Possibility of radiative polarization of high-energy electron-positron beams in curved single crystals

V. L. Mikhalev and R. A. Rzaev

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We present an explicit form of the equation of motion of the spin vector in an electromagnetic field in the quasiclassical approximation with inclusion of radiative damping in any order in the parameter $\chi = (\hbar/mc)(|\mathbf{w}|/c^2)\gamma^2$, where $\gamma = \varepsilon/mc^2$; $\mathbf{w}$ and ε are the acceleration and energy of the particle. For particles captured in planar channeling in traversing a slightly curved single crystal we find the solution of this equation in the approximation $\chi \ll 1$. Averaging over the levels of the transverse energy is carried out for a thin crystal. The influence of the angular divergence of the beam on its polarization is estimated. We demonstrate the possibility of obtaining polarized beams of electrons (degree of polarization $\approx 40\%$) and positrons (degree of polarization $\approx 65\%$). Estimates are made of the degree of polarization of the beams for the case $\chi \gg 1$.

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1. It is well known that in contemporary proton accelerators electron and positron beams are produced in the energy region inaccessible for existing electron accelerators. The possibility of polarization of these beams would permit substantial extension of the set of experimental investigations which can be performed.

The present work is devoted to discussion of one of the possible methods of obtaining polarized electrons or positrons of high energy, which was first mentioned in Ref. 1: radiative polarization in traversal of curved single crystals by electrons in the planar channeling mode. Effects which influence the establishment of polarization of the beams are considered.

We shall start from the equation of motion of the spin vector in electromagnetic fields, which we shall write for an arbitrary value of the quantum parameter of the radiation

$$\chi = \frac{\hbar}{mc}\frac{|\mathbf{w}|}{c^2}\gamma^2,$$

where $\gamma = \varepsilon/mc^2$; $\mathbf{w}$ and ε are the acceleration and energy of the particle. The necessity of such a discussion is due to the fact that for a number of crystals, as will be shown below, a value $\chi \gtrsim 1$ is reached already in the energy region of several hundred GeV.

Using the quasiclassical method,[2] this equation can be written in the form

$$\frac{d\boldsymbol{\zeta}}{dt} = \frac{ce}{\varepsilon}\left[\boldsymbol{\zeta}\times\left(\frac{\mu'}{\mu}\mathbf{H}_R + \mathbf{H}_E\right)\right] + \mathbf{G}. \qquad (1)$$

Here $\boldsymbol{\zeta}$ is the average value of the spin operator in the rest system of the particle;

$$\mathbf{H}_E = \mathbf{H} + \frac{1}{1+1/\gamma}\left[\mathbf{E}\times\frac{\mathbf{v}}{c}\right]$$

is the effective field acting on the inherent magnetic moment of the particle;

$$\mathbf{H}_R = \gamma\left\{\mathbf{H} - \frac{\mathbf{v}}{c}\left(\frac{\mathbf{v}}{c}\mathbf{H}\right)\frac{1}{1+1/\gamma} + \left[\mathbf{E}\times\frac{\mathbf{v}}{c}\right]\right\}$$

is the effective field acting on the anomalous part of the magnetic moment μ'; $\mathbf{v}$ is the particle velocity and ε is its energy; $\mathbf{H}$ and $\mathbf{E}$ are the magnetic and electric field strengths.

The first term in the right-hand side of Eq. (1) describes the precession of the spin vector in electromagnetic fields with allowance for the appearance in the particle of an anomalous magnetic moment (as a consequence of the interaction with the radiation field). Here the magnitude of the field must satisfy the condition $H/H_0 \ll 1$ ($E/E_0 \ll 1$), where $H_0 = E_0 = m^2c^3/e\hbar = 4.41\cdot 10^{13}$ Oe $= 1.32\cdot 10^{6}$ V/m, which for interatomic crystal fields is satisfied quite well.

The second term in the right-hand side of Eq. (1), which we have denoted by G, determines the change of $\boldsymbol{\zeta}$ as a consequence of radiation events with spin flip. Using the quasiclassical method of Ref. 2, we can show that $\mathbf{G}\boldsymbol{\zeta} = -2W^{\zeta}$, where W^{ζ} is the probability per unit of a radiative transition with spin flip. The expression for W^{ζ} which is exact in χ has the form

$$W^{\zeta} = \frac{\alpha m^2c^4}{2\cdot 3^{1/2}\pi\hbar\varepsilon}\int_0^\infty \frac{u^2\,du}{(1+u)^3}\left\{\left[1-\left(\zeta\frac{\mathbf{v}}{c}\right)^2\right]K_{2/3}\left(\frac{2u}{3\chi}\right) + \left(\zeta\frac{\mathbf{v}}{c}\right)^2\int_{2u/3\chi}^{\infty} K_{1/3}(x)\,dx + \left(\zeta\left[\frac{\mathbf{v}}{c}\times\mathbf{s}\right]\right)K_{1/3}(2u/3\chi),\right.$$

where $\alpha = e^2/\hbar c$, $\mathbf{s} = \mathbf{w}/|\mathbf{w}|$ and $K_\nu(x)$ is the MacDonald function. From this for arbitrary χ we obtain for $\mathbf{G}$

$$\mathbf{G} = -\frac{9\cdot 3^{1/2}\alpha m^2c^4}{8\pi\hbar\varepsilon}\chi^3\left\{\boldsymbol{\zeta}A(\chi) - \frac{\mathbf{v}}{c}\left(\boldsymbol{\zeta}\frac{\mathbf{v}}{c}\right)B(\chi) + \left[\frac{\mathbf{v}}{c}\times\mathbf{s}\right]C(\chi)\right\},$$

$$A(\chi) = \int_0^\infty \frac{y^2}{(1+{}^3/_2\chi y)^3}K_{2/3}(y)\,dy, \quad C(\chi) = \int_0^\infty \frac{y^2}{(1+{}^3/_2\chi y)^3}K_{1/3}(y)\,dy, \qquad (2)$$

$$B(\chi) = A(\chi) - \int_0^\infty \frac{y^2}{(1+{}^3/_2\chi y)^3}\left[\int_y^\infty K_{1/3}(x)\,dx\right]dy.$$

In the case $\chi \ll 1$, assuming

$$A(\chi) \approx A(0) = 5\pi/9,\ B(\chi) \approx B(0) = 10\pi/81,\ C(\chi) \approx C(0) = 8\cdot 3^{1/2}\pi/27,$$

we arrive at the well known equation of motion of the spin vector. In the limiting case $\chi \gg 1$ it is necessary to expand the function $K_\nu(y)$ in series in y, taking into account that in a substantial region of the integration

$y \sim 1/\chi \ll 1$. Retaining the leading terms in powers of $1/\chi$, we obtain

$$A(\chi) \approx \frac{16\pi}{81 \cdot 3^{1/6}} \Gamma\left(\frac{2}{3}\right) \frac{1}{\chi^{2/3}}, \quad B(\chi) \approx A(\chi) - \frac{8\pi \ln \chi}{27 \cdot 3^{1/2} \chi^{3}},$$
$$C(\chi) \approx \frac{40\pi}{243 \cdot 3^{1/6}} \Gamma\left(\frac{1}{3}\right) \frac{1}{\chi^{4/3}}.$$

2. Let us turn to discussion of the characteristics of a beam of particles captured in the planar channeling mode.

The energy of the transverse motion $\varepsilon_\perp$ of a particle in an interplanar potential $U(x)$ is given by the expression

$$\varepsilon_\perp = U(x) + \varepsilon\theta^2/2,$$

where θ is the angle between the direction of motion of the particle and the crystal planes, from which we obtain for the critical channeling angle the estimate

$$\theta_c\,[\mathrm{rad}] \approx (2U_0/\varepsilon)^{1/2} \approx 4.47 \cdot 10^{-5} (U_0\,[\mathrm{eV}]/\varepsilon[\mathrm{GeV}])^{1/2},$$

where U_0 is the depth of the potential well ($U_0 \sim 10$–100 eV).

A quasiclassical evaluation of the number of levels in the potential well gives

$$N \sim \left(\frac{2\varepsilon U_0}{c^2}\right)^{1/2} \frac{d}{2\pi\hbar} \gtrsim \gamma^{1/2} \gg 1,$$

where $d \approx 1$–3 Å is the interplanar distance. This demonstrates the validity of a classical discussion of the motion of particles in a channel.

Choice of a specific form of the interplanar potential for electrons (e^-)

$$U(x) = U_0\left[1 - \left(1 - \left|\frac{x}{d/2}\right|\right)^2\right]$$

and for positrons (e^+)

$$U(x) = U_0\left(\frac{x}{d/2}\right)^2,$$

where $-d/2 \leqslant x \leqslant d/2$, is justified by the following considerations. The region of energy which is interesting for realization of the radiative polarization effect in interaction of a beam of particles with curved crystals, as will be shown below, begins with energies several tens of GeV. At these energies the synchrotron radiation conditions ($\theta_c \gg 1/\gamma$) is well satisfied and, in contrast to dipole radiation, this depends only slightly on the specific structure of the potential. Defining the quantity

$$\rho_0 \equiv (\theta_c\gamma)^2 = 2U_0\varepsilon/m^2c^4 \approx 8 \cdot 10^{-3} U_0\,[\mathrm{eV}]\,\varepsilon[\mathrm{GeV}],$$

we can write the synchrotron radiation condition in the form $\rho_0 \gg 1$. For example, for crystals of Si and Au the condition $\rho_0 \gtrsim 1$ is satisfied beginning at energies 4.2 and 1.4 GeV, respectively. Accordingly it is also not necessary to consider the influence of thermal and zero-point vibrations of the atoms of the lattice on the form of the interplanar potential. An estimate of the magnitude of these effects can be found in Refs. 3 and 4: $\max(\Delta U/U_0) \sim 10\%$.

The dynamic characteristics of the beam during its interaction with the crystal will depend substantially on the nature of the distribution of the particles in the transverse energy levels. We shall find the form of the distribution function for two cases:

1) the beam on entry has a uniform distribution in the angle θ and the coordinate x;

2) a beam without angular divergence is uniformly distributed over the coordinate and is parallel to the crystal planes on entry.

Comparison of these two cases will permit us also to evaluate the influence of the angular divergence of the beam on the effect of interest here.

The uniform distribution of the beam over the coordinate x is determined by the fact that, generally speaking, we cannot trace the coordinate of the particle (the beam width is significantly greater than the interplanar distance). Usually the width of the angular distribution of the beam is $(\overline{\theta^2})^{1/2} \gg \theta_c$, and therefore as a rule we have the case 1. In what follows we shall consider only rather thin crystals whose length does not exceed the characteristic dechanneling lengths, and the distribution over the transverse energy levels is determined by the conditions of entry into the crystal. Using the specific form of the potentials for e^+ and e^-, we obtain the respective distributions in $\varepsilon_\perp$: in case 1

$$dV_{e^-} = \ln\left[\frac{1 + (\varepsilon_\perp/U_0)^{1/2}}{(1 - \varepsilon_\perp/U_0)^{1/2}}\right] d\left(\frac{\varepsilon_\perp}{U_0}\right), \quad dV_{e^+} = d\left(\frac{\varepsilon_\perp}{U_0}\right); \tag{3a}$$

in case 2

$$dV_{e^-} = \frac{1}{2} \frac{1}{(1 - \varepsilon_\perp/U_0)^{1/2}} d\left(\frac{\varepsilon_\perp}{U_0}\right), \quad dV_{e^+} = \frac{1}{2}\left(\frac{\varepsilon_\perp}{U_0}\right)^{-1/2} d\left(\frac{\varepsilon_\perp}{U_0}\right). \tag{3b}$$

The dependence of $\varepsilon_\perp$ on the level number n is given by the expressions

$$\left(\frac{\varepsilon_\perp}{U_0}\right)_{e^-} \approx \left[\frac{2\pi\hbar c(n + 1/2)}{d}\right]^{1/2} \frac{1}{(2\varepsilon U_0)^{1/4}}, \quad \left(\frac{\varepsilon_\perp}{U_0}\right)_{e^+} \approx \frac{2\pi\hbar c(n + 1/2)}{d(2U_0\varepsilon)^{1/2}}. \tag{4}$$

On the basis of solutions of the equations of motion in the selected potentials we shall find the minimal crystal bending radius $R_{\min}$ at which motion of particles in the planar channeling mode is still possible. From the condition $c^2/R_{\min} = \max \ddot{x}$ we have

$$R_{\min\, e^-}\,[\mathrm{cm}] = R_{\min\, e^+} = R_{\min} = \frac{d}{4} \frac{\varepsilon}{U_0} \approx 2.5d\ [\text{Å}]\, \frac{\varepsilon\,[\mathrm{GeV}]}{U_0\,[\mathrm{eV}]}.$$

However, on approach to $R_{\min}$ the number of particles captured in the channeling mode will fall off. Carrying out the appropriate calculations in the approximation $R_{\min}/R \ll 1$, we obtain for the efficiency F of capture into a channel in case 1:

$$F_{e^-} \approx [1 - (\ln 2) R_{\min}/R], \quad F_{e^+} \approx 1 - 2R_{\min}/R;$$

in case 2 we obtain

$$F_{e^-} \approx 1 - (2R_{\min}/R)^{1/2}, \quad F_{e^+} \approx 1 - R_{\min}/R.$$

It follows from this that the discussion of the problem is meaningful only for the condition

$$R_{\min}/R \ll 1. \tag{5}$$

On the basis of the chosen potentials and the equations (3) we shall find the average over the trajectory and over the particle distribution in transverse energy of the parameter $\langle\chi\rangle$ in case 1:

$$\langle\bar\chi\rangle_{e^-}=\frac{\pi}{2}\langle\bar\chi\rangle_{e^+}\approx\frac{2\lambda_c}{3R_{min}}\gamma^2\approx4{,}2\cdot10^{-5}\varepsilon\,[\text{GeV}]\,\frac{U\,[\text{eV}]}{d[\text{Å}]},\quad \lambda_c=\frac{\hbar}{mc}.$$

The bar over the χ denotes averaging over the particle trajectory for a fixed value of $\varepsilon_\perp$, and the symbol $\langle\ldots\rangle$ denotes averaging over the levels of the transverse energy.

3. The polarization which arises in transverse and is directed along the vector $\mathbf{s}\times\mathbf{v}/c$, along which we also shall direct the z axis. The equation for ζ_z takes the form

$$\frac{d\zeta_z}{dt}=-\frac{9\cdot3^{1/2}\alpha m^2c^4}{8\pi\hbar\varepsilon}\chi^3\left\{\zeta_zA(\chi)+\left[\frac{\mathbf{v}}{c}\times\mathbf{s}\right]_zC(\chi)\right\}. \tag{6}$$

The solution of this equation is the function

$$\zeta_z=\zeta_z^0\exp\left\{-\int_0^tA_1(\chi)\,dt'\right\}-\exp\left\{-\int_0^tA_1(\chi)\,dt'\right\}\times\left[\int_0^tC_1(\chi)\exp\left\{\int_0^{t'}A_1(\chi)\,dt''\right\}dt'\right], \tag{7}$$

$$A_1(\chi)=\frac{9\cdot3^{1/2}\alpha m^2c^4}{8\pi\hbar\varepsilon}\chi^3A(\chi),\quad C_1(\chi)=\frac{9\cdot3^{1/2}\alpha m^2c^4}{8\pi\hbar\varepsilon}\chi^3\left[\frac{\mathbf{v}}{c}\times\mathbf{s}\right]_zC(\chi),$$

where ζ_z^0 is the initial polarization.

The dependence $A_1(\chi)$, $C_1(\chi)$, and χ on the time enters through the acceleration $\mathbf{w}(t)$, which with inclusion of the condition $R_{min}/R\ll1$ we shall represent in the form of the sum

$$w(t)=c^2/R+a(t),$$

where $a(t)$ is the acceleration in a planar channel of an uncurved crystal. The condition (5) is equivalent to the condition $\langle|\overline{\mathbf{w}(t)}|\rangle\gg c^2/R$, and therefore over a substantial region of integration in equation (7) it is permissible to expand all functions in series in the small parameter $R_{min}/R\ll1$. Since the length of the crystal contains many periods of oscillation of the particle in the channel, in the integration we can use the method of averaging over the trajectory for a fixed value of $\varepsilon_\perp$. There is no dependence on the polarization characteristics of the beam on the initial phase. In calculation of the terms characterizing the rate of establishment of the polarization, a small correction ($\sim R_{min}/R$) will be dropped completely, and in the expression which determines the magnitude of the final, established polarization, only the first power of this correction will be retained.

Performing the concrete calculations, we obtain

$$\zeta_z(t)=\zeta_z^0\exp(-\bar Bt)+\chi\left(\frac{c^2}{R}\right)\frac{\bar D}{\bar B}[1-\exp(-\bar Bt)],$$

$$B(\chi)=\frac{9\cdot3^{1/2}\alpha m^2c^4\chi^3}{8\pi\hbar\varepsilon}\int_0^\infty\frac{y^2}{(1+{}^3/_2\chi y)^3}K_{2/3}(y)\,dy, \tag{8}$$

$$D(\chi)=\frac{27\cdot3^{1/2}\alpha m^2c^4\chi^2}{8\pi\hbar\varepsilon}\int_0^\infty\frac{y^2}{(1+{}^3/_2\chi y)^4}K_{1/3}(y)\,dy.$$

In the case $\chi\ll1$ we have

$$B(\chi)=\frac{5\cdot3^{1/2}\alpha m^2c^4}{8\hbar\varepsilon}\chi^3,\quad D(\chi)=\frac{3\alpha m^2c^4}{\hbar\varepsilon}\chi^2.$$

After averaging, for electrons we obtain

$$\zeta_z(t)=\zeta_z^0\exp\{-k_1tf_1(\varepsilon_\perp/U_0)\}+k_2f_2(\varepsilon_\perp/U_0)\,[1-\exp\{-k_1tf_1(\varepsilon_\perp/U_0)\}],$$

$$k_1=\frac{80\cdot3^{1/2}\alpha\hbar^2}{3m^3c^7}\left(\frac{U_0}{d}\right)^3\gamma^2,\quad k_2=\frac{6\cdot3^{1/2}R_{min}}{5R}, \tag{9}$$

$$f_1(x)=x^{1/2}(3-2x)/\ln\frac{1+x^{1/2}}{1-x^{1/2}},\ f_2(x)=\left[(1-x)\ln\frac{1+x^{1/2}}{1-x^{1/2}}+2x^{1/2}\right]\Big/x^{1/2}(3-2x).$$

For positrons we have correspondingly

$$\zeta_z(t)=\zeta_z^0\exp\{-k_1t(\varepsilon_\perp/U_0)^{1/2}\}+k_2(\varepsilon_\perp/U_0)^{-1/2}[1-\exp\{-k_1t(\varepsilon_\perp/U_0)^{1/2}\}],$$

$$k_1=\frac{160\,\alpha\hbar^2}{3^{1/2}\pi m^3c^7}\left(\frac{U_0}{d}\right)^3\gamma^2,\quad k_2=\frac{3^{1/2}\pi R_{min}}{5R}.$$

Finally we shall average the obtained formulas over the transverse energy levels with the appropriate distribution function (3). As a result we have

$$\zeta_z(t)=\zeta_z^0\varphi_1(k_1t)+\zeta_z^{est}\varphi_2(k_1t) \tag{10}$$

for the conditions $\varphi_1(0)=1$, $\varphi_2(0)=0$, $\varphi_1(\infty)=0$, $\varphi_2(\infty)=1$, where ζ_z^{est} is the magnitude of the established polarization of the beam. For electrons in case 1 we have

$$\varphi_1(k_1t)=\int_0^1\exp[-k_1tf_1(x)]\ln\left[\frac{1+x^{1/2}}{(1-x)^{1/2}}\right]dx,$$

$$\varphi_2(k_1t)=1-\left\{\int_0^1f_2(x)\ln\left[\frac{1+x^{1/2}}{(1-x)^{1/2}}\right]\exp[-k_1tf_1(x)]\,dx\right\}\Big/\int_0^1f_2(x)\times\ln\left[\frac{1+x^{1/2}}{(1-x)^{1/2}}\right]dx,$$

$$\zeta_z^{est}=\frac{6\cdot3^{1/2}R_{min}}{5R}\int_0^1f_2(x)\ln\left[\frac{1+x^{1/2}}{(1-x)^{1/2}}\right]dx$$

and correspondingly in case 2:

$$\varphi_1(k_1t)=\frac{1}{2}\int_0^1\frac{1}{(1-x)^{1/2}}\exp[-k_1tf_1(x)]\,dx,$$

$$\varphi_2(k_1t)=1-\left\{\int_0^1\frac{f_2(x)}{(1-x)^{1/2}}\exp[-k_1tf_1(x)]\,dx\right\}\Big/\int_0^1\frac{f_2(x)}{(1-x)^{1/2}}dx,$$

$$\zeta_z^{est}=\frac{6\cdot3^{1/2}R_{min}}{10R}\int_0^1\frac{f_2(x)}{(1-x)^{1/2}}dx.$$

The results of computer calculations for the functions $\varphi_1(x)$ and $\varphi_2(x)$ are shown in Fig. 1, and the values of ζ_z^{est} are as follows: in case 1

$$\zeta_z^{est}=1.779\frac{6\cdot3^{1/2}R_{min}}{5R} \tag{11a}$$

and in case 2

$$\zeta_z^{est}=1.780\frac{6\cdot3^{1/2}R_{min}}{5R}. \tag{11b}$$

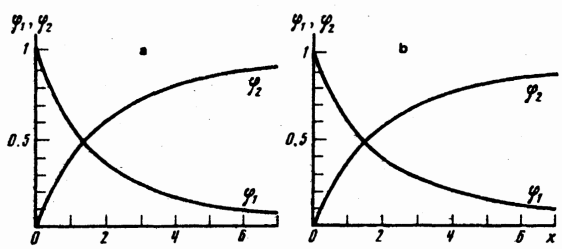

FIG. 1. Plots of functions characterizing the damping of the initial polarization $\varphi_1(x)$ of electrons and the establishment of their final polarization $\varphi_2(x)$: a—for the initial beam of case 1; b—for case 2.

For positrons in case 1 we have

$$\varphi_1(k_1t)={}^2/_3\gamma({}^2/_3,k_1t),\quad \varphi_2(k_1t)=1-\gamma({}^1/_3,k_1t)/3(k_1t)^{1/3},$$

$$\zeta_z^{\text{est}}=2k_2=\frac{6\cdot 3^{1/3}\pi R_{min}}{5R}, \tag{12}$$

where $\gamma(\alpha,x)$ is the incomplete Gamma function.[5]

In the corresponding calculation for case 2 we encounter a logarithmic divergence of several integrals at the lower limit, which is explained by the violation of the condition (5) in the limit $\varepsilon_\perp/U_0\to 0$. The condition $R_{\min}/R\ll 1$ is equivalent to the following inequality: $|\mathbf{w}(t)|\gg c^2/R$, or to the equivalent condition

$$\frac{\varepsilon_\perp}{U_0}\gg\frac{\pi^2}{4}\left(\frac{R_{min}}{R}\right)^2. \tag{13}$$

This leads to the result that for $\varepsilon_\perp/U_0\to 0$ the degree of polarization grows as $(\varepsilon_\perp/U_0)^{-1/2}$ [see Eq. (9)], while its limiting value cannot exceed the well known value $8/5\sqrt{3}\approx 0.92$. Therefore it is necessary to limit the rise of the polarization to this limiting value, i.e.,

$$k_2\left(\frac{\varepsilon_\perp}{U_0}\right)^{-1/2}\leqslant\frac{8}{5\cdot 3^{1/2}},\quad \frac{\varepsilon_\perp}{U_0}\geqslant\left(\frac{9\pi R_{min}}{8R}\right)^2.$$

Here the condition (13) also will be satisfied. As a result for positrons in case 2 we can assume

$$\varphi_1(k_1t)=\gamma({}^1/_3,k_1t)/3(k_1t)^{1/3},\quad \varphi_2(k_1t)\approx 1-\exp\{-k_1t(9\pi R_{min}/8R)^2\},$$

$$\zeta_z^{\text{est}}=\frac{3^{1/2}\pi R_{min}}{5R}\left[1-\ln\frac{9\pi R_{min}}{8R}\right]. \tag{14}$$

Plots of the functions $\varphi_1(x)$ and $\varphi_2(x)$ for positrons are shown in Fig. 2. We recall that all of the results are valid for thin crystals in which the distribution function of the particles over the transverse energy levels depends only the conditions of entry into the crystal.

4. Let us analyze the effects which influence the form of the distribution function and find the characteristic lengths of crystals for which this discussion is justified. References 6–8 are devoted to the problem of finding the distribution over the transverse energy levels in rather thick crystals. We shall restrict the discussion to two principal effects: radiative transitions between levels, and multiple scattering by atoms of the lattice (and also by atomic electrons).

The rate of change of $\varepsilon_\perp$ as the result of radiative transitions is related to the energy loss of the particle by the equation[9]

$$\frac{d\varepsilon}{dt}=\left(\frac{1}{\gamma^2}+\frac{q\varepsilon_\perp}{\varepsilon}\right)^{-1}\frac{d\varepsilon_\perp}{dt},$$

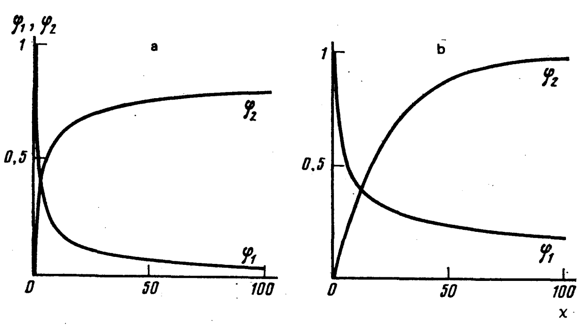

FIG. 2. The same as in Fig. 1 but for positrons.

where q is the exponent which determined the relation $\varepsilon_\perp\sim\varepsilon^{-q}$. In the case of planar channeling $q=1/4$ for e^- and $1/2$ for e^+. For the intensity of radiation $I=-d\varepsilon/dt$ we can use the classical expression averaged over the particle trajectory for a fixed value of $\varepsilon_\perp$. As a result we obtain the following equations for e^+ and e^- respectively:

$$\frac{d\varepsilon_\perp}{dt}=-\frac{16}{3}\left(\frac{1}{\gamma^2}+\frac{\varepsilon_\perp}{2\varepsilon}\right)\frac{e^2}{m^2c^3}\left(\frac{U_0}{d}\right)^2\gamma^2\frac{\varepsilon_\perp}{U_0},$$

$$\frac{d\varepsilon_\perp}{dt}=-\frac{16}{3}\left(\frac{1}{\gamma^2}+\frac{\varepsilon_\perp}{4\varepsilon}\right)\frac{e^2}{m^2c^3}\left(\frac{U_0}{d}\right)^2\gamma^2\left\{1-\frac{\varepsilon_\perp}{U_0}+2\left(\frac{\varepsilon_\perp}{U_0}\right)^{1/2}\Big/\ln\left[\frac{1+(\varepsilon_\perp/U_0)^{1/2}}{1-(\varepsilon_\perp/U_0)^{1/2}}\right]\right\}. \tag{15}$$

Proceeding from the expressions for $\langle d\varepsilon_\perp/dt\rangle$ let us determine the characteristic lengths of change of the distribution function in the transverse energy as the result of radiative transitions. In case 1 for electrons and positrons we have respectively:

$$l_{\text{rad}}^{e-}[\text{cm}]\approx\frac{3m^2c^4}{16e^2\gamma^2}\left(\frac{d}{U_0}\right)^2\left(\frac{1}{\gamma^2}+0.139\frac{U_0}{\varepsilon}\right)^{-1}\approx\frac{6.394\cdot 10^4}{(U_0\,[\text{eV}]/d[\text{Å}])^2\varepsilon\,[\text{GeV}]},$$

$$l_{\text{rad}}^{e+}[\text{cm}]\approx\frac{3m^2c^4}{8e^2\gamma^2}\left(\frac{d}{U_0}\right)^2\left(\frac{1}{\gamma^2}+0.333\frac{U_0}{\varepsilon}\right)^{-1}\approx\frac{5.328\cdot 10^4}{(U_0\,[\text{eV}]/d[\text{Å}])^2\varepsilon\,[\text{GeV}]}. \tag{16}$$

We shall give an estimate of the characteristic lengths as the result of multiple scattering by lattice atoms on the basis of the well known formula for the mean square multiple scattering angle in an amorphous material:

$$\overline{\theta_x^2}=\frac{\varepsilon_s^2 l}{\varepsilon^2 L_{\text{rad}}},$$

where $\varepsilon_s=14.85$ MeV, L_{rad} is the radiation length of the crystal material, and l is the path length in the crystal. The main fraction of the particles in channeling pass a substantial part of the time in the space between crystal planes under the influence of the interplanar potential. Multiple scattering by atoms occurs only on traversal of the rather thin layer containing the crystal plane. We shall take the thickness of the layer to be

$$a_c=(a_0^2+u_1^2)^{1/2},$$

where $a_0\approx 0.885a_BZ^{-1/3}$ is the Thomas-Fermi screening radius, $a_B=\hbar^2/me^2$, and u_1 is the amplitude of thermal vibrations of the lattice atoms. We shall assume that multiple scattering in the layer is described by the formula for $\overline{\theta_x^2}$ in an amorphous material. In the calculations it is necessary to take into account that the density of atoms in the layer is $d/2a_c$ times greater than the average density of atoms in the crystal. Performing the calculations with use of the equality $d\varepsilon_\perp/dl=(\varepsilon/2)d\overline{\theta_x^2}/dl$ and averaging over the trajectory and the transverse energy levels, in case 1 we obtain respectively for e^- and e^+:

$$l_a[\text{cm}]\approx 0.61\frac{\varepsilon U_0}{\varepsilon_s^2}L_{\text{rad}}\approx 2.77\cdot 10^{-6}U_0\,[\text{eV}]\,\varepsilon[\text{GeV}]L_{\text{rad}}[\text{cm}], \tag{17}$$

$$l_a[\text{cm}]\approx 1.18\left(\frac{d}{a_c}\right)^{1/2}\frac{U_0\varepsilon}{\varepsilon_s^2}L_{\text{rad}}\approx 5.35\cdot 10^{-6}\left(\frac{d}{a_c}\right)^{1/2}U_0\,[\text{eV}]\,\varepsilon[\text{GeV}]L_{\text{rad}}[\text{cm}].$$

Let us consider multiple scattering by atomic electrons. Taking the cross section for scattering by free electrons[10]

$$d\sigma=(8\pi e^4/\varepsilon^2\theta^3)\,d\theta,$$

we have for the mean square angle of multiple scattering

$$\frac{\overline{d\theta^2}}{dl}=\int_{\theta_{min}}^{\theta_{max}} n_e\theta^2 d\sigma(\theta)=n_e\frac{8\pi e^4}{\varepsilon^2}\ln\frac{\theta_{max}}{\theta_{min}},$$

where n_e is the density of electrons and the value of $\theta_{\max}$ is obviously determined by the limiting channeling angle: $\theta_{\max}=\theta_c=(2U_0/\varepsilon)^{1/2}$; $\theta_{\min}=\hbar\omega_p/\varepsilon$, and $\omega_p=4\pi n_e e^2/m$ is determined by the polarization of the electron gas.[11] Taking into account the influence only of the nearest crystal plane, for $n_e(x)$ we shall write

$$n_e(x)=2\pi Nd\int_0^\infty n_a(x^2+\rho^2)^{1/2}\rho d\rho,$$

where N is the concentration of atoms in the crystal and n_a is the density of atomic electrons. Carrying out the calculations with use of the Thomas-Fermi model, we find

$$n_e(x)=\left(\frac{4}{3\pi}\right)^{2/3} Z^{4/3}N\frac{d}{a_B}\left[-\frac{d\varkappa}{dt}\bigg|_{t=x/a_0}\right].$$

Here $\varkappa(t)$ is the universal Thomas-Fermi function.[12]

Substituting the function $n_e(x)$ into the formula for $d\theta^2/dl$ and averaging the resulting expression over the particle trajectory for a fixed value of $e_\perp$, we have

$$\frac{d(\varepsilon_\perp/U_0)}{dl}=\frac{4\pi e^4}{\varepsilon U_0}\left(\frac{4}{3\pi}\right)^{2/3}\frac{d}{a_B}\ln\frac{\theta_{max}}{\theta_{min}}\left[-\frac{d\varkappa}{dt}\bigg|_{t_0}\right]NZ^{4/3};$$

$$t_0=\frac{d}{2a_0}\left[1-\left(1-\frac{\varepsilon_\perp}{U_0}\right)^{1/2}\right]\quad\text{for } e^-,\qquad t_0=\frac{d}{2a_0}\left(\frac{\varepsilon_\perp}{U_0}\right)^{1/2}\quad\text{for } e^+.$$

Then, averaging this expression over the transverse energy levels with the appropriate distribution function, we obtain for the characteristic length due to multiple scattering by atomic electrons

$$l_{el}\approx\left(\frac{3\pi}{4}\right)^{2/3}\frac{\varepsilon U_0 a_B}{4\pi e^4 NZ^{4/3}d}\left[\ln\frac{\theta_{max}}{\theta_{min}}\right]^{-1}\left\langle-\frac{d\varkappa}{dt}\bigg|_{t_0}\right\rangle^{-1}$$
$$\approx 2\cdot10^{21}\frac{U_0[\text{eV}]\,\varepsilon\,[\text{GeV}]}{NZ^{4/3}d[\text{Å}]}\left\langle-\frac{d\varkappa}{dt}\right\rangle^{-1}. \tag{18}$$

With increase of Z the quantity $\langle d\varkappa/dt\rangle$ varies in the range from -0.2 to -0.1 for electrons and from -0.08 to -0.04 for positrons.

5. Let us consider the problem in the limiting case $\chi\gg1$. As is well known, in this case the radiative polarization falls off with increase of χ as $\chi^{-1/3}$. As a result we shall not search for a solution of Eq. (8), for which it is necessary to use expressions for $B(\chi)$ and $D(\chi)$ which are exact in χ, but shall make an asymptotic estimate of the results. For sufficiently large values of χ, where the principal fraction of particles channeled in a curved planar channel of a crystal spend a significant part of their time in the region $\chi\gg1$, we can use approximate expressions for the functions $B(\chi)$ and $D(\chi)$ for large values of the argument:

$$B(t)\approx\frac{2\Gamma(2/3)\alpha m^{2/3}}{9\cdot3^{1/3}\hbar^{1/3}}\gamma^{2/3}|\mathbf{w}|^{2/3},$$
$$D(t)\approx\frac{5\cdot3^{2/3}\Gamma(1/3)\alpha m^{4/3}c^4}{81\hbar^{4/3}}\gamma^{-1/3}|\mathbf{w}|^{-2/3},$$

from which according to Eq. (8) for the final polarization value and correspondingly for the parameter characterizing the rate of establishment of the polarization we obtain

$$\zeta_z^{\text{est}}\approx\frac{5\cdot3^{2/3}\Gamma(1/3)m^{2/3}c^3}{18\cdot\Gamma(2/3)\hbar^{2/3}\gamma^{2/3}R}\overline{|\mathbf{w}|^{-2/3}}\left(\overline{|\mathbf{w}|^{4/3}}\right)^{-1}$$
$$\approx\frac{5\cdot3^{2/3}\Gamma(1/3)}{18\Gamma(2/3)}k^{-4/3}\gamma^{-2/3}\left(\frac{R_{min}}{\lambda_c}\right)^{2/3}\frac{R_{min}}{R} \tag{19}$$
$$\approx29.34d^{2/3}[\text{Å}]\,\varepsilon^{2/3}(\text{GeV})\,k^{-4/3}U_0^{-2/3}[\text{eV}]\,\gamma^{-2/3}R_{min}/R\sim\gamma^{-2/3};$$
$$\frac{R_{min}}{R}=\text{const},\qquad \bar{B}t=\frac{2\Gamma(2/3)k^{2/3}\alpha\gamma^{2/3}l}{9\cdot3^{1/3}\lambda_c^{2/3}R_{min}^{2/3}}\approx30.6\left(\frac{U_0[\text{eV}]}{d[\text{Å}]}\right)^{2/3}\frac{k^{2/3}}{\varepsilon^{1/3}[\text{GeV}]}l[\text{cm}].$$

Here we have made use of the expression $|\bar{\mathbf{w}}|=kc^2/R_{\min}$, where $k=4/3\pi$ for e^+ and $k=2/3$ for e^- in case 1.

TABLE I.

Type of crystal	u_1, Å (293 K)	Concentration of atoms, 10^{22} cm^{-3}	U_0, eV	$d(110)$, Å	δ	a_c, Å	L_{rad}, cm
C	0.04	17.6	26.0	1.26	1.4	0.261	27.8
Si	0.075	5.00	30.0	1.92	2.5	0.208	9.0
Ge	0.085	4.42	54.0	2.0	3.0	0.171	2.1
Ag	0.093	5.85	56.2	1.44	2.3	0.16	0.9
W	0.050	6.30	160.0	2.24	3.8	0.123	0.35
Au	0.087	5.90	89.0	1.44	2.5	0.139	0.35

6. For numerical estimates we shall use the crystal parameters given in Table I.[3,4,13,14] The depth of the potential well can be calculated according to the approximate formulas

$$U_0=6\pi Ze^2Nda_0e^{-\delta}[\text{ch}\,\delta-1],\quad \delta=d/2a_s,$$
$$a_s\approx0.01643[Z^{2/3}-9.975654Z^{1/3}+42.21546]\ [\text{Å}].$$

As follows from Figs. 1 and 2, the characteristic values of the arguments of the functions $\varphi_1(x)$ and $\varphi_2(x)$ are $x\approx2$ for e^- and $x\approx10$ for e^+, from which we can obtain the length for establishment of the polarization respectively for e^- and e^+:

$$l_n[\text{cm}]\approx1.3\cdot10^8(U_0[\text{eV}]/d[\text{Å}])^{-3}\varepsilon^{-2}[\text{GeV}],$$
$$l_n[\text{cm}]\approx1.1\cdot10^9(U_0[\text{eV}]/d[\text{Å}])^{-3}\varepsilon^{-2}[\text{GeV}]. \tag{20}$$

Using Eqs. (16)–(18) and (20), we find the minimal energy $E_{\min}$ at which $l_n\leqslant\min[l_a,l_{el},l_{rad}]$. The results of the numerical calculations, which are given in Table II, show that for electrons

$$l_n\leqslant\min[l_a,l_{el},l_{rad}]=l_a,$$

while for positrons

$$l_n\leqslant\min[l_a,l_{el},l_{rad}]=l_{rad}.$$

The quantity $E_{\min}$ obtained characterizes the minimal energy value at which the distribution over the transverse levels is determined by the conditions of entry into the crystal, *not* the lower limit of existence of the radiative polarization effect in curved crystals. From comparison of Fig. 1 for electrons and Fig. 2 for posi-

TABLE II.

		C	Si	Ge	Ag	W	Au
E_χ, (GeV) $\langle\bar{\varkappa}\rangle\approx1$	e^-	1150	1500	880	610	330	380
	e^+	1800	2400	1380	960	520	600
E_{min}, (GeV)	e^-	593	359	277	251	132	186
	e^+	990	1310	757	524	286	330
l_n, cm	e^-	0.042	0.27	0.087	0.035	0.02	0.016
	e^+	0.126	0.17	0.096	0.067	0.036	0.042

trons we can see a weak dependence of the polarization characteristics of the beam on the form of the distribution function over the transverse energy levels. Consequently the radiative polarization effect can be observed in a number of crystals beginning at energies of a few hundred GeV for positrons and $E \sim 50-100$ GeV for electrons.

In the case $\chi \ll 1$ the final polarization value does not depend on the energy for $R_{\min}/R = \text{const}$ and, as follows from Eqs. (11), (12), and (14), for example for $R_{\min}/R = 0.1$, it is equal to 37% in cases 1 and 2 for electrons and 65.3% and 66.6% in cases 1 and 2 for positrons.

In the limiting case $\chi \gg 1$ the final polarization value and the characteristic length for its establishment behave with increase of the energy as

$$\zeta_z^{\text{est}} \sim \gamma^{-1/3}, \quad l_n \sim \gamma^{1/3}$$

for $R_{\min}/R = \text{const}$. At an energy $E = 3000$ GeV, as follows from Table II, the case $\chi \gg 1$ is realized for crystals of W and Au, for example. For W at this energy and $R_{\min}/R = 0.1$ we have $\zeta_z^{\text{est}} = 8.4\%$ and $l_n \approx 0.036$ cm for e^-, and $\zeta_z^{\text{est}} \approx 15.4\%$, $l_n \approx 0.048$ cm for e^+.

In conclusion we note that the length for establishment of polarization behaves with increase of energy as $l_n \sim 1/\varepsilon^2$, while the period of oscillation of a particle in the interplanar channel increases as $L \sim \varepsilon^{1/2}$. Accordingly there are grounds for hope of observing the radiative polarization effect in surface channeling, where l_n will be comparable with the period of the motion L. Here also the energy loss of the particles being polarized will be significantly less.

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