Law of approach of the magnetization to saturation in amorphous ferromagnets[1)]

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On the basis of the phenomenological theory of amorphous and microcrystalline magnetic systems developed by V. A. Ignatchenko *et al.* [Sov. Phys. JETP **45**, 526 (1977); **47**, 725 (1978); **48**, 328 and 726 (1978); Izv. Akad. Nauk SSSR, Ser. Fiz. **44**, 1434 (1980)], an analytical expression is obtained for the law of approach of the magnetization to saturation (LAS) in such substances. The LAS is studied experimentally for amorphous and microcrystalline Co-P alloys with various concentrations of P. It is shown that from the form of the LAS it is possible to determine such characteristics of a magnetic stochastic system as the value of the effective local magnetic anisotropy and the correlation radius of the fluctuations of direction of the axes of this anisotropy. These characteristics are determined for an amorphous ferromagnet for the first time, and their variations with concentration are investigated. It is shown that the value of the local magnetic anisotropy for the alloys studied does not undergo any abrupt changes at the transition (with respect to concentration) to the amorphous state. The principal contribution to the value of the local anisotropy in Co-P alloys comes from the magnetocrystalline anisotropy, a minor contribution from the anisotropy of elastic stresses. The correlation radius of the fluctuations of anisotropy decreases abruptly at the concentrational transition to the amorphous state but remains much larger than the correlation radius of exchange fluctuations.

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INTRODUCTION

In sufficiently strong magnetic fields, when the processes of domain-wall displacement in a ferromagnet terminate, the magnetization approaches saturation, with increase of magnetic field, by processes of rotation of the magnetic moments. In a single crystal magnetized otherwise than along an anisotropy axis, this may be a process of uniform rotation; in a structurally nonuniform system (a polycrystal, a crystal with elastic inhomogeneities or dislocations, etc.), a process of spatially nonuniform magnetic-moment rotation occurs. The energy of the external magnetic field is thereby expended in overcoming the energy of local effective magnetic anisotropy: the crystallographic anisotropy of each crystallite in a polycrystal, the effective magnetic anisotropy of elastic stresses and dislocations, etc.

This process of approach of the magnetization to saturation has been used, beginning with Refs. 2–5, to obtain certain information about the structural inhomogeneities of a magnetic material. For this purpose, the magnetization in high fields was written as an expansion in powers of the magnetic field ("law of approach of the magnetization to saturation"):

$$\frac{M_s}{M}=1-\frac{a_1}{H^{1/2}}-\frac{a_2}{H}-\frac{a_3}{H^{3/2}}-\frac{a_4}{H^2}-\dots . \tag{1}$$

Each term of this expansion was identified with a corresponding source of magnetic inhomogeneities. By measuring the coefficients a_i, experimenters determined the "strength" of the corresponding source. The law of approach of the magnetization to saturation (LAS) was used most effectively for measurement of the magnetocrystalline anisotropy on polycrystalline specimens.[6,7]

An amorphous magnet is also a structurally inhomogeneous system, and therefore it is natural that study of the LAS in such materials should give certain information about these inhomogeneities. The LAS was first investigated for a series of amorphous alloys in papers of Kronmüller *et al.*[8–11] They developed a phenomenological[2)] description of the LAS (that is, one based on the Landau-Lifshitz[12] equation), taking into account the exchange and magnetic-dipole interaction of the magnetic inhomogeneities. The authors started from the traditional concept of the construction of an *a priori* model: sources of magnetic inhomogeneities, and determinations of the contribution of each source to the coefficients a_i of the series (1).

The present paper developed another approach to the interpretation of the LAS in amorphous magnets, based on the ideas of Refs. 13–16. We shall briefly review these ideas.

For a uniaxial crystalline ferromagnet, the phenomenological Hamiltonian has the form

$$\mathcal{H}=\tfrac{1}{2}\alpha(\nabla\mathbf{M})^2-\tfrac{1}{2}\beta(\mathbf{Ml})^2-\mathbf{HM}-\tfrac{1}{2}\mathbf{H}_m\mathbf{M}, \tag{2}$$

where α is the exchange constant, β is the anisotropy constant, $\mathbf{l}$ is the unit vector along the direction of the anisotropy axis, $\mathbf{H}$ is the external magnetic field, and $\mathbf{H}_m$ is the magnetic-dipole field, determined by Maxwell's equations. In a phenomenological description of an amorphous magnet, it is assumed that the form of the Hamiltonian (2) is retained but that the parameters that occur in it become random functions of the spatial coordinates. The characteristics of these random functions—the mathematical expectation and the correlation functions—are assumed to be arbitrary. Calculation of them is a problem of microscopic theory. But the problem of phenomenological theory is to look for phenomena and effects that would enable experimenters to measure these characteristics. Thus it was shown in Refs. 13–16 that the dispersion relation for spin waves is the most informative: it in principle contains information about all the correlation functions of an amor-

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phous magnet. On the basis of this theory it has already been possible to detect a modification of the dispersion law corresponding to exchange fluctuations, and to measure for the first time the principal elements of the correlation function[17] of these fluctuations: the mean square deviation and the correlation radius.

In the present paper, the analogous problem is posed for fluctuations of the axis of effective magnetic anisotropy: from the experimentally studied process of magnetic saturation, to determine the principal elements of the correlation function—the value of the local magnetic anisotropy and its correlation radius. We emphasize the paramount value of knowledge of the correlation radius (the region of relative orderliness of the corresponding parameter) for understanding of the real structure of an amorphous magnet. The urgent necessity for determining this characteristic forces us to abandon the traditional simplified description of the LAS in the form (1) and to obtain a more rigorous analytical expression for the LAS; also required are a formulation of the experiment and a processing of the experimental data that are somewhat different from the traditional ones. These special features of our approach to the investigation of the LAS in amorphous magnets will be analyzed in detail at appropriate places in the paper.

A comparison of our approach with the approach developed in Refs. 8–11 is conveniently made in the Conclusion.

THEORY

The theory of stochastic magnetic structure in amorphous ferromagnets was developed in Refs. 13 and 14; here, therefore, we shall merely recall the principal results briefly and make some necessary additions.

In an amorphous ferromagnet described by the Hamiltonian (2), all the parameters may fluctuate in space: α, β, the modulus M of the magnetization, and the direction $\mathbf{l}$. In fluctuations of α, β, and M (if the signs of α and β do not change), the ground state (in magnetic fields larger than the field for technical saturation) is a state with spatially uniform orientation of the magnetization vector. Only fluctuations of the anisotropy axis, $\mathbf{l}=\mathbf{l}(\mathbf{r})$, lead in this case to the establishment of a spatially nonuniform ground state, a stochastic magnetic structure.[3)] As has already been indicated in Ref. 19, where this approach was proposed for description of a stochastic magnetic structure—"magnetization ripple"—in thin polycrystalline films, magnetic anisotropy is here understood in a generalized sense: both as a real local crystallographic anisotropy, and as the anisotropy of elastic stresses or other defects, to the degree to which the action of these factors can be represented in the form of the anisotropic term in the Hamiltonian (2) or of a term similar to it (the last assertion will be made more precise below).

The general expression for the LAS in an amorphous ferromagnet has the form

$$M_z/M=(1-2d_m)^{1/2},$$
$$d_m=\frac{1}{2}\left(\frac{\langle M_\perp\rangle}{M}\right)^2=\frac{1}{M^2}\int S_m(\mathbf{k})\,d\mathbf{k}, \qquad (3)$$

where d_m is the relative dispersion, and where $S_m(\mathbf{k})$ is the spectral density of the correlation function of the fluctuations of the transverse components of the magnetization.

In the linear approximation, $S_m(\mathbf{k})$ was determined in Ref. 14. In a spherical coordinate system, after integration over the azimuthal angle φ, it has the form

$$S_m(k,\theta)=\pi\left(\frac{H_a}{\alpha}\right)^2\left[\frac{1}{(k^2+k_H^2)^2}+\frac{1}{(k^2+k_H^2+k_M^2\sin^2\theta)^2}\right]S(k), \qquad (4)$$

where $H_a=\beta M$ is the local-anisotropy field, $k_H=(H/\alpha M)^{1/2}$ is the characteristic wave number of the interaction of the magnetization with the magnetic field, $k_M=(4\pi/\alpha)^{1/2}$ is the characteristic wave number of the dipole-dipole interaction, and $S(k)$ is the spectral density of the correlation function for the random function $\rho_{xx}(\mathbf{r})=l_x(\mathbf{r})l_x(\mathbf{r})$ of the fluctuation of the anisotropy axis (because of the magnetic symmetry in the xy plane, the correlation functions of the random functions ρ_{xz} and ρ_{yz} are the same).

The experimental problem consists in the determination of the value of H_a and of the form of the spectral density $S(k)$. This is the maximum information that can be obtained from investigations of the stochastic magnetic structure. In the expression for the LAS, however, $S_m(k,\theta)$ occurs under an integral sign. Therefore in practice, the inverse problem can be solved only approximately: having postulated a general form of the desired function $S(k)$, substitute it in (4) and subsequently in (3) and carry out the integration. Then it will be possible to determine from the LAS the values of the arbitrary parameters by means of which we characterized the approximating function $S(k)$. Following Refs. 13 and 14, we choose the simplest form of the correlation function:

$$K(\eta)=D\exp(-k_l\eta), \quad S(k)=\frac{Dk_l}{\pi^2(k_l^2+k^2)^2}, \qquad (5)$$

where $\eta=|\mathbf{r}-\mathbf{r}'|$, D is the dispersion of the function ρ_{xx}, k_l is the correlation wave number, and $r_l=k_l^{-1}$ is the correlation radius. For the case of equiprobable fluctuation of the orientation of the unit vector $\mathbf{l}$ in all directions, $D=1/15$.

Carrying out the integration over k in (3) by means of the theory of residues, and transforming the remaining integral over θ by a change of functions, we have

$$d_m=\frac{D}{2}\left(\frac{H_a}{\alpha M}\right)^2\left[\frac{1}{k_H(k_l+k_H)^3}+\frac{1}{k_B^3k_M}J\right]; \qquad (6)$$
$$J=\int_q^1\frac{dx}{(x+p)^3(1-x^2)^{1/2}}; \quad p=\frac{k_l}{k_B}, \quad q=\frac{k_H}{k_B}, \quad k_B^2=k_H^2+k_M^2.$$

The remaining integral is easily evaluated; it has different analytical expressions, depending on the relative locations of the correlation number k_l and of the characteristic number k_B of the magnetic induction:

$$J=\frac{(1-3pq-4p^2)}{2(1-p^2)^2(q+p)^2}+\frac{1+2p^2}{2|1-p^2|^{3/2}}\begin{cases}\ln\dfrac{1+(\eta u)^{1/2}}{1-(\eta u)^{1/2}}, & p<1\\ \left[\dfrac{\pi}{2}-\arcsin\left(\dfrac{1+qp}{q+p}\right)\right], & p>1\end{cases} \qquad (7)$$

where $\eta=(1-p)/(1+p)$, $u=(1-q)/(1+q)$. The poles at $p\to 1$ are only apparent: the expression for J contains no divergences, and for $p=1$ we have

$$J=1/4[1/3u^{1/2}+2/3u^{3/2}+u^{1/2}]. \tag{7'}$$

In order to visualize more graphically the structure of the LAS, we consider what form the exact expressions (6) and (7) take for the case $k_l \ll k_M$:

$$d_m=\frac{DH_a^2}{2H^{1/2}(H_l^{1/2}+H^{1/2})^3}[1+f(H,B)], \quad f(H,B)\approx\frac{H(B+2H+B^{1/2}H^{1/2})}{2B^{1/2}(B^{1/2}+H^{1/2})}, \tag{8}$$

where $H_l=\alpha M k_l^2$ is the correlation magnetic field, and where $B=H+4\pi M$.

The LAS corresponding to the case $k_l \ll k_M$ ($H_l \ll 4\pi M$) is shown schematically in Fig. 1, in logarithmic coordinates. The asymptotes a, b, and c in this figure correspond to the limiting expressions for the LAS at different values of the magnetic field:

$$d_m\approx DH_a^2\begin{cases}1/2\,H_l^{2/3}H^{1/3}, & H\ll H_l\ll 4\pi M\\ 1/2\,H^2, & H_l\ll H\ll 4\pi M.\\ 1/H^2, & H_l\ll 4\pi M\ll H\end{cases} \tag{9}$$

It is evident from Fig. 1 that the LAS has two characteristic points along the field scale: H_l and $4\pi M$. In the vicinity of the correlation field H_l, there occurs a change of the functional dependence, caused by the expression in front of the square brackets in formula (8): the function $f(H,B)$ here is close to zero. In the vicinity of $H=4\pi M$, the numerical coefficient of the functional dependence H^{-2} changes from 1 to 2 because of the change of the function $f(H,B)$ (in the shcematic Fig. 1, this change is shown, for clarity, in exaggerated form). It is evident that when $H\gg H_l$, the expression (9) reduces to the usual classical LAS, in which the mean-square fluctuations of the direction of the magnetization are determined solely by the ratio of the anisotropy field to the external magnetic field. On decrease of the field, an ordering effect of the exchange interaction manifests itself in the region $H\sim H_l$; this partly suppresses the spatial fluctuations of the orientation of the magnetization, and this in turn leads to a change of the law for $H\leq H_l$.

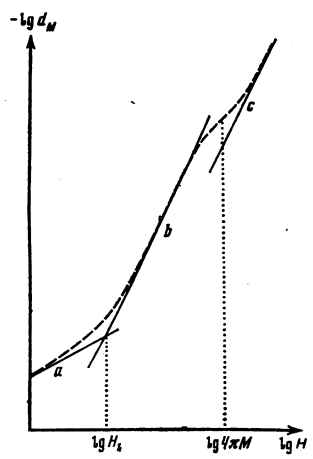

FIG. 1. Schematic representation of the law of approach to saturation in logarithmic coordinates. The asymptotes a, b, and c correspond to the expressions (9).

Before doing the further analysis, we shall make one generalization of the expressions (7)–(9). We obtained these expressions for a definite form of the magnetic-anisotropy term in the Hamiltonian (2). What will change if the anisotropy is described by a term of different form? It can be shown that in the theory linear with respect to D_M, within the framework of which all expressions here are obtained, nothing will change except the value of the constant D, which characterizes the dispersion of the easy axes of anisotropy in an equiprobable distribution of their orientations.

We shall now give a simple method of calculating this constant for an arbitrary form of the anisotropy term. For this purpose, it is sufficient to derive the law of approach to saturation in the simplest case of formal neglect of the exchange and dipole-dipole interactions. For the uniaxial case considered above, this means that in the Hamiltonian (2) we restrict ourselves to the second and third terms. Since we now have to do with an ensemble of noninteracting regions (crystallites) with different directions l, we can go over from the methods of the theory of random functions to the simpler methods of direct averaging over an ensemble with a known distribution function (in our case, one uniform with respect to the orientations l). For an arbitrary form of the magnetic anistropy, this averaging is carried out more simply in a system of coordinates rigidly attached to the anisotropy axis; then the members of the ensemble differ in the direction of the magnetic field H. Therefore, we shall proceed in the same way also for uniaxial anisotropy, where the problem is equally simple in both coordinate systems.

Thus let l be directed along the z axis, and H in an arbitrary direction described by polar angle θ and azimuthal angle φ. The direction of the magnetization is then described by certain angles θ_1 and φ_1, determined by the static Landau-Lifshitz equations.[12] Supposing the magnetic field to be sufficiently large, and linearizing the problem with respect to the small quantities $\eta=\theta_1-\theta$ and $\xi=\varphi_1-\varphi$, we get

$$\eta=-(H_a/H)\sin\theta\cos\theta, \quad \xi=0. \tag{10}$$

The projection of the magnetization on the direction of the magnetic field, independently of the form of the expression

$$m_h=1-1/2(\eta^2+\xi^2\sin^2\theta). \tag{11}$$

Substituting here the values of η and ξ and carrying out an averaging over all values of the spatial angle, we obtain an expression for the law of approach to saturation in the form

$$\langle m_h\rangle\approx 1-1/15(H_a/H)^2, \quad H_a=\beta M. \tag{12}$$

Comparing this expression with the bottom formula (9), we see that they agree and differ from the middle formula (9) by the coefficient $\frac{1}{2}$. This coefficient results from allowance for dipole-dipole interaction. Its origin is easily traced from the structure of the expression (4): here on formally setting $k_M=0$, we obtain directly, for $H\gg H_a$, the limiting case (12). In fact, of course, neglect of the long-range action is not permissible, and in the range $H\lesssim 4\pi M$ the correct expression is (9); but

understanding of the essence of the matter allowed us to find easily that for a uniaxial crystal, $D=1/15$.

We shall carry out the same procedure for a cubic crystal:

$$\mathcal{H}=-\mathbf{HM}+K(m_x^2m_y^2+m_y^2m_z^2+m_z^2m_x^2). \tag{13}$$

In this case, we have by linearization of the static Landau-Lifshitz equation with respect to η and ξ

$$\eta=\frac{H_a}{H}\sin\theta\cos\theta\left[\frac{1}{4}\sin^2\theta(7+\cos 4\varphi)-1\right], \quad \xi=-\frac{H_a}{4H}\sin^2\theta\sin 4\varphi. \tag{14}$$

Substituting this solution in (11) and averaging, we get the LAS in the form

$$\langle m_h\rangle\approx 1-\frac{2}{105}\left(\frac{H_a}{H}\right)^2, \quad H_a=\frac{2K}{M}. \tag{15}$$

It is evident that now $D=2/105$. In the same way, the value of this constant can be easily determined for an arbitrary law of magnetic anisotropy, whether crystallographic or caused, for example, by nonuniform elastic stresses.

We turn now to a discussion of the general expressions (6)–(9) for the LAS. Within the framework of the approximations used in deriving them [consideration of linear terms only, in the equations for the transverse components of the magnetization; approximation of the unknown correlation function by the simplest expression, (5)], the expressions (6)–(9) are of quite universal character. Specifically, they are invariant with respect to the analytic form of the local anisotropy energy. Change of the symmetry of the anisotropy (or allowance for higher degrees of the expansion in the anisotropic term) leads only to a renormalization of the constant D in the expressions (6)–(9).

It is evident that for solution of the inverse problem that is of interest to us, it is necessary to do experimental measurements in the range $H\sim H_l$. This differs significantly from the usual experimental situation in classical investigations of the LAS, which are restricted to the range $H>4\pi M$ (sufficient for finding H_a, but giving no information about H_l). The necessity for making experimental measurements in fields $H<4\pi M$ leads to specific difficulties in the processing of the experimental data; these must now be discussed.

We approximated the unknown correlation function with the expression (5) and thereby decreased the generality of the derived expressions (6)–(9). The asymptotes (9), however, are independent of the form of the approximating function. Therefore if one carries out investigations of the LAS at $H\ll H_l$ and at $H\gg H_l$, one can determine H_l, independently of the form of the approximating function. Here, however, it is necessary that in fields $H\ll H_l$ the inequality $8d_m\gg 1$, which limits the range of applicability of the linear theory of stochastic magnetic structure,[19] shall be satisfied. This ideal situation can be realized extremely rarely. For the Co-P alloys investigated in this paper, as will be evident below, only those experimental curves that correspond to fields $H>H_l$ may be compared with the linear theory. Furthermore, the inequality $H_l\ll 4\pi M$ is also not satisfied sufficiently strictly for all the alloys. This means that the characteristic points in Fig. 1 are drawn together and the asymptote b is "washed out."

Thus the processing of the experimental data must be carried out not on the basis of the asymptotic expressions (9), which as a rule are not attained, but on the basis of a finite segment of the exact expression (7). But the expression (7), in contrast to the asymptotes, depends on the choice of the approximating correlation function. In order to test how large an error is introduced into the determination of H_l and H_a by the indeterminacy of the functional form of $K(\eta)$, it would be necessary to obtain expressions for the LAS by use of different approximating correlation functions and to compare the results with each other. So far, we have carried out this program only for the function (5) and for a function of the form

$$K(\eta)=D(1+k_l'\eta)\exp(-k_l'\eta), \quad S(k)=\frac{4Dk_l'^3}{\pi^2(k_l'^2+k^2)^3}. \tag{16}$$

The correlation wave number k_l' that occurs in these expressions can be related to the correlation wave number k_l of the expressions (5) by the linear relation $k_l'=\nu k_l$, with an undetermined numerical coefficient ν. On substituting (16) in (4) and then in (3) and carrying out the integration over k and θ, we get a new expression for the LAS. For the case $H_l\ll 4\pi M$, it takes the form

$$d_m=\frac{DH_a^2(4\nu H_l^{1/2}+H^{1/2})}{2H^{1/2}(\nu H_l^{1/2}+H^{1/2})^4}[1+f(H,B)], \tag{17}$$

where $H_l=\alpha Mk_l^2$ is the same correlation field as in the expression (8).

If we choose as the value of the numerical coefficient $\nu=(4)^{1/3}$, then all the asymptotes of the expression (17) coincide with the corresponding asymptotes of the expression (8). Despite the functional difference between the expressions (8) and (17), the laws of approach to saturation described by them differ little from each other. In the processing of the experimental data according to formulas (8) and (17), the discrepancy in the determination of the desired parameters did not exceed 7%.

It can now be stated more precisely what information can be extracted from experimental study of the law of approach of the magnetization to saturation (6)–(9). This law contains only two parameters characterizing a certain random function of the coordinates, an effective magnetic anisotropy that fluctuates in direction: the value of the root-mean-square fluctuation of the anisotropy field $D^{1/2}H_a$ and its correlation field H_l. Further information can be extracted from these two parameters only if additional information about the magnetic system is known from independent experiments. Thus knowledge of the symmetry of the local magnetic anisotropy (that is, knowledge of the numerical value of the constant D) permits determination of the effective local magnetic anistropy field H_a. Knowledge of the value of the magnetization and of the exchange makes it possible to calculate from H_l a fundamental characteristic of the local anisotropy, the correlation radius r_l.

EXPERIMENT AND DISCUSSION

The law of approach of the magnetization to saturation was studied experimentally on microcrystalline and amorphous Co-P alloys. Co-P alloys prepared by the method of chemical deposition in the form of thin films, with P concentration from 1 to 13 at.%, have now been quite thoroughly investigated at the Institute of Physics, Siberian Branch, Academy of Sciences, USSR[20] and therefore are suitable objects for experimental application of the method of extracting the stochastic characteristics of the fluctuations of local anisotropy that was theoretically validated in the preceding section. We shall present here some information[20,21] that will be needed in the interpretation of the results obtained in the present paper. It is well known that pure Co has two polymorphic modifications: the hexagonal, stable at low temperatures, and the cubic, stable at high temperatures (the temperature of the polymorphic transition for Co films usually lies in the range 400–420°C). But Co-P alloys, depending on the phosphorus concentration and the conditions of deposition may be obtained either in the crystalline (hexagonal or cubic modification) or in the amorphous state. For the crystalline solid solution of P in Co, the concentrational transition from hexagonal to cubic symmetry is observed in the range 4–5 at.% P. With further increase of the P concentration, a concentrational transition to the amorphous state occurs in the neighborhood of 9 at.%.

We investigated the law of approach of the magnetization to saturation with a vibration magnetometer, at room temperature, on specimens that were rectangles of dimensions 2 × 4 mm, of thickness 10 to 30 μm. Certification of the specimens was carried out by the methods of x-ray spectroscopic, x-ray diffraction, and thermographic analyses.[20] The magnetic field was applied in the plane of the layer, along the smaller side of the specimen; the value of the magnetic field was varied from 50 Oe to 33 kOe. The error of measurement of the field with an NMR probe was 0.01%; the error of measurement of the change of magnetization M_z/M did not exceed 0.05%. The results of the measurements are shown in Fig. 2 (here, for convenience of perception, the field axis is terminated at the value 3 kOe). As is seen from the figure, the $M_z(H)$ curves are grouped in three families, corresponding to three concentration ranges: 1) a solid solution with hexagonal Co lattice (x = 1 to 4 at.% P); 2) a solid solution with cubic Co lattice (x = 6 to 8 at.% P); 3) an amorphous alloy ($x \geq 9$ at.% P). The curve corresponding to $x = 5$ at.% P reflects the transitional nature of this state.

For comparison with the theory set forth in the preceding section, it is convenient to represent the experimental curves of Fig. 2 in the form of $\log d_m$ vs $\log H$ relations (Fig. 3). The simple theoretical laws corresponding to the conditions (9) are plotted in this figure as dashed lines: the line a corresponds to $d_m \sim H^{-1/2}$, the line b to $d_m \sim H^{-2}$. Change of the value of $D^{1/2}H_a$ leads to a parallel displacement of both these lines along the axis of ordinates, change of H_i to a similar displacement of the line a alone.

We shall discuss the correspondence between theory and experiment. It is evident that it is possible by parallel displacement to match the curve b with the high-field sections of the experimental curves corresponding to the hexagonal phase (x = 1 to 4 at.% P); at the same time, comparison of the curve b with the experimental curves for the cubic and amorphous phases shows that the high-field sections of these curves no longer fit the H^{-2} law. We shall now compare the curve a with the low-field sections of the experimental curves. It is possible by parallel displacement to make the line a tangent to all the curves corresponding to crystalline states. But this occurs already far beyond the limits of applicability of the linear theory: only experimental data corresponding to $\log d_m < -1$ may be compared with this theory (see Ref. 19, where the relation $8d_m \ll 1$, which determines the limits of applicability of the linear theory of stochastic magnetic structure, is derived). Therefore in a comparison with the theory developed in the preceding section, only those sections of the experimental curves have significance that lie above the straight line $\log d_m = -1$ in Fig. 3. But these

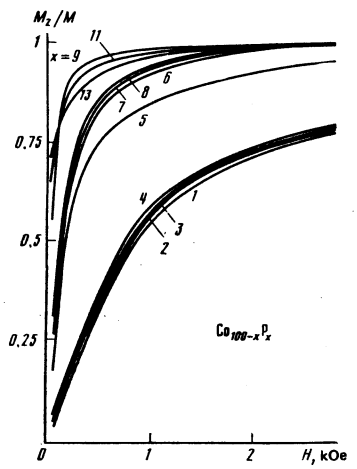

FIG. 2. Variation of the magnetization with the value of the magnetic field for specimens of Co-P alloys with various concentrations of P; x is the concentration of P in at.%.

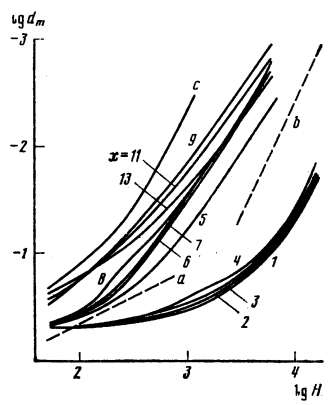

FIG. 3. Variation of the dispersion d_m of the stochastic magnetic structure of microcrystalline and amorphous Co–P alloys with the value of the magnetic field, in a log-log coordinate system. The curve c is the similar relation for an amorphous Fe–Ni–B alloy, plotted from the data of Ref. 11; a and b are the limiting theoretical relations.

sections correspond (to judge by a comparison of them with the lines a and b) to the range $H \gtrsim H_l$. Thus the asymptotic expressions (9) for the law of approach to saturation cannot be used for extraction of the parameters $D^{1/2}H_a$ and H_l in our case.

The processing of the experimental data was carried out in the following manner. With a computer, a set of curves was constructed that corresponded to the theoretical relations (6) and (7) for various values of H_l. The experimental curve was superposed on this grid and was displaced along the axis of ordinates until the section of it corresponding to $|\log d_m| > 1$ coincided with one of the curves of the grid. Thus the value of the correlation field H_l was determined. The root-mean-square fluctuation of the anisotropy field $D^{1/2}H_a$ was estimated from the value of the segment formed by the zeroes of the coordinates of the theoretical grid and of the experimental graph (to different values of $D^{1/2}H_a$ there corresponds a displacement of the whole grid of curves along the axis of ordinates). The parameters thus determined are shown in Fig. 4 as functions of the phosphorus concentration (at.%). (In the processing of the magnetization curves, no account was taken of the demagnetizing field, which in these fields varies from 25 to 30 Oe. But the amount of this systematic error is significantly smaller than the error of the proposed graphical method, for which the error of determination of $D^{1/2}H_a$ and H_l amounts to 10%.)

In the range $x \approx 5$ a transition occurs from the hexagonal structure of the cobalt lattice to the cubic; the cubic structure persists in the form of short-range order even in the amorphous state. Describing the hexagonal phase approximately with uniaxial anisotropy, we find that for $x < 5$ the constant $D = 1/15$, while for $x \geq 5$, $D = 2/105$; knowledge of this constant enables us to determine the value of the local magnetic anisotropy field H_a. In order to find from the concentration dependence of H_l the concentration dependence of $r_l(x)$, it is necessary to know the concentration dependences $M(x)$ and $\alpha(x)$. Such investigations were made in Ref. 22; their results are shown in Fig. 5.

The concentration dependences $H_a(x)$ and $r_l(x)$ that we obtained are shown in Fig. 6.

It is seen that with decrease of x, the anisotropy field H_a approaches the known value (~12 kOe) corresponding to the magnetocrystalline anisotropy of a hexagonal single crystal of Co. In the vicinity of $x \approx 5$ there is a sharp decrease of H_a, and then there is a slow, practically linear increase with increase of x. As far as we know, this is the first measurement of the local anisotropy of cubic solid solutions and amorphous states of Co-P alloys. Conventional extrapolation to $x = 0$ of the linear concentration dependence of H_a found for $x > 5$ leads to satisfactory agreement with the known[23] (approximately) value of the magnetocrystalline anisotropy of cubic Co, $H_a \approx 900$ Oe.

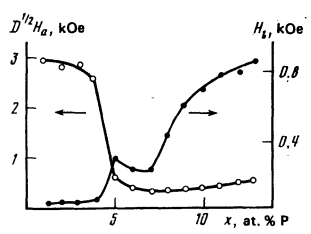

FIG. 4. Concentration dependence of the root-mean-square fluctuation of the anisotropy field $D^{1/2}H_a$ and of the correlation field H_l for Co—P alloys.

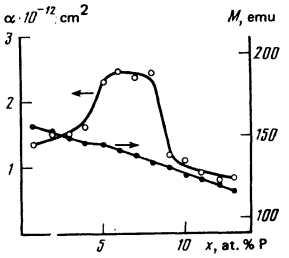

FIG. 5. Concentration dependence of the saturation magnetization M and of the exchange constant α for Co—P alloys, according to the data of Ref. 22. On the $\alpha(x)$ curve, limiting concentrations are evident: 9 at.% P separates solid-solution states from amorphous; 5 at.% P separates solid solutions of different symmetries.

It is seen from the $H_a(x)$ curve that the value of the local anisotropy is insensitive to the concentrational phase transition of the solid solution to the amorphous state. This in general is not surprising, if the value of H_a is principally due to magnetocrystalline anisotropy; then it is determined by short-range order, which does not change in such a transition.

We shall now discuss the concentration dependence of the correlation radius $r_l(x)$. This quantity characterizes the size of the region of uniform orientation of the anisotropy axis. In a polycrystal, $2r_l$ corresponds to the mean size of a crystallite; in an amorphous material, to the size of some oriented cluster. Thus the abrupt change of r_l at the polymorphic transition of the hexagonal Co-P solid solution to cubic ($x \approx 5$ at.% P) corresponds simply to a decrease of the size of the crystallites. The decrease of r_l beginning with $x = 8$ is due to the process of amorphization of the substance. This is the first measurement of the correlation radius of the fluctuations of direction of the axis of anisotropy in an amorphous magnet. The general tendency of the

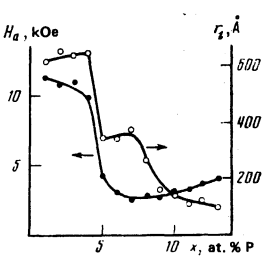

FIG. 6. Concentration dependence of the values of the local effective magnetic anisotropy field H_a and of the correlation radius r_l of the fluctuations of the axis of magnetic anisotropy, for Co—P alloys, calculated from experimental LAS curves (Fig. 2) according to the theory developed in the present paper. The errors of determination of H_a and of r_l are 10% and 20%.

behavior of $r_l(x)$—an abrupt drop on transition to the amorphous state, and thereafter a slow decrease with increase of the amorphizing ingredients (which increase both the chemical and the structural disorder)—agrees qualitatively with the intuitive ideas about the amorphous state that have been put together so far. An unexpected fact is the large value of the correlation radius of the fluctuations of the anisotropy axis in the amorphous state: $r_l \sim 100$ Å. We recall that for the correlation radius of exchange fluctuations in these same alloys one gets $r_\alpha \sim 20$ to 30 Å (Ref. 17), and that the structural correlation radius for all known amorphous substances is still shorter and corresponds to 6 to 8 Å (Ref. 24).

As we have already noted above, the local anisotropy field H_a is here understood to mean both the magnetic anisotropy proper of the immediate environment, and the anisotropy of elastic stresses. Therefore the following experiment was done: for one of the compositions ($x = 9$), a prolonged (10 h) anneal was carried out at temperature 125 °C, substantially below the crystallization temperature (240 °C). After the first 4 h of the anneal, the value of H_a had decreased by about 30% from the initial value; further annealing for 6 h did not lead to any change of the value of H_a. The anneal had no effect on the value of the correlation radius r_l. These results can apparently be interpreted as removal by the anneal of nonuniform elastic stresses. Then the initial effective local magnetic anisotropy that we determined for the amorphous Co-P alloy (Fig. 6) is the sum of the crystallographic anisotropy (~70%) and the anisotropy of elastic stresses (~30%).

Thus from the curves of Fig. 2 and from certain auxiliary data, we have determined for each x the numerical values of two fundamental parameters that characterize the effective magnetic anisotropy: H_a and r_l. That is, we have solved the inverse problem; that was the goal of this paper.

CONCLUSION

A fundamental characteristic of any spatially fluctuating parameter of the spin system of an amorphous (or microcrystalline) magnet is its correlation function $K_i(\eta)$ [or the Fourier transform $S_i(k)$ of the correlation function]. In the phenomenological theory developed in Refs. 13–16, the correlation functions occur in general form, as certain unknown functions that must be determined from experiment. This approach has maximum generality, since it is not connected with a choice of any model. At the same time, it permits systemization of the experimental data, showing how the same fundamental characteristics of a stochastic system can be determined from different experiments; accumulation of such experimental data will make it possible hereafter to approach the construction of concrete models on a firmer basis.

A number of parameters of an amorphous magnet can be represented, in a phenomenological theory, in the form of an effective magnetic anisotropy fluctuating in direction. This is first of all the magnetocrystalline anisotropy, and then the effect of the action of inhomogeneous elastic stresses or of any other defects and inhomogeneities, which in the phenomenological Hamiltonian are representable by anisotropic terms of arbitrary form and symmetry. The principal difference of these anisotropic parameters from the isotropic (see footnote 3) is that they produce an orientationally nonuniform state of magnetization—a stochastic magnetic structure.

The spectral density $S_m(\mathbf{k})$ of the stochastic magnetic structure depends on the characteristics of the effective local anisotropy: the quantity H_a, the symmetry (which determines the parameter D), and the spectral density $S(k)$ of the fluctuations of the anisotropy axis.

The problem of the present paper was theoretical and experimental validation of a method of determination of some of these characteristics by investigation of the LAS (since the latter is one of the indirect methods of studying stochastic magnetic structure). Since the quantity $M_s(H)$ measured by this method is expressed in terms of an integral (3) of $S_m(\mathbf{k})$, in order to obtain an analytic expression for the law of approach to saturation it is necessary to assume some analytic form for the desired function $S(\mathbf{k})$. Here we used the two simplest correlation functions, each of which is characterized by a single parameter: an arbitrary correlation radius r_l.

Within the framework of these approximations, an analytic expression for the LAS was obtained; it has a quite universal character:

a) it describes (formally) polycrystalline, amorphous, and any clustered classical systems;

b) it is invariant with respect to the analytical form of the local magnetic anisotropy energy.

It has been shown theoretically and experimentally that two characteristics of a spatially fluctuating magnetic anisotropy can be determined from the LAS: the value of the root-mean-square fluctuation of the anisotropy field $D^{1/2}H_a$ and the correlation field H_l. Supplementary information, obtained from independent experiments (the symmetry of the anisotropy, the values of the exchange and of the magnetization), make it possible to calculate from these characteristics the effective local magnetic anisotropy field H_a and its correlation radius r_l.

Both amorphous and polycrystalline systems are described in terms of these fundamental characteristics. Therefore the quantity r_l responds sharply both to a polymorphic transition accompanied by change of size of the crystallites, and to a transition between the polycrystalline and the amorphous states. It has been shown for the first time experimentally that on transition to the amorphous state, the value of the local anisotropy experiences no abrupt changes.

We shall now consider the difference of the approach proposed here from the approach developed in the papers of Kronmüller and coworkers.[8–11] We shall demonstrate this difference in the example of a calculation of the fluctuations of the magnetic anisotropy and of the elastic stresses that can determine the law of

approach to saturation. (Refs. 8 and 9 also analyzed the contribution to the LAS from fluctuations of the isotropic characteristics, the modulus of the magnetization and the exchange parameter, and it was shown that it was negligibly small; we shall not return to this question here.)

The principal difference of the approach of Refs. 8–11 from ours is the traditional construction, from the very beginning, of *a priori* models for each fluctuating parameter: models, in our view, often not justified by anything (and, from the point of view of our approach, superfluous). Thus in analyzing the contribution to the LAS from fluctuations of the magnetic anisotropy, the authors of Refs. 9–11 assume from the very beginning that in the phenomenological theory one must consider its correlation radius to be zero (that is, that the correlation dies off at distances of the order of the lattice parameter). As a result, they obtain for the LAS an expression in the form of a sum of two terms:

$$d_m = A/H^{1/2} + B/H^2. \qquad (18)$$

The first term of this expression corresponds in our approach to the first term of the expansion of the LAS in small magnetic fields, $H \ll H_i$; see (9). The coefficient A corresponds approximately to the special value of the analogous coefficient in (9) obtained by replacing the correlation radius r_i in the latter by the lattice parameter.

The second term in the expression (18) has a field dependence corresponding in our case to sufficiently large fields. But here the coincidence is purely accidental. The $1/H^2$ dependence in Refs. 9–11 appeared as a result of the fact that the value of $\langle \rho \rangle$ (in our notation) is considered to be different from zero. But this means that the authors of Refs. 9–11 are considering magnetization of a textured magnet by a magnetic field whose direction does not coincide with the texture axis; this problem simply has no relation to the LAS problem. Therefore the coefficient B in the expression (18), obtained in Refs. 9 and 10, does not correspond to the coefficient of the H^{-2} term in any of our limiting cases (9).

In the limiting cases of small and large magnetic fields, the expression (18) behaves completely oppositely to our expression for the LAS: for us, the $H^{-1/2}$ dependence corresponds to small and the H^{-2} dependence to large fields, whereas for the expression (18) the opposite is true. It is therefore natural that the authors of Refs. 10 and 11 were unable to describe their experimental results with this expression. From this they concluded that the LAS observed by them was not formed by fluctuations of the magnetic anisotropy. In actuality, from the disagreement of (18) with experiment it follows only that the fluctuations of anisotropy cannot be characterized by zero correlation radius.

Further, the authors of Refs. 10 and 11 in the same way, i.e. by starting from a concrete model, investigate the effect of fluctuations of elastic stresses on the form of the LAS. They postulate the existence of dislocation dipoles of length L, with a mean distance $\mathscr{D}$ between them, and a geometry of distribution of these dipoles. As a result of mathematical processing of this model, the authors find an integral expression for the LAS, from which, in the limiting cases of small and of large fields, analytical expressions are obtained for the LAS; they correspond to the expressions (9). These expressions correspond qualitatively to the experimental relations obtained by the authors; therefore they conclude that the LAS is determined by nonuniform elastic stresses. Further, from a comparison of their theory with experiment, they estimate the mean distance $\mathscr{D}$ between the hypothetical dipoles.

But in actuality, the qualitative agreement of theory with experiment came about not because elastic stresses, dislocation dipoles, etc. were considered, but because in the solution of this problem the authors tacitly allowed the existence of an arbitrary (nonzero) correlation radius of the fluctuations. And only the value of this correlation radius (and the value of the effective magnetic anisotropy field) can be determined from experimental investigation of the LAS. In Fig. 3, the curve c was plotted from the experimental data of Ref. 11, in which the LAS was investigated for an amorphous Fe-Ni-B alloy. Processing of it according to our theory gives the following results: $H_a \approx 750$ Oe, $r_i \approx 8800$ Å. And this is all that can be obtained from the LAS.

It is impossible to determine from the $M_s(H)$ curve alone what contribution to H_a is made by magnetocrystalline anisotropy and what contribution is made by elastic stresses or other defects. For this purpose it is necessary to investigate at least the concentration or temperature dependence of the LAS. Such investigations, made by us for Co-P alloys, enabled us to conclude that for these alloys the principal contribution to H_a comes from the magnetocrystalline anisotropy.

[1)] A short exposition of this work was published in Ref. 1.

[2)] The authors of Refs. 8–11 use the term "micromagnetic". This term, introduced by W. F. Brown at the end of the thirties, is still used, in accordance with tradition, by many authors, although at the present time the use of the prefix "micro" for a purely classical (nonquantal) problem seems somewhat archaic. [*Translator's Note*: I introduced the term "micromagnetics" in my paper at the 1958 Grenoble Conference, published in 1959. I do not recall encountering the adjective "micromagnetic" earlier, although Quimby used the term "micromagnetization" before 1936].

[3)] Because of the anisotropic character of the magnetic-dipole interaction, isotropic fluctuations of the modulus of the magnetization also lead to orientational oscillations of the magnetization vector; but an estimate[8,9] shows the negligible smallness of this effect. In ferrimagnets, fluctuations of exchange, of magnetization, and of the value of the anisotropy may lead to the occurrence of a stochastic magnetic structure comparable in intensity with that produced by fluctuations of the anisotropy axis.[18]

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Translated by W. F. Brown, Jr.