Phonon emission by rotons and relaxation of a quasiparticle gas

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We investigate emission (absorption) of phonons by rotons, due to the energy uncertainty of the latter. The temperature Green's function method is used to obtain the photon-roton relaxation time for the process in question. The contributions of the obtained time to the phonon viscosity and to the thermal conductivity at various pressures are calculated.

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The principles of the kinetics of superfluid helium were developed by Landau and Khalatnikov.[1-3] According to the cited papers, establishment of equilibrium between phonons and rotons is determined by the Rayleigh scattering of the phonons by the rotons. It will be shown in the present article that in addition to this mechanism, relaxation in the quasiparticle gas is also possible and is due to emission (absorption) of phonons by rotons.

It is known that at momenta p smaller than and equal to the momentum p_0 corresponding to the minimum of the energy Δ, a roton cannot emit a phonon. A for the region of energetic rotons with $p > p_c = p_0 + \mu c$, the possibility of this process cannot be unequivocally deduced from the experimental data. (Here μ is the roton effective mass and c is the speed of sound). However, as will be shown below, if account is taken of the finite lifetime of the rotons, the energy uncertainty permits the emission and absorption of phonons to take place in the entire range of roton momenta, and this leads to additional relaxation in the quasiparticle gas. A similar situation, but connected with the broadening of the phonon energy spectrum, arises when the absorption of ultrasound by thermal lattice vibrations in a solid is considered.[4]

According to Ref. 1, the Hamiltonian of the considered system can be written in the form

$$H = \sum \omega_{\mathbf{k}} b_{\mathbf{k}}^{+} b_{\mathbf{k}} + \sum E_{\mathbf{p}} B_{\mathbf{p}}^{+} B_{\mathbf{p}} + \sum (B_{\mathbf{p}-\mathbf{k}/2}^{+} B_{\mathbf{p}+\mathbf{k}/2} b_{\mathbf{k}}^{+} V_3(\mathbf{p}, \mathbf{k}) + \text{H.c.}) + \frac{1}{2} \sum B_{\mathbf{p}_1} B_{\mathbf{p}_2}^{+} V_4(\mathbf{p}_1 \mathbf{p}_2 | \mathbf{p}_1' \mathbf{p}_2') B_{\mathbf{p}_1'} B_{\mathbf{p}_2'} + H_{phph}. \tag{1}$$

The first two terms describe here the gas of interacting phonons and rotons, the third describes the phonon-roton interaction, and the fourth and fifth respectively the roton-roton and phonon-phonon interactions. (The explicit form of H_{phph} is of no importance to us.)

We start with the temperature Green's function for phonons:

$$G(x, x') = -\langle T b(x) \bar{b}(x') \rangle, \tag{2}$$

whose Fourier transform we write in the form

$$G(\mathbf{k}, i\omega) = [i\omega - \omega_{\mathbf{k}} - \Sigma(\mathbf{k}, i\omega)]^{-1}.$$

The self-energy part, irreducible in the phonon lines, which enters in the mass operator Σ and correspond to the process of emission (absorption) of a phonon by a roton, can be represented graphically (see Fig. 1) in the form of a loop made up of two complete roton Green's functions and having two vertices, one of which V_3 is simple and the other is unrenormalized. As shown in the diagram, the second vertex can be expressed in terms of the simple vertex and the interaction between the rotons. In the upshot we have

$$\Sigma_{phr} = -\int V_3(\mathbf{p}, \mathbf{k}) F(\mathbf{p}, \mathbf{p}' | \mathbf{k}, i\omega) V_3(\mathbf{p}', \mathbf{k}) \frac{d\mathbf{p} d\mathbf{p}'}{(2\pi)^3}, \tag{3}$$

where $F(\mathbf{p}, \mathbf{p}' | \mathbf{k}, i\omega)$ is the Fourier transform of the temperature correlator

$$\langle T \delta f_{\mathbf{pk}}(\tau) \overline{\delta f}_{\mathbf{p}'\mathbf{k}}(\tau') \rangle_{\mathrm{irr}}.$$

The subscript "irr" means here that only the irreducible parts are to be taken, and

$$\delta f_{\mathbf{pk}}(\tau) = \bar{B}_{\mathbf{p}-\mathbf{k}/2}(\tau) B_{\mathbf{p}+\mathbf{k}/2}(\tau)$$

is the operator analog of the fluctuation of the distribution function in the temperature representation.

To obtain the phonon reciprocal lifetime t^{-1}, due to the considered process, we must carry out in (3) an analytic continuation ($i\omega \to \omega + i\delta$), which is carried out here in standard fashion (see, e.g., Ref. 5). As a result we have for $t^{-1} = 2 \operatorname{Im} \Sigma_{phr}$

$$t^{-1} = -2 \operatorname{Im} \int V_3(\mathbf{p}, \mathbf{k}) F(\mathbf{p}, \mathbf{p}' | \mathbf{k}, \omega) V_3(\mathbf{p}', \mathbf{k}) \frac{d\mathbf{p}' d\mathbf{p}}{(2\pi)^3}. \tag{4}$$

Here $\omega = ck$ is the phonon energy and $F(\mathbf{p}, \mathbf{p}' | \mathbf{k}, \omega)$ is the Fourier transform of the function of the linear response of the roton to an external perturbation of the type $U_p \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$. This function is expressed in terms of the mean value of the commutators δf and δf^{+} in the form

$$F(\mathbf{p}, \mathbf{p}' | \mathbf{k}, t) = -i\theta(t) \langle [\delta f_{\mathbf{pk}}(t), \delta f_{\mathbf{p}'\mathbf{k}}^{+}(0)] \rangle. \tag{5}$$

Differentiating the function (5) with respect to time we obtain

$$i \frac{\partial F}{\partial t} = \Delta f \delta(\mathbf{p} - \mathbf{p}') \delta(t) + \theta(t) \left\langle \left[\frac{\partial \delta f_{\mathbf{pk}}(t)}{\partial t}, \delta f_{\mathbf{p}'\mathbf{k}}^{+}(0) \right] \right\rangle, \tag{6}$$

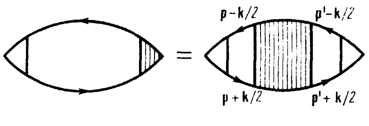

FIG. 1.

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where $\Delta f = f(\mathbf{p}-\mathbf{k}/2) - f(\mathbf{p}+\mathbf{k}/2)$ is the difference between the roton distribution functions:

$$f(\mathbf{p}\pm\mathbf{k}/2) = \langle B^{+}_{\mathbf{p}\pm\mathbf{k}/2} B_{\mathbf{p}\pm\mathbf{k}/2}\rangle.$$

The operator analog of the linearized kinetic equation for δf can be written in the form

$$\left(i\frac{\partial}{\partial t} - \mathbf{kv} - iI\right)\delta f_{\mathbf{pk}}(t) = 0, \tag{7}$$

where I is the linearized collision integral and $\mathbf{v} = \partial E/\partial \mathbf{p}$ is the roton velocity. (We take into account here and everywhere that $p \gg k$.) Using (6) and (7), we obtain an equation for the Fourier transform of the linear-response function:

$$(\omega - \mathbf{kv} - iI)F = \Delta f \delta(\mathbf{p}-\mathbf{p}'). \tag{8}$$

Substituting (8) in (4) and integrating with respect to $\mathbf{p}'$ we obtain

$$t^{-1} = -2\,\mathrm{Im}\int V_3(\mathbf{p},\mathbf{k})(\omega - \mathbf{kv} - iI)^{-1} V_3(\mathbf{p},\mathbf{k})\Delta f \frac{d\mathbf{p}}{(2\pi)^3}. \tag{9}$$

If we disregard the dependence of the parameters of the roton spectrum on the density then, following Ref. 1, we can write V_3 in the form

$$V_3(\mathbf{p},\mathbf{k}) = (ck/2\rho)^{1/2}\mathbf{pk}/k. \tag{10}$$

Relation (9) yields then, accurate to terms of order $(v/c)^2$,

$$t^{-1} = -\frac{c^2}{\rho\omega}\,\mathrm{Im}\int \mathbf{pk}\left(\frac{1}{\omega - iI} + \frac{1}{\omega - iI}\mathbf{kv}\frac{1}{\omega - iI}\right)\mathbf{pk}\Delta f\frac{d\mathbf{p}}{(2\pi)^3}. \tag{11}$$

According to the law of momentum conservation in collisions, the first term under the integral sign makes no contribution to t^{-1}, and the second can be rewritten in the form

$$t^{-1} = -\frac{c^2}{\rho\omega^2}\,\mathrm{Im}\int(\mathbf{pk})(\mathbf{vk})(\omega - iI)^{-1}(\mathbf{pk})\Delta f\frac{d\mathbf{p}}{(2\pi)^3}. \tag{12}$$

In the hydrodynamic limit t^{-1} can be expressed exactly (in the considered approximation) in terms of the first and second roton viscosities. To this end it is necessary to average in (12) over the direction of the vector $\mathbf{k}$ and symmetrize the obtained expression. The result is the known formula (see, e.g., Ref. 6) for the absorption of first sound in a roton gas:

$$t^{-1} = \frac{\omega^2}{\rho c^2}\left(\frac{4}{3}\eta + \zeta\right), \tag{13}$$

where the first viscosity is

$$\eta = -\frac{1}{10T}\int p_\alpha v_\beta I^{-1}\left(p_\alpha v_\beta - \frac{1}{3}\delta_{\alpha\beta}pv\right)f(\mathbf{p})\frac{d\mathbf{p}}{(2\pi)^3},$$

and the second viscosity

$$\zeta = -\frac{1}{3T}\int pvI^{-1}\left(\frac{pv}{3} - \frac{S_r}{c_r}E\right)f(\mathbf{p})\frac{d\mathbf{p}}{(2\pi)^3}.$$

Here S_r and c_r are respectively the entropy and specific heat of the roton gas.

The character of the interaction between rotons is not well known, so that we continue the calculation in the τ_r approximation. In the upshot we obtain according to (12) for arbitrary $\omega\tau_r$

$$t^{-1} = \frac{p_0^2 N}{5\rho\mu c^2}\frac{\omega^2\tau_r}{1+\omega^2\tau_r^2}, \quad N = \frac{2p_0^2(\mu T)^{1/2}}{(2\pi)^{3/2}}e^{-\Delta/T}. \tag{14}$$

Here N is the roton density and τ_r is a parameter of the problem and is brought about by the roton-roton interaction. In the hydrodynamic limit ($\omega\tau_r \ll 1$) the parameter τ_r is determined by dissipative processes in the roton gas. At $\omega\tau_r \gg 1$ it is due to broadening of the roton spectrum. Thus, the upper limit τ_η of the parameter τ_r and be obtained from the roton viscosity, and the lower τ_r from the measured width of the roton line. It follows from most experiments that $\tau_\eta \approx \tau_r$ (see, e.g., Ref. 7 and the literature there). Exceptions are the results of Ref. 8 in which the ratio $\tau_\eta/\tau_r \approx 10$ at $T = 1.2$ K.

The linear-response function can, of course, be found in standard fashion, by solving an equation of the type (8) in the τ_r approximation. Its pole corresponds, as it should, to the roton second sound.[9] The result obtained in this manner coincides naturally with (14). Here, however, this approach turns out to be more cumbersome. In addition, it presupposes the use of the τ_r approximation at the very outset and does not yield expression (13).

The foregoing results presuppose smallness of $(v/c)^2$. For rotons with momenta $p > p_c$ this parameter, according to available experimental data, is not small, therefore the contribution of rotons with $p > p_c$ to the quasiparticle-gas relaxation calls for a separate analysis. The starting point is expression (9). Just as in Ref. 10, the dispersion law for the rotons will be assumed in this region to be close to linear:

$$E = E_c + c(p-p_c)[1-\alpha(p-p_c)], \quad E_c = \Delta + \mu c^2/2. \tag{15}$$

To estimate the relaxation time at $\omega\tau_r \gg 1$ we use in (9) the τ_r approximation. As a result we get

$$t^{-1} = \frac{p_c^2\omega\tau_r}{\rho}(1-e^{-\omega/T})\int\frac{\theta(p-p_c)f(\mathbf{p})}{1+(\omega-\mathbf{kv})^2\tau_r^2}\frac{d\mathbf{p}}{(2\pi)^3}. \tag{16}$$

To obtain the last expression we made use of the presence of a sharp maximum of the integrand at small angles between the vectors $\mathbf{k}$ and $\mathbf{v}$. After integration, expression (16) takes the form

$$t^{-1} = \frac{p_c^2 N_c}{2\rho}(1-e^{-\omega/T})\,\mathrm{arctg}\frac{2\omega\tau_r}{1+2\alpha\overline{(p-p_c)}\omega^2\tau_r^2}. \tag{17}$$

Here $N_c = (p_c^2 T/2\pi^2 c)\exp(-E_c/T)$ is the density of rotons with momenta larger than p_c, and $(\overline{p-p_c}) = T/c$ is the mean value with the distribution function $f(\mathbf{p})$ at $p > p_c$.

If $2|\alpha|Tc^{-1}\omega^2\tau_r^2 \ll 1$ (but $\omega\tau_r \gg 1$), we obtain

$$t^{-1} = (\pi p_c^2/4\rho)N_c(1-e^{-\omega/T}). \tag{18}$$

We note that the result (18) does not depend on whether we have or have not a decay spectrum, and everything is determined by the broadening of the roton energy.

In the inverselimiting case $2|\alpha|Tc^{-1}\omega^2\tau_r^2 \gg 1$ (but $|\alpha|T/c \ll 1$) we have for a non-decay spectrum ($\alpha > 0$)

$$t^{-1} = \frac{p_c^2 N_c}{2\rho}(1-e^{-\omega/T})\,\mathrm{arctg}\frac{c}{\alpha T\omega\tau_r}, \tag{19}$$

and for a decay spectrum ($\alpha < 0$)

$$t^{-1} = \frac{p_c^2 N_c}{2\rho}(1-e^{-\omega/T})\left(\pi - \mathrm{arctg}\frac{c}{|\alpha|T\omega\tau_r}\right). \tag{20}$$

We note that in this case the result depends substantially on the character of the spectrum. If it is assumed that $|\alpha|Tc^{-1}\omega\tau_r \gg 1$ but $\omega/T > 1$, then (20) coincides in fact with the result obtained in Ref. 11 directly from the collision integral.

In the hydrodynamic case ($\omega\tau_r \ll 1$) the contribution of the rotons with momenta $p > p_c$ to the relaxation process can be easily obtained by putting (12) $v = c$ and using the τ_r approximation:

$$t^{-1} = (p_c^2 N_c / 5\rho T)\,\omega^2 \tau_r. \tag{21}$$

Numerical estimates show that the obtained time is 5–7 times larger (depending on the temperature) than the time given by expression (14).

Unfortunately, the available experimental data do not make it possible to determine the parameter α with sufficient accuracy. It is therefore difficult to estimate the contribution made to the quasiparticle-gas relaxation by the rotons with $p > p_c$. In the subsequent calculations we confine ourselves therefore to the time given in expression (14).

We note that we can obtain in similar fashion the roton relaxation time due to the considered processes. In this case we must start out from the roton Green's function. The corresponding irreducible self-energy part, which enters in the roton mass operator, can be represented by a loop made up of the complete roton and phonon Green's functions with simple vertices V_3. The relaxation time obtained in this manner is much longer than τ_r.

We calculate now the contributions of the obtained time to the phonon viscosity and to the thermal conductivity at various pressures. The collision integral of the kinetic equation for the phonon distribution function n, following Ref. 12, can be written in the form

$$\left(\frac{\partial n}{\partial t}\right)_c = -(n - n_0)/t_i - [n - (e^{\beta\omega} - 1)^{-1}]/t_l, \tag{22}$$

where $n_0 = (e^{\omega/T} - 1)^{-1}$ is the equilibrium distribution function; $t_i^{-1} = t_{phr}^{-1} + t_{ph}^{-1} + t^{-1}$ is the total collision frequency, in which

$$t_{phr}^{-1} = p_0^4 N \omega^4 / 100\pi\rho^2\mu^2 c^7 \tag{23}$$

is due to the Rayleigh scattering of the phonons by the rotons, t^{-1} is due to emission (absorption) of photons by rotons, t_{ph}^{-1} is due to phonon-phonon large-angle scattering; t_l is the time connected with small-angle phonon scattering.

The first term in (22) describes relaxation to the equilibrium distribution function, and the second the establishment of equilibrium in a specified direction with a temperature β^{-1} that depends on $\mathbf{j} = \mathbf{k}/k$. The parameter $\beta(\mathbf{j})$ is determined by the law of energy conservation in phonon-phonon collisions in the specified direction:

$$\int \frac{1}{t_l} [n - (e^{\beta\omega} - 1)^{-1}]\,\omega k^2\,dk = 0. \tag{24}$$

If small-angle scattering does not establish equilibrium in the number of phonons, it will be necessary to introduce in the second term of (22) also the chemical potential $\mu(\mathbf{j})$, which is determined by the conservation of the number of phonons in collisions at a fixed $\mathbf{j}$. However, as will be made clear by the calculations that follow, allowance for $\mu(\mathbf{j})$ is of no importance.

The deviation from the equilibrium state is assumed, as usual, to be small:

$$n = n_0 + n_1, \quad \beta = T^{-1} + \beta_1.$$

To obtain the viscosity due to the phonons we must use a kinetic equation with a collision integral (22) and with the condition (24). As a result we have

$$n_1 = \left(j_\alpha j_\beta - \frac{1}{3}\delta_{\alpha\beta}\right) V_{\alpha\beta} \frac{\omega n_0'}{2T} \frac{t_i t_l}{t_i + t_l} \left(1 + \frac{1}{t_l}\frac{J_1}{J_2}\right), \tag{25}$$

where the prime of n_0' denotes differentiation with respect to the argument; $V_{\alpha\beta}$ is the shear tensor in the kinetic equation;

$$J_1 = \int_0^\infty \frac{t_i \omega^4 n_0'}{t_i + t_l}\,d\omega, \quad J_2 = \int_0^\infty \frac{\omega^4 n_0'}{t_i + t_l}\,d\omega.$$

Substituting (25) in the momentum-flux tensor, we obtain for the first-viscosity coefficient

$$\eta_{ph} = -\frac{1}{30\pi^2 c^3 T}\left(\frac{J_1^2}{J_2} + J_3\right), \tag{26}$$

where

$$J_3 = \int_0^\infty \frac{t_i t_l}{t_i + t_l}\,\omega^4 n_0'\,d\omega.$$

We have similarly for the thermal-conductivity coefficient

$$\varkappa_{ph} = -\frac{1}{6\pi^2 c T^2}\left(1 - \frac{ST}{\rho_n c^2}\right)\left(\frac{J_1^2}{J_2} + J_3\right), \tag{27}$$

where S is the entropy and ρ_n the density of the normal component. We note that expressions (26) and (27) depend in like fashion on the relaxation times. It suffices therefore hereafter to consider only the viscosity.

We confine ourselves to the temperature region $T \gtrsim 0.9$ K. At saturated-vapor pressure, the shortest is the small-angle relaxation time $t_l = t_{1\to 2}$, which corresponds to three-phonon processes.[13] In first-order approximation $t_{1\to 2} = 0$, and then

$$\eta_{ph} = -\frac{4\pi^4}{75}\rho_{nph} c^2 \left(\int_0^\infty t_i^{-1} x^4 n_0'\,dx\right)^{-1}, \tag{28}$$

where ρ_{nph} is the phonon part of the normal density, and $x = \omega/T$ is a dimensionless parameter.

We note that in this case the viscosity, in accord with Ref. 2, is found to be inversely proportional to the mean value of the collision frequency. Since $t_{phr}^{-1} \sim x^4$, the main contribution upon averaging comes from the energetic phonons with $x \approx 2\pi$. This is a reflection of the fact that, according to Refs. 1–3, the relaxation in the phonon gas proceeds in two stages. An equilibrium is first established between the roton gas and the high-energy phonons with $x \approx 2\pi$, and then all the remaining phonons attune themselves instantaneously ($t_{1\to 2} \to 0$) to the energetic ones. The dissipative coefficients for finite $t_{1\to 2}$ were obtained in Ref. 14.

Integration in (28) yields

$$\eta_{ph} = {}^1/_5\,\rho_{nph} c^2 \tau_{phr} (1 + \tau_{phr}/t_a)^{-1}, \tag{29}$$

where $\tau_{phr} = t_{phr}$ at $x = 2\pi$, and t_a is determined by (14) with $\omega\tau_r \gg 1$. We present the numerical value of the time ratio in (29):

$$\tau_{phr}/t_a=3.7\cdot 10^{-12}T^{-4}\tau_r^{-1}. \tag{30}$$

If τ_r is taken to be its upper bound, obtained from the roton viscosity, then $\tau_{phr}/t_a \ll 1$ in the entire considered temperature range and relation (29) coincides in fact with the result given in Ref. 15. If, however, τ_r is calculated from the data of Ref. 8, then the ratio (30) at $T \approx 1.2$ K is of the order of unity and must be taken into account.

The situation changes markedly with increasing pressure. It is known that at $P \gtrsim 18$ atm the phonon spectrum is non-decaying. Consequently $t_{1-2} \to \infty$ and all the remaining phonon times increase by two orders of magnitude (because of the change in the parameters of He II with increasing pressure). An exception is the time t given by (14), which decreases with increasing pressure. Estimates show that at a pressure $P=24$ atm the times t_a and t_{phr} are the shortest. Taking this circumstance into account, we should retain in (16) only the times t_{phr} and t_a. In the upshot we get

$$\eta_{ph}=-\frac{3}{4\pi^4}\rho_{nph}c^2\int_0^\infty t_i x^4 n_0' dx. \tag{31}$$

We note that in this case η_{ph}, in contrast to (28), is proportional to the mean value of the characteristic time, just as in the usual gaskinetic theory. The convergence of the integral in (31) is then ensured at small x by the time t_a, since $t_{phr} \sim x^{-4}$. From the mathematical point of view the results (28) and (31) are two limiting cases of the general expression (26). From the physical point of view this means that at increased pressures equilibrium in the phonon gas is ensured only by the phonon-roton collisions. In contrast to the case of saturated-vapor pressure, we have no establishment of equilibrium in two stages of which one is purely phonon.

Relation (31) can be rewritten in the form

$$\eta_{ph}=-\frac{3}{4\pi^4}\rho_{nph}c^2 t_a\int_0^\infty \frac{x^4 n_0'}{1+t_a/t_{phr}}dx. \tag{32}$$

If $P \approx 24$ atm, we have

$$t_a/t_{phr}=2.3\cdot 10^7 (xT)^4\tau_r. \tag{33}$$

At all possible values of τ_r, the main contribution to the integral that enters in (32) is given by $x=3$ to 4. We have therefore with accuracy sufficient for our purpose

$$\eta_{ph}={}^1/_5\rho_{nph}c^2 t_a(1+t_a/t_{phr}(4))^{-1}, \tag{34}$$

where $t_{phr}(4)=t_{phr}$ at $x=4$. If τ_r is taken to be its lower value obtained from Ref. 8, then $t_a/t_{phr}(4) \ll 1$ and the phonon viscosity is determined in fact by t_a. On the other hand if it is assumed that τ_r is governed by the roton viscosity, the ratio in the considered temperature interval is $t_a/t_{phr}(4) \lesssim 1$ and must be taken into account.

It follows from the results that the numerical value and the temperature dependence of the phonon viscosity and of the thermal conductivity should change substantially with increasing pressure. At the present time, unfortunately, the experimental data on the viscosity and thermal conductivity under pressure are extremely scanty. In addition, the available results obtained by different experimenters vary greatly. A detailed comparison of theory and experiment can hardly be of use at present. It can only be indicated that in order of magnitude the theoretical value (34) is close to the experimentally observed one.

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