Dynamics of the A-phase of ^3He at low pressures

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The strong-coupling corrections to the BCS theory for superfluid ^3He become small at low pressures. Since subsystems with different spin projections are independent in the A-phase in the BCS approximation, the region in which the order parameter changes in the A-phase widens and additional quasi-Goldstone modes arise. Linear dynamic equations for the A-phase at low pressures have been derived by the semiphenomenological method used earlier to obtain exact equations for the orbital dynamics of the A-phase at $T=0$, taking account of additional soft variables. The equations indicate the existence of nine normal modes having a spectrum that coincides, in the limit when Fermi liquid and strong coupling effects are absent, with the nine-phonon mode spectrum obtained by Alonso and Popov [Sov. Phys. JETP **46**, 760 (1977)] by the exact path integral method in the BCS model. Since the new modes are coupled to the oscillations of longitudinal magnetization, it is proposed to use the longitudinal NMR technique to observe them.

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1. INTRODUCTION

In the BCS model the A-phase has a higher energy than the B-phase at all temperatures (see the review by Leggett[1]). The existence of the A-phase is connected with the so-called strong-coupling effects which are outside the scope of the BCS model. The strong-coupling effects are large at high pressure where, thanks to them, the A-phase is energetically more favored than the B-phase. On reducing the pressure, they decrease and cannot stabilize the A-phase, which can therefore exist at low pressures in the presence of a magnetic field. Measurements of the heat capacity jump $\Delta C/C_N$, which is sensitive to the strong coupling effects, in the transition from the normal to the superfluid state showed[2] that at pressures close to zero $\Delta C/C_N$ hardly differs from the value 1.43 which is characteristic of the BCS model. This indicates that the effects of strong coupling are small at zero pressure.

Suppression of the effects of strong coupling leads to a number of interesting phenomena. As is known, in the BCS model (or in other words in the weak-coupling model) the A-phase is described by an order parameter of more complicated form than in the general case when effects of strong coupling are present.[3] In the general case the order parameter is

$$A_{\mu i}=Cd_\mu e_i, \tag{1.1}$$

where d_μ is a unit real vector in spin space and e_i is the orbital triad,

$$\mathbf{e}=\Delta'+i\Delta'',\quad \mathbf{l}=[\Delta'\times\Delta''], \tag{1.2}$$

$\mathbf{l}$ is the direction of the orbital momentum and $C=\text{const}$.

In the case of weak coupling

$$A_{\mu i}={}^1/_2C(e_\mu e_i^{(1)}+e_\mu^* e_i^{(2)}). \tag{1.3}$$

Here e_μ is the spin triad,

$$e_\mu=d_\mu'+id_\mu'',\quad \mathbf{s}=[\mathbf{d}'\mathbf{d}''], \tag{1.4}$$

$\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ are two different orbital triads of the form of Eq. (1.2)

The order parameter (1.3) describes two subsystems. The first subsystem, described by the first term, consists of pairs with spin directed along the $\mathbf{s}$ axis; these pairs have an orbital momentum directed along $\mathbf{l}^{(1)}=[\Delta'^{(1)}\Delta''^{(1)}]$. In the second subsystem the spins of the pairs are oriented along $\mathbf{s}$ and the orbital moments along $\mathbf{l}^{(2)}=\Delta'^{(2)}\times\Delta''^{(2)}$. In the weak-coupling approximation the relative orientation of $\mathbf{l}^{(1)}$ and $\mathbf{l}^{(2)}$ can be arbitrary. The effects of strong coupling lead to a pinning of the angular momenta:

$$F_{sc}={}^1/_2\gamma(\mathbf{l}^{(1)}-\mathbf{l}^{(2)})^2; \tag{1.5}$$

as a result, at equilibrium $\mathbf{l}_0^{(1)}=\mathbf{l}_0^{(2)}\equiv\mathbf{l}_0$, and Eq. (1.3) goes over into Eq. (1.1). The order parameter (1.3) also includes within itself the planar phase $\mathbf{l}_0^{(1)}=-\mathbf{l}_0^{(2)}$.

The order parameter (1.3) has thus a large number of degrees of freedom and the number of Goldstone modes in the weak-coupling approximation is thereby increased. In the general case the order parameter (1.1) varies in a five-dimensional manifold, which leads to five Goldstone modes. The order parameter (1.3) assumes its values in eight-dimensional space in the weak-coupling model. However, the exact microscopic calculation of the mode spectrum in the weak-coupling model, carried out by Alsonso and Popov,[4] shows the existence of nine phonon modes for $\mathbf{l}_0^{(1)}=\mathbf{l}_0^{(2)}$. The disparity in the number of phonon modes and in the dimensionality of the space indicates that this space is not homogeneous. It was just because of this fact that the study of the topology of stable defects in the weak-coupling model required the application of non-standard methods.[5]

Our problem is to obtain dynamic equations for the A-phase at low pressures, taking account of the new degrees of freedom which arise. Wartak[6] was the first to consider this problem phenomenologically, but he obtained only two and not four additional modes. In the third section, by using the semi-phenomenological method developed by Volovik and Mineev,[7,8] we obtain a closed system of equations for the linear dynamics of the A-phase at low temperatures. These equations give nine normal modes with a spectrum which goes over to the spectrum of nine phonon modes obtained by Alonso and Popov,[4] in the limit when Fermi-liquid and strong-coupling effects are absent. In the second sec-

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tion we discuss the structure of the A-phase degeneracy space.

2. STRUCTURE OF THE A-PHASE DEGENERACY SPACE IN THE WEAK-COUPLING LIMIT

All values of the order parameter (1.3) can be sorted out if the orientations of the orbital triads and the direction of the spin momentum **s** (1.4) are given. The degeneracy space R in the weak coupling approximation is thus eight-dimensional (two triplets of Euler angles giving the orientation of the triads, and two angles determining the **s** axis). The R-space is not homogeneous; to see this we consider the subspace $\tilde{R}\subset R$, specified by the equality of the orbital momenta $\mathbf{l}^{(1)}=\mathbf{l}^{(2)}$. Since we have superimposed two couplings, the subspace $\tilde{R}$ should be six-dimensional in the case of homogeneity of the R space. In fact, for $\mathbf{l}^{(1)}=\mathbf{l}^{(2)}$ we obtain an order parameter (1.1) with five-dimensional degeneracy space. This results from the fact that rotations of the spin space around the **d** axis in Eq. (1.1) do not alter the order parameter. While for $\mathbf{l}^{(1)}=\mathbf{l}^{(2)}$ different values of the order parameter correspond to different orientation of **s**, for $\mathbf{l}^{(1)}=\mathbf{l}^{(2)}$ all **s** axes lying in one plane correspond to one and the same order parameter, i.e. the dimensionality of the $\tilde{R}$ space is lowered to unity.

The number of Goldstone modes depends on whether the system is in $\tilde{R}$ or in $R/\tilde{R}$. If the state of the system $A^0_{\mu i}\notin\tilde{R}$, the number of Goldstone modes must be equal to the dimensionality of the R-space, i.e., eight, but if $A^0_{\mu i}\in\tilde{R}$ then the number of modes is equal to nine. This latter can be shown on the basis of the simplest considerations expounded in the next section. Here we give the result of studying the quadratic form obtained by expanding the energy in terms of small departures $\delta A_{\mu i}$ from the initial value of the order parameter $A^0_{\mu i}$ (see the Appendix). We consider the Ginzburg-Landau region where the energy in the weak-coupling approximation has the form (in dimensionless units)

$$F=-A_{\mu i}A_{\mu i}^*+(A_{\mu i}A_{\mu i}^*)^2-{}^1/_2|A_{\mu i}A_{\mu i}|^2$$
$$+A_{\mu i}A_{\nu i}^*A_{\nu k}A_{\mu k}^*-A_{\mu i}A_{\nu i}^*A_{\nu k}^*A_{\mu k}+A_{\mu i}A_{\nu i}A_{\nu k}^*A_{\mu k}^*. \qquad (2.1)$$

The eigenvalues λ of the matrix of the quadratic form $\delta^2F\{\delta A_{\mu i}\}$ at the point $A_{\mu i}=A^0_{\mu i}$ depend on the point $A^0_{\mu i}$ through a single parameter, the angle φ between $\mathbf{l}_0^{(1)}$ and $\mathbf{l}_0^{(2)}$:

$$\cos\varphi=\mathbf{l}_0^{(1)}\mathbf{l}_0^{(2)}. \qquad (2.2)$$

For all non-zero $\varphi (0<\varphi\leq\pi)$ there are eight zero eigenvalues λ_{1-8} corresponding to eight Goldstone modes. The remaining eigenvalues λ_{9-18} are positive for $0<\varphi<\varphi_{cr}$, where $\cos\varphi_{cr}=3-2\sqrt{3}$. One of these eigenvalues $\lambda_9(\varphi)\to 0$ as $\varphi\to 0$, giving the ninth Goldstone mode for $\varphi=0$. For $\varphi=\varphi_{cr}$ another of the positive eigenvalues $\lambda_{10}(\varphi)$ goes to zero and then becomes negative for $\varphi>\varphi_{cr}$, indicating the loss of local stability of the A-phase at these φ. Starting from φ_{cr}, the system can go over to the B-phase with a monotonic lowering of the energy.

In the presence of a magnetic field **H** the spin vector **s** is oriented along H at $\varphi>0$ due to the magnetic anisotropy energy. Two degrees of freedom are thereby fixed and the degeneracy space becomes six-dimensional. In this case the order parameter (1.3) can be parametrized in the following way:

$$e_\mu=\hat{x}_\mu+i\hat{y}_\mu,\quad \hat{\mathbf{z}}\parallel\mathbf{H},$$
$$\mathbf{e}^{(1)}=\vec{R}^{(1)}(\hat{\mathbf{x}}+i\hat{\mathbf{y}}),\quad \mathbf{e}^{(2)}=\vec{R}^{(2)}(\hat{\mathbf{x}}+i\hat{\mathbf{y}}). \qquad (2.3)$$

Here $\vec{R}^{(1)}$ and $\vec{R}^{(2)}$ are matrices of three-dimensional rotations. The degeneracy space R_H is the product of two groups of three-dimensional rotations (see the Appendix):

$$R_H=SO_3^{(1)}\times SO_3^{(2)} \qquad (2.4)$$

and is a homogeneous space. As a result of this, the number of Goldstone variables is six for any angle φ between $\mathbf{l}^{(1)}$ and $\mathbf{l}^{(2)}$. This agrees with the results of Alonso and Popov,[4] obtained in two limiting cases: for the planar phase ($\mathbf{l}_0^{(1)}=-\mathbf{l}_0^{(2)}$) and for the A-phase of the form of Eq. (1.1) ($\mathbf{l}_0^{(1)}=\mathbf{l}_0^{(2)}$). The angle φ_{cr}, starting from which the A-phase becomes unstable, increases with increasing field and becomes equal to π for $H=H_c$, where

$$\mu^2H_c^2=2\pi^2T_c(T_c-T)/7\zeta(3),$$

(see also Alonso and Popov[4]). For $H>H_c$ the A-phase is locally stable in the entire R_H region. Finally, the effects of strong coupling [Eq. (1.5)] which fix $\mathbf{l}^{(1)}-\mathbf{l}^{(2)}=0$, contract R_H to a four-dimensional space

$$\tilde{R}_H=SO_3\times U(1)/Z_2.$$

In the following section we obtain the dynamic equations describing all the nine modes and find their spectrum, taking strong-coupling effects into account.

3. LINEAR DYNAMIC EQUATIONS OF THE A-PHASE AT $T=0$

We shall look for dynamic equations for all degrees of freedom in the R space, taking account of strong-coupling effects. We choose an initial equilibrium orientation of the order parameter; the variables are the deviations from this orientation. At equilibrium $\mathbf{l}_0^{(1)}=\mathbf{l}_0^{(2)}\equiv\mathbf{l}_0$, and the order parameter is given by Eq. (1.1), with $\mathbf{d}_0\perp\mathbf{H}$ at equilibrium. At equilibrium the spin quantization axis $\mathbf{s}_0$ can be oriented in any direction in the plane perpendicular to $\mathbf{d}_0$. We shall first fix the $\mathbf{s}_0$ axis. In the weak-coupling approximation the system consists of two independent components with spins parallel and antiparallel to $\mathbf{s}_0$. Each of these components has its orbital order parameter. The orbital dynamics of each component is described by a Lagrangian which differs by a factor $\frac{1}{2}$ from the Lagrangian obtained by Volovik and Mineev[7,8] which describes the orbital dynamics in the usual case when the orbital order parameters of both components move in phase. The action S has in the quadratic approximation the following form:

$$S=\int d^3x\,dt(\mathscr{L}^{(1)}+\mathscr{L}^{(2)}+F_{sc}), \qquad (3.1)$$

where, according to Volovik and Mineev[8]

$$\mathscr{L}^{(a)}={}^1/_2\varepsilon(\delta\mathbf{l}^{(a)};\mathbf{v}_s^{(a)})+{}^1/_2\varepsilon(2\rho^{(a)})-{}^1/_4\chi_{orb}(\dot{\mathbf{l}}^{(a)})^2$$
$$-{}^1/_8(\rho-C_0)\delta\dot{\theta}^{(a)}\delta\mathbf{l}^{(a)}+{}^1/_2\delta\dot{\Phi}^{(a)}\delta\rho^{(a)},\quad a=1,2. \qquad (3.2)$$

Here $\rho^{(1)}$ and $\rho^{(2)}$ are the density of each of the components, $\rho^{(1)}+\rho^{(2)}=\rho$; $\delta\theta^{(1)}$ and $\delta\theta^{(2)}$ are the angles of rotation of the orbital order parameters:

$$\delta\mathbf{l}^{(a)}=[\delta\theta^{(a)}\times\mathbf{l}_0],\quad \delta\Phi^{(a)}=-\mathbf{l}_0\delta\theta^{(a)},\quad \mathbf{v}_s^{(a)}=\hbar\nabla\Phi^{(a)}/2m; \tag{3.3}$$

$\frac{1}{2}\chi_{orb}$ and $\frac{1}{4}(\rho-C_0)$ are respectively the orbital susceptibility and the spontaneous orbital momentum of each component; the energies $\varepsilon(\delta\mathbf{l}^{(a)};\mathbf{v}_s^{(a)})$ and $\varepsilon(\rho)$ are the same as in the usual A-phase:

$$\varepsilon(\delta\mathbf{l};\mathbf{v}_s)={}^1/_2\rho\mathbf{v}_s^2-(\hbar/2m)C_0(\mathbf{l}_0\mathbf{v}_s)(\mathbf{l}_0\operatorname{rot}\delta\mathbf{l})$$
$$+{}^1/_2K_1(\nabla\delta\mathbf{l})^2+{}^1/_2K_2(\mathbf{l}_0\operatorname{rot}\delta\mathbf{l})^2+{}^1/_2K_3[\mathbf{l}_0\times\operatorname{rot}\delta\mathbf{l}]^2. \tag{3.4}$$

The values of the coefficients in Eq. (3.4) are given by Cross.[9] The action (3.1) is described by the dynamics of six orbital variables $\delta\theta^{(1)}$ and $\delta\theta^{(2)}$. If $A^0_{\mu k}\notin\tilde{R}$, the deviation of $\mathbf{s}_0$ from the equilibrium state must be added to these six variables, i.e., there are two more variables.

The situation is different in the case considered, $A^0_{\mu k}\in\tilde{R}$, since the state does not depend on the direction of $\mathbf{s}_0$ in the plane perpendicular to $\mathbf{d}_0$. We now choose a different direction for the $\mathbf{s}_0$ axis; the order parameter at equilibrium does not change then. However, some of the variables transform then and the Lagrangian (3.1) becomes noninvariant relative to a rotation of $\mathbf{s}_0$. In fact, the variable $\delta S=\frac{1}{2}(\delta\rho^{(1)}-\delta\rho^{(2)})$, which is the spin density in the $\mathbf{s}_0$ direction, transforms as a vector on rotating $\mathbf{s}_0$, while the variable $\delta\rho=\delta\rho^{(1)}+\delta\rho^{(2)}$, which is the change of the total density, does not transform. Therefore, $\frac{1}{2}(\delta\rho^{(1)}-\delta\rho^{(2)})$ is one of the components of a two-component vector S_μ defined in the plane perpendicular to $\mathbf{d}_0$. This also applies to the variables $\frac{1}{2}(\delta\theta^{(1)}-\delta\theta^{(2)})$ which must be replaced by the corresponding two-component spin vectors $\delta\theta_\mu$; in the same way the number of Goldstone variables effectively increases by three and becomes equal to nine.

Separating out both sorts of combination in the action (3.1), we write it in a form invariant with respect to transformation of the $\mathbf{s}_0$ axis:

$$S=\int d^3x\,dt\{\varepsilon(\delta\mathbf{l};\mathbf{v}_s)+\varepsilon(\rho)-{}^1/_2\chi_{orb}\dot{\mathbf{l}}^2+{}^1/_2\delta\dot{\Phi}\delta\rho-{}^1/_4(\rho-C_0)\mathbf{l}_0[\delta\mathbf{l}\times\delta\dot{\mathbf{l}}]$$
$$+\int d^3x\,dt\{\varepsilon(\delta\mathbf{l}_\mu;\mathbf{v}_{s\mu})+S_\mu^2/2\chi-{}^1/_2\chi_{orb}\delta\dot{\mathbf{l}}_\mu^2-[\mathbf{d}\times\delta\dot{\mathbf{d}}]_\mu S_\mu$$
$$-{}^1/_4(\rho-C_0)(\mathbf{l}_0,[\delta\dot{\mathbf{l}}_\mu\times\delta\mathbf{l}_\mu])+2\gamma(\delta\mathbf{l}_\mu)^2\}. \tag{3.5}$$

Here $\delta\mathbf{l}=\frac{1}{2}(\delta\mathbf{l}^{(1)}+\delta\mathbf{l}^{(2)})$ describes the in-phase oscillations of the orbital momentum of a pair; $\delta\Phi=\frac{1}{2}(\delta\Phi^{(1)}+\delta\Phi^{(2)})$ is the change in overall phase of the order parameter; $\mathbf{v}_s=(\hbar/2m)\nabla\Phi$ is the usual superfluid velocity; the variables $\delta\mathbf{l}_\mu=\delta\theta_\mu\times\mathbf{l}_0$ is a vector generalization of the variables $\frac{1}{2}(\delta\mathbf{l}^{(1)}-\delta\mathbf{l}^{(2)})$ for fixed $\mathbf{s}_0$; $\delta\mathbf{d}=-\delta\Phi\times\mathbf{d}_0$, where $\delta\Phi_\mu$ is the vector generalization of the variable $\frac{1}{2}(\delta\Phi^{(1)}-\delta\Phi^{(2)})$;

$$\mathbf{v}_{s\mu}=(\hbar/2m)e_{\mu\lambda\nu}d_\lambda\nabla\delta d_\nu, \tag{3.6}$$

χ is the spin susceptibility equal to $\frac{1}{4}(\partial^2\varepsilon/\partial\rho^2)^{-1}$. The condition of being orthogonal to the vector $\mathbf{d}_0$ is imposed on the spin variables:

$$S_\mu d_{0\mu}=\delta\mathbf{l}_\mu d_{0\mu}=\mathbf{v}_{s\mu}d_{0\mu}=0. \tag{3.7}$$

The action (3.5) consists of three independent actions. The first part (in the first curly bracket) describes the usual orbital dynamics of the A-phase. The remaining part of the action describes the independent dynamics of two projections of the spin vectors (along $\mathbf{H}$ and along $\mathbf{d}_0\times\mathbf{H}$). In the absence of strong coupling and of a magnetic field all three actions are identical.

Varying the action (3.5) leads to the following system of linear equations:

$$\frac{\partial\rho}{\partial t}+\nabla\mathbf{j}=0,\quad \mathbf{j}=\frac{\partial\varepsilon}{\partial\mathbf{v}_s}; \tag{3.8}$$

$$\frac{\partial\mathbf{v}_s}{\partial t}=-\nabla\frac{\partial\varepsilon}{\partial\rho}; \tag{3.9}$$

$$\chi_{orb}\left[\mathbf{l}\times\frac{\partial^2}{\partial t^2}\mathbf{l}\right]+\frac{1}{2}(\rho-C_0)\frac{\partial\mathbf{l}}{\partial t}=-\left[\mathbf{l}\times\frac{\delta\varepsilon}{\delta\mathbf{l}}\right]; \tag{3.10}$$

$$\frac{\partial S_\mu}{\partial t}+\nabla\mathbf{j}_\mu=0,\quad S_\mu d_\mu=0,\quad \mathbf{j}_\mu=\frac{1}{2}\frac{\delta\varepsilon}{\delta\mathbf{v}_{s\mu}}; \tag{3.11}$$

$$\frac{\partial\mathbf{d}}{\partial t}=\frac{1}{\chi}[\mathbf{S}\times\mathbf{d}]; \tag{3.12}$$

$$\chi_{orb}\left[\mathbf{l}\times\frac{\partial^2}{\partial t^2}\delta\mathbf{l}_\mu\right]+\frac{1}{2}(\rho-C_0)\frac{\partial\delta\mathbf{l}_\mu}{\partial t}=-\left[\mathbf{l}\times\frac{\delta\varepsilon}{\delta l_\mu}\right]+4\gamma[\mathbf{l}\times\delta\mathbf{l}_\mu],\quad d_\mu\delta\mathbf{l}_\mu=0. \tag{3.13}$$

The first three equations (3.8) to (3.10) describe the usual orbital dynamics. Equations (3.11) and (3.12) describe the usual spin dynamics, $\mathbf{j}_\mu$ is the spin current. The four equations (3.13) are related to the four additional degrees of freedom which arise in the weak-coupling limit.

The action (3.5) is written without taking Fermi-liquid effects into account. However, the structure of the equations should not change when these effects are taken into account. It is only necessary to take into account the fact that the energy ε, the variational derivative of which enters into the equations, has the most general form consistent with the symmetry conditions. The coefficients also change: for example, the magnetic susceptibility χ is no longer equal to $\frac{1}{4}(\partial^2\varepsilon/\partial\rho^2)^{-1}$.

The spectrum of all modes can be obtained from Eqs. (3.8) to (3.13). We will not write it out in the general case, but confine ourselves to one particular case: suppose ε is represented in the form of a sum of $\varepsilon(\rho)$, $\varepsilon(\delta\mathbf{l};\mathbf{v}_s)$ and $\varepsilon(\delta\mathbf{l}_\mu;\mathbf{v}_{s\mu})$ as in the action (3.5); we neglect the magnitude of the spontaneous angular momentum $\frac{1}{2}(\rho-C_0)\ll\rho$; we use the fact that as $T\to0$ the coefficient K_s in the energy (3.4) becomes much larger than the other coefficients.[9] In this case we have the following spectrum:

$$\omega_1=c_1q,\quad c_1^2=\rho\partial^2\varepsilon/\partial\rho^2;\quad \omega_{2,3}=c_2q,\quad c_2^2=\rho/4\chi;$$
$$\omega_{4,5}{}^2=c_3^2(\mathbf{ql})^2,\quad c_3^2=K_3/\chi_{orb};\quad \omega_{6-9}{}^2=c_3^2(\mathbf{ql})^2+4\gamma/\chi_{orb}. \tag{3.14}$$

Here c_1, c_2 and c_3 are the velocities of, respectively, ordinary sound, spin waves, and orbital waves in the usual A-phase.

The remaining four modes ω_{6-9} are connected with the extra spatial degeneracy of the states which arise when the coefficient γ is small. Our result for these modes differs from Wartak's result[6] obtained for a wave vector $\mathbf{q}=0$, in that he found only two such modes (taking account of polarization) and not four, and he also had a coefficient 2 instead of 4 before γ. The latter is evidently due to an incorrect determination by Wartak of the orbital susceptibility for each of the spin components. In the absence of strong-coupling and Fermi-liquid effects (in the gas-like approximation $c_1 = c_2 = v_F\sqrt{3}$, $c_3 = v_F$) the Alonso and Popov result[4] is obtained, i.e., three modes with a spectrum $\omega = v_F|\mathbf{q}\cdot\mathbf{l}|$.

The expression for the spectrum of modes $\omega_{6\text{-}9}$ is valid for the gap in the spectrum

$$(4\gamma/\chi_{orb})^{1/2} \ll \Delta_0, \tag{3.15}$$

where Δ_0 is the amplitude of the gap in the Fermi excitation spectrum. The quantity γ/χ_{orb} is of the order of $\delta\Delta_0^2$, where δ is the spin-fluctuation parameter for strong coupling (see, for example, Cross[10]). Condition (3.15) is apparently poorly fulfilled at high pressures but is well fulfilled at low pressures, when $\delta \ll 1$.

In addition, Eq. (3.14) only applies at sufficiently low temperatures $T \ll T_c$. The A-phase can only exist at low pressures and low temperatures in strong magnetic fields $H > 5$ kOe. In a magnetic field a gap [see Eq. (A8)] of the order of the Larmor frequency appears in the spectrum of the three modes ω_3 and $\omega_{8,9}$ whose spin variables $\delta\boldsymbol{\theta}_\mu$ are transverse to the magnetic field (i.e. $\delta\boldsymbol{\theta}_\mu H_\mu = 0$). This gap is comparable with Δ_0 in those strong fields in which the A-phase exists at low temperatures. The spectrum of the remaining six modes does not change in a magnetic field.

Since one of the longitudinal modes $\omega_{6,7}$ in the general case becomes coupled with fluctuations of longitudinal magnetization ω_2 because of the term $(\mathbf{l}\cdot \mathrm{curl}\,\delta\mathbf{l}_\mu)(\mathbf{l}\cdot\mathbf{v}_{s\mu})$ in the action (3.5), a noticeable absorption line can be expected in longitudinal NMR at a frequency $(4\gamma/\chi_{orb})^{1/2}$.

In conclusion the authors consider it their pleasant duty to thank V. P. Mineev for valuable discussions.

APPENDIX

We shall consider the quadratic form obtained by expanding the free energy (2.1) in terms of the deviation of the order parameter $A_{\mu i}$ from its equilibrium value $A^0_{\mu i}$:

$$\delta A_{\mu i} = u_{\mu i} + i v_{\mu i}. \tag{A1}$$

We parametrize the order parameter $A^0_{\mu i}$ (1.3), which takes on its values in the expanded eight-dimensional R space, in the following form

$$\mathbf{e}^{(1)} = \hat{\mathbf{x}} + i\alpha(\hat{\mathbf{y}} + \hat{\mathbf{z}}\cos\theta), \quad \mathbf{e}^{(2)} = \hat{\mathbf{x}} + i\alpha(\hat{\mathbf{y}} - \hat{\mathbf{z}}\cos\theta), \\ e = \hat{\mathbf{x}} + i\hat{\mathbf{y}}, \quad \alpha^2 = 1/(1+\cos^2\theta). \tag{A2}$$

We have here separated the single parameter $\boldsymbol{\theta}$, connected with the angle φ [Eq. (2.2)] between the orbital momenta $\mathbf{l}_0^{(1)}$ and $\mathbf{l}_0^{(2)}$ by the relation

$$\cos\varphi = \sin^2\theta/(1+\cos^2\theta). \tag{A3}$$

The remaining seven parameters are the angles of rotation of the spin and orbital spaces and the phase variable. They do not influence the result because of the symmetry of the energy (2.1) relative to transformations connected with these parameters. Equations (A2) and (A3) hold for $0 \leqslant \varphi \leqslant \pi/2$. For $\pi/2 \leqslant \varphi \leqslant \pi$ a different parametrization must be used:

$$\mathbf{e}^{(1)} = \hat{\mathbf{x}} + i\alpha(\mathbf{y}\cos\theta + \hat{\mathbf{z}}), \\ \mathbf{e}^{(2)} = \hat{\mathbf{x}} + i\alpha(\hat{\mathbf{y}}\cos\theta - \hat{\mathbf{z}}), \quad \mathbf{e} = \hat{\mathbf{x}} + i\hat{\mathbf{y}}, \tag{A4}$$

where θ is connected with φ by the relation

$$\cos\varphi = -\sin^2\theta/(1+\cos^2\theta). \tag{A5}$$

We also switch on a magnetic field $\mathbf{H}$ in which there is an additional term added to the energy (2.1)

$$\frac{\nu}{H^2}|H_\mu A_{\mu i}|^2, \quad \nu = \frac{7\zeta(3)}{4\pi^2}\left(\frac{\mu H}{T_c}\right)^2\left(1 - \frac{T}{T_c}\right)^{-1}, \tag{A6}$$

and direct it along the $\hat{z}$ axis in order that the states (A2) and (A4) should correspond to the equilibrium state in the magnetic field. The value of C in Eq. (1.3) then turns out to be $\frac{1}{2}$.

The quadratic form sought has the following form:

$$2\delta^2 F = 3u_{11}^2 + v_{11}^2 + u_{21}^2 + 3v_{21}^2 + v_{32}^2 + \beta^2(3v_{12}^2 + u_{12}^2 + 3u_{22}^2 + v_{22}^2 + u_{31}^2) \\ + \bar\beta^2(3u_{23}^2 + u_{13}^2 + 3v_{13}^2 + v_{32}^2 + v_{33}^2) + (4-\beta^2)v_{31}^2 + (4\beta^2 - 1)u_{32}^2 \\ + 2\beta\bar\beta(u_{12}v_{23} + u_{13}v_{22} - 3u_{23}v_{12} - 3u_{22}v_{13}) \\ + 2\bar\beta(v_{23}v_{11} - u_{23}u_{11} + v_{21}v_{13} - u_{21}u_{13}) \\ + 2\beta(u_{11}v_{12} + u_{12}v_{11} - u_{21}v_{22} - u_{31}v_{32} - u_{22}v_{21} - v_{32}v_{31}) + 2\nu(u_{3k}^2 + v_{3k}^2), \\ \beta = \begin{cases} \alpha, & \varphi \leqslant \pi/2 \\ \alpha\cos\theta, & \varphi \geqslant \pi/2 \end{cases}, \quad \bar\beta = \begin{cases} \alpha\cos\theta, & \varphi \leqslant \pi/2 \\ \alpha, & \varphi \geqslant \pi/2 \end{cases}. \tag{A7}$$

This form can easily be diagonalized. The eigenvalue λ and the eigenvectors corresponding to them are written out below, and it is pointed out as well which hydrodynamic variables are related at $\varphi = 0$ to these vectors:

$$\lambda_1 = 0, \quad u_{12} - \beta v_{11}, \quad \delta\Phi; \\ \lambda_2 = 0, \quad v_{22} + \beta u_{21}, \quad \delta\mathbf{d} \perp \mathbf{H}; \\ \lambda_3 = 2\nu, \quad u_{31} + \beta v_{32}, \quad \delta\mathbf{d} \parallel \mathbf{H}; \\ \lambda_4 = \lambda_5 = 0, \quad \beta u_{13} - \bar\beta v_{22}, \quad \beta v_{13} + \bar\beta u_{22}, \quad \delta\mathbf{l}; \\ \lambda_6 = \lambda_7 = 0, \quad v_{23} - \bar\beta v_{11}, \quad \beta u_{23} + \bar\beta v_{12}, \quad \delta\mathbf{l}_\mu, \quad e_{\mu\nu\varkappa}\delta\mathbf{l}_\mu H_\nu = 0; \\ \left.\begin{matrix} \lambda_8 = 2\nu, & u_{33} \\ \lambda_9 = 2\nu + 4(1-\beta^2), & v_{33} \end{matrix}\right\} \delta l_\mu, \quad \delta l_\mu H_\mu = 0; \\ \lambda_{11} = \lambda_{12} = \lambda_{13} = \lambda_{14} = 2 \quad \begin{cases} v_{11} + \beta u_{12} + \bar\beta v_{23}, & u_{21} - \bar\beta u_{13} - \beta v_{22}, \\ u_{11} - \beta v_{12} + \bar\beta u_{23}, & v_{21} - \bar\beta v_{13} + \beta u_{22}; \end{cases} \\ \lambda_{15} = \lambda_{16} = 4, \quad u_{11} + \beta v_{12} - \bar\beta u_{23}, \quad v_{21} + \bar\beta v_{13} - \beta u_{22}; \\ \lambda_{17} = 2\nu + 1 + \beta^2, \quad \beta u_{31} - v_{32}. \tag{A8}$$

For u_{32} and v_{31} the characteristic equation is of the following form:

$$(\lambda - 2\nu)^2 - 3(1+\beta^2)(\lambda - 2\nu) + 16\beta^2 - 4\beta^4 - 4 = 0. \tag{A9}$$

This equation gives the eigenvalues λ_{10} and λ_{18}. One of the solutions λ_{18} is always positive, while λ_{10} becomes negative for a certain angle φ_{cr} equal to

$$\cos\varphi_{cr} = 3 + 3\nu/2 - (12 + 18\nu + 25\nu^2/4)^{1/2}. \tag{A10}$$

This φ_{cr} becomes equal to π for $\nu = \frac{1}{2}$. In stronger fields ($\nu > \frac{1}{2}$) all the eigenvalues λ are non-negative. In the absence of a magnetic field there are eight zero eigenvalues $\lambda_{1\text{-}8}$ for all $\varphi > 0$. There is one more eigenvalue $\lambda_9 = 0$ for $\varphi = 0$.

We also note that twelve eigenvalues $\lambda_{1,2}$, $\lambda_{4\text{-}7}$, $\lambda_{11\text{-}16}$, whose eigenvectors orthogonal to the field ($u_{\mu i}H_\mu = v_{\mu i}H_\mu = 0$), are independent of φ. This results from the fact that for the $A_{\mu i}$ matrices orthogonal to the magnetic field ($A_{\mu i}H_\mu = 0$), the energy functional (2.1) is symmetric relative to rotations of each of the orbital spaces. Its symmetry group is

$$G = U^{(1)}(1) \times SO_3^{(1)} \times U^{(2)}(1) \times SO_3^{(2)}. \tag{A11}$$

The symmetry group of the order parameter in a magnetic field is of the form

$$H=U(1)\times U(1).$$

As a result, the R_H space, which is the region of the change in the order parameter in the magnetic field, is a homogeneous space G/H and is given by Eq. (2.4). It should also be realized that for $\mathbf{l}^{(1)}\neq\mathbf{l}^{(2)}$ all states in a magnetic field are doubly degenerate since the spin axis $\mathbf{s}$ can be directed both along the field and in the opposite direction. These states are indistinguishable in the subspace $\mathbf{l}^{(1)}=\mathbf{l}^{(2)}$. The space of the states in the magnetic field therefore consists of two spaces R_H with different directions of the spin axis joined along the subspace $\mathbf{l}^{(1)}=\mathbf{l}^{(2)}$.

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