field.[5,6]

We consider neutron capture by heavy nuclei with mass $M=Am\gg m$. Then, in accordance with Refs. 7 and 8, we can neglect the change of the mass of the nucleus upon capture of the neutron. Assume that at $t=-\infty$ the field contains N photons with frequency ω, wave vector $\boldsymbol{\varkappa}$, and polarization $\boldsymbol{\varepsilon}$, while the neutron has a momentum $\hbar\mathbf{k}_0$ and a spin projection μ_0. The initial state of the system is described by the wave function

$$|a\rangle=|v_s^0\rangle|N_{\varkappa\varepsilon}\rangle\Psi_{I_0M_0}(\zeta)e^{i\mathbf{k}_0\mathbf{R}}\Phi_{\mathbf{k}_0\mu_0}(\rho), \tag{6}$$

where $|v_s^0\rangle$ is the wave fuction of the crystal, v_s^0 is the initial number of photons, $|N_{\varkappa\varepsilon}\rangle$ is the function of the field, $\Psi_{I_0M_0}$ is that of the nucleus, and I_0 is the spin of the nucleus. The distorted plane wave

$$\Phi_{\mathbf{k}_0\mu_0}(\rho)=\psi_{\mathbf{k}_0}(\rho)\chi_{1/2\mu_0}$$

describes the motion of a slow neutron in the field $U(\rho)$:

$$\psi_{\mathbf{k}_0}(\rho)=\left[e^{i\mathbf{k}_0\rho}+\frac{1}{2ik_0}(e^{2i\delta_0}-1)\frac{e^{ik_0\rho}}{\rho}\right], \tag{7}$$

where δ_0 is the phase shift of the potential scattering of the neutron in the field $U(\rho)$ (for simplicity we neglect its dependence on the spins). The function (6) is an eigenfunction of the operator $\hat{\mathscr{H}}_0$:

$$\hat{\mathscr{H}}_0|a\rangle=E_a|a\rangle, \tag{8}$$

where the system energy is

$$E_a=\sum_s\hbar\omega_s(v_s^0+1/2)+N\hbar\omega+E,\quad E=\hbar^2k_0^2/2m, \tag{9}$$

ω_s is the frequency of the s-th oscillator of the crystal.

The ground-state energy of the nucleus is set equal to zero.

The coherent wave emitted by the laser is described by the wave function $|\alpha\rangle$, which is a superposition of the states of the field $|N\rangle$ with different numbers of photons.[9] The probability that N photons are contained in the state $|\alpha\rangle$ is determined by the Poisson distribution

$$P(N)=\frac{(\bar{N})^N}{N!}e^{-\bar{N}}, \tag{10}$$

where $\bar{N}$ is the average number of photons. Since $\bar{N} \gg 1$, the variance $[(N-\bar{N})^2]^{1/2}/\bar{N} \ll 1$. The averaging of our final formulas over N reduces therefore simple to replacement of N by $\bar{N}$.

It is known[10] that the scattering is determined by the operator

$$\hat{T}=\hat{V}+\hat{V}\hat{G}(E_a+i\eta)\hat{V}, \tag{11}$$

$$\hat{G}(E_a+i\eta)=(E_a+i\eta-\hat{\mathscr{H}})^{-1}, \quad \eta\to+0. \tag{12}$$

The resonant scattering of a neutron through the p level is determined by the term $\hat{V}_r\hat{G}(E_a+i\eta)\hat{V}_r$. The first factor on the right determines here the transitions of the system with emission or absorption of one optical photon $(\varkappa,\boldsymbol{\varepsilon})$ in the intermediate states

$$|b\rangle=|v_s\rangle\Psi_{IM}(\zeta,\rho)|(N\mp1)_{\varkappa\varepsilon}\rangle,$$

where $\Psi_{LM}^{(c)}$ is the wave function of the p-level of the compound nucleus.

Choosing the quantization axis z along the light-polarization unit vector $\boldsymbol{\varepsilon}$ and taking into account the equality[6]

$$\langle f|\hat{\mathbf{p}}|i\rangle=im\omega_{fi}\langle f|\mathbf{r}|i\rangle, \tag{13}$$

we obtain the matrix element for the $E1$ transition with emission of a photon $(\varkappa,\boldsymbol{\varepsilon})$:

$$\langle b|\hat{V}_r|a\rangle=\left(\frac{E-E_0}{\hbar\omega}\right)\langle v_s|e^{i\mathbf{k}_0\mathbf{R}}|v_s^0\rangle ie_{eff}(2\pi\hbar\omega n_{\varkappa\varepsilon})^{1/2}\langle\Psi_{IM}^{(c)}|\rho_z|\Psi_{I_0M_0}\Phi_{\mathbf{k}_0\mu_0}\rangle, \tag{14}$$

where $n_{\varkappa\varepsilon}=N_{\varkappa\varepsilon}/L^3$ is the photon density and $e_{\text{eff}}=eZ/(A+1)$ is the effective charge of the neutron.[11]

The matrix the determines the (n,n') or (n,γ) scattering through the p resonance is of the form

$$T_{a'a}=\sum_b\frac{\langle a'|\hat{V}_r|b\rangle\langle b|\hat{V}_r|a\rangle}{E-E_r-\sum_s\hbar\omega_s(v_s-v_s^0)+i\Gamma/2}, \tag{15}$$

where $E_r=E_0\mp\hbar\omega$ is the energy of the nuclear resonance shifted by an amount equal to the light-photon energy (the sign of $\hbar\omega$ is chosen to satisfy the resonance condition $E\approx E_r$), Γ is the total width

$$\Gamma=\Gamma_0+\Delta\Gamma, \tag{16}$$

Γ_0 is the width of the p-level of the compound nucleus in the absence of the laser, and $\Delta\Gamma$ is its broadening due to the interaction with the laser radiation.

The total width is determined by the known formula[5,10]

$$\Gamma=2\pi\sum_{a''}|\langle a''|\hat{V}|b\rangle|^2\delta(E_a-E_{a''}), \tag{17}$$

where $|a''\rangle$ are all the states to which the decay can occur. In the derivation of (15) we have assumed an incident-neutron energy $E\gg\hbar\bar{\omega}_s$ ($\bar{\omega}_s$ is the characteristic frequency of the phonons), so that the neutron energy differs little from E in all the final states $|a''\rangle$. Then, inasmuch as

$$\sum_{v_s''}\langle v_s'|e^{-i\mathbf{k}\mathbf{R}}|v_s''\rangle\langle v_s''|e^{i\mathbf{k}\mathbf{R}}|v_s\rangle=\delta_{v_s v_s'},$$

the Green's-function matrix $\hat{G}(E_a+i\eta)$ is diagonal in the phonons, and the width Γ is independent of the phonons. The broadening $\Delta\Gamma$ is determined by the same equation (17) with $\hat{V}$ replaced by $\hat{V}_r$ and with $|a''\rangle$ standing for all the states of the system into which an $E1$ transition takes place from the state $|b\rangle$ with emission or absorption of a photon $(\varkappa,\boldsymbol{\varepsilon})$ and emission of a neutron. Taking (14) into account, we have

$$\Delta\Gamma=\frac{mk_0}{2\pi\hbar^2}e_{eff}^2\frac{S}{c}\sum_{M_0'',\mu''}\int d\Omega_k|\langle\Psi_{I_0M_0''}\Phi_{\mathbf{k}\mu''}|\rho_z|\Psi_{IM}^{(c)}\rangle|^2, \tag{18}$$

where $d\Omega_k=\sin\beta d\beta d\alpha$; k_0, β, and α are the spherical coordinates of the vector $\mathbf{k}$, and S is the average laser-radiation energy flux density:

$$S=\hbar\omega\bar{n}_{\varkappa\varepsilon}c. \tag{19}$$

We note that $\Delta\Gamma$ in (18) coincides with the induced width of the p-level of the free nucleus.

3. CROSS SECTION FOR INDUCED ABSORPTION

The neutron-scattering amplitude is connected in the following manner with the scattering matrix:

$$f_{a'a}=-\frac{m}{2\pi\hbar^2}T_{a'a}. \tag{20}$$

The total cross section is, in accord with the optical theorem, given by

$$\sigma_t=\frac{4\pi}{k_0}\frac{1}{2(2I_0+1)}\sum_{M_0\mu_0}\sum_{v_s^0}g(v_s^0)\operatorname{Im}f_{aa}, \tag{21}$$

where $g(v_s^0)$ is the Gibbs distribution over the crystal states $|v_s^0\rangle$ and f_{aa} is the amplitude of the zero-angle elastic scattering.

A relation similar to the Wigner-Eckart theorem holds:

$$\langle\Psi_{IM}^{(c)}|\rho_z|\Psi_{I_0M_0}\Phi_{\mathbf{k}_0\mu_0}\rangle=(1^1/_2 0\mu_0|j\mu_0)(jI_0\mu_0M_0|IM)\langle I\|\rho_z\|I_0{}^1/_2\rangle, \tag{22}$$

where $(j_1j_2m_1m_2|jm)$ are Clebsch-Gordan coefficients, and $\langle\|\ \|\rangle$ is a reduced matrix element independent of the magnetic quantum numbers. Recognizing also that

$$(1^1/_2 0\mu_0|j\mu_0)^2=(2j+1)/6,$$
$$\sum_{M_0\mu_0}(jI_0\mu_0M_0|IM)^2=1, \tag{23}$$

we can express the numerator in f_{aa} in terms of $\Delta\Gamma$. As a result we obtain the following expression for the total cross section for induced capture of neutrons to the p-levels:

$$\sigma_t=\pi k_0^{-2}g\Delta\Gamma\Gamma w(E), \tag{24}$$

where the spin factor $g=(2I+1)/2(2I_0+1)$ and

$$w(E)=\sum_{v_s v_s^0} g(v_s^0)\frac{|\langle v_s|\exp(i\mathbf{k}_0\mathbf{R})|v_s^0\rangle|^2}{(E-E_r-\sum_s \hbar\omega_s(v_s-v_s^0))^2+(\Gamma/2)^2}. \tag{25}$$

The dependence of the cross section (24) on the phonons is determined by the same factor $w(E)$ as for ordinary neutron capture to the s-levels.

In the simplest case, when the probability of creation (annihilation) of several phonons is low, the cross section is equal to the sum of the zero-phonon cross section $\sigma_t(0)$ (the analog of the Mössbauer line) and the one-phonon cross section $\sigma_t^{(1)}$ (Refs. 7 and 8). The zero-phonon cross section is of the form

$$\sigma_t^{(0)}=\frac{\pi}{k_0^2}ge^{-2W(k_0)}\frac{\Gamma\Delta\Gamma}{(E-E_r)^2+(\Gamma/2)^2}, \tag{26}$$

where $\exp(-2W(k_0))$ is the Debye-Waller factor, $2W(k_0)=\langle(\mathbf{k}_0\cdot\mathbf{u})^2\rangle$, $\mathbf{u}$ is the displacement of the nucleus from the equilibrium position, and the angle brackets denote quantum-mechanical and statistical averaging over the phonons. The single-phonon cross section is simple in form in the case of weak coupling, when $\Gamma/2+(\mathscr{R}\overline{\mathscr{E}})^{1/2}\gg k_B\Theta$, where $\mathscr{R}=\hbar^2k_0^2/2M$ is the nuclear recoil energy, $\overline{\mathscr{E}}$ is the average energy per lattice oscillator, k_B is the Boltzmann constant, and Θ is the Debye temperature. Then

$$\sigma_t^{(1)}=\frac{4\pi}{k_0^2}g\frac{\Delta\Gamma}{\Gamma}\psi(\xi,x), \tag{27}$$

$$x=2(E-E_r-\mathscr{R})/\Gamma,\quad \xi=\Gamma/\Delta,\quad \Delta=2(\mathscr{R}\overline{\mathscr{E}})^{1/2}, \tag{28}$$

$$\psi(\xi,x)=\int_0^\infty \cos xy e^{-y-y^2/\xi^2}dy. \tag{29}$$

The absorption cross section (27) in the crystal at a temperature T coincides with the cross section for absorption by a gas[1-3] at a temperature $\overline{\mathscr{E}}/k_B$. Further simplifications of the cross section (27) were discussed in Refs. 7 and 8. Calculations of the function (25) for multiphonon processes were made in Refs. 12–14.

4. INDUCED WIDTH

Following Ref. 1, we consider the induced capture of neutrons to a bound level of a compound nucleus, with negative energy E_0. We consider first a highly idealized model, assuming that this bound state is a single-particle bound state of a neutron in a square well, and the core is not excited in this case. This approach, as will be shown later, imposes an upper bound on the induced width $\Delta\Gamma$ at a given laser power. We shall analyze the deviations from the estimates of Refs. 1–3, and present estimates for a realistic model of the compound nucleus.

Thus, we take the potential $U(\rho)$ in the form of a rectangular well of depth E_0 and radius a: $U(\rho)=-U_0$ at $0\leqslant\rho\leqslant a$ and $U(\rho)=0$ at $\rho>a$. The bound p-state of a neutron in this square well is described by the wave function

$$\Phi_{jm}(\rho)=\varphi_p(\rho)\sum_{\lambda,\mu}(1{}^1\!/_2\lambda\mu|jm)Y_{1\lambda}(\theta,\varphi)\chi_{^1\!/_2\mu}. \tag{30}$$

The radial function $\varphi_p(\rho)$ extends outside the nucleus to a distance $\rho_0=\hbar/(2m|E_0|)^{1/2}$. For shallow levels with binding energy $|E_0|\sim 1$ eV we have $\rho_0\sim 10^{-9}$ cm, i.e., $\rho_0\gg a$. In the approximation $\rho_0\gg a$ the radial function is

$$\varphi_p(\rho)\approx\left(\frac{2a}{3}\right)^{1/2}\frac{1}{a\rho}\left[\frac{\sin(K\rho)}{K\rho}-\cos(K\rho)\right],\quad 0\leqslant\rho\leqslant a, \tag{31}$$

$$\varphi_p(\rho)\approx\left(\frac{2a}{3}\right)^{1/2}\frac{1}{\rho_0\rho}\left[1+\frac{\rho_0}{\rho}\right]e^{-\rho/\rho_0},\quad \rho\geqslant a,$$
$$K=[2m(U_0+E_0)]^{1/2}/\hbar. \tag{32}$$

If the well depth is such that there exists only one p-level, than at $\rho_0\gg a$ we have

$$K\approx\pi+\pi^{-1}(a/\rho_0)^2. \tag{33}$$

In this single-particle approach, the compound-nucleus wave function is made up of the function (30) and of the wave function of the ground state of the core:

$$\Psi_{IM}(\zeta,\rho)=\sum_{M_0,m}(I_0jM_0m|IM)\Psi_{I_0M_0}(\zeta)\Phi_{jm}(\rho). \tag{34}$$

Substitution of (34) in (18) yields

$$\Delta\Gamma=\frac{mk_0e_{eff}^2}{\hbar^2}\frac{4\pi(2j+1)}{9}\frac{S}{c}\left|\int_0^\infty\rho^3d\rho\varphi_p(\rho)\psi_{k_0}(\rho)\right|^2. \tag{35}$$

In the simplest case $k_0\rho_0\ll 1$ we have hence

$$\Delta\Gamma\approx\frac{8\pi(2j+1)}{3}\frac{mk_0e_{eff}^2}{\hbar^2}\frac{S}{c}a\rho_0^4. \tag{36}$$

We used this formula to estimate the laser-energy flux density needed to obtain an induced width $\Delta\Gamma\approx 0.1$ eV. Choosing $j=\frac{1}{2}$, $k_0=10^8$ cm^{-1}, $e_{eff}=0.5\,e$, $a=10^{-12}$ cm, and $\rho_0=10^{-9}$ cm we obtain $S=3\times10^{18}$ W/cm^2.

The exact wave function of the bound state of the compound nucleus is an expansion in the single-particle functions of the neutron, with factors that describe different states of the core. Addition of deeper neutron states in the square well to the function (34) leads to localization of the neutron in a smaller volume. The required value of S can only increase in this case.

The obtained value of S is much larger than the estimates given in Ref. 1. To determine the cause of this discrepancy, we track the derivation of $\Delta\Gamma$ by the method of Ref. 1. We use the operator identity

$$\hat{\mathbf{p}}\hat{H}_0-\hat{H}_0\hat{\mathbf{p}}=-i\hbar\hat{\nabla}_\rho U(\rho), \tag{37}$$

where $\hat{\mathbf{p}}=-i\hbar\nabla_\rho$ is the momentum operator and $H_0=\hat{p}^2/2m+U(\rho)$. The calculation of the matrix for (37) on the eigenfunctions of the operator $\hat{H}_0$ leads to the equation

$$\left\langle f\left|\frac{\partial}{\partial\rho}\right|i\right\rangle=-\frac{1}{E_f^{(0)}-E_i^{(0)}}\left\langle f\left|\frac{\partial U}{\partial\rho}\right|i\right\rangle, \tag{38}$$

where $E^{(0)}$ are the corresponding eigenvalues of $\hat{H}_0$. In our problem the eigenfunctions of $\hat{H}_0$ are the function (30) for the bound p-level in a square well and the function (7) that describes the neutron scattering by this well. Accordingly, δ_0 is the phase shift of the scattering of an S wave by a square well containing a shallow p level. This phase shift is determined by the expression[6]

$$\operatorname{tg}\delta_0=\frac{k_0D-\operatorname{tg}(k_0a)}{1+k_0D\operatorname{tg}(k_0a)}, \tag{39}$$

$$D=(K')^{-1}\,\mathrm{tg}\,(K'a),\quad K'=[2m(U_0+E)]^{1/2}/\hbar. \tag{40}$$

Taking (32) and (33) into account, we obtain from this

$$\frac{\delta_0}{k_0}\approx -a+\frac{a}{2\pi^2}\left[3\left(\frac{a}{\rho_0}\right)^2+(k_0a)^2\right]. \tag{41}$$

Transforming the integral in (35) with the aid of (13) and (38), and recognizing also that

$$\partial U/\partial\rho=-U_0\delta(\rho-a), \tag{42}$$

we obtain

$$\Delta\Gamma\approx\frac{4\pi(2j+1)}{9}\frac{S}{c}\frac{k_0e_{eff}^2U_0^2a^2}{m\hbar^2\omega^4}\varphi_p^2(a)(a+\delta_0/k_0)^2. \tag{43}$$

Recalling (32), (33), and (41), we can show that Eq. (43) goes over into (36) in the case $k_0\rho_0 \ll 1$. Introducing the neutron width Γ_n, we obtain from (43) the result of Ref. 1:

$$\Delta\Gamma\sim\frac{\pi e_{eff}^2U_0^2}{3(\hbar\omega)^4}\frac{S}{c}\Gamma_n(a+\delta_0/k_0)^2. \tag{44}$$

The broadening $\Delta\Gamma$ was estimated in Ref. 1 under the assumption that $(a+\delta_0/k_0)\sim 10^{-12}$ cm. According to (41), however, this quantity is $\sim 10^{-18}$ cm for $|E_0|\sim 1$ eV. Therefore the value of $\Delta\Gamma$ given in Ref. 1 should be decreased by 12 orders of magnitude.

The difference between the scattering length (41) and the experimental value is due to the fact that to obtain the upper bound we have considered an idealized model of potential scattering (without excitation of the core). Accordingly δ_0 in our formulas is the phase shift of only the potential scattering. In the traditional model of resonant scattering of neutrons (see, e.g., Ref. 15), it is assumed that the energy transferred by the neutron to the nucleus becomes redistributed among many nucleons, therefore the radius of the compound nucleus changes insignificantly compared with the radius of an ordinary nucleus. Following Refs. 15 and 16, we estimate $\Delta\Gamma$ by putting $\varphi_p=$const at $0<\rho<a$ and $\varphi_p=0$ at $\rho>a$. Then $\Delta\Gamma$ is determined by expression (36), in which ρ_0^4 is replaced by $a^4/32$. Consequently, $\Delta\Gamma\sim 0.1$ eV only at $S\sim 10^{31}$ W/cm^2.

5. ROLE OF POLARITONS

We consider an ionic crystal with two ions per unit cell and with optical isotropy. Its dielectric constant is[17]

$$\varepsilon(\omega)=\varepsilon_\infty+\frac{\varepsilon_0-\varepsilon_\infty}{1-(\omega/\omega_0)^2-i(\omega/\omega_0)(\gamma/\omega_0)}, \tag{45}$$

where γ is the damping of the optical oscillation in sec^{-1}, ε_0 and ε_∞ are respectively the static and high-frequency dielectric constants, and ω_0 is the dispersion frequency. The refractive index $n=[\varepsilon(\omega)]^{1/2}$. If a wave with electric component

$$\mathbf{E}^{(inc)}(\mathbf{r},t)=\mathbf{E}_0^{(inc)}\cos(\varkappa\mathbf{r}-\omega t)$$

is incident on this crystal, then a wave

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}_0\cos(\mathbf{K}\mathbf{r}-\omega t),\quad \mathbf{E}_0=2\mathbf{E}_0^{(inc)}/(n+1),$$

propagates inside the crystal and the wave vector is $\mathbf{K}=\{\varkappa_x,\varkappa_y,n\varkappa_z\}$, where z axis is directed inward perpendicular to the plate surface.

The electromagnetic wave excites in the ionic crystal transverse optical oscillations. According to classical polariton theory, the displacement of a positively or negatively charged ion from the equibrium position, due to these oscillations, is

$$\mathbf{u}_\pm=\pm\frac{(\bar M v_0)^{1/2}}{M_\pm}\left(\frac{\varepsilon_0-\varepsilon_\infty}{4\pi}\right)^{1/2}\frac{1}{\omega_0}\frac{\mathbf{E}(\mathbf{r},t)}{1-(\omega/\omega_0)^2-i(\omega/\omega_0)(\gamma/\omega_0)}, \tag{46}$$

where v_0 is the volume of the unit cell, $M_\pm$ are the masses of the ions, and $\bar M=M_+M_-/(M_++M_-)$ is their reduced mass. It is seen from (46) that $\mathbf{u}=\mathbf{A}\cos(\omega t+\varphi)$, where φ is an inessential phase shift.

We consider now resonant scattering of neutrons by the nucleus of an ion made to oscillate by the laser. Following Ref. 4, we take into account only the role of the transverse optical oscillations. Using the formal expansion of the S matrix in powers of the interaction, we can show that in resonant scattering from $|a\rangle$ into $|a'\rangle$ we have

$$S_{a'a}=\delta_{a'a}-2\pi i\sum_{n,n'=-\infty}^{\infty}\frac{M'Mi^{n'}J_{n-n'}(\mathbf{k}'\mathbf{A})J_n(\mathbf{k}_0\mathbf{A})}{E-E_0-n\hbar\omega+i\Gamma/2}\delta(E_{a'}-E_a-n'\hbar\omega), \tag{47}$$

where $J_n(x)$ is a Bessel function of n-th order, M and M' are the matrix elements that determine the capture of a neutron by a free nucleus to the s or p level and its subsequent decay. In the derivation of (47) we took into account the known equation

$$e^{iz\cos\omega t}=\sum_{n=-\infty}^{\infty}i^nJ_n(z)e^{in\omega t}. \tag{48}$$

From (47) we obtain the cross sections. In particular, the doubly differential neutron-scattering cross section is

$$\frac{d^2\sigma}{d\Omega dE'}=\sum_{n'=-\infty}^{\infty}\left(\frac{m}{2\pi\hbar^2}\right)^2\frac{k'}{k_0}|M'|^2|M|^2$$

$$\times\left|\sum_{n=-\infty}^{\infty}\frac{J_{n-n'}(\mathbf{k}'\mathbf{A})J_n(\mathbf{k}_0\mathbf{A})}{E-E_0-n\hbar\omega+i\Gamma/2}\right|^2\delta(E'-E-n'\hbar\omega), \tag{49}$$

where E' is the energy of the scattered neutron. The total cross section is

$$\sigma_t=\frac{\sigma_0\Gamma^2}{4}\sum_{n=-\infty}^{\infty}\frac{J_n^2(\mathbf{k}_0\mathbf{A})}{(E-E_0-n\hbar\omega)^2+(\Gamma/2)^2}, \tag{50}$$

where σ_0 is the cross section for absorption at resonance. It is seen from (49) and (50) that the resonance energy can be shifted by an amount that is a multiple of the laser photon energy $\hbar\omega$. It is therefore possible to observe resonant levels with negative energy E_0.

The intensity of the additional peaks in (50) with $n\neq 0$ is determined by the parameter $\mathbf{k}_0\cdot\mathbf{A}$. When $\mathbf{k}_0\cdot\mathbf{A}\ll 1$, their intensity decreases rapidly with increasing n, since $J_n(x)\to(x/2)^n/n!$ as $x\to 0$. Apart from a few details, our equations agree at $\mathbf{k}_0\cdot\mathbf{A}=k_0A\ll 1$ with the results of the theory.[4]

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