Polarization of continuous radiation of a weakly ionized gas in a magnetic field

B. A. Zon and V. Ya. Kupershmidt

Voronezh State University
(Submitted 12 June 1981; resubmitted 8 December 1981)
Zh. Eksp. Teor. Fiz. **82**, 1393–1406 (May 1982)

The polarization characteristics of radiation emitted by slow electrons scattered by neutral atoms in a magnetic field, for free-free and free-bound transitions, are calculated. Analytic expressions are obtained for the anti-Hermitian part of the dielectric tensor and it is shown that the quantum corrections to the elements of this tensor describe in the high-frequency limit the dichroism of a weakly ionized magnetoactive gas.

PACS numbers: 51.70. + f, 34.80.Dp, 51.60. + a

1. INTRODUCTION

Continuous emission from a plasma is determined by the bremsstrahlung due to scattering by ions and neutral particles, as well as by recombination and photoattachment processes. For a low-temperature plasma, in which the ion density is much less than the density of the neutral atoms, the ratio of the intensities, for example of the bremsstrahlung in scattering by ions and atoms, is estimated by the formula[1]

$$\frac{S_i}{S_a}=\frac{5\pi}{16}\frac{e^4}{(kT)^2\sigma_{tr}}\left(\frac{g_i}{g_a}\right)^{1/2}\left(\frac{\pi m^3k^3T^3}{18\hbar^6N_a^2}\right)^{1/4}e^{-I/2kT}, \tag{1}$$

where $g_{i,a}$ are the statistical weights of the ion and the atom, I and N_a are the ionization potential and the density of the neutral atoms, and σ_{tr} is the transport cross section for elastic scattering of a slow electron by an atom.

Substituting the numerical values of the constants in (1) we find that at

$$N_a[\text{cm}^{-3}]\,\sigma_{tr}^2[10^{-16}\,\text{cm}^2]>2\cdot10^{26}T^{-5/2}[\text{K}]\,e^{-I/2kT}$$

the bremsstrahlung of a low-temperature plasma is determined by scattering from neutral atoms.

The emission spectrum of a weakly ionized gas has been the subject of a large number of studies. Thus, bremsstrahlung was investigated in Refs. 1 and 2, in which the interaction of the electron with the atom was simulated by a short-range potential. The results of Refs. 1 and 2 have found extensive application, in particular, in astrophysical research. Bremsstrahlung with allowance for the possible contribution of partial waves with higher moments to the cross section was considered in Ref. 4.

At the same time, the polarization characteristics of the continuous radiation have not been sufficiently investigated, although the appropriate investigations can yield additional useful information on the physical processes in a weakly ionized gas. It is obvious that if the electrons have an isotropic distribution in the gas the radiation cannot be polarized. An external magnetic field, however, introduces anisotropy into the system, so that one can expect polarization of the continuous radiation. This question is indeed the subject of the present paper.

In Sec. 2 we present general formulas for the Stokes parameters of the continuous radiation and determine a coordinate frame in which the polarization takes on the simplest form. In Sec. 3 we calculate the amplitude of the bremsstrahlung of an electron moving in a constant magnetic field when scattered by a neutral atom. In Sec. 4 is considered continuous radiation in the case of radiative attachment. The equations obtained for the elementary cross sections and the Stokes parameters are averaged over the Maxwellian distribution of the electrons in Sec. 5. This averaging is carried out in a quasiclassical approximation that is valid when the electron energy in the initial and final states greatly exceeds the distance between the Landau levels. In Sec. 6 are analyzed certain limiting and particular cases of the general formulas, and numerical estimates of the radiation polarization are presented.

The results on the polarization of continuous radiation can be used also to analyze the polarization dependence of an absorption of an electromagnetic field by a weakly ionized gas. In Sec. 7 we derive analytic expressions for the anti-Hermitian part of the dielectric tensor. In particular, we calculate the quantum corrections to the off-diagonal elements of this tensor, which are found to determine the dichroism of the gas in the high-frequency limit.

Since we are considering here the interaction of the field with only electrons, the field frequency ω has a lower bound

$$\omega\gg(\omega_{Bi}\omega_B)^{1/2}, \tag{2}$$

where ω_{Bi} and ω_B are respectively the ion and electron cyclotron frequencies.[5] The upper bound of the frequency is connected with the neglect of the internal degrees of freedom of the gas atoms:

$$\omega\ll\omega^*, \tag{3}$$

where ω^* is the first characteristic frequency of the atom. We neglect also spatial-dispersion effects. The corresponding restrictions take the form[5]

$$k_\perp v_T\ll\omega_B,\quad |\omega-n\omega_B|\gg k_\parallel v_T,\quad n=1,\,2,\ldots, \tag{4}$$

where $k_{\perp,\parallel}$ are the perpendicular and parallel (to the external magnetic field) components of the wave vector of the electromagnetic wave, and v_T is the electron thermal velocity.

We use a system of units in which $c=\hbar=1$ and the temperature is measured in energy units.

 0038-5646/82/050809-07$04.00

2. STOKES PARAMETERS

The probability of dipole emission of a photon with polarization $\mathbf{e}$ and frequency ω into a solid angle $d\Omega$ when an electron goes over between the states i and f is given by[6]

$$dW_{fi}=\frac{\omega^3}{2\pi}|\mathbf{e}^*\mathbf{d}_{fi}|^2 d\Omega, \tag{5}$$

where $\mathbf{d}$ is the dipole moment of the electron. Expression (5) for the emission probability corresponds to neglect of the interaction of the atomic electrons with the electromagnetic field. It is shown in Ref. 7 that this neglect is permissible if condition (3) is satisfied.

We direct the z axis along the external magnetic field $\mathbf{B}$ and introduce the real polarization vectors

$$\mathbf{e}_1=[\mathbf{k}\times\boldsymbol{\zeta}]/|[\mathbf{k}\times\boldsymbol{\zeta}]|,\quad \mathbf{e}_2=[\mathbf{k}\times\mathbf{e}_1]/\omega, \tag{6}$$

where $\mathbf{k}$ is the wave vector of the radiated photon and $\boldsymbol{\zeta}$ is a unit vector along the z axis. The polarization matrix of the radiation density takes, apart from a normalization, the form

$$\rho_{ab}\sim(\mathbf{e}_a\mathbf{d}_{fi})(\mathbf{d}_{if}\mathbf{e}_b),\quad a,\ b=1,\ 2.$$

On the other hand, this matrix can be expressed in terms of the Stokes parameters ξ:

$$\rho_{ab}=\frac{1}{2}\begin{pmatrix}1+\xi_3 & \xi_1-i\xi_2\\ \xi_1+i\xi_2 & 1-\xi_3\end{pmatrix}.$$

Comparing these two formulas, we obtain explicit expressions for the Stokes parameters

$$\begin{gathered}\xi_1=2A^{-1}\operatorname{Re}\{(\mathbf{e}_1\mathbf{d}_{fi})(\mathbf{e}_2\mathbf{d}_{if})\},\quad \xi_2=-2A^{-1}\operatorname{Im}\{(\mathbf{e}_1\mathbf{d}_{fi})(\mathbf{e}_2\mathbf{d}_{if})\},\\ \xi_3=A^{-1}\{|\mathbf{d}_{fi}\mathbf{e}_1|^2-|\mathbf{d}_{fi}\mathbf{e}_2|^2\},\quad A=|\mathbf{d}_{fi}\mathbf{e}_1|^2+|\mathbf{d}_{fi}\mathbf{e}_2|^2=|\mathbf{d}_{fi}|^2-|\mathbf{d}_{fi}\mathbf{k}|^2/\omega^2.\end{gathered} \tag{7}$$

For the polarization vectors (6) we readily obtain

$$\mathbf{d}\mathbf{e}_1=[\mathbf{d}\times\mathbf{k}]_z/k_\perp,\quad \mathbf{d}\mathbf{e}_2=[(\mathbf{d}\mathbf{k})k_z-d_z k^2]/(\omega k_\perp). \tag{8}$$

Since the observed radiation is connected with transitions of many electrons, it is necessary to average in (7) over the initial states i. The quantity A should be averaged here independently of the other factors contained in ρ_{ab}, since it determines the normalization of ρ. Averaging of the Stokes parameters presupposes that the radiation detector is quadratic in the electromagnetic-field amplitude. Namely, if the polarization matrix of the sensitivity of the detector is ρ_d, the signal from the detector is proportional to $\operatorname{Tr}(\rho\rho_d)$.

If the radiation is connected with several physical processes, e.g., with bremsstrahlung and radiative attachment, the Stokes parameters are determined by the equations

$$\xi_i=\frac{A_b}{A}\xi_i^b+\frac{A_r}{A}\xi_i^r,\quad A^{-1}=A_b^{-1}+A_r^{-1},$$

in which $\xi_i^{b,r}$ and $A_{b,r}$ are given by Eqs. (7) for the corresponding process. We note that the quantities $A_{b,r}$ are proportional to the spectral intensities of the emissions into a specified solid angle, and are therefore of independent interest.

3. BREMSSTRAHLUNG AMPLITUDE

Let the electron move in a magnetic field $\mathbf{B}$ and in the field of a neutral atom located at the origin. The potential $V(\mathbf{r},\nu)$ of the interaction of the electron with the atom depends on the radius vector of the electron and on the internal coordinates of the atom ν.

Neglecting the electron spin, we write down the system wave function in the form[8]

$$\psi_\lambda^{(\pm)}(\mathbf{r},\nu)=\chi_\lambda(\mathbf{r},\nu)+\sum_{\lambda'}\frac{\langle\lambda'|T^{(\pm)}|\lambda\rangle\chi_{\lambda'}(\mathbf{r},\nu)}{E_\lambda-E_{\lambda'}\pm i0}. \tag{9}$$

Here

$$\langle\lambda'|T^{(\pm)}|\lambda\rangle=\langle\chi_{\lambda'}|V|\psi_\lambda^{(\pm)}\rangle \tag{10}$$

is the amplitude for the scattering of the electron by an atom in a magnetic field, χ_λ is the wave function of the electron and the atom without allowance for the interaction between them:

$$\chi_\lambda(\mathbf{r},\ \nu)=\varphi_l(\nu)\Phi_{nMp}(r),\quad \lambda=\{n,\ M,\ p,\ l\}, \tag{11}$$

φ_l is the wave function of the atom in the state l with energy $\mathscr{E}_l$, and Φ is the wave function of the electron in the magnetic field. In cylindrical coordinates[9] we have

$$\begin{gathered}\Phi_{nMp}(\mathbf{r})=\frac{1}{\sqrt{2\pi}}\exp[i(pz+M\varphi)]R_{nM}(\rho),\\ R_{nM}(\rho)=(a_B^{1+|M|}|M|!)^{-1}\left[\frac{(n+|M|)!}{2^{|M|}n!}\right]^{1/2}\\ \times\exp\left(-\frac{\rho^2}{4a_B^2}\right)\rho^{|M|}F\left(-n,1+|M|,\frac{\rho^2}{2a_B^2}\right),\\ a_B=(m\omega_B)^{-1/2},\quad \omega_B=|e|B/m,\end{gathered} \tag{12}$$

F is a confluent hypergeometric function. The corresponding energy values are

$$\begin{gathered}E_\lambda=\mathscr{E}_l+\mathscr{E}_{nMp},\quad \mathscr{E}_{nMp}=\mathscr{E}_{nM}+p^2/2m,\\ \mathscr{E}_{nM}=\omega_B(N+1/2),\quad N=n+(|M|+M)/2.\end{gathered} \tag{13}$$

The "±" signs in (9) correspond to waves that diverge or converge relative to the z axis.

Assuming, as before, that the electron energy is much lower than the excitation energy of the atom, we can neglect in (9) the excited states of the atom, assuming that the function φ_l corresponds to the ground state. We shall leave out the index l hereafter.

With the aid of (9)–(12) we can calculate the dipole matrix elements

$$\begin{gathered}\langle\psi_f^{(-)}|d_-|\psi_i^{(+)}\rangle=ea_B\Big\{\operatorname{sign}(M_i)\Big[\frac{N_i^{1/2}}{\omega-\omega_B}\langle N_i-1,M_i-1,p_i|T^{(-)}|f\rangle^*\\ -\frac{(N_i+1-M_i)^{1/2}}{\omega}\langle N_i,M_i-1,p_i|T^{(-)}|f\rangle^*\Big]-\operatorname{sign}(M_f)\Big[\frac{(N_f+1)^{1/2}}{\omega-\omega_B}\\ \times\langle N_f+1,M_f+1,p_f|T^{(+)}|i\rangle-\frac{(N_f-M_f)^{1/2}}{\omega}\langle N_f,M_f+1,p_f|T^{(+)}|i\rangle\Big]\Big\},\\ \langle\psi_f^{(-)}|d_+|\psi_i^{(+)}\rangle=ea_B\Big\{\operatorname{sign}(M_i)\Big[\frac{(N_i+1)^{1/2}}{\omega+\omega_B}\langle N_i+1,M_i+1,p_i|T^{(-)}|f\rangle^*\\ -\frac{(N_i-M_i)^{1/2}}{\omega}\langle N_i,M_i+1,p_i|T^{(-)}|f\rangle^*\Big]-\operatorname{sign}(M_f)\Big[\frac{N_f^{1/2}}{\omega+\omega_B}\\ \times\langle N_f-1,M_f-1,p_f|T^{(+)}|i\rangle-\frac{(N_f+1-M_f)^{1/2}}{\omega}\langle N_f,M_f-1,p_f|T^{(+)}|i\rangle\Big]\Big\},\\ \langle\psi_f^{(-)}|d_0|\psi_i^{(+)}\rangle=\frac{ie}{\omega}\Big\{\frac{p_i}{m\omega}\langle i|T^{(-)}|f\rangle^*-\frac{p_f}{m\omega}\langle f|T^{(+)}|i\rangle\\ -\frac{\partial}{\partial p_i}\langle i|T^{(-)}|f\rangle-\frac{\partial}{\partial p_f}\langle f|T^{(+)}|i\rangle\Big\},\\ d_\pm=\frac{d_x\pm id_y}{2^{1/2}},\quad d_0=d_z,\quad |i\rangle\equiv|N_i,M_i,p_i\rangle,\ |f\rangle\equiv|N_f,M_f,p_f\rangle.\end{gathered} \tag{14}$$

In the calculation of the matrix elements (14), no account was taken of transitions between the unperturbed functions Φ. These transitions correspond to the well-investigated synchrotron radiation, which is concentrated for nonrelativistic electrons near the frequency ω_B. According to bremsstrahlung theory[6] the initial state should correspond to diverging waves and the final to converging ones; this was in fact taken into account in the presented formulas.

In the subsequent calculations of the Stokes parameters we shall confine ourselves to the quasiclassical approximation: $\mathscr{E}_i, \mathscr{E}_i - \omega \gg \omega_B$. In addition, we recognize that in scattering of slow electrons by a neutral atom the main contribution is made by states with small values of M. Assuming also that the scattering amplitude changes slowly over an energy interval of the order of ω, we can neglect the terms $\sim\partial/\partial p_i$ and $\partial/\partial p_f$ in the last formula of (14).

Substituting (14) in (7) and (8), we write down the Stokes parameters accurate to quantities of order $\omega M/\mathscr{E}_{i,f} \ll 1$, assuming that in the quasiclassical approximation $N_{i,f} \gg 1$:

$$\xi_1=0,\quad \xi_2=A^{-1}(v_-^{\,r}-v_+^{\,r})\cos\theta,\quad \xi_3=A^{-1}\left(\frac{v_-^{\,b}+v_+^{\,b}}{2}-v_0^{\,b}\right)\sin^2\theta,$$
$$A=\frac{v_+^{\,b}+v_-^{\,b}}{2}(1+\cos^2\theta)+v_0^{\,b}\sin^2\theta, \tag{15}$$

$$v_\pm^{\,b}=\frac{1}{(\omega\pm\omega_B)^2}\{N_i|\langle N_i, M_i\pm1, p_i|T^{(-)}|f\rangle|^2+N_f|\langle N_f, M_f\mp1, p_f|T^{(+)}|i\rangle|^2$$
$$-2\,\mathrm{sign}(M_iM_f)N_i^{1/2}N_f^{1/2}\,\mathrm{Re}(\langle N_f, M_f\mp1, p_f|T^{(+)}|i\rangle\langle N_i, M_i\pm1, p_i|T^{(-)}|f\rangle)\},$$
$$v_0^{\,b}=\frac{a_B^2}{\omega^2}\{p_i^2|\langle i|T^{(-)}|f\rangle|^2+p_f^2|\langle f|T^{(+)}|i\rangle|^2-2p_ip_f\,\mathrm{Re}(\langle i|T^{(-)}|f\rangle\langle f|T^{(+)}|i\rangle)\}. \tag{16}$$

Here θ is the angle between **k** and **B**, the common factors of the quantities A and v are left out, since they do not change the values of the Stokes parameters.

4. AMPLITUDE OF RADIATIVE ATTACHMENT

Besides the free-free transitions, continuous radiation of a weakly ionized gas is produced also by free-bound transitions. By way of example of such a transition, we consider radiative attachment—the formation of a negative ion in electron-atom collisions accompanied by photon emission.

We write down the wave function of the valence electron in the s state of the negative ion in the form[10]

$$u(r)=b\frac{\varkappa^{1/2}}{(2\pi)^{1/2}}\frac{e^{-\varkappa r}}{r}, \tag{17}$$

where $\mathscr{E}_0=\varkappa^2/2m$ is the ion binding energy. We assume here that

$$\varkappa a_B \gg 1. \tag{18}$$

Therefore the influence of the magnetic field on the bound state can be neglected. The coefficient b is determined from the condition that the formula (17) coincide with the asymptotic more correct expression for the wave function of an electron bound in a negative ion.

Assuming that the attachment of the electron to the atom does not change the wave function of the ground state of the atom, we calculate the matrix elements of the transition

$$\langle u|d_\pm|\psi_i^{(+)}\rangle=\langle u|d_\pm|\Phi_i\rangle=be(2\varkappa)^{1/2}\delta_{M,\mp1}\int_0^\infty K_0(\rho(p^2+\varkappa^2)^{1/2})R_{nM}(\rho)\rho^2d\rho,$$
$$\langle u|d_0|\psi_i^{(+)}\rangle=\langle u|d_0|\Phi_i\rangle=b\frac{iep(2\varkappa)^{1/2}}{(p^2+\varkappa^2)^{1/2}}\delta_{M,0}\int_0^\infty K_1(\rho(p^2+\varkappa^2)^{1/2})R_{nM}(\rho)\rho^2d\rho. \tag{19}$$

Here K is a Macdonald function, the index i of the quantum numbers n, M, and p is left out for brevity. Equations (19) do not contain T-matrix elements, since the potential V is assumed to be spherically symmetrical.

To calculate the integral, we take into account the inequality (8) and the exponential damping of the function $K_{0,1}(x)$ at $x \gg 1$. Consequently, the argument of the functions R_{nM} takes on small values, whereas large values are of importance for the radial quantum number $n \sim T/\omega_B$ (T is the effective electron temperature). For the hypergeometric function in (12) we can therefore use the approximation[11]

$$R_{nM}(\rho)\approx a_B^{-1}[(n+|M|)!/(n!n^{|M|})]^{1/2}J_{|M|}((2n)^{1/2}\rho/a_B). \tag{20}$$

Here J is a Bessel function, and we assume M to be bounded because (19) contains $M=0,\pm1$. We assume likewise that the quantity $n\rho^2/a_B^2$, which is of the order of $T/\mathscr{E}_0$, to be bounded.

Substitution of (20) in (19) leads to tabulated integrals:

$$\langle u|d_\pm|\psi_i^{(+)}\rangle=\frac{4eb\omega_B(\varkappa(n+1))^{1/2}}{(M\omega_B-2\omega)^2}\delta_{M,\mp1},$$
$$\langle u|d_0|\psi_i^{(+)}\rangle=\frac{iebp(\varkappa\omega_B)^{1/2}}{m^{3/2}\omega^2}\delta_{M,0}.$$

Substituting these equations in (7) and in (8) we obtain for the Stokes parameters the expressions (15), in which

$$v_\pm^{\,r}=\frac{16e^2\omega_B^2\varkappa(n_i+1)}{(\omega_B\mp2\omega)^4}\delta_{M_i,\mp1},\quad v_0^{\,r}=\frac{e^2p_i^2\varkappa\omega_B}{m^3\omega^4}\delta_{M_i,0}. \tag{21}$$

It should be noted that the constant b drops out of the Stokes parameters. We note also that the calculations presented here are not valid for negative ions with valence p-electrons.

5. AVERAGING OVER THE ELECTRON DISTRIBUTION

It is seen from (15) that averaging of the Stokes parameters corresponds to averaging of the quantities v. Summing also over all the final states, we write

$$\langle v_k\rangle=\sum_{N_i,N_fM_iM_f}\int\frac{dp_i\,dp_f}{(2\pi)^2}F(\mathscr{E}_i)v_k\delta(\mathscr{E}_i-\mathscr{E}_f-\omega),$$

where $F(\mathscr{E}_i)$ is a distribution function which we assume to depend only on the electron energy.

We consider first bremsstrahlung. The calculations can be carried through to conclusion in two cases. First, when the main contribution to the cross section of the low-energy scattering is made by one of the partial waves. This case, with s-scattering predominant, takes place in particular for noble-gas atoms. Under this assumption we can neglect in (16) the interference terms.

Second, if the terms with different partial moments make comparable contributions to the cross section, the calculations can be carried out in the long-wave approximation: $\omega \ll \mathscr{E}_{i,f}$. The results is then expressed in terms of a certain quantity which goes over in the absence of the magnetic field into the transport cross section.

We assume next that for a spherically symmetrical scattering potential the T-matrix elements summed over the magnetic quantum numbers are determined in the quasiclassical limit only by the electron energy. In the Born approximation, this result follows from Ref. 12, and in the case of a δ-potential it is exact and can be obtained with the aid of the wave functions given in Ref. 13 for an electron in the field of a short-range potential and in a magnetic field.

The foregoing assumption allows us to use the results of Ref. 14 to connect the T-matrix elements with the total elastic-scattering cross section:

$$\sum_{M_iM_f} |\langle N_i, M_i\pm 1, p_i|T^{(-)}|f\rangle|^2 = \sum_{M_iM_f} |\langle N_f, M_f\mp 1, p_f|T^{(+)}|i\rangle|^2 = (4\pi m^2 a_B^4)^{-1}\sigma(\mathscr{E}, B). \quad (22)$$

The appearance of the factor a_B^{-4} in (22) is due to the different normalization of the wave functions of the free motion and of the motion in a magnetic field.

Taking (22) into account and replacing in the quasiclassical approximation the summation over N_i and N_f by integration, after first separating the singularities of the radicand by the method described in Ref. 15, we obtain:

1) If the interference effects are neglected,

$$\langle v_+^b + v_-^b\rangle = \frac{2m}{3\pi^3\omega_B}\frac{\omega^2+\omega_B^2}{(\omega^2-\omega_B^2)^2}\int_{\bar{\mathscr{E}}}^{\infty} (\mathscr{E}(\mathscr{E}-\omega))^{1/2}\sigma(\mathscr{E};B)F(\mathscr{E}) \times\left[\mathscr{E}\left(1+\frac{3}{4}\left(\frac{\omega_B}{\mathscr{E}\Delta}\right)^{1/2}\right)\left(1+\frac{1}{2}\left(\frac{\omega_B}{(\mathscr{E}-\omega)\Delta'}\right)^{1/2}\right) + (\mathscr{E}-\omega)\left(1+\frac{3}{4}\left(\frac{\omega_B}{(\mathscr{E}-\omega)\Delta'}\right)^{1/2}\right)\left(1+\frac{1}{2}\left(\frac{\omega_B}{\mathscr{E}\Delta}\right)^{1/2}\right)\right]d\mathscr{E}, \quad (23)$$

$$\langle v_0^b\rangle = \frac{m}{3\pi^3\omega^2\omega_B}\int_{\bar{\mathscr{E}}}^{\infty}(\mathscr{E}(\mathscr{E}-\omega))^{1/2}\sigma(\mathscr{E};B)F(\mathscr{E}) \times\left[\mathscr{E}\left(1+\frac{1}{2}\left(\frac{\omega_B}{(\mathscr{E}-\omega)\Delta'}\right)^{1/2}\right)+(\mathscr{E}-\omega)\left(1+\frac{1}{2}\left(\frac{\omega_B}{\mathscr{E}\Delta}\right)^{1/2}\right)\right]d\mathscr{E}, \quad (24)$$

$$\bar{\mathscr{E}} = \omega + \omega_B/2,$$

where Δ is the fractional part of $\mathscr{E}/\omega_B - 1/2$ and

$$\Delta' = \begin{cases}\Delta-\delta, & \Delta>\delta\\ 1+\Delta-\delta, & \Delta<\delta\end{cases}, \quad (25)$$

δ is the fractional part of ω/ω_B.

Equations (23) and (24) contain singular terms of two types, namely, proportional to $\Delta^{-1/2}$, $(\Delta')^{-1/2}$, and to the product $(\Delta\Delta')^{-1/2}$. The former make a small contribution upon averaging, whereas the latter lead to a logarithmic divergence even after the averaging. In the calculations that follow we shall therefore retain only these last singular terms.

2) In the long-wave approximation, the cross section for elastic scattering is replaced in (23) and (24) respectively by a transverse and a longitudinal transport cross section. Leaving out the terms $\Delta^{-1/2}$ and $(\Delta')^{-1/2}$, we obtain accurate to $\sim\omega/\mathscr{E}$

$$\langle v_+^b + v_-^b\rangle = \frac{2m}{3\pi^3\omega_B}\frac{\omega^2+\omega_B^2}{(\omega^2+\omega_B^2)^2}\int_0^{\infty}\mathscr{E}^2\sigma_\perp^*(\mathscr{E};B)F(\mathscr{E}) \times\left[1+\frac{3}{4}\frac{\omega_B}{\mathscr{E}\sqrt{\Delta\Delta'}}\right]d\mathscr{E}, \quad (26)$$

$$\langle v_0^b\rangle = \frac{m}{3\pi^3\omega^2\omega}\int_0^{\infty}\mathscr{E}^2\sigma_\parallel^*(\mathscr{E};B)F(\mathscr{E})d\mathscr{E},$$

$$\sigma_\perp^* = \sigma(\mathscr{E};B) - \frac{3\pi\omega_B}{2}\sum_{M_iM_f}\operatorname{sign}(M_iM_f)\int dN_i\,dN_f\frac{N_i^{1/2}N_f^{1/2}}{\mathscr{E}^2} \times[(\mathscr{E}-N_i\omega_B)(\mathscr{E}-N_f\omega_B)]^{-1/2}\operatorname{Re}[\langle N_f, M_f-1, p_f|T^{(+)}|i\rangle\langle N_i, M_i+1, p_i|T^{(-)}|f\rangle],$$

$$\sigma_\parallel^* = \sigma(\mathscr{E};B) - \frac{3\pi}{8}\int\frac{p_ip_f}{m\mathscr{E}^2}\sum_{M_iM_f}dN_i\,dN_f[(\mathscr{E}-N_i\omega_B)(\mathscr{E}-N_f\omega_B)]^{-1/2} \times\operatorname{Re}[\langle i|T^{(-)}|f\rangle\langle f|T^{(+)}|i\rangle]. \quad (27)$$

The values of $\sigma^*_{\perp,\parallel}$ coincide as $B\to 0$ and are equal to the transport scattering cross section in the absence of a magnetic field. To prove this fact, we note that the radiation intensity is expressed in terms of the parameters v^b in accord with Eq. (15):

$$\frac{dI}{d\omega\, d\Omega_k} \sim \frac{1}{2}\langle v_+^b + v_-^b\rangle(1+\cos^2\theta) + \langle v_0^b\rangle\sin^2\theta. \quad (28)$$

At $B=0$ the intensity is independent of the angle θ, therefore, comparing expressions (25) and (27) we arrive at the equation $\sigma_\perp^* = \sigma_\parallel^*$ at $B=0$. The total radiation intensity coincides in this case with the radiation intensity calculated in Refs. 1 and 4, if it is assumed that

$$\sigma^*_{\perp,\parallel} = \sigma_{tr}, \quad B=0. \quad (29)$$

Before we integrate with respect to $d\mathscr{E}$ in (23)–(26), we write, in accord with Ref. 15:

$$\Delta^{-1/2} = 2^{1/2}\sum_{M=1}^{\infty}\frac{(-1)^M}{M^{1/2}}\cos\left(\frac{2\pi M\mathscr{E}}{\omega_B} - \frac{\pi}{4}\right).$$

Discarding in the product $(\Delta\Delta')^{-1/2}$ the rapidly oscillating terms, which make a small contribution upon averaging, we write

$$(\Delta\Delta')^{-1/2} = \sum_{M,M'}^{\infty}\frac{(-1)^{M+M'}}{(M'M)^{1/2}}\cos\left[\frac{2\pi\mathscr{E}(M-M')}{\omega_B} + 2\pi M'\frac{\omega}{\omega_B}\right].$$

The logarithmic divergence in this expression is the result of the terms with $M=M'$. At $\delta\ll 1$ or $1-\delta\ll 1$ we obtain

$$(\Delta\Delta')^{-1/2} = -\ln[2\pi\delta(1-\delta)] = J(\delta). \quad (30)$$

If we assume for $F(\mathscr{E})$ a Maxwellian distribution and regard σ as a quantity that depends weakly on the energy, so that it can be taken outside the integral sign, then the remaining integrals can be obtained from the tables

$$\langle v_+^b + v_-^b\rangle = \left(\frac{2}{mT}\right)^{1/2}\frac{\bar{\sigma}_\perp^*\omega^2 e^{-\bar{\mathscr{E}}/T}}{3\pi\omega_B}\frac{\omega^2+\omega_B^2}{(\omega^2-\omega_B^2)^2}H_1(\beta,\delta),$$

$$\langle v_0^b\rangle = \left(\frac{2}{mT}\right)^{1/2}\frac{\bar{\sigma}_\parallel^*\omega e^{-\bar{\mathscr{E}}/T}}{6\pi\omega_B}H_2(\beta),$$

$$\langle v_-^b - v_+^b\rangle = \frac{2\omega\omega_B}{\omega^2+\omega_B^2}\langle v_+^b + v_-^b\rangle, \quad (31)$$

$$H_1(\beta,\delta) = H_2(\beta) + \frac{3}{4\sqrt{\pi}}\frac{(2+\beta)J(\delta)}{n\beta^2}$$

$$H_2(\beta) = \beta\psi(5/2, 5;\beta), \quad \beta=\omega/T, \quad \bar{\sigma}^*_{\perp,\parallel} = \sigma^*_{\perp,\parallel}(\mathscr{E}_{av}; B),$$

n is the number of the cyclotron-resonance harmonic and is equal to the integer part of ω/ω_B, and ψ is a confluent hypergeometric function.[11]

In writing down (31) we used the fact that the total and transport cross sections are equal if scattering with a definite partial moment coincide. We shall therefore use everywhere the transport cross section. Substituting expressions (31) in (15), we obtain for the Stokes parameters the expressions

$$\begin{gathered}\xi_1{}^b=0,\quad \xi_2{}^b=\frac{4\omega\omega_B D(\beta,\delta)}{(\omega^2+\omega_B{}^2)\mathcal{A}_b}\cos\theta,\\ \xi_3{}^b=\frac{D(\beta,\delta)-1}{\mathcal{A}_b}\sin^2\theta,\quad \mathcal{A}_b=D(\beta,\delta)\,(1+\cos^2\theta)+\sin^2\theta,\\ D(\beta,\delta)=D_0(\omega)\gamma\frac{H_1(\beta,\delta)}{H_2(\beta)},\\ D_0(\omega)=\frac{\omega^2(\omega^2+\omega_B{}^2)}{(\omega^2-\omega_B{}^2)^2},\quad \gamma=\frac{\bar\sigma_\perp{}^*}{\bar\sigma_\parallel{}^*}.\end{gathered}\tag{32}$$

Similar calculations can be carried out also for continuous radiation corresponding to radiation attachment. Here, however, it suffices to retain quantities linear in ω_B/ω, since $\omega_B \ll \mathscr{E}_0$ even for a binding energy $\mathscr{E}_0 \approx 0.01$ eV at $B \ll 10^6$ G.

As a result we obtain

$$\begin{gathered}\xi_1{}^r=0,\quad \xi_2{}^r=\frac{4\omega_B}{\omega\mathcal{A}_p}\cos\theta,\quad \xi_3{}^r=\frac{3}{2}\left(\frac{\omega_B}{\omega-\mathscr{E}_0}\right)^{1/2}\mathcal{A}_r{}^{-1}\sin^2\theta,\\ \mathcal{A}_r=\left(1+\frac{3}{2}\left(\frac{\omega_B}{\omega-\mathscr{E}_0}\right)^{1/2}\right)(1+\cos^2\theta)+\sin^2\theta.\end{gathered}\tag{33}$$

6. LIMITING CASES AND NUMERICAL ESTIMATES

Equations (33), which determine the Stokes parameters of the radiation in radiative attachment, are quite simple and require no further analysis. We consider therefore limiting and particular cases, when the rather cumbersome expressions (32) can be simplified.

The parameter ξ_2^b, which determines the degree of circular polarization of the bremsstrahlung, takes a simple form if the radiation is absorbed along the magnetic field:

$$\xi_2{}^b=\pm\frac{2\omega\omega_B}{\omega^2+\omega_B{}^2},$$

where the upper sign corresponds to $\theta = 0$ and the lower to $\theta = \pi$.

For the other observation angles, the Stokes parameters are determined by the function $D(\beta, \delta)$. This function becomes simpler at $\omega \ll T$, when $\beta \ll 1$. It is known that in this frequency band the radiation has the highest intensity. Using the expansion of the function $\psi(a, c; \beta)$ at $\beta \ll 1$ (Ref. 11), we write

$$D(\beta,\delta)=\gamma D_0(\omega)\left[1+\frac{3}{8}\frac{\omega J(\delta)}{nT}\right].\tag{34}$$

The second term in (34) gives rise to oscillations at frequencies that are multiples of the cyclotron frequency (cf. the theory of magnetophonon oscillations[16]). If the quantum oscillations are neglected, then

$$\begin{gathered}\xi_2{}^b=\frac{4\omega_B\omega\gamma D_0(\omega)}{(\omega^2+\omega_B{}^2)\mathcal{A}_b}\cos\theta,\quad \xi_3{}^b=\frac{\gamma D_0(\omega)-1}{\mathcal{A}_b}\sin^2\theta,\\ \mathcal{A}_b=\gamma D_0(\omega)\,(1+\cos^2\theta)+\sin^2\theta.\end{gathered}\tag{35}$$

For example, putting $\gamma = 1$, we have $\xi_2^b \approx 0.38$ for B = 500 G and ω = 10 GHz at $\theta = \pi/2$.

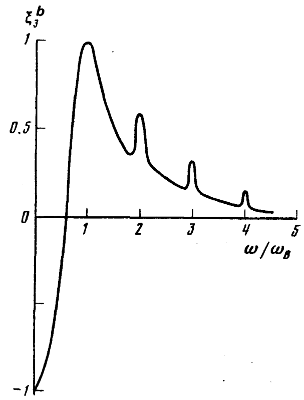

FIG. 1. Dependence of the Stokes parameter ξ_3^b on the radiation frequency at $\delta_{min} = 0.001$, $\omega_B/T = 0.2$, and $\theta = \pi/2$.

At $\omega \ll \omega_B$ it follows from (35) that $\xi_2^b \approx 0$ and $\xi_3^b \approx -1$. Thus, at frequencies much lower than the cyclotron frequency, the bremsstrahlung is almost completely linearly polarized, and its directivity pattern is $\sin^2\theta$, just as radiation from a classical electric dipole.

For frequencies $\omega_B \ll \omega \ll T$ we have

$$\xi_2{}^b\approx 2\omega_B\omega^{-1}\cos\theta,$$

and the Stokes parameter ξ_3^b can take on in this frequency region a maximum value determined by the amplitude of the oscillations at the multiple harmonics:

$$(\xi_3{}^b)_{max}\approx\frac{(\pi\gamma\omega/8nT)\ln\delta_{min}}{1+\gamma+(\pi\gamma\omega/8nT)\ln\delta_{min}},$$

where $\delta_{min} = \delta\omega/\omega_B$ is the minimum value of the detuning, determined by various broadening mechanisms, which are discussed in the next section. The dependence of ξ_3^b on the frequency is shown in Fig. 1.

7. DICHROISM OF WEAKLY IONIZED GAS

The results obtained in the preceding sections for the polarization and angular distribution of the radiation of a weakly ionized gas can be used to analyze similar characteristics of absorption by the gas.

The coefficient of absorption of radiation with polarization **e** and with wave vector **k** is given by

$$\varkappa(\mathbf{k},\mathbf{e})=\frac{N_aN_e\omega}{I_{\mathbf{k}e}}\langle W_{fi}^{(abs)}(\mathbf{k},\mathbf{e})-W_{fi}^{(ind)}(\mathbf{ke})\rangle,\tag{36}$$

where $N_{e,a}$ are the densities of the electrons and neutral atoms, $I_{\mathbf{k}e}$ is the intensity of the incident radiation, $W^{(abs,\,ind)}$ are the probabilities of the absorbed and induced radiation; the averaging over i and f is understood in the same sense as in Sec. 5.

It is known[6] that the quantities $W_{if}^{(abs)}$ and $W_{fi}^{(ind)}$ are equal and are connected with the spontaneous-emission probability (5) by the relation

$$W_{if}^{(abs)}=W_{fi}^{(ind)}=\frac{8\pi^3}{\omega^3}I_{\mathbf{k}e}W_{fi}.\tag{37}$$

If we consider absorption due to free-free transitions, then

$$\langle W_{fi}^{(\mathrm{abs})}(\mathbf{k},\mathbf{e})\rangle=\frac{m}{2\pi^2}\int\limits_{\omega_{B/2}}^{\infty}d\mathscr{E}_iF(\mathscr{E}_i)\sum_{N_i=0}^{\bar{N}_i}\sum_{N_f=0}^{\bar{N}_f}\left\{\left[\mathscr{E}_i-\left(N_i+\frac{1}{2}\right)\omega_B\right]\right.$$
$$\left.\times\left[\mathscr{E}_i+\omega-\left(N_f+\frac{1}{2}\right)\omega_B\right]\right\}^{-1/2}W_{fi}^{(\mathrm{abs})}(\mathbf{k},\mathbf{e}),$$
$$\langle W_{fi}^{(\mathrm{ind})}(\mathbf{k},\mathbf{e})\rangle=\frac{m}{2\pi^2}\int^{\infty}d\mathscr{E}_iF(\mathscr{E}_i)\sum_{N_i=0}^{\bar{N}_i}\sum_{N_f=0}^{\bar{N}_f}\left\{\left[\mathscr{E}_i-\left(N_i+\frac{1}{2}\right)\omega_B\right]\right.$$
$$\left.\times\left[\mathscr{E}_i-\omega-\left(N_f+\frac{1}{2}\right)\omega_B\right]\right\}^{-1/2}W_{fi}^{(\mathrm{ind})}(\mathbf{k},\mathbf{e}),\tag{38}$$

where $\bar{N}_{i,f}$ are the integer parts of the numbers $\mathscr{E}_i/\omega_B-\frac{1}{2}$ and $(\mathscr{E}_i-\omega)/\omega_B-\frac{1}{2}$, respectively.

Making in (38) the substitution $\mathscr{E}_i\to\mathscr{E}_i-\omega$ and redesignating the summation indices $N_i\leftrightarrow N_f$, we obtain after substituting (37) and (38) in (36)

$$\varkappa(\mathbf{k},\mathbf{e})=\frac{8\pi^3}{\omega^2}N_aN_e(e^\beta-1)\langle W_{fi}(\mathbf{k},\mathbf{e})\rangle.\tag{39}$$

A Maxwellian distribution was assumed here for the function $F(\mathscr{E})$.

We introduce the coefficients of linear and circular dichroism[17]:

$$\gamma_l=2\frac{\varkappa(\mathbf{k},\mathbf{e}_1)-\varkappa(\mathbf{k},\mathbf{e}_2)}{\varkappa(\mathbf{k},\mathbf{e}_1)+\varkappa(\mathbf{k},\mathbf{e}_2)},$$
$$\gamma_c=2\frac{\varkappa(\mathbf{k},\mathbf{e}_-)-\varkappa(\mathbf{k},\mathbf{e}_+)}{\varkappa(\mathbf{k},\mathbf{e}_-)+\varkappa(\mathbf{k},\mathbf{e}_+)},$$

where $\mathbf{e}_\pm=(\mathbf{e}_1\pm i\mathbf{e}_2)/2^{1/2}$, and $\mathbf{e}_1$ and $\mathbf{e}_2$ are defined in (6).

With the aid of (7) and (39) we obtain

$$\gamma_l=2\xi_3{}^b,\quad \gamma_c=2\xi_2{}^b.\tag{40}$$

Precisely the same relations between the Stokes parameters and the dichroism coefficients can be obtained also for the "photodetachment–radiative attachment" process. We can therefore use for the dichroism coefficients the equations obtained in the preceding sections for the Stokes parameters.

Certain interest attaches also to the absorption coefficient (36), which determines the anti-Hermitian part of the dielectric tensor, a part connected with the collisions of the electrons with the neutral component. Usually this tensor is written in the form

$$\varepsilon_{ij}^{(a)}=\begin{pmatrix}\varepsilon_\perp{}^a & ig^a & 0\\ -ig^a & \varepsilon_\perp{}^a & 0\\ 0 & 0 & \varepsilon_\parallel{}^a\end{pmatrix},\tag{41}$$

$$\varepsilon_\perp{}^a=\frac{i\omega_p{}^2(\omega^2+\omega_B{}^2)}{\omega(\omega^2-\omega_B{}^2)^2}\nu_{\mathrm{eff}},\quad \varepsilon_\parallel{}^a=\frac{i\omega_p{}^2}{\omega^3}\nu_{\mathrm{eff}},$$
$$g^a=\frac{2i\omega_p{}^2\omega_B}{(\omega^2-\omega_B{}^2)^2}\nu_{\mathrm{eff}},$$

$\omega_p=(4\pi e^2N_e/m)^{1/2}$ is the electron plasma frequency, and ν_{eff} is the effective collision frequency. The tensor (41) is written in a coordinate system in which the z axis is directed along B and the x axis along $\mathbf{e}_1$.

The absorption coefficients (36) are expressed in terms of the components of the tensor ε_{ij}^a with the aid of the following formulas:

$$\varkappa(\mathbf{k},\mathbf{e}_-)-\varkappa(\mathbf{k},\mathbf{e}_+)=2i\omega g^{a*}\cos\theta,\quad \varkappa(\mathbf{k},\mathbf{e}_1)=i\omega\varepsilon_\perp{}^{a*},$$
$$\varkappa(\mathbf{k},\mathbf{e}_2)=i\omega(\varepsilon_\perp{}^{a*}\cos^2\theta+\varepsilon_\parallel{}^{a*}\sin^2\theta).\tag{42}$$

Using the calculations performed in the preceding sections to find the quantities (36), and comparing the result with Eqs. (32), we can obtain expressions for the frequency of the collisions due to absorption in free-free transitions:

$$\nu_{\perp,\parallel}(\omega)=\nu_{\mathrm{eff}}\lambda_{\perp,\parallel}\beta^2(1-e^{-\beta}),\quad \nu_{\mathrm{eff}}=\frac{8}{3\pi^{1/2}}\bar{\sigma}_{tr}N_a\left(\frac{2T}{m}\right)^{1/2},$$
$$\lambda_\perp=\frac{\pi^{1/2}}{8}\frac{\overline{\sigma_\perp{}^*}(B)}{\bar{\sigma}_{tr}}H_1(\beta,\delta),\quad \lambda_\parallel=\frac{\pi^{1/2}}{8}\frac{\overline{\sigma_\parallel{}^*}(B)}{\bar{\sigma}_{tr}}H_2(\beta),\tag{43}$$

σ_{tr} is the transport cross section in the absence of a magnetic field and corresponds to the average energy $\mathscr{E}_{av}$. The quantity ν_{eff} was chosen to be the effective collision frequency; in most papers the numerical coefficient of ν_{eff} is as a rule not determined (see, e.g., Ref. 10).

As follows from the foregoing equations, the collision frequency outside the main cyclotron resonance line differs from that used in the τ approximation by two factors: by the frequency dependence that leads to a difference between the cyclotron-resonance line shape from a Lorentzian, and the anisotropy connected with the magnetic field. The quantum effects lead also to oscillatory singularities of $\nu_\perp$. The amplitude of the oscillations is determined by the line-broadening mechanisms, most important among which for the problem considered here are the following.[18]

1) The broadening connected with the motion and recoil of the gas atoms. This mechanism leads to the following limitation on the quantity $\delta_{\min}^{(1)}$ that determines the oscillation amplitude

$$\delta_{\min}^{(1)}=\left(\frac{m}{M}n\frac{T_a}{\omega_B}\right)^{1/2},$$

where M is the mass of the atom and T_a is the temperature of the atoms.

2) Collision broadening, which leads to finite lifetime of the electron on a given Landau level. In this case

$$\delta_{\min}^{(2)}=(\omega_B\tau_{\mathrm{coll}})^{-1}.$$

We note that the Doppler broadening $\sim nv_T/c$, which is connected with condition (4), as well as the radiative broadening, turns out to be much less.

If the gas density is low enough, so that the following

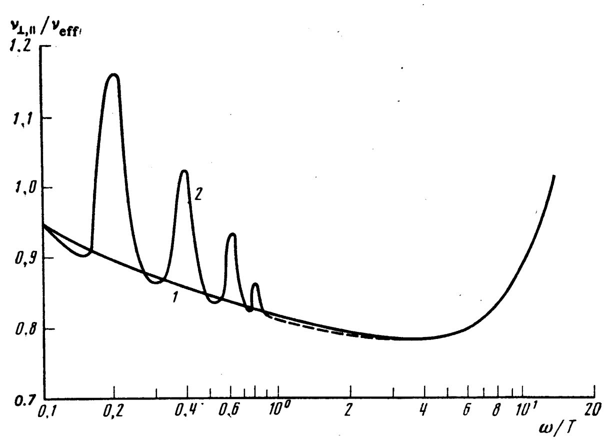

FIG. 2. Dependence of the transverse (curve 2) and longitudinal (curve 1) collision frequencies of radiation frequency in semilog scale.

condition is satisfied

$$\frac{m}{\bar{\sigma}_{tr}N_a}\left(\frac{T_a}{T_e}\frac{\omega_B}{M}n\right)^{1/2}\gg 1,$$

then the broadening mechanism connected with the motion and the recoil of the atoms is found to predominate. For example, at $T_e\approx T_a$, $B\approx 10^4$ G, $N_a\approx 10^{16}$ cm^{-3}, and $\bar{\sigma}_{tr}=10^{-15}$ cm^2 the reduced ratio turns out to be ~10^3.

Taking the foregoing into account and carrying out estimates similar to those in Ref. 19, it is easy to verify that the amplitude of the oscillations of the quantities $\varepsilon_\perp^a$ and g^a decreases with increasing number of the harmonic like ~n^{-2}.

Using the asymptotic value of the function ψ at small and large values of the argument,[11] we obtain at $\omega \ll T$

$$\nu_\perp(\omega)=\nu_{\text{eff}}\frac{\bar{\sigma}_\perp^*(B)}{\bar{\sigma}_{tr}}\left[1-\frac{\beta^2}{4}-\frac{3}{8}\frac{\beta}{n}\ln\left(\frac{m}{M}n\frac{T_a}{\omega_B}\right)^{1/2}\right],\quad \nu_\parallel(\omega)=\nu_{\text{eff}}\frac{\bar{\sigma}_\parallel^*(B)}{\bar{\sigma}_{tr}}\left[1-\frac{\beta^2}{4}\right], \tag{44}$$

and at $\omega \gg T$

$$\nu_{\perp,\parallel}=\frac{\pi^{1/2}}{8}\nu_{\text{eff}}\frac{\bar{\sigma}_{\perp,\parallel}^*(B)}{\bar{\sigma}_{tr}}\beta^{1/2}. \tag{45}$$

Putting $\bar{\sigma}_{\perp,\parallel}^*=\bar{\sigma}_{tr}$, we can easily verify that the asymptotic form (45) coincides with the asymptotic form that follows from the results of Ref. 4. The collision-frequency growth given by expression (45) continues up to frequencies limited by the condition (3).

Figure 2 shows the calculated values of $\nu_{\perp,\parallel}$ at $\omega_B/T = 0.2$, $\delta_{\min}^{(1)}=0.01$, and $\bar{\sigma}_{\perp,\parallel}=\bar{\sigma}_{tr}$.

[1]O. B. Firsov and M. I. Chibisov, Zh. Eksp. Teor. Fiz. **39**, 1770 (1960) [Sov. Phys. JETP **12**, 1235 (1961)].

[2]T. Ohmura and H. Ohmura, Phys. Rev. **121**, 513 (1961).

[3]A. Dalgarno and N. F. Lane, Astrophys. J. **145**, 623 (1966).

[4]V. A. Kas'yanov and A. N. Starostin, Zh. Eksp. Teor. Fiz. **48**, 295 (1965) [Sov. Phys. JETP **21**, 193 (1965)].

[5]E. M. Lifshitz and L. P. Pitaevskiĭ, Physical Kinetics, Pergamon, 1981, Chap. V.

[6]V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, Kvantovaya elektrodinamika (Quantum Electrodynamics), Nauka, 1980, Chap. V.

[7]B. A. Zon, Zh. Eksp. Teor. Fiz. **77**, 44 (1979) [Sov. Phys. JETP **50**, 21 (1979)].

[8]M. L. Goldberger and K. M. Watson, Collision Theory, Wiley, 1964.

[9]L. D. Landau and E. M. Lifshitz, Quantum Mechanics, Nonrelativistic Theory, Pergamon, 1978, §112.

[10]B. M. Smironov, Fizika slaboionizovannogo gaza (Physics of Weakly Ionized Gas), Nauka, 1972.

[11]A. Erdelyi, ed., Higher Transcendental Functions, McGraw, 1953.

[12]R. Goldman, Phys. Rev. **133**, A647 (1964).

[13]Yu. N. Demkov and V. N. Ostrovskiĭ, Metod potentsialov nulevogo radiusa v atomnoĭ fizike (Method of Zero-Radius Potentials in Atomic Physics), Leningrad Univ. Press, 1975.

[14]I. Ventura, Phys. Rev. **A8**, 3021 (1973).

[15]Transl. in: Voprosy kvantovoĭ teorii neobratimykh protcessov (Problems in Quantum Theory of Irreversible Processes), IIL, 1961.

[16]I. M. Tsidil'kovskiĭ, Élektrony i dyrki v poluprovodnikakh (Electrons and Holes in Semiconductors), Nauka, 1972, Chap. 6.

[17]M. Born, Optik, Springer, 1933; Edwards Bros. 1944.

[18]G. G. Pavlov and A. N. Panov, Zh. Eksp. Teor. Fiz. **71**, 572 (1976) [Sov. Phys. JETP **44**, 300 (1976)].

[19]Yu. A. Gurvich, *ibid.* **61**, 1120 (1971) [**34**, 598 (1972)].

Translated by J. G. Adashko