P-odd van der Waals forces

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(Submitted 10 November 1981)
Zh. Eksp. Teor. Fiz. **82**, 1026–1031 (April 1982)

Two-photon exchange between atoms with allowance for parity nonconservation leads to a P-odd potential that is expressed in terms of the vector and tensor polarizabilities of the atom, does not depend on the velocity, and decreases like some power of the distance. This interaction splits the energies of the crystals with right- and left-helical spin structure.

PACS numbers: 34.20.Fi

1. Spatial parity nonconservation in atomic phenomena is being studied at present by many groups. The first experimental success in this field was reached in Novosibirsk, where a weak interaction between electrons and atoms, due to neutral currents, was discovered by observing the optical activity of atomic bismuth vapor (Ref. 1, see also Refs. 2–4). Neutral currents are not the only source of P-parity violation. Parity nonconservation in nucleon interaction leads to the appearance of multipoles of "wrong" parity in the electromagnetic vector potential of the nucleus. The resultant unusual interaction of the electron with the nucleus is one more reason for parity nonconservation in atoms.[5] However, both the weak interaction and the induced electromagnetic interaction are of the contact type.

A peculiar phenomenon takes place in μ-mesic atoms. The radiative corrections to the weak interaction of the muon with the nucleus lead there to the appearance of a nonlocal increment in this interaction.[5] The radius of this "long range action," however, is limited by the Compton wavelength of the electron.

It is shown in the present paper that parity violation leads, on account of two-photon exchange, to P-odd forces that have a power-law dependence on the distance.

It is useful to explain qualitatively what distinguishes, in this sense, the two-photon and one-photon exchanges. Owing to nonconservation of spatial parity, a helical spin structure and toroidal currents appear in the system.[6-8] The field produced by these currents is obviously localized inside the system. This is also the cause of the locality of the P-odd electromagnetic interaction in the one-photon approximation. (For a more rigorous proof of this fact see Refs. 5 and 9–13). In the case of two-photon exchange (see Fig. 1) the intermediate states of the interacting systems, say atoms, differ from the initial and final ones. Upon restructuring of the system, the field produced by the toroidal currents emerges to the outside. This is perfectly clear even in the classical approach, since a change of the magnetic field inside a toroidal winding leads to the appearance of an external field simply by virtue of Maxwell's equations. The experimental proof that off-diagonal P-odd matrix elements of the current give rise to an external field is the circular polarization of the radiation, discovered in Refs. 1. In this situation, however, there is every reason for expecting parity-violating nonlocal forces to appear in the two-photon approximation. It is natural to call them P-odd van der Waals forces.

2. The interacting atoms considered will be assumed to be at rest and separated by large distances. In analogy with the calculation of ordinary van der Waals forces (see Ref. 14) we seek the potential energy of a P-odd interaction of two atoms in the form

$$U(\mathbf{R})=i\langle S(\mathbf{R})\rangle/t, \tag{1}$$

where $\langle S(\mathbf{R})\rangle$ is the S operator averaged over the wave functions of the states of both atoms, given the coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$ of their nuclei, as well as over the photon vacuum. It is equal to

$$S=\frac{(-i)^4}{4!}\int dt_1dt_2dt_3dt_4T\{V(t_1)V(t_2)V(t_3)V(t_4)\}. \tag{2}$$

Here T is the T-product symbol and $V(t)$ is the time-dependent electromagnetic-interaction operator, which can be written in our case in the form

$$V=-\mathbf{E}(\mathbf{r}_1)\mathbf{d}_1-\mathbf{E}(\mathbf{r}_2)\mathbf{d}_2-\mathbf{H}(\mathbf{r}_1)\boldsymbol{\mu}_1-\mathbf{H}(\mathbf{r}_2)\boldsymbol{\mu}_2; \tag{3}$$

E and **H** are the operators of the electric and magnetic fields; d and μ are the dipole (electric and magnetic) moments of the atoms. We use a system of units in which $\hbar=c=1$. Averaging the operator (2) over the photon vacuum and taking (3) into account, we obtain for its P-odd part

$$\begin{gathered}\langle S(\mathbf{R})\rangle={}^1/_2\int dt_1dt_2dt_3dt_4F(\mathbf{R},t_1,t_2,t_3,t_4); \\ \mathbf{R}=\mathbf{r}_1-\mathbf{r}_2,\quad F(\mathbf{R},t_1,\ldots,t_4)=\langle TE_i(t_1,\mathbf{r}_1)H_k(t_2,\mathbf{r}_2)\rangle \\ \times\langle TE_l(t_3,\mathbf{r}_2)E_m(t_4,\mathbf{r}_1)\rangle\langle Td_i^{(1)}(t_1)d_m^{(1)}(t_4)\rangle\langle T\mu_k^{(2)}(t_2)d_l^{(2)}(t_3)\rangle \\ +\langle TH_i(t_1,\mathbf{r}_1)E_k(t_2,\mathbf{r}_2)\rangle\langle TE_l(t_3,\mathbf{r}_2)E_m(t_4,\mathbf{r}_1)\rangle\langle T\mu_i^{(1)}(t_1)d_m^{(1)}(t_4)\rangle \\ \times\langle Td_k^{(2)}(t_2)d_l^{(2)}(t_3)\rangle+\langle TE_i(t_1,\mathbf{r}_1)E_k(t_2,\mathbf{r}_2)\rangle\langle TH_l(t_3,\mathbf{r}_2)E_m(t_4,\mathbf{r}_1)\rangle \\ \times\langle Td_i^{(1)}(t_1)d_m^{(1)}(t_4)\rangle\langle Td_k^{(2)}(t_2)\mu_l^{(2)}(t_3)\rangle+\langle TE_i(t_1,\mathbf{r}_1)E_k(t_2,\mathbf{r}_2)\rangle \\ \times\langle TE_l(t_3,\mathbf{r}_2)H_m(t_4,\mathbf{r}_1)\rangle\langle Td_i^{(1)}(t_1)\mu_m^{(1)}(t_4)\rangle\langle Td_k^{(2)}(t_2)d_l^{(2)}(t_3)\rangle.\end{gathered} \tag{4}$$

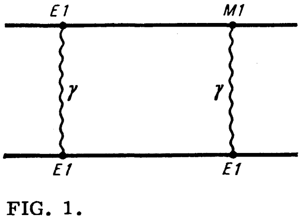

FIG. 1.

0038-5646/82/040601-03$04.00

We confine ourselves, naturally, to terms linear in μ. It is convenient to express the mean value over the photon vacuum in terms of the Green's function $D_{ik}(\omega, \mathbf{R})$ of the photon in the mixed $(\omega, \mathbf{R})$ representation:

$$\langle TE_i(t_1,\mathbf{r}_1)E_k(t_2,\mathbf{r}_2)\rangle = -i\int\frac{d\omega}{2\pi}\omega^2 D_{ik}(\omega,\mathbf{R})\,e^{-i\omega(t_1-t_2)},$$

$$\langle TE_i(t_1,\mathbf{r}_1)H_k(t_2,\mathbf{r}_2)\rangle = -\varepsilon_{knm}\nabla_n\int\frac{d\omega}{2\pi}\,\omega D_{im}(\omega,\mathbf{R})\,e^{-i\omega(t_1-t_2)},$$

$$\langle TH_i(t_1,\mathbf{r}_1)E_k(t_2,\mathbf{r}_2)\rangle = -\varepsilon_{inm}\nabla_n\int\frac{d\omega}{2\pi}\,\omega D_{mk}(\omega,\mathbf{R})\,e^{-i\omega(t_1-t_2)}, \qquad (5)$$

$$D_{ik}(\omega,\mathbf{R}) = -4\pi\left(\delta_{ik}+\frac{\nabla_i\nabla_k}{\omega^2}\right)\int\frac{d^3k}{(2\pi)^3}$$

$$\times\frac{e^{i\mathbf{k}\mathbf{R}}}{\omega^2-k^2+i0} = \left(\delta_{ik}+\frac{\nabla_i\nabla_k}{\omega^2}\right)\frac{1}{R}e^{iR|\omega|}. \qquad (6)$$

We use a gauge $A_0 = 0$, in which

$$\mathbf{E} = -\partial\mathbf{A}/\partial t, \qquad \mathbf{H} = \operatorname{rot}\mathbf{A}.$$

We expand the mean values over the states of the atoms

$$\alpha_{ik}(t_1-t_2) = i\langle Td_i(t_1)\,d_k(t_2)\rangle, \qquad \beta_{ik}(t_1-t_2) = i\langle Td_i(t_1)\,\mu_k(t_2)\rangle \qquad (7)$$

in Fourier integrals

$$\alpha_{ik}(t) = \int\frac{d\omega}{2\pi}\alpha_{ik}(\omega)\,e^{-i\omega t}; \quad \beta_{ik}(t) = \int\frac{d\omega}{2\pi}\beta_{ik}(\omega)\,e^{-i\omega t}. \qquad (8)$$

Besides the usual polarizability

$$\alpha_{ik}(\omega) = \sum_n\left\{\frac{\langle 0|d_i|n\rangle\langle n|d_k|0\rangle}{\omega_{n0}-\omega-i0}+\frac{\langle 0|d_k|n\rangle\langle n|d_i|0\rangle}{\omega_{n0}+\omega-i0}\right\} \qquad (9)$$

the answer contains the P-odd polarizability

$$\beta_{ik}(\omega) = \sum_n\left\{\frac{\langle 0|d_i|n\rangle\langle n|\mu_k|0\rangle}{\omega_{n0}-\omega-i0}+\frac{\langle 0|\mu_k|n\rangle\langle n|d_i|0\rangle}{\omega_{n0}+\omega-i0}\right\}. \qquad (10)$$

In these expressions $\omega_{n0} = E_n - E_0$, where E_0 and E_n are the energies of the ground and excited atomic states.

Substituting (5) and (8) in (4) and carrying out trivial integrations with respect to time and frequency, we arrive at the following general expression for $U(\mathbf{R})$

$$U(\mathbf{R}) = \varepsilon_{nkm}\int\omega^3 D_{ls}(\omega,\mathbf{R})\,\nabla_n D_{im}(\omega,\mathbf{R})$$

$$\times[\alpha_{is}^{(1)}(\omega)\,\beta_{lk}^{(2)}(\omega)-\beta_{lk}^{(1)}(\omega)\,\alpha_{is}^{(2)}(\omega)]\frac{d\omega}{2\pi}. \qquad (11)$$

To make this equation more lucid and more detailed, we must dwell in detail on the structures of the tensors $\alpha_{is}(\omega)$ and $\beta_{lk}(\alpha)$.

3. If the level widths are neglected, the polarizability tensor $\alpha_{ik}(\omega)$ must be Hermitian. Its most general form is therefore

$$\alpha_{ik}(\omega) = \delta_{ik}\alpha_s(\omega)+i\varepsilon_{ikl}j_l\alpha_v(\omega)+Q_{ik}\alpha_t(\omega). \qquad (12)$$

Here j is the angular momentum of the system:

$$Q_{ik} = j_ij_k+j_kj_i-2\delta_{ik}j(j+1)/3;$$

the scalar $\alpha_s(\omega)$, vector $\alpha_v(\omega)$, and tensor $\alpha_t(\omega)$ polarizabilities are real:

$$\alpha_{s,v,t}(\omega) = \alpha^*_{s,v,t}(\omega).$$

From the very definition (9) it follows that $\alpha_{ik}(\omega) = \alpha_{ki}(-\omega)$, so that $\alpha_{s,t}(\omega)$ are even functions of ω, and $\alpha_v(\omega)$ is odd:

$$\alpha_s(\omega) = \alpha_s(-\omega), \quad \alpha_v(-\omega) = -\alpha_v(\omega), \quad \alpha_t(\omega) = \alpha_t(-\omega). \qquad (13)$$

Whereas the scalar and tensor polarizabilities α_s and α_t are quite familiar characteristics of the system, the vector polarizability α_v has become the subject of discussions relatively recently (see, e.g., Refs. 15–18). Moreover, it is directly stated in the literature that this structure is not present in $\alpha_{ik}(\omega)$. The reasoning is that the basis of the states of the system can always be chosen real, so that no imaginary term can arise in $\alpha_{ik}(\omega)$. However, if the spin-orbit interaction is taken into account, the wave functions of states with a given angular moment j at $l \neq 0$ cannot be chosen real at all. Therefore if the fine structure of the level is resolved, the vector polarizability of a state with given j and with $l \neq 0$ not only exists, but contains also a certain additional smallness compared with $\alpha_{s,t}$. Fine splitting of the excited levels gives rise to α_v even in the state with $l = 0$.

The tensor $\beta_{ik}(\omega)$ differs from zero only because of parity violation in the atom. If we neglect the level widths, it follows from the definition (10) that

$$\beta_{ik}^*(\omega) = \beta_{ik}(-\omega). \qquad (14)$$

Using the invariance to time reversal, we can readily show that

$$\beta_{ik}(\omega) = -\tilde{\beta}_{ik}(-\omega), \qquad (15)$$

where $\bar{\beta}_{ik}(\omega)$ is the polarizability tensor in a state with inverted angular-momentum projection. [We note that expression (12) for $\alpha_{ik}(\omega)$ satisfies automatically the condition $\alpha_{ik}(\omega) = \bar{\alpha}_{ki}(\omega)$, that follows from the T-invariance requirement.] The conditions (14) and (15) mean that in the expression

$$\beta_{ik}(\omega) = i\delta_{ik}\beta_s(\omega)+\varepsilon_{ikl}j_l\beta_v(\omega)+iQ_{ik}\beta_t(\omega) \qquad (16)$$

the functions $\beta_{s,v,t}(\omega)$ have the properties

$$\beta^*_{s,v,t}(\omega) = \beta_{s,v,t}(\omega), \qquad (17)$$

$$\beta_s(-\omega) = -\beta_s(\omega), \quad \beta_v(\omega) = \beta_v(-\omega), \quad \beta_t(-\omega) = -\beta_t(\omega). \qquad (18)$$

4. Substituting expressions (12) and (16) in (11) and taking (13) and (18) into account we can transform the P-odd potential $U(\mathbf{R})$ into

$$U(\mathbf{R}) = U_v(\mathbf{R})+U_t(\mathbf{R});$$

$$U_v(\mathbf{R}) = 2i\frac{\mathbf{n}[\mathbf{j}_1\times\mathbf{j}_2]}{R^3}\int\omega^3\left(1+\frac{2i}{|\omega|R}-\frac{1}{\omega^2R^2}\right)e^{2i|\omega|R}$$

$$\times[\alpha_v^{(1)}(\omega)\,\beta_v^{(2)}(\omega)+\beta_v^{(1)}(\omega)\,\alpha_v^{(2)}(\omega)]\frac{d\omega}{2\pi}, \qquad (19)$$

$$U_t(\mathbf{R}) = -i\varepsilon_{kni}Q_{is}^{(1)}Q_{lk}^{(2)}\int[\alpha_t^{(1)}(\omega)\,\beta_t^{(2)}(\omega)+\beta_t^{(1)}(\omega)\,\alpha_t^{(2)}(\omega)]$$

$$\times\omega^3 D_{ls}(\omega,\mathbf{R})\,\nabla_n\frac{1}{R}e^{i|\omega|R}\frac{d\omega}{2\pi}, \quad \mathbf{n} = \mathbf{R}/R. \qquad (20)$$

Further simplification is obtained by considering the interaction at "short" ($a \ll R \ll \lambda_0 \sim 1/\omega_0$) and "long" ($R \gg \lambda_0$) distances. Here a are the characteristic atomic dimensions and ω_0 are the characteristic atomic frequencies.

Let $a \ll R \ll \lambda_0$. Since $\omega \sim \omega_0$, we also have $\omega R \ll 1$. In this case

$$U_v(\mathbf{R}) = 2\frac{\mathbf{n}[\mathbf{j}_1\times\mathbf{j}_2]}{R^5}\cdot\frac{1}{2\pi i}\int[\alpha_v^{(1)}(\omega)\,\beta_v^{(2)}(\omega)+\beta_v^{(1)}(\omega)\,\alpha_v^{(2)}(\omega)]\,\omega\,d\omega, \qquad (21)$$

$$U_t(\mathbf{R}) = \varepsilon_{kni}\frac{n_n}{R^5}(\delta_{ls}-3n_ln_s)\,Q_{is}^{(1)}Q_{ik}^{(2)}$$

$$\times\frac{1}{2\pi i}\int[\alpha_t^{(1)}(\omega)\,\beta_t^{(2)}(\omega)+\beta_t^{(1)}(\omega)\,\alpha_t^{(2)}(\omega)]\,\omega\,d\omega. \qquad (22)$$

In the case $R \gg \lambda_0 \sim 1/\omega_0$ the important values are $\omega \ll \omega_0$, for at $\omega \gtrsim \omega_0$ the integrals are cancelled out by the oscillating factor $\exp(2i\omega R)$. The polarizabilities can then replaced by their values as $\omega \to 0$:

$$\alpha_v(\omega) \to \omega\alpha_v'(0), \quad \beta_v(\omega) \to \beta_v(0),$$
$$\alpha_t(\omega) \to \alpha_t(0), \quad \beta_t(\omega) \to \omega\beta_t'(0),$$

where the prime denotes differentiation. The integration is then elementary, and we obtain from (19) and (20) respectively

$$U_v(\mathbf{R}) = -\frac{7}{2\pi}\frac{\mathbf{n}[\mathbf{j}_1 \times \mathbf{j}_2]}{R^8}[\alpha_v'^{(1)}(0)\beta_v^{2)}(0) + \beta_v^{(1)}(0)\alpha_v'^{(2)}(0)], \qquad (23)$$

$$U_t(\mathbf{R}) = -\frac{35}{8\pi R^8}\varepsilon_{kni}n_n\left(\delta_{ls} - \frac{9}{5}n_l n_s\right)Q_{is}^{(1)}Q_{lk}^{(2)}$$
$$\times[\alpha_t^{(1)}(0)\beta_t'^{(2)}(0) + \beta_t'^{(1)}(0)\alpha_t^{(2)}(0)]. \qquad (24)$$

We note that in the region $a \ll R \ll \lambda_0$ the P-odd van der Waals interaction turns out to have a longer range than the usual interaction.

5. The calculation of the discussed P-odd interaction is a task even more complicated than in the case of ordinary van der Waals forces. A simple estimate of the corresponding constant, however, can be obtained immediately. This constant contains two additional smallnesses compared with the usual van der Waals one. First, the P-odd weak interaction gives rise to the factor (see, e.g., Ref. 19)

$$\sim \frac{1}{\pi} G m^2 \alpha^2 Z^2 R,$$

where $G = 10^{-5} m_p^{-2}$ is the Fermi weak-interaction constant, m_p is the proton mass, Z is the charge of the nucleus, and R is the relativistic enhancement factor, which increases rapidly with Z in heavy atoms and reaches values ~ 10 at $Z \sim 80$. Second, the presence one $M1$ amplitude in place of $E1$ yields an additional factor. In the upshot, even for heavy atoms the P-odd constant amounts to only about 10^{-11} of the usual van der Waals constant.

The interaction discussed should lead to a curious phenomenon, observation of which does not seem hopeless at present. There are known crystals, mainly of rare-earth elements, which have a helical spin structure (see, e.g., Ref. 20). Owing to the interactions (21) and (22), the energies of such crystals will be different for right-hand and left-hand spin helices. This is in essence the analog of the discussed difference between the energies of the right-hand and left-hand molecules (or crystals), due to weak interaction. Here, however, the role of the coordinate helix is assumed by the spin helix. The roughest estimate for such an energy difference per unit lattice site yields at $Z \sim 70$

$$\Delta E \sim 10^2 \text{ Hz}.$$

An accuracy $\sim 10^5$ Hz is quite usual in measurements of spin-interaction constants by paramagnetic-resonance methods (see Ref. 21).

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Translated by J. G. Adashko