Dynamic damping of domain walls in a ferromagnet with dislocations

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The motion of a domain wall in a ferromagnet with defects is investigated. It is shown that the presence of static defects of any kind not only produces a coercive force (static friction force) but contributes also to the dynamic damping forces. The contributions to the damping force from the various interactions between magnons and the wall are analyzed. The main contribution comes from the emission of a single magnon localized near the wall. It is shown that the interaction between a moving domain wall and the inhomogeneities can cause the wall to have a negative differential mobility. The physical consequences of this circumstance are analyzed.

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Most real crystals always contain point defects (impurities, vacancies, etc.) as well as extended defects — dislocations. Both types of defect can influence substantially the dynamic properties of the domain walls (DW). The influence of the crystal defects leads primarily to the appearance of a coercive force (static friction force[1]). Dynamic damping (viscous friction), due to additional energy transfer to the moving DW by the thermal magnons, is also possible on account of the change in the character of the interaction of the DW with the magnons in the presence of static defects that have no internal degrees of freedom.

We consider in this paper dynamic damping of DW in a ferromagnetic (FM) crystal with randomly disposed defects. It is shown that the presence of inhomogeneities, particularly dislocations, must be taken into account not only in the calculation of the coercive force but also in the analysis of the dynamic damping of DW in FM, by virtue of the following circumstances. First, if the crystal contains inhomogeneities, free magnons can be radiated at any DW velocity. We recall that in an ideal crystal the radiation can take place only at velocities exceeding the minimum phase velocity of the spin waves. The physics of this phenomenon is the following: the produced magnon can acquire from the DW an energy $\mathbf{q}\cdot\mathbf{v}$ only together with a momentum $\mathbf{q}$, and at the same time the magnon can obtain from the defect a momentum $\varkappa$. The energy conservation law corresponding to this process can be written in the form [see Eqs. (10) below]

$$\mathbf{q}\mathbf{v}=\varepsilon(\varkappa+\mathbf{q}),$$

where $\varepsilon(\mathbf{k})$ is the energy of a magnon with momentum $\mathbf{k}$. In the absence of defect (without allowance for $\varkappa$) this process proceeds, naturally, only at $v \geqslant \min[\varepsilon(k)/k]$, i.e., at a DW velocity exceeding the minimum phase velocity of the spin waves. Analysis shows, however, that at $\varkappa \neq 0$ the radiation condition becomes much less stringent, namely, at any value of v there exists a value of $\varkappa$ such that this process is possible and contributes to the dynamic damping of the DW. This contribution is particularly large in radiation of so-called surface magnons localized near the DW. The contribution of these processes to the DW damping force does not depend on the temperature and is the main contribution at low temperatures.

The presence of inhomogeneities can affect the contribution of two-magnon processes to the damping force. First, two-particle processes with participation of surface magnons, which are absent from an ideal crystal, become possible. Second, the nonreflecting character of the interaction of the bulk magnons with the DW, which is inherent in an ideal crystal,[2] is violated and leads to an additional contribution to the DW mobility.

We calculate in this paper the contribution of one- and two-magnon processes to the damping of DW and obtain the dependences of the damping force on the temperature and on the velocity and the density of the dislocations. A specific feature of the contribution of defects to the dynamic damping force is the possible appearance of DW velocity intervals in which a negative differential mobility B is realized, $B = dF(v)/dv < 0$. We discuss the dependence of the velocity of the steady-state motion of the DW on the external force in this situation.

1. INTERACTION OF DW WITH CRYSTAL DEFECTS

To describe the interaction of DW with dislocations, we represent the functional of the FM energy in the form of a sum of the energy of an ideal FM and the energy of the interaction of the magnetization $\mathbf{M}$ with the dislocation field:

$$W=\frac{1}{2}\int d\mathbf{r}\left\{\alpha\left(\frac{\partial M_i}{\partial x_k}\right)^2+\beta(M_0^2-M_z^2)+\rho M_x^2\right\}+W_{int}, \tag{1}$$

where $\mathbf{M}$ is the FM magnetization; $\mathbf{M}^2 = M_0^2$; α is the exchange constant; β and ρ are the anisotropy constants. The contribution of the magnetic dipole interaction to the FM energy is disregarded and it is assumed that $\beta, \rho \gg 4\pi$.

The energy of the interaction of the DW with an inhomogeneous strain is determined by the magnetostriction energy, which we write in a form typical of a uniaxial crystal[3,4]:

$$W_{int}=\int d\mathbf{r}\{\lambda_1 M_i M_k+\lambda_2(\mathbf{Mn})M_i n_k+\lambda_3(\mathbf{Mn})^2\delta_{ik}+\lambda_4(\mathbf{Mn})^2 n_i n_k\}\frac{\partial u_i}{\partial x_k}. \tag{2}$$

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Here λ_i are the magnetostriction constants and $\mathbf{n}$ is a unit vector along the z axis. If we neglect the change of the elastic strains due to the motion of the DW (a "pinned" system of defects — "external" force for the magnetic system), we can substitute for the strain tensor in (2) its value produced by a specified defect configuration. We consider hereafter DW motion with velocity much lower than the known Walker limit v_w, $v \ll v_w$ (Ref. 5).

The magnetization in the DW is described by the angles θ and φ,

$$M_z=M_0 \cos \theta, \; -M_y+iM_x=M_0 \sin \theta e^{i\varphi}.$$

To calculate the coefficient of v^2 in the damping force [see (10) below] we must take into account the dependence of the DW structure on the velocity. We assume $\rho/\beta \ll 1$; it suffices then to take θ and φ in the form

$$\cos \theta = \text{th}\,[(x-vt)/x_0], \quad \varphi=-v/2v_W, \tag{3}$$

where $x_0 = (\alpha/\beta)^{1/2}$ is the thickness of the DW at rest.

We consider small fluctuations of the magnetization in the FM with the DW, and express them in terms of the Holstein–Primakoff operators.[4] The Hamiltonian of the spin waves in FM with moving DW can be represented by the series

$$H=H_0+H_1+H_2+\ldots, \tag{4}$$

where H_0 does not contain the Holstein-Primakoff operators a^+ and a; H_1 is linear and H_2 is quadratic in these operators, etc.; the operators H_i depend on the form of $\theta(\mathbf{r})$ and $\varphi(\mathbf{r})$. Substitution of the solution (3), which describes the motion of a DW in an ideal crystal, yields $H_1 = 0$ (Ref. 2); consequently, in our case H_1 is determined only by the inhomogeneities, i.e., H_1 describes single-magnon processes determined by the inhomogeneities; H_2 can be represented in the form $H_0 + H_{2\text{int}}$, where H_0 is the Hamiltonian of the magnons in an ideal crystal, and $H_{2\text{int}}$ describes the defect-induced scattering of magnons by the DW and the emission of a pair of magnons by a moving DW.

We shall use the Hamiltonian (4) to describe the interaction of the DW with magnons in the presence of a system of randomly displaced defects. This Hamiltonian depends explicitly on the time because of the motion of the DW, leading to inelastic processes in the magnon subsystem and as a consequence to the transfer of the DW energy to the magnons, i.e., to the damping of the DW.

We calculate the damping force by perturbation theory. In this case, naturally, there are two possibilities. We can use as the zeroth approximation the state (3), which describes the DW motion without allowance for dislocations. In the second approach we can start from the exact solution of the Landau-Lifshitz equation for the DW motion with the defects taken into account. This motion is in principle not uniform, i.e., it includes the deviation of the shape of the DW from planar and the non-uniform DW motion. If we start from the exact solution, then $H_1 = 0$.[2] Analysis shows, however, that this solution contains deviations of the magnetization from the equilibrium value, and these deviations do not decrease far from the DW (as $x \to \pm\infty$). These deviations can be represented as sums of two terms, one independent of time and determined only by the defects, and the other determined by the interaction of the DW with defect and dependent on the time. The latter term leads to a nonzero energy flux far from the DW and hence to dissipation of the energy of the moving DW.

It is much simpler, however, to describe dissipation by starting from the first approach. In this case $H_1 \neq 0$, and hence $\langle a(\mathbf{r},t)\rangle \neq 0$, i.e., the vacuum state for the operators $a(\mathbf{r},t)$ is not the true vacuum of the system: a peculiar dynamic condensate is produced in the system. This result describes an obvious fact: we are starting not from the exact ground state of the system, and the produced "condensate" describes the corrections to the magnetization distribution (3) in the DW, due to the inhomogeneities. At low deformation density it can be assumed that the condensate amplitude is small, and the physical characteristics of the system can be described by starting from (4) with $H_1 \neq 0$. It is simplest, in particular, to calculate in this manner the rate of dissipation of the DW energy, and the results obtained by both methods agree in first order in the small parameter $(\partial u_i/\partial x_k) \ll 1$.

In addition, the presence of $\langle a(\mathbf{r},t)\rangle \neq 0$, or more accurately of a part of $\langle a(\mathbf{r},t)\rangle$ localized near the DW, describes exactly the above-noted non-uniformity of the DW motion. Consequently, both approaches are equivalent both for the calculation of the damping force and for the description of the non-uniform motion of the DW. The method chosen by us, however, is much simpler and leads more readily to a result for the DW damping force.

We choose below, as the zeroth approximation, the known abbreviated equation that describes the DW motion in an ideal crystal. We expand the operators $a^+(\mathbf{r},t)$ and $a(\mathbf{r},t)$ in the total orthonormalized set of Winter states a_k and $a_{k\perp}$ (Ref. 6)

$$a(\mathbf{r},t)=\frac{1}{\Omega^{1/2}}\sum_{k}\frac{\text{th}\,\xi-ik_x x_0}{(1+k_x^2 x_0^2)^{1/2}}e^{i\mathbf{kr}}a_k+\frac{1}{(2Sx_0)^{1/2}}\sum_{k_\perp}\frac{1}{\text{ch}\,\xi}e^{i\mathbf{k}_\perp \mathbf{r}_\perp}a_{k_\perp}, \tag{5}$$

where $\xi = (x - vt)/x_0$, $\mathbf{k}_\perp$ is the wave vector in the DW plane, S is the DW area, and Ω is the volume of the system. The operator $a_{k\perp}$ corresponds to a magnon localized near the DW (surface magnon).

Substituting (5) in (4), we obtain

$$H_0=\sum_k\left[A_k a_k^+ a_k+\frac{1}{2}B_k(a_k^+a_{-k}^+ + a_k a_{-k})\right] + \sum_k\left[A_{k_\perp}a_{k_\perp}^+ a_{k_\perp}+\frac{1}{2}B_{k_\perp}(a_{k_\perp}^+ a_{-k_\perp}^+ + a_{k_\perp}a_{-k_\perp})\right], \tag{6}$$

where

$$A_k=\varepsilon_0(1+\rho/2\beta+x_0^2\mathbf{k}^2), \quad A_{k_\perp}=\varepsilon_0(\rho/2\beta+x_0^2\mathbf{k}_\perp^2), \quad B_k=B_{k_\perp}=\varepsilon_0\rho/2\beta,$$
$$\varepsilon_0=2\mu_0\beta M_0,$$

μ_0 is the Bohr magneton, $\mu_0 > 0$. The Hamiltonian (6) can be diagonalized by the standard Bogolyubov uv transformation (see Ref. 4), $c_k = u_k a_k + v_k^* a_{-k}^+$. In terms of c_k and $c_{k\perp}$ the Hamiltonian H_0 takes the form

$$H_0=\sum_k \varepsilon_k c_k^+ c_k+\sum_{k_\perp}\varepsilon_{k_\perp}c_{k_\perp}^+ c_{k_\perp}, \tag{7}$$

where $\varepsilon_k = (A_k^2 - B_k^2)^{1/2}$, i.e.,

$$\varepsilon_k=\varepsilon_0[(1+x_0^2\mathbf{k}^2)(1+\rho/\beta+x_0^2\mathbf{k}^2)]^{1/2},$$
$$\varepsilon_{k\perp}=\hbar v_0|\mathbf{k}_\perp|(1+\beta x_0^2\mathbf{k}_\perp^2/\rho)^{1/2},\quad v_0=2\mu_0 M_0(\alpha\rho)^{1/2}/\hbar.$$

Here c_k and $c_{k\perp}$ are the operators of the volume and surface magnons.

In terms of c_k and $c_{k\perp}$, H_1 takes the form of two terms that describe the creation and annihilation of one surface (s) and one volume (v) magnon:

$$H_1=\sum_{\varkappa}\frac{1}{\Omega_x^{1/2}}U_s(\varkappa)\exp(-i\varkappa_x vt)c^+_{\varkappa_\perp}+\frac{1}{\Omega_x}\sum_{\varkappa,q}U_v(\varkappa,q)e^{-iqvt}c_k^+ +\text{H.c.},\tag{8}$$

where

$$\mathbf{k}=\varkappa_\perp+k_x\mathbf{e}_x=\varkappa+q\mathbf{e}_x,\quad q=k_x-\varkappa_x,\quad U_s=M_0(\tfrac{1}{2}\mu_0 x_0 M_0)^{1/2}\Psi,\tag{9}$$

$$\Psi_s(\varkappa)=\frac{\varkappa}{\operatorname{sh}(\pi\varkappa/2)}\Big\{u_{zz}\lambda\varkappa(u-v)+u_{xx}[-\lambda_1(u+v)\sin 2\varphi$$
$$+(\lambda_3-\lambda_1\sin^2\varphi)\varkappa(u-v)]+u_{yy}[\lambda_1(u+v)\sin 2\varphi+(\lambda_3-\lambda_1\cos^2\varphi)\varkappa(u-v)]$$
$$+2u_{xy}\lambda_1[(u+v)\cos 2\varphi+\varkappa(u-v)\sin 2\varphi]\Big\}-(2\lambda_1+\lambda_2)\frac{i\varkappa}{\operatorname{ch}(\pi\varkappa/2)}$$
$$\times\{u_{yz}[(u+v)\sin\varphi-\varkappa(u-v)\cos\varphi]+u_{xz}[(u+v)\cos\varphi+\varkappa(u-v)\sin\varphi]\}.$$

Here $\varkappa = \varkappa_x x_0$; u and v are the coefficients of the uv transformation: $u_{ik} = u_{ik}(\varkappa)$ are the Fourier components of the tensor of the strain produced by the defects:

$$u_{ik}(\mathbf{r})=\frac{1}{\Omega^{1/2}}\sum_{\varkappa}u_{ik}(\varkappa)e^{i\varkappa\mathbf{r}}.$$

The radiation amplitude $U_v(\varkappa, q)$ of the volume magnon is similar in structure

$$\Psi_v(\varkappa,q)\sim\lambda u_{ik}(\varkappa)\begin{cases}1, & \varkappa_x x_0\leqslant 1\\ \exp(-\pi\varkappa_x x_0), & \varkappa_x x_0\gg 1.\end{cases}$$

The Hamiltonian H_1 leads to the appearance of inelastic transitions in the magnon system, so that the energy of the moving DW is transferred to the magnon thermostat. The damping force $F(v)$ per unit area of the DW is determined by the energy dissipation rate $\dot{Q}$, namely $F(v) = \dot{Q}/Sv$, where S is the DW area. In second-order perturbation theory in H_1 we have $\dot{Q} = \dot{Q}_s + \dot{Q}_v$, and these terms are governed by processes in which one surface and volume magnon take part, respectively. For $F(v) = F_s(v) + F_v(v)$ we easily obtain

$$F_s=(2\pi/\Omega)\sum_{\varkappa}\varkappa_x\overline{|U_s(\varkappa)|^2}\delta(\varepsilon_{\varkappa_\perp}-\varkappa_x v),\tag{10a}$$

$$F_v=(2\pi/\Omega\Omega_x)\sum_{\varkappa,q}q\overline{|U_v(\varkappa,q)|^2}\delta(\varepsilon_{\varkappa+q}-qv).\tag{10b}$$

The superior bar in Eqs. (10) denotes averaging over the system of defects. An important property of the single-magnon radiation processes is that their contribution to the damping force does not contain the magnon occupation numbers, and is therefore independent of temperature and can assume a leading role at low temperatures.

2. INHOMOGENEITY PRODUCED BY DISLOCATION SYSTEM

We assume that dislocation loops are randomly distributed in the crystal, with average radius R and at a distance d. The averaging is carried out as in Ref. 7, and it is assumed that $d \gg R$ and $d \gg x_0$.

The functions $\overline{|U|^2}$ are quite complicated, including powers of $\varkappa_i$ and Bessel functions of argument $\varkappa R$. Noting, however, that only small $\varkappa_\perp$ contribute to (10a) at low velocities, and that the main contribution is always made by $\varkappa \lesssim 1/x_0$, we replace the Bessel functions in the calculation of $F_s(v)$ by their asymptotic forms, assuming that $\varkappa R \ll 1$ at $R \ll x_0$ and $\varkappa R \gg 1$ at $R \gg x_0$. In the upshot we get[1)]

$$F_s=F_0+Dv^2=\frac{\pi(\mu_0 M_0 b)^2 sRc^{2/3}}{3\varepsilon_0 a^3}\left[\Lambda^2+L\left(\frac{v}{v_W}\right)^2\right],\tag{11}$$

where in the case when $R \gg x_0$

$$\Lambda^2=\Lambda_0^2=8\lambda_1^2+16[2\lambda_3(\gamma^2-2)-\lambda_2-\lambda_4]^2+2(\lambda_2+\lambda_4)^2+(2\lambda_1+\lambda_2)^2,$$
$$L=-0.2\{16[4\lambda_3(\gamma^2-2)-\lambda_2-\lambda_4]^2+8(2\lambda_1+\lambda_2+\lambda_4)^2\tag{12}$$
$$+7(2\lambda_1+\lambda_2)^2\}+4\lambda_1^2+2\lambda_1\lambda_3[(8\gamma^2-13)^2-11]$$
$$+(2\lambda_1+\lambda_2)^2+8\lambda_1(\lambda_2+\lambda_4)(5-4\gamma^2),$$

and at $R \ll x_0$

$$\Lambda^2=\zeta(5)(R/x_0)^3[24\Lambda_0^2+21(2\lambda_1+\lambda_2)^2],$$
$$L=-(63/2\pi^2)\zeta(7)(R/x_0)^2\{128[4\lambda_3(\gamma^2-2)-\lambda_2-\lambda_4]^2\tag{13}$$
$$+64(2\lambda_1+\lambda_2+\lambda_4)^2+63(2\lambda_1+\lambda_2)^2\}$$
$$+\zeta(5)(R/x_0)^3\{96\lambda_1^2+48\lambda_1\lambda_3[(8\gamma^2-13)^2-11]$$
$$+45(2\lambda_1+\lambda_2)^2+192\lambda_1(\lambda_2+\lambda_4)(5-4\gamma^2)\}.$$

Here b is the mean value of the Burgers vector, $c = (N/\Omega)^{2/3}$ is the dislocation density, N is the number of dislocation loops, $\gamma^2 = \tilde{\mu}/(2\tilde{\mu} + \tilde{\eta})$, $\tilde{\mu}$ and $\tilde{\eta}$ are Lamé coefficients, and s is the spin of the atom. The calculated values of $F(v)$ are presented in the first-order approximation in the parameter ρ/β.

The first term of (11) determines the static friction force, i.e., $\dot{Q}_0 = F_0 Sv$ determines the rate of energy dissipation in the case of infinitely slow motion of the DW. It is not clear from general consideration how $\dot{Q}$ varies with increasing velocity; it can only be stated that $\dot{Q} > 0$ at all values of the velocity, but there are no premises that would yield the sign of D, which can be either positive or negative, depending on the values and signs of λ_i. It is seen from (12) and (13) that as a rule $D < 0$ (e.g., at $\lambda_1 \gg \lambda_2$ to λ_4). This leads to a decrease of $F_s(v)$ with increasing v, a result whose consequences will be discussed below.

We proceed to the calculation of the DW dragging force that results from emission of a volume magnon. It follows from the conservation law in (10b) that a contribution to the dragging is made only by $q \sim \omega_0/v \gg 1/x_0$, but at the same time $|\varkappa_\perp| \ll 1/x_0$. If $x_0 \ll R$ we must use for the Bessel function only its asymptotic value at $\varkappa R \gg 1$. If, however, $x_0 \gg R$, two velocity regions appear: the asymptotic $\varkappa R \gg 1$ is valid as before at $v \ll R\omega_0$, but at $\omega_0 R \ll v \ll x_0\omega_0$ the asymptotic $\varkappa R \ll 1$ is valid.

Changing from summation over the wave vectors to integration, and calculating F_v with allowance for the inequalities indicated above, we obtain

$$F_v=\frac{2\pi(\mu_0 bM_0)^2 sRc^{3/2}}{\varepsilon_0 a^3}\left(\frac{\omega_0 x_0}{v}\right)^{3/2}\exp\left(-\frac{\pi\omega_0 x_0}{v}\right)\times\begin{cases}\Lambda_1^2, & v\ll\omega_0\max(R,x_0)\\ \Lambda_2^2(R/x_0)^3(\omega_0 x_0/v)^3, & R\omega_0\ll v\ll\omega_0 x_0\end{cases}.\tag{14}$$

The effective constants Λ_i^2 are of the order of λ^2 and are too unwieldy to write out here. The function $F_v(v)$ is not analytic as $v \to 0$, with $F_v/v \to 0$ as $v \to 0$, i.e., emission of a volume magnon does not contribute to either the static friction force or the mobility of the DW. It will be shown below that this property (nonanalytic behavior of the damping force as a function of velocity) is typical of the emission of any number of quasiparticles, provided that several (n) of them have activation; in this case

$$F \sim (\omega_0 x_0/v)^{1/n} \exp\{-n\omega_0 x_0/v\}.$$

The contribution of the emission of several magnons to the damping force increases with temperature, but is less than $F_s(v)$ at all values of the temperature $T < T_c$.

It can thus be concluded that in the entire range of temperatures $T < T_c$ and velocities $v \lesssim v_W$ the effect of the dislocations on the dynamic damping of DW is due to the contribution made to the damping force by the emission of one surface magnon.

3. POINT DEFECTS

The contributions of defects of other types can be analogously investigated if their role reduces to the production of an inhomogeneous strain by virtue of the damping. The main regularities remain the same as before, namely, the damping force that is determined by the emission of one surface magnon can be represented in the form (11), in which it is necessary to make the substitution

$$(\Lambda^2, L)\, b^2 R c^{3/2} \to (\bar{\lambda}^2, l)\, (v_0/x_0)^2 c_{imp}, \tag{15}$$

where c_{imp} is the number of inhomogeneities per unit volume, v_0 is the average volume of the inhomogeneity (we have assumed that $v_0 \ll x_0^3$), $\bar{\lambda}^2 \sim \lambda_i^2 > 0$, $l \sim \lambda_i^2$, and the sign of the constant l can be arbitrary. Is the inhomogeneity is produced by impurities, we have $v_0 \sim a^3$, where a is the lattice constant.

This means that dislocation loops having an average radius R and a density c produce the same effect as impurities with density c_{imp}^{ef}, where the equivalent value c_{imp}^{ef} is defined by

$$c_{imp}^{ef} = c^{3/2} \begin{cases} (R x_0^3/a^4), & R \gg x_0, \\ (R/a)^4, & R \ll x_0. \end{cases} \tag{16}$$

It follows from the equation for c_{imp}^{ef} that the dislocations make on the whole a larger contribution to the damping force than the impurities. For example, if $R \sim x_0 \sim 10^{-5}$ cm, then one dislocation loop is equivalent to 10^{12} impurity atoms. A much larger contribution, as seen from (15), can be made by macroscopic inhomogeneities such as inclusions of another phase.

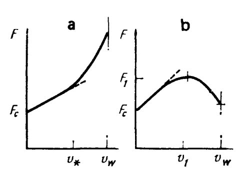

FIG. 1. Damping force vs velocity: a) at $D > 0$, b) at $D < 0$, F_c is the coercive force.

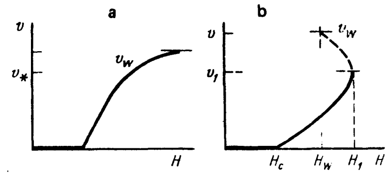

FIG. 2. Induced DW velocity vs external field: a) at $D > 0$, b) at $D < 0$. The dashed lines indicate the unstable section of $v(H)$, corresponding to a negative differential mobility.

4. CHARACTER OF INDUCED MOTION OF THE WALL IN THE PRESENCE OF INHOMOGENEITIES

We compare now the contributions of the defects to the dynamic damping of DW with the damping force due to two- or three-magnon terms, a force present at $T \neq 0$ in an ideal crystal and having at low velocities the form $F = B(T)v$ (Refs. 2 and 8). Since $B_2 \propto \exp(-\varepsilon_0)/T$ and $B_3 \propto \exp(-2\varepsilon_0/T)$ as $T \to 0$, we have $F_{2,3} \ll F_s$ at low temperature and at finite domain wall velocity, since F_s is independent of temperature [see (11)]. Let us compare the contribution of the inhomogeneities F_s with F_3 at[2)] $T \gg \varepsilon_0$. Using Eq. (43) of Ref. 2 we find that $B_3(T)v < |D| v^2$ if $v < v_*(T)$. Assuming $\beta \sim \rho \sim 1$, we obtain

$$v_*(T) = \frac{B(T)}{|D|} = \frac{v_W}{9(2\pi)^3} \left(\frac{\hbar v_W}{\lambda \mu_0 M_0 s x_0}\right) \frac{a^3}{b^2 c^{3/2} R x_0^3} \frac{T^2 \ln^2(T/\varepsilon_0)}{T_c^{3/2} \varepsilon_0^{1/2}}. \tag{17}$$

Putting $\varepsilon_0 \sim 0.3$ K, $T_c \sim 10^3$ K, $a \sim b \sim 10^{-8}$ cm, $R \sim 10^{-4}$ cm, and $x_0 \sim 10^{-5}$ cm we obtain $v_*(T) < u_W$ at $T < T_*$ $\approx 10^{-5} T_c [c]^{3/4}$, where $[c]$ is the dislocation density in cm^{-2}. It is seen that at $c = 10^8$ cm^{-2} we have $v_*(T) < v_W$ at all $T < T_c$. If, e.g., $c \sim 10^4$ cm^{-2}, a value typical of very pure samples, then $T_* \sim 10^{-2} T_c \sim 10$ K, i.e., the contribution of dislocation to DW damping is important at low temperatures even in pure samples.

We discuss now the influence of defects on the dependence of the DW velocity on the driving force (the external field H_z). Equating the magnetic pressure on the DW to the damping force (see Fig. 1) we get

$$2M_0 H = F_0 + B(T)v + Dv^2. \tag{18}$$

It is seen that motion takes place at low DW velocities if H exceeds the coercive field $H_c = F_0/2M_0$. We consider now the laws of motion at a velocity that is not low.

A graphic solution of Eq. (18) is shown in Fig. 2. The form of the function $v(H)$ depends on the sign of D. If $D > 0$, allowance for the inhomogeneities leads to a nonlinear $v(H)$ dependence at $v > v_*$ [see Fig. 2(a)], and then $v = (2M_0/H/D)^{1/2}$. A $v \sim H^{1/2}$ dependence was observed in Ref. 10. As already noted, the situation $D < 0$ is no less probable. In this case it is easy to verify that at $v > v_1(T) = v_*(T)/2$ the DW motion is characterized by a negative differential mobility [see Fig. 1(b)]. The $v(H)$ dependence then becomes non-single-valued. [See Fig. 2(b).] The upper branch of $v(H)$ corresponds to stable motion of the DW, and the Walker limit w_W is not reached. The role of the limiting velocity of the stationary motion is assumed by $v_1(T)$, which depends strongly on the temperature, has nothing in common with v_W, and is determined by the dislocation density and by the EW mobility.

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1)Analysis shows that the result $F_s = F_0 + Dv^2$ is general and is not connected with the defect model chosen here. An insufficiently consistent analysis (e.g., without allowance for the change of the DW structure) can yield the formula $F_s = F_0 + Bv$.

2)The DW mobility is determined by B_2 or B_3, depending on the crystal parameters.[8] We shall assume $B_2 < B_3$; this is typical, e.g., of ferrite films with large so-called quality factor Q (see Ref. 9).

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