Alteration of a vortex lattice at a defect and pinning in type-II superconductors

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It is shown that a lattice instability develops very rapidly (with respect to the coupling constant) near defects whose interaction with the vortex lattice impedes the entrance of the vortex core into the defects. If the interaction exceeds a numerically small threshold value, metastable states will be formed at the defect. Such states cannot be described by elasticity theory.

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1. INTRODUCTION

A magnetic field penetrates into type-II superconductors in the form of quantized vortices. In ideal superconductors, the vortices form a regular triangular lattice.[1] The flow of current in such superconductors produces motion of the vortex lattice as a whole and leads to energy dissipation. Various defects, which are always present in superconductors, lead to the anchoring of the vortex lattice. As a result, a nondissipative current of finite amplitude can flow through the superconductive.

In many cases the critical current density in the experiment is proportional to the defect concentration. As has been shown by Labusch,[2] this requires the satisfaction of a rather strict criterion for the strength of the interaction of the defect with the vortex lattice. The essence of the Labusch criterion is that the magnitude of the displacement of the vortex lattice at the defect site should be of the order of the radius of action of the pinning forces. Upon satisfaction of this condition, formation of metastable states at the defect is possible. The change in the free energy on going from one metastable state to another also determines the magnitude of the critical current.[3]

However, in addition to the metastable states with a smooth deformation, the abrupt change of state of the vortex lattice, of the type of a structural transition, also turns out to be possible at the defect. The resultant state is not described by elasticity theory. There exists a physical reason for the relative ease of onset of such a transition: the lattice vortex and the energies of the triangular and square lattices differ by no more than 2% near the critical field H_{c2}. As will be shown below, such a change in state is also realized at weak defects of small radius. In contrast to the Labusch deformation instability, for which the sign of the interaction does not play a role, a change in the lattice structure takes place upon repulsion. A structural transition also arises in the case of attraction, but the interaction with the defect in this case should be greater by two orders of magnitude than in the repulsion case.

The simplest case of a weak defect of small radius will be considered below, a defect that is strongly drawn out along the magnetic field. Here we limit ourselves to consideration of magnetic fields close to H_{c2}, and to a temperature T close to T_c. The latter limitation is unimportant and the analysis can easily be generalized to the case of arbitrary temperature.

2. VORTEX LATTICES IN A SUPERCONDUCTOR WITH A DEFECT

We consider a superconductor with a defect of cylindrical shape and small transverse dimensions. Near the transition temperature, the free energy of such a superconductor can be represented in the form[4]

$$\begin{aligned}\mathcal{F}=&\nu\int d^3\mathbf{r}\left\{-\tau|\Delta|^2+\frac{7\zeta(3)}{16\pi^2T^2}|\Delta|^4+\frac{\pi D\eta}{8T}|\partial_-\Delta|^2\right\}\\&+\frac{1}{8\pi}\int d^3\mathbf{r}\,(H^2-2H_0H)+\delta\mathcal{F};\\ \delta\mathcal{F}=&\frac{\pi\nu}{8T}\int d^3\mathbf{r}\,\delta(\eta D)\,|\partial_-\Delta|^2+\nu\int d^3r\,g_1(r)\,|\Delta|^2,\end{aligned}\tag{1}$$

where $\nu=mp_0/2\pi^2$ is the density of states on the Fermi surface, $\zeta(3)$ is the Riemann zeta function, $D=vl_{tr}/3$ is the diffusion coefficient, H_0 is the external magnetic field, $\partial_-=\partial/\partial\mathbf{r}-2ie\mathbf{A}$, $\mathbf{A}$ is the vector potential,

$$\eta(T)=1-\frac{8T\tau_{tr}}{\pi}\left[\psi\left(\frac{1}{2}+\frac{1}{4\pi T\tau_{tr}}\right)-\psi\left(\frac{1}{2}\right)\right],\tag{2}$$

$\psi(x)$ is the psi function. For superconductors with small free path length of the electrons, the quantity $\eta(T)=1$.

The function $\delta\mathcal{F}$ is the free-energy change associated with the presence of the defect. It is assumed here that there is little change in the electron free path length and the effective electron-electron interaction constant in the vicinity of the defect:

$$g^{-1}(\mathbf{r})=g_m^{-1}+g_1(\mathbf{r}),\tag{3}$$

where g_m is the effective interelectron interaction constant in the superconducting matrix.

Near the critical field H_{c2} the order parameter Δ can be expanded in a series in the eigenfunctions of the operator ∂_-^2 with the smallest eigenvalue. We choose a special gauge of the vector potential:

$$\mathbf{A}_0=H_0(0,\ x,\ 0).\tag{4}$$

In this gauge, the order parameter Δ can be represented in the form[5]

$$\Delta=\tilde{\Delta}_0+\sum_{m=0}^{\infty}C_m\rho^m\exp\left[ixy+im\varphi-\frac{x^2+y^2}{2}\right].\tag{5}$$

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Here we have transformed to the dimensional variable

$$\rho=(eH)^{1/2}\mathbf{r}, \quad \rho=(x, y); \tag{6}$$

ρ is a two-dimensional vector in a plane perpendicular to the magnetic field. In formula (5), $\tilde{\Delta}_0$ is the solution corresponding to a regular triangular lattice:

$$\tilde{\Delta}_0=C\sum_{n=-\infty}^{\infty}\exp\left[\frac{i\pi n^2}{2}+in\beta+2iy\left(\frac{na}{2}+\alpha a\right)-\left(x-\frac{na}{2}-\alpha a\right)^2\right], \tag{7}$$

where

$$a^2=\frac{2\pi}{3^{1/2}}, \quad |C|^2=\frac{1-H_0/H_{c2}}{1-1/2\varkappa^2}\frac{8\pi^2T^2\tau}{7\cdot 3^{1/4}\zeta(3)\beta_A};$$
$$\varkappa^2=\frac{63\zeta(3)}{2\pi^3e^2p^2v^3\tau_{tr}{}^2\eta^2}, \quad \beta_A=\sum_{N,M}\exp[-2\pi(N^2+M^2-NM)/3^{1/2}]=1.1596, \tag{8}$$

the parameters α and β fix the position of the lattice relative to the defect, $\varkappa^2$ is the Ginzburg-Landau parameter.

As will be shown below, the structural transition first arises at the position where the null of the unperturbed lattice coincides with the defect. We therefore choose the parameters α and β in the form

$$\alpha=-1/4-\delta x(3^{1/2}/2\pi)^{1/2}, \quad \beta=\pi/2+(2\pi/3^{1/2})^{1/2}\delta y. \tag{9}$$

With such a choice of parameters α and β, the zero of the unperturbed solution is located at the point $(\delta x, \delta y)$. In what follows, we shall be interested in the case of the small values

$$|\delta x|, |\delta y|\ll 1. \tag{10}$$

With account of formula (5), the expression (1) for the free energy reduces to the form

$$\mathscr{F}=\nu\int d^2r\left\{\frac{7\zeta(3)}{16\pi^2T^2}|\Delta|^4(1-1/2\varkappa^2)-\frac{e\pi D\eta}{4T}(H_{c2}-H_0)|\Delta|^2\right.$$
$$\left.+g_1(r)|\Delta_0|^2+\frac{\pi}{8T}\delta(\eta D)|\partial_-\Delta_1|^2\right\}, \tag{11}$$

where Δ_0 and Δ_1 are components of the order parameter Δ, and are proportional to the function in the right side of formula (5) with $m=0.1$. For defects of small size, the interaction with components with $m\geqslant 2$ is small.

It is convenient to introduce new variables in place of the variables C_m through the formula

$$C_m=CD_m/\Gamma^{1/2}(m+1), \tag{12}$$

where $\Gamma(x)$ is the Euler gamma function. The equations for the coefficients D_m are obtained from the extremum condition for the free energy $\mathscr{F}$

$$\delta\mathscr{F}/\delta D_m{}^*=0,$$

and can be written in the form

$$\frac{1}{3^{1/4}\beta_A}\times\left\{\sum_{m_1,m_2=0}^{\infty}\frac{\pi\Gamma(m_1+m_2+1)}{2^{m_1+m_2+1}}\frac{D_{m_1}D_{m_2}D^*_{m_1+m_2-m}}{[\Gamma(m+1)\Gamma(m_1+1)\Gamma(m_2+1)\Gamma(m_1+m_2-m+1)]^{1/2}}\right.$$
$$+\sum_{m_1=0}^{\infty}\left[2I(m,m_1)D_{m_1}+D_{m_1}^*I(m+m_1)\left(\frac{\Gamma(m+m_1+1)}{\Gamma(m+1)\Gamma(m_1+1)}\right)^{1/2}\right]$$
$$+\sum_{m_1,m_2=0}^{\infty}\left[M(m_2+m-m_1)\right.$$
$$\times\frac{D_{m_1}D_{m_2}^*}{2^{m_2+m}}\frac{\Gamma(m_2+m+1)}{[\Gamma(m+1)\Gamma(m_1+1)\Gamma(m_2+1)\Gamma(m_2+m-m_1+1)]^{1/2}}$$
$$+M^*(m_1+m_2-m)$$
$$\left.\left.\times\frac{D_{m_1}D_{m_2}}{2^{m_1+m_2+1}}\frac{\Gamma(m_1+m_2+1)}{[\Gamma(m+1)\Gamma(m_1+1)\Gamma(m_2+1)\Gamma(m_1+m_2-m+1)]^{1/2}}\right]\right\}$$
$$-\pi D_m+Z(D_1+M(1)/\pi)\delta_{m,1}+Z_1(D_0+M(0)/\pi)\delta_{m,0}=0, \tag{13}$$

where the parameters Z and Z_1 are determined by the interaction of the lattice with the defect and are equal to

$$Z=\frac{\pi eH_{c2}}{4T\tau(1-H_0/H_{c2})}\int d^2\rho\,\delta(\eta D), \quad Z_1=\frac{1}{\tau(1-H_0/H_{c2})}\int d^2\rho g_1. \tag{14}$$

The matrix elements $M(m)$, $I(m)$ and $I(m, m_1)$, which enter into the set of equations (13), are equal to

$$M(m)=\frac{1}{C\Gamma^{1/2}(m+1)}\int d^2\rho\tilde{\Delta}_0\rho^m\exp\left[-im\varphi-\frac{\rho^2}{2}-ixy\right]$$
$$=\pi\sum_{N=-\infty}^{\infty}\sum_{l=0}\frac{\Gamma^{1/2}(m+1)(-1)^l}{2^l\Gamma(l+1)\Gamma(m+1-2l)}\left[\left(\frac{2\pi}{3^{1/2}}\right)^{1/2}(N+2\alpha)\right]^{m-2l}$$
$$\times\exp\left[\frac{i\pi}{2}N^2+iN\beta-\frac{\pi}{2\cdot 3^{1/2}}(N+2\alpha)^2\right];$$
$$I(m)=\sum_N\frac{M(N)M(m-N)}{2^{m+1}\pi}\left[\frac{\Gamma(m+1)}{\Gamma(N+1)\Gamma(m-N+1)}\right]^{1/2}; \tag{15}$$
$$I(m,m_1)=\sum_N\frac{M(N)M^*(m_1+N-m)}{2^{m_1+N+1}\pi}$$
$$\times\frac{\Gamma(m_1+N+1)}{[\Gamma(m+1)\Gamma(m_1+1)\Gamma(N+1)\Gamma(m_1+N-m+1)]^{1/2}}.$$

It is natural to expect, and calculation confirms this expectation that the structural transition with respect to the parameters Z and Z_1 takes place first at the point at which the zero of the unperturbed order parameter coincides with the defect.

The order parameter $|\tilde{\Delta}_0|^2$ has a six fold symmetry axis passing through the zeros of $\tilde{\Delta}_0$. This important property leads to a special form of the matrix elements $M(m)$ and to a breakup of the set of equations (13) into several weakly coupled subsystems.

The matrix elements $M(m)$ can be expanded in powers of δx and δy in the vicinity of the point $(\delta x, \delta y)=0$. The expansion coefficients are defined by sums of the form

$$S(m)=-\sum_{n=1}^{\infty}\left(n-\frac{1}{2}\right)^m\exp\left[\frac{i\pi}{2}(n^2+n)-\frac{\pi}{2\cdot 3^{1/2}}\left(n-\frac{1}{2}\right)^2\right]. \tag{16}$$

The presence of the sixfold symmetry axis leads to an infinite series of relations for the sums $S(m)$. Some of them are:

$$S(3)/S(1)=3^{1/2}/2\pi; \quad S(5)/S(1)=45/4\pi^2;$$
$$S(9)/S(1)=-2835(3^{1/2}/2\pi)^4+(18\cdot 3^{1/2}/\pi)S(7)/S(1), \tag{17}$$
$$S(11)/S(1)=-93555(3^{1/2}/2\pi)^5+990(3^{1/2}/2\pi)^2S(7)/S(1).$$

Using the relations (17), we reduce the matrix elements $M(m)$ in the vicinity of the point $(\delta x, \delta y)=0$ to the form

$$M_{1+6K}(\delta x, \delta y)=B(1+6K)P(1+6K, \delta x, \delta y),$$
$$M_{6K}(\delta x, \delta y)=-B(6K)(2\pi/3^{1/2})^{1/2}(\delta x+i\delta y)P(6K, \delta x, \delta y), \tag{18}$$
$$M_{2+6K}(\delta x, \delta y)=-B(6K+2)(2\pi/3^{1/2})^{1/2}(\delta x-i\delta y)P(2+6K, \delta x, \delta y),$$

where $K=0, 1, 2, \ldots$;

$$P(m,\delta x,\delta y)=\left\{1-\left(m+\frac{1}{2}\right)[(\delta x)^2\right.$$
$$\left.+(\delta y)^2]\right\}\exp\left[\frac{i}{2}\left(\frac{2\pi}{3^{1/2}}\right)^{1/2}\delta y+i\delta x\delta y\right]. \tag{19}$$

The remaining matrix elements are proportional to the square of the departure from the point $(\delta x, \delta y)=0$.

We can easily remove the exponential phase factor in (19) in the matrix elements by introducing the new variables

$$D_m(\delta x, \delta y) = \exp\left[-\frac{i}{2}\left(\frac{2\pi}{3^{1/2}}\right)^{1/2}\delta y + i\delta x\delta y\right]\tilde{D}_m(\delta x, \delta y) \quad (20)$$

in place of the quantities D_m. In what follows, we shall assume that this substitution has been carried out.

The coefficients $B(m)$ in formula (18) are real numbers. We write out the first few such coefficients:

$$B(1) = -6.99864;\ B(7) = 10.1705;\ B(13) = 5.66386;$$
$$B(0) = 3.67455;\ B(2) = -5.1966;\ B(16) = -14.128; \quad (21)$$
$$B(8) = 15.1035;\ B(12) - 10.722;\ B(14) = 11.1267.$$

It follows from formulas (15) and (18) for the matrix elements that in the vicinity of the point $(\delta x, \delta y) = 0$ the set of equations (13) breaks up into the four weakly coupled subsystems

$$\{1+6K\},\ \{6K, 2+6K\},\ \{3+6K, 5+6K\},\ \{4+6K\};\ K=0,1,2,\dots \quad (22)$$

Analysis shows that a structural instability in the vicinity of the defect arises in the system $\{6K, 6K+2\}$ and induces weak transitions in the remaining subsystems. Only the coupling with the subsystem $\{1+6K\}$ is significant here near the transition point.

We now proceed to a detailed analysis of two important special cases: In the first the defect is a region with a changed value of the interelectron interaction constant ($Z=0$, $Z_1 \neq 0$). In the second case, there are regions in the superconductor with a changed value of the mean free path of the electrons ($Z \neq 0$, $Z_1 = 0$).

3. STRUCTURAL TRANSITION ON A DEFECT WITH CHANGED VALUE OF THE INTERELECTRON INTERACTION CONSTANT ($Z = 0, Z_1 \neq 0$)

At $Z=0$, $Z_1 \neq 0$, the system of equations (13) turns out to be simplest to study. As has been noted above, a structural transition takes place in the subsystem $\{6K, 6K+2\}$. The critical value Z_1^c at which the structural transition occurs is determined from the condition

$$\det\{S(m, m_1) + 3^{1/4}\beta_A Z_1^c \delta_{m,0}\delta_{m_1,0}\} = 0, \quad (23)$$

where

$$m = 2K+6L,\ m_1 = 2K_1 + 6L_1,\ K=0,1,\ K_1=0,1,\ L, L_1 = 0,1,2\dots,$$
$$S(m, m_1) = 2I(m, m_1) + I(m+m_1)\,[\Gamma(m+m_1+1)/ /\Gamma(m+1)\Gamma(m_1+1)]^{1/2} - 3^{1/4}\pi\beta_A\delta_{m,m_1}. \quad (24)$$

Here the matrix elements of M must be taken at the point $(\delta x, \delta y) = 0$. Numerical calculations yield the value

$$Z_1^c = -0.3417. \quad (25)$$

The eigenvector corresponding to the eigenvalue Z_1^c is equal to

$$(D_0; D_2^*; D_6; D_8^*; D_{12}; D_{14}^* \dots) = = Y(0.531;\ -0.561;\ -0.486;\ 0.398;\ -0.056;\ 0.076;\dots). \quad (26)$$

The quantity Y, at values of Z_1 close to Z_1^c, is proportional to the square root of the supercriticality. For the determination of the value of Y and also of the radius of the circle within which the structural transition takes place, it is necessary to take into account the nonlinear terms in the set of equations (13) and also the connection with the subsystem $\{1+6K\}$. As is easily seen from the set of equations (13),

$$D_{1+6K} = |Y|^2 U_K, \quad K=0,1,2\dots. \quad (27)$$

The vector U_K is equal to

$$U_K = \{0.133;\ -0.0659;\ 0.00226;\ 0.0031;\ -0.0022\dots\}. \quad (28)$$

Multiplying the subsystem $\{6K, 2+6K\}$ of the set of equations (13) on the left by the column vector $(D_0^*; D_2; D_6^*; D_8 \dots)$, with account of the formulas (18) and(25)–(27), we obtain a cubic equation for the quantity Y:

$$0.093\tilde{Y}^3 + 0.43(Z_1 - Z_1^c)\tilde{Y} + 0.617\rho - 8.95\rho\tilde{Y}^2 = 0, \quad (29)$$

where

$$\delta x + i\delta y = \rho e^{i\varphi};\quad Y = \tilde{Y}e^{i\varphi}; \quad (30)$$

$\tilde{Y}$ is a real quantity.

The last term in Eq. (29) is small in terms of the subcriticality parameter, but it determines the value of the jump in the free energy. It follows from Eq. (29) that an abrupt change in the state of the vortex lattice in the neighborhood of the defect takes place at $Z_1 < Z_1^c$ (the structural transition). The distance ρ at which the break occurs, and also the initial $\tilde{Y}_{br}$ and final $\tilde{Y}_{fin}$ states, are easily found from Eq. (29):

$$\rho_{br} = 0.57(-\delta Z_1)^{3/2},\quad \tilde{Y}_{br} = 1.24(-\delta Z_1)^{1/2} + 32\rho,$$
$$\tilde{Y}_{fin} = -2.48(-\delta Z_1)^{1/2} + 32\rho, \quad (31)$$

where $\delta Z = Z_1 - Z_1^c$.

We note that the structural transition sets in very early in the case of negative value of the interaction constant Z_1. The negative sign corresponds to ejection of the vortex from the region of the defect. In the case of a positive sign of the interaction, no transition of the kind considered takes place.

4. CHANGE OF STATE OF THE VORTEX LATTICE AT INHOMOGENEITIES IN THE FREE PATH LENGTH OF THE ELECTRONS ($Z \neq 0, Z_1 = 0$)

The study of this case turns out to be somewhat more complicated. The solution of the set of equations (13) has the form

$$\{D_1; D_7; D_{13}; \dots\} = \{Y_2\} + |Y|^2\{U\},$$
$$\{D_0; D_2^*; D_6; D_8^*; D_{12}; D_{14}^*; \dots\} = Y\{Y_1\}, \quad (32)$$

where $\{Y_2\}$, $\{U\}$, $\{Y_1\}$ are real vectors. The critical value of Z_c is determined from the condition

$$\det\{S(m, m_1) + S_1(m, m_1)\} = 0, \quad (33)$$

where $\{m, m_1\}$ and the matrix S are determined by the formulas (23) and (24), and the matrix S_1 is generated by the vector $\{Y_2\}$ and is equal to

$$S_1(m, m_1) = \sum_{N=0}^{\infty} \left\{ \frac{M(m_2+m-m_1)}{2^{m_2+m}} Y_2(1+N) \right.$$

$$\times \frac{\Gamma(m_2+m+1)}{[\Gamma(m+1)\Gamma(m_1+1)\Gamma(m_2+1)\Gamma(m_2+m-m_1+1)]^{1/2}} + \frac{M(m_1+m_2-m)}{2^{m_1+m_2}}$$

$$\times Y_2(1+N) \frac{\Gamma(m_1+m_2+1)}{[\Gamma(m+1)\Gamma(m_1+1)\Gamma(m_2+1)\Gamma(m_1+m_2-m+1)]^{1/2}} \tag{34}$$

$$+ \frac{M(m_1+m-m_2)}{2^{m_1+m}}$$

$$\left. \times Y_2(1+N) \frac{\Gamma(m_1+m+1)}{[\Gamma(m+1)\Gamma(m_1+1)\Gamma(m_2+1)\Gamma(m_1+m-m_2+1)]^{1/2}} \right\},$$

where $m_2 = 1 + 6N$.

As a result of numerical calculation, we find $Z_c = 0.1505$,

$$\begin{gathered} \{Y_2\} = 10^{-2}\{6.55;\ 0.692;\ -0.319;\ -0.0449;\ 0.0517;\ 0.0035\ldots\}, \\ \{Y_1\} = \{0.481;\ -0.555;\ -0.516;\ 0.428;\ -0.0678;\ 0.08;\ldots\}, \\ \{U\} = \{0.114;\ -0.0976;\ 0.0562;\ 0.0252;\ -0.0186;\ldots\}. \end{gathered} \tag{35}$$

The value of Y is determined from a cubic equation. This equation can be obtained by multiplication of the subsystem (13) $\{6K, 6K+2\}$ by the vector: $\{Y_1\}$. With account of formulas (18) and (35) we find

$$\begin{gathered} 0.08\tilde{Y}^3 - 0.839(Z-Z_c)\tilde{Y} + 0.7\rho - 9.05\rho\tilde{Y}^2 = 0, \\ \delta x + i\delta y = \rho e^{i\varphi}, \quad Y = \tilde{Y}e^{i\varphi}, \end{gathered} \tag{36}$$

where $\tilde{Y}$ is a real function.

The equation (36) has the same form as Eq. (29). The break point and the solutions in the initial $\tilde{Y}_{br}$ and final $\tilde{Y}_{fin}$ states are equal to

$$\begin{gathered} \rho_{br} = 1.5(\delta Z)^{3/2}, \quad \tilde{Y}_{br} = 1.87(\delta Z)^{1/2} + 37.5\rho; \\ \tilde{Y}_{fin} = -3.74(\delta Z)^{1/2} + 37.5\rho, \end{gathered} \tag{37}$$

where $\delta Z = Z - Z_c$.

We note that in repulsion ($Z > 0$) an abrupt change in the state of the vortex lattice (structural instability) set in at a numerically small value of the coefficient Z. In attraction, the structural instability arises at a large value, $Z = 10.3$. Here it turns out that, both in the case of a defect in which the transition temperature differs from the transition temperature in the superconductor, and in the case of a defect in which only the free path length of the electrons differs from its value in the superconducting matrix, if the sign of the interaction corresponds to attraction of the core of the vortex to the defect the presence of the defect leads initially to stabilization of the lattice, increasing its stability. And only at relatively large values of the interaction constant Z does the structural instability again arise.[5]

5. JUMP IN THE FREE ENERGY IN STRUCTURAL TRANSITION AT A DEFECT AND THE DENSITY OF THE CRITICAL CURRENT

Formulas (31) and (37) allow us to find the jump in the free energy in the case of a structural transition at a defect. First, by using Eq. (13), we reduce the expression (11) for the free energy to the form

$$\frac{\mathcal{F}_1 - \mathcal{F}_0}{\nu/eH} = \frac{E}{2} \left\{ \frac{1}{3^{1/4}\beta_A} \sum_{m_1, m_2} \left[2I(m_1, m_2) D_{m_1}^* D_{m_2} \right. \right.$$

$$\left. + \frac{1}{2} \left[\frac{\Gamma(m_1+m_2+1)}{\Gamma(m_1+1)\Gamma(m_2+1)} \right]^{1/2} (D_{m_1} D_{m_2} I^*(m_1+m_2) + D_{m_1}^* D_{m_2}^* I(m_1+m_2)) \right]$$

$$+ \frac{1}{2} \sum_{m_1, m_2, m_3} \frac{\Gamma(m_2+m_3+1)}{2^{m_2+m_3+1}[\Gamma(m_1+1)\Gamma(m_2+1)\Gamma(m_3+1)\Gamma(m_2+m_3-m_1+1)]^{1/2}}$$

$$\times (D_{m_1} D_{m_2}^* D_{m_3} M(m_2+m_3-m_1) + M^*(m_2+m_3-m_1) D_{m_1}^* D_{m_2} D_{m_3})$$

$$- \pi \sum_{m_1} |D_{m_1}|^2 + Z\left[|D_1|^2 + \frac{3}{2\pi}(D_1 M^*(1) + D_1^* M(1)) + \frac{2|M(1)|^2}{\pi^2} \right]$$

$$\left. + Z_1 \left[|D_0|^2 + \frac{3}{2\pi}(M(0) D_0^* + D_0 M^*(0)) + \frac{2|M(0)|^2}{\pi^2} \right] \right\}, \tag{38}$$

where

$$E = \tau\left(1 - \frac{H_0}{H_{c2}}\right)\left[\frac{1 - H_0/H_{c2}}{1 - 1/2\varkappa^2} \frac{8\pi^2 T^2 \tau}{7 \cdot 3^{1/4} \zeta(3) \beta_A}\right]. \tag{39}$$

Using the expressions (26) and (27) that we obtained for the coefficients D_m at $\{Z = 0, Z_1 \neq 0\}$, and (32) and (35) in the case $\{Z \neq 0, Z_1 = 0\}$, we reduce the expression (38) for the jump in the free energy to the form

$$\begin{gathered} \delta\left(\frac{\mathcal{F}_1 - \mathcal{F}_0}{\nu/eH}\right) = \frac{E}{2}\delta\{0.28\delta Z_1 |\tilde{Y}|^2 + 1.2\rho\tilde{Y}\}, \quad \{Z = 0, Z_1 \neq 0\}, \\ \delta\left(\frac{\mathcal{F}_1 - \mathcal{F}_0}{\nu/eH}\right) = \frac{E}{2}\delta\{-0.55\delta Z |\tilde{Y}|^2 + 1.38\rho\tilde{Y}\}, \quad \{Z \neq 0, Z_1 = 0\}. \end{gathered} \tag{40}$$

The jump in the values of $\tilde{Y}$ and $|\tilde{Y}|^2$ in the transition is determined by formulas (31) and (37). Substituting the value of the jump found from these formulas in the expression (40) we reduce the expression for the change in the free energy in a structural transition at a defect to the form

$$\begin{gathered} \delta\left(\frac{\mathcal{F}_1 - \mathcal{F}_0}{\nu/eH}\right) = -1.9(-\delta Z_1)^2 E, \quad \{Z = 0, Z_1 \neq 0\}; \\ \delta\left(\frac{\mathcal{F}_1 - \mathcal{F}_0}{\nu/eH}\right) = -8.7(\delta Z)^2 E, \quad \{Z \neq 0, Z_1 = 0\}. \end{gathered} \tag{41}$$

If an individual defect is capable of bringing about the formation of metastable states, then, in the approximation of low concentration of defects, the critical current density is proportional to the jump in the free energy at the transition from one metastable state to another and to the defect concentration n:[3]

$$j_c B = 2nL\delta\mathcal{F}\rho_{br}(eH)^{1/2}/\pi \tag{42}$$

where L is the mean length of the defect in the direction of the magnetic field, $\delta\mathcal{F}$ is the jump in the free energy per unit length [formula (41)], and ρ_b is the radius at which the break occurs [formulas (31) and (37)].

In the cubic equations (29) and (36), the coefficient in front of the cubic term is small. As a result, the value of the lattice deformation, which is determined by the quantity $\tilde{Y}$, increases very rapidly upon increase in the supercriticality δZ and δZ_1. The numerical smallness of the coefficient of $\tilde{Y}^3$ leads to the result that the region of s applicability of formulas (31) and (37)) is small in terms of the value of the supercriticality. An estimate of the magnitude of the region can be obtained from the condition that the terms proportional to ρ in formulas (31) and (37) should be small in comparison with the first term. From this we find

$$\begin{gathered} |\delta Z_1/Z_1^c| < 0.07, \quad \{Z = 0, Z_1 \neq 0\}, \\ |\delta Z/Z^c| < 0.03, \quad \{Z \neq 0, Z_1 = 0\}. \end{gathered} \tag{43}$$

Upon increase in the supercriticality, saturation first occurs. The character of the solution upon further increase in the supercriticality has not been investigated.

6. CONCLUSION

The critical current in many cases is proportional to the defect concentration.[6] This circumstance indicates the formation of metastable states at an individual defect. At the same time, the Labusch criterion is too stringent and is never satisfied for defects of small size.[4,6] We have shown that, in addition to the metastable states with smooth deformations, which are also given by the Labusch criterion, an abrupt change in the state of the lattice vortex near the defect (structural transition) takes place at defects whose interaction with the vortex lattice impedes the entry of the vortex into the defect. Here the structural transition sets in the case of a numerically small interaction. It follows from formula (14) that upon approach to H_{c2}, in addition to the numerically small coefficient, the condition of onset of a structural transition is eased because of the factor $1-H_0/H_{c2}$. The very rapid increase in the value of the deformation with increase in the supercriticality is extremely important. It is natural to suppose that saturation of the deformation is already taking place at the boundary determined by formula (43). The character of the solution for further increase in the interaction has not been investigated. It can be expected that if the defect is a superconductor region with increased value of T_c, then the critical current will be an oscillating function of the parameter Z_1, similar to what took place in the previous work.[5] The presence of scatter in the characteristics of the defect leads to a smoothing of the dependence of the critical current on the value of the magnetic field. As follows from formulas (39) and (42), the quantity $j_c B$ here is proportional to

$$j_c B \sim \tau^{1.5}(1-H_0/H_{c2})^2. \tag{44}$$

We note that formula (44) has a universal character and does not depend on the strength of the interaction of the vortex lattice with the defect.[5] If the interaction of the defects with the vortex lattice is sufficiently weak, then, as follows from formula (14), on approaching the critical field H_{c2}, we fall into the region of a strong field dependence of the critical field density (the region of the peak effect).

On the rising portion, this dependence reflects the distribution of defects with respect to the interaction strength and does not have a universal character. Upon further approach to H_{c2}, the dependence of the critical current density becomes the universal law determined by formula (44).

The sign of the interaction is extremely important for the start of the structural instability, in contrast to the instability of the Labusch type. The structural instability sets in early only in the case of repulsion of the vortex from the defect. In the case of attraction, the defect initially stabilizes the lattice. Only in the region of large values of the interaction constant Z does the vortex lattice again lose stability. Such a sensitivity to the sign of the interaction appears on defects of large size.[3]

We emphasize again that the physical reason for the anomalously rapid loss of stability of the lattice and the appearance of the structural transition near the defect are connected with the looseness of the vortex lattice and the small (2%) difference between the energies of the triangular and square lattices.

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