Permittivity, molar volume, and critical phenomena near the λ-point of liquid ^{4}He

V. I. Panov and V. A. Khvostikov

Moscow State University

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Measurements have been made of the permittivity, ε, molar volume, V, and thermal expansion coefficient at saturated vapor pressure, β_s, of liquid ^{4}He in the temperature range from 1.4 to 4.2 K. It is shown that near the λ-transition, β_s can be described to high accuracy by a logarithmic dependence. The nature of the divergence in the isobaric expansion coefficient, β_p, for $1\times 10^{-5} < |T - T_\lambda|/T_\lambda < 3\times 10^{-2}$ was studied and values were found for the critical exponents, $\alpha = -0.006\pm 0.0025$ at $T > T_\lambda$ and $\alpha' = 0.000\pm 0.0025$ at $T < T_\lambda$. This [contrary to other results (Ref. 8, G. Ahlers, Phys. Rev. **A3**, 696, 1971; **A8**, 530, 1973. Ref. 9, K. H. Mueller, G. Ahlers, F. Pobbell, Phys. Rev. Lett. **34**, 513, 1975; Phys. Rev. **B14**, 2096, 1976)] confirms the logarithmic form of the divergence of the thermodynamic quantities in ^{4}He near the λ-transition.

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1. INTRODUCTION

If the polarizability of a material α_E is known, its molar volume V and density can be determined with high accuracy from measurement of the permittivity ε, and a detailed study of the behavior of the thermal expansion coefficient $\beta = -\rho^{-1}d\rho/dT \sim d\varepsilon/dT$ can also be made by using the Clausius-Mosotti relation:

$$\frac{(\varepsilon-1)}{(\varepsilon+2)} = \frac{4\pi\rho\alpha_E}{3\mu} = \frac{4\pi\alpha_E}{3V}; \qquad (1)$$

here μ is the molar mass. This method is most advantageous in the case of cryogenic liquids for which Eq. (1) is sufficiently well satisfied and the use of superconducting resonators increases appreciably the measurement accuracy.[1-3]

Among cryogenic liquids, ^{4}He is of special interest and its properties have been long and intensively studied (see, for example, Refs. 4–6). However, there is still no consensus on the nature of the divergence of the thermodynamic quantities in ^{4}He near the λ-transition, and there is a scatter in the experimental results on the basic parameters V, ρ, and β, which makes it difficult to compare them with the theory.

In the present work, therefore, values of the molar volume, V, coefficient of thermal expansion under saturated vapor pressure, β_s, and isobaric thermal expansion coefficient, β_p, have been calculated with high accuracy on the basis of a detailed study of the temperature dependence on the permittivity of pure liquid ^{4}He. The form of the divergence of the thermodynamic quantities near T_λ has also been studied. It turned out that the values of molar volume obtained agree well with the results of direct dilatometric measurements[7] only in the region $T \leq T_\lambda$. There is an appreciable (~0.3%) discrepancy between the dilatometric results[7] and the present measurements for $T > T_\lambda$.

The coefficient of thermal expansion under saturated vapor pressure, β_s, near the λ-transition is described to high accuracy by the logarithmic relation $\beta_s \sim A \ln|T/T_\lambda - 1| + B$ with a small difference in the values of A below and above $T_\lambda (A/A' = 1.029,\ B = 4.77\times 10^{-2}\ \mathrm{K}^{-1}$ and $B - B' = 3.59\times 10^{-2}\ \mathrm{K}^{-1})$.

To study the nature of the divergence of the isobaric expansion coefficient, β_p, near the λ-transition, comparison was made between the experimental values of β_p and the analytical expression

$$\beta_p \sim \frac{A}{\alpha}\left|\frac{T-T_\lambda}{T_\lambda}\right|^{-\alpha} + B.$$

The critical exponents obtained from the experiment,

$$\alpha = -0.006\pm 0.0025\ (\text{for } T > T_\lambda) \text{ and } \alpha' = 0.000\pm 0.0025\ (\text{for } T < T_\lambda)$$

confirm the logarithmic divergence of the thermodynamic quantities for ^{4}He near the λ-transition (as distinct from the results of Ahlers[8] and Mueller *et al.*[9], where $\alpha \approx \alpha' \approx -0.026$).

2. EXPERIMENTAL APPARATUS AND MEASURING TECHNIQUE

The apparatus for studying the permittivity of liquid helium is shown in Fig. 1. A coaxial type superconducting microwave cavity with well defined electrical capacitance is placed inside a copper chamber which is filled with liquid helium (^{4}He). The resonator made from a solid piece of super-pure niobium, had a natural frequency $f_0 \approx 2.876$ GHz and $Q \approx 3.8\times 10^6$ at a temperature $T = 4.2$ K. The inside diameter of the cavity, 1.6

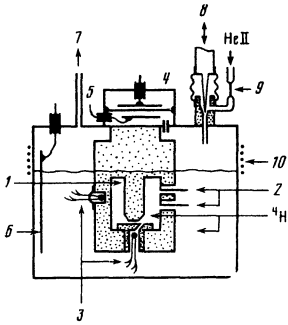

FIG. 1. Arrangement for measuring the permittivity of ^{4}He. 1—Measuring resonator, 2—inductive coupling loop, 3—thermometers, 4—pressure gauge, 5—calibration capacitance, 6—liquid ^{4}He level indicator, 7—capillary, 8—valve, 9—superleak, 10—heater.

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cm, was equal to its height and the capacitive gap between the central rod and the lower wall of the resonator was 0.9 mm and was used as the measuring capacitance. The orifice in the lower part of the resonator between the capacitance gap and the supplementary volume in which the principal thermometer was housed served to reduce the temperature gradient between the portion of helium being studied and the thermometer. It was shaped to exclude action of the microwave power on the semiconductor thermometer. The external sinusoidal microwave power and the electronic circuit for processing the signal produced by the resonator were connected to the latter by inductive coupling loops placed in its side wall. The resonator temperature was measured by thermometers which were also used to determine the deviations of its resonant frequency as functions of temperature in the absence of helium in the chamber.

A capacitor plate was placed 2 mm from the wall to determine and monitor the liquid helium level in the chamber, and the position of the ^{4}He liquid-vapor boundary could be measured to an accuracy of ~1 mm. The helium vapor was pumped through a 0.05 mm diameter capillary placed in the upper part of the measuring chamber. There was also a diaphragm capacitive manometer there with a capacitor plate for independent electrostatic pressure calibration, which was used to measure and monitor the absolute value of the pressure and its variation.

The liquid helium under study was fed to the measuring chamber from an external volume at a temperature $T \approx 1.6$ to 1.9 K through a superleak using a cut-off valve, reducing appreciably the possible presence of microscopic ^{3}He impurity in the specimen being investigated. The chamber was connected through thermal bridges to an isothermal copper platform that conducted away the heat dissipated in the resonator and regulated the rate of change of temperature in the chamber when measuring the ^{4}He permittivity near T_λ.

Measurement of the specimen temperature was made with a resistance thermometer made from a $2\times1\times0.3$ mm wafer of doped germanium. Either standard GaAs semiconductor thermometers or carbon resistance thermometers were used as additional thermometers to measure the resonator temperature. All thermometers were calibrated against the T_{62} and T_{58} ^{3}He and ^{4}He vapor pressure scales. The systematic error in determining the absolute temperature during the permittivity measurements was within $\approx 1\times10^{-4}$ K and was determined by the calibration accuracy over the whole temperature range. This accuracy was several times better near T_λ and corresponded to the accuracy in fixing the break in the permittivity curve on passing through the λ-point. The shift in T_λ as a result of hydrostatic pressure and electrostriction effects could be neglected for this as it did not exceed 3×10^{-6} K.

The reproducibility of the readings of the main thermometer for a single heating lay within the accuracy of the calibration. However, the readings of the thermometers were checked against ^{4}He vapor pressure during each measurement run to prevent errors. The relative variations near T_λ were recorded in steps of $\approx 0.7\times10^{-5}$ K, which were determined by the digital temperature recording arrangement and by the sensitivity of the resistance thermometer at the given temperature. The actual temperature resolution was increased by more than an order of magnitude by using a method of measurement with which the change in ^{4}He permittivity near T_λ was recorded continuously between two readings of the temperature measuring system. Corrections in the determined value of the temperature, due both to slight non-linearity in the recording system and to self-heating of the thermometers, were taken into account in the measurements and did not exceed 30% for the best resolution. The monotonic drift in the readings of the main thermometer, equal to 0.6×10^{-5} $\mathrm{K\cdot h^{-1}}$, were also subtracted.

The electronic measuring system consisted of an external microwave sinusoidal oscillator stabilized by a solid-state superconducting resonator[10] with a relative frequency instability $\langle \Delta f/f_0\rangle \lesssim 1\times10^{-10}$ per hour, and of a low-noise device to detect small changes in the power in the measuring resonator due to shifts of its resonance frequency. The microwave power dissipated in the measuring resonator during an experiment did not exceed 5×10^{-5} W. The frequency change of the resonator during measurements of the permittivity ε of liquid ^{4}He over a wide temperature range (from 1.4 to 4.2 K) was recorded with a Pound circuit.[11] Near T_λ these measurements were carried out by the method used earlier.[12] Relative permittivity variations were recorded with an accuracy of $\approx 1\times10^{-9}$ at $T > T_\lambda$ and $\approx 3\times10^{-10}$ at $T < T_\lambda$ regardless of the method of measurement.

The error in the measured absolute value of ε was deduced from the relation

$$\Delta\varepsilon_{\rm abs} \approx \beta(\varepsilon-1)\Delta T_{\rm abs} + \frac{2\Delta f_{\rm abs}}{f_0}, \qquad (2)$$

where $\Delta f_{\rm abs}$ is the systematic error in determining the absolute frequency shift of the measuring resonator when filled with liquid ^{4}He; $\Delta T_{\rm abs}$ is the accuracy in the absolute determination of the temperature.

It can be seen from Eq. (2) that $\Delta\varepsilon_{\rm abs}$ depended on the temperature but its maximum value did not exceed 1×10^{-7} since $\Delta f_{\rm abs}/f_0 \lesssim 1.5\times10^{-8}$, while temperature calibration against T_λ in separate measurements gave values of $\Delta T_{\rm abs}$ 2 to 4 times smaller than for calibration against ^{4}He vapor pressure.

When processing the results of the measurements, a correction was made to the resonator frequency for the mechanical effect of the pressure of the saturated vapor and of the liquid ^{4}He, equal to $\Delta f = -4.72\ \mathrm{Hz\cdot torr^{-1}}$. Measurement of the permittivity ε and of the thermal expansion coefficient $\beta = -\rho^{-1}d\rho/dT \sim d\varepsilon/dT$ were carried out not less than ~1 hour after filling the chamber with pure liquid ^{4}He to 4 to 6 mm above the resonator, after a stationary temperature had been established. This ^{4}He level was maintained in the chamber within an accuracy of ±1.5 mm during each measurement run. Individual values of $\varepsilon(T_i)$ and $d\varepsilon(T_i)/dT$ were determined at the saturated vapor pressure not less than 3 to 4 minutes after the temperature T_i had been established in the measuring volume. Each value of $\varepsilon(T_i)$ was obtained by averaging over n points ($n=4$ to 10) both on cooling and on warming the specimen over the whole temperature range. The mean square deviation for $\varepsilon(T_i)$ did not exceed the accuracy $\Delta\varepsilon_{abs}$ in determining the absolute value of the permittivity.

The measurement of β was carried out in two stages. In the range $|\tau|=|T-T_\lambda|/T_\lambda \leq 5\times10^{-3}$ continuous recording of $\varepsilon(T)$ was carried out with simultaneous fixing of the temperature with an interval $T_j - T_{j+1}=0.7\times10^{-5}$ K. For these measurements the rate of change of temperature could be regulated from 5×10^{-5} K·min^{-1} to 1×10^{-2} K·min^{-1}. In the range $|\tau|>5\times10^{-3}$, the value of $d\varepsilon/dT$ was defined for separate fixed values of temperature as

$$\lim \frac{\Delta\varepsilon(T)}{\Delta T} \quad \text{as} \quad \Delta T\to 0.$$

The systematic error in determining β depended mainly on the accuracy in recording the temperature interval ΔT and was $\leq 0.3\%$ for $|\tau|<0.5\times10^{-4}$. The random error was then at least an order of magnitude less, and did not exceed 0.03%.

3. PERMITTIVITY AND MOLAR VOLUME OF LIQUID ^{4}He

The molar volume and density of liquid ^{4}He have been measured previously by a variety of methods,[2,7,13-15] of which only dilatometric measurements[7] give directly the values of V and ρ. However, there are appreciable errors in this method at temperatures $T>T_\lambda$, due to the presence of the gas phase and boiling of the ^{4}He near the surface, the position of which has to be recorded in the experiment. In addition, for a number of delicate experiments with ^{4}He and for various theoretical calculations, these parameters must be known with a high accuracy that can only be obtained by measurement of its permittivity.

The accuracy achieved previously in determining the permittivity of liquid ^{4}He was improved by an order of magnitude by using the method described above, based on the use of high-stability oscillators and superconducting resonators with small gap capacitance, so that V_E and ρ_E could be deduced from the Clausius-Mosotti relation with great accuracy.

The permittivity was measured at saturated vapor pressure in the temperature range from 1.4 to 4.2 K. The value of $\varepsilon(T)$ was determined from the difference between the resonant frequencies of the measuring resonator when empty and filled with helium, using the expression

$$\varepsilon(T)=[f_0(T)/f_E(T)]^2, \tag{3}$$

where $f_0(T)$ is the resonant frequency of the empty resonator at temperature T and $f_E(T)$ is the resonator frequency when filled with ^{4}He at the same temperature.

Measured values of ε for different temperatures T are given in Table I. The maximum value of the permittivity $\varepsilon_m=1.0575945$, corresponding to the minimum molar volume of ^{4}He, measured under saturated vapor pressure, was reached at a temperature $T_m=2.1785\pm0.0001$ K, which agrees with the results of Peshkov and Borovikov.[16] The permittivity at $T=T_\lambda$, $\varepsilon_\lambda=1.0575919$ was obtained by averaging ε in the temperature range $|\Delta T|\approx 1.5\times10^{-5}$ K near the λ-point.

TABLE I.

T, K	ε	V_E	$(V-V_E)\cdot10^3$
1.498	1.0572126	27.574509	1.7
1.554	1.0572245	27.568872	2.6
1.617	1.0572371	27.562907	1.6
1.694	1.0572569	27.553539	0.9
1.761	1.0572792	27.542996	−0.2
1.824	1.0573040	27.531279	−1.6
1.927	1.0573548	27.507312	−5.0
2.049	1.0574383	27.467961	−8.9
2.123	1.0575153	27.431866	−9.7
2.167	1.0575785	27.402274	−9.2
2.171	1.0575886	27.397551	−9.0
2.1720	1.0575919	27.396008	−9.0
2.1785	1.0575945	27.394791	−8.9
2.192	1.0575891	27.397316	−8.5
2.213	1.0575737	27.404519	−8.1
2.241	1.0575483	27.416406	−7.0
2.310	1.0574528	27.461121	−6.1
2.433	1.0572287	27.566883	−6.9
2.502	1.0570701	27.642184	−15.1
2.820	1.0562109	28.057498	−29.2
3.040	1.0554741	28.423894	−23.2
3.220	1.0547869	28.774507	1.9
3.485	1.0536235	29.388565	52.9
3.808	1.0518835	30.358353	84.5
4.104	1.0499716	31.501822	62.3

It is possible to determine the molar volume or density of ^{4}He from the values obtained for the permittivity using Eq. (1) only if the polarizability α_E is specified. Several calculated values of α_E are known[2,13,17,18] for ^{4}He, and the most accurate of these is

$$\alpha_E=\alpha_0+\frac{\alpha_1}{V}=0.123396\ \text{cm}^3/\text{mole}-\frac{0.005604}{V}\text{cm}^6/\text{mole},$$

obtained by direct measurements of polarizability.[18] The molar volume V_E of lqiuid ^{4}He at saturated vapor pressure was calculated by using this value of α_E. The calculated values of V_E are shown in Table I. Also shown is the difference between the molar volume V obtained in direct dilatometric measurements[7] and the results of the present work. A graph of the difference $V-V_E$ is shown in Fig. 2 and it can be seen that there is good agreement between these results in the temperature region $T<2.5$ K.

The maximum difference between the direct dilatometric measurements and the values of molar volume obtained from measurements of the permittivity is $\approx 0.27\%$ at $T\approx3.8$ K. Such a difference between V and V_E at $T>2.5$ K is rather large, but it must be pointed out that no direct measurements of molar volume at

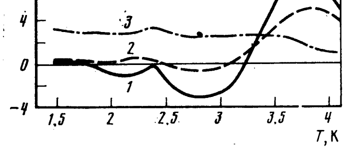

FIG. 2. Difference between molar volumes $V-V_E$: 1, 2, 3—comparison with the results of Refs. 7, 14, 15.

TABLE II.

T_m	V_m	ε_m	
2.1785	27.39479	1.0575945	Present work
2.1777	27.3859	1.0576136	[7]
2.1796	27.3982	1.0576370	[14]
2.1779	27.4238	1.0575326	[15]

$T > 2.8$ K were carried out,[7] but the results of earlier, less accurate, experiments were used. The values obtained should also be compared with other results[14,15] calculated on the basis of measurements of permittivity (see Fig. 2). It can be seen from the comparison that at $T < 3.0$ K the calculated values of V_E agree rather well with the molar volumes obtained by Van Degrift and Pellam.[14] The disagreement at $T \gtrsim 3.0$ K evidently derives from the fact that only one measurement of V was made above 2.5 K, while the remaining molar-volume values were taken (as in Ref. 7) from earlier investigations.

A comparison with Kierstead's results[15] is of greater interest since he carried out detailed measurements of molar volume over a wide temperature range from 0.7 to 4.2 K, and the permittivity ε was determined to an accuracy of $\approx 1.5 \times 10^{-6}$. The $V(T)$ temperature dependence given by Kierstead agrees very well with the $V_E(T)$ dependence obtained by us, but there is a constant difference between them of $\approx 0.1\%$ which cannot be explained by systematic experimental error.

The molar volume was measured with high accuracy in the region $T \approx T_m$, where the thermal expansion coefficient $\beta \to 0$, and was $V_m = 27.394791\ \text{cm}^3 \cdot \text{mole}^{-1}$. For comparison with other results, values of T_m, V_m, and ε_m obtained by different authors[7,14,15] are given in Table II.

It follows from Table II that, regardless of the relative accuracy achieved in each of the experiments, there is appreciable divergence in the values of T_m, V_m, and ε_m between the results of different authors, which cannot be explained by systematic or random errors.

4. STUDIES OF CRITICAL PHENOMENA IN ^{4}He NEAR THE λ-POINT

In the region of the λ-transition, thermodynamic quantities display pronounced anomalies. Much theoretical and experimental work has been devoted to studying these anomalies (see, for example, Refs. 6 and 19 and the literature cited in them).

According to current theoretical ideas,[20] the heat capacity C_p and thermal expansion coefficient have singularities as $T \to T_\lambda$ of the form

$$\beta_p \sim C_p \sim \frac{A}{\alpha}|\tau|^{-\alpha} + B, \quad \tau > 0, \tag{4}$$

$$\beta_p \sim C_p \sim \frac{A'}{\alpha'}|\tau|^{-\alpha'} + B', \quad \tau < 0. \tag{5}$$

Within the framework of accepted theoretical models, the values of the critical exponents α and α' should be the same, while the magnitudes of the coefficients A, A' and B, B' can in general be different.

Additional correction terms are introduced into Eqs. (4) and (5) by some authors.[21,9] These expressions then take the form

$$\beta_p \sim C_p \sim \frac{A}{\alpha}|\tau|^{-\alpha}(1 + D|\tau|^{\Delta}) + B, \quad \tau > 0, \tag{6}$$

$$\beta_p \sim C_p \sim \frac{A'}{\alpha'}|\tau|^{-\alpha'}(1 + D'|\tau|^{\Delta}) + B', \quad \tau < 0, \tag{7}$$

where in accordance with theoretical estimates[9,22,23] the critical exponent Δ is usually taken as 0.5.

In the particular case when $\alpha = 0$, the temperature dependence of the heat capacity and of the expansion coefficient are represented by the logarithmic law:

$$\beta_p \sim C_p \sim A \ln|\tau| + B, \quad \tau > 0, \tag{8}$$

$$\beta_p \sim C_p \sim A' \ln|\tau| + B', \quad \tau < 0; \tag{9}$$

here $A = A'$ but $B \neq B'$.

Theoretical calculations by the renormalization group method show that the magnitude of $\alpha = \alpha'$ is very small and an estimate of it[22,23] gives $\alpha_{\text{th}} \approx -0.026$.

Experimental investigations of the temperature dependences of the heat capacities C_p, at constant pressure, and C_s, at saturated vapor pressure, and of the coefficient of thermal expansion for ^{4}He near the λ-point have until now not provided a final identification of the nature of the divergence of these thermodynamic quantities. According to existing experimental results one can only assert that the critical exponents α and α' lie within the limits from small positive quantities to $\{\alpha, \alpha'\} \approx \alpha_{\text{th}}$.[8,9,24-26] Results obtained recently[26] in studies of the temperature dependence of C_s in the range $10^{-5} < |\tau| < 10^{-8}$ indicate a logarithmic divergence of the thermodynamic quantities rather than the dependence described by Eqs. (4) to (7) with a value of α close to α_{th}.

Because of what we have said above and also because there are no sufficiently accurate values of the expansion coefficient of ^{4}He at saturated vapor pressure near T_λ, we carried out a detailed experimental investigation of the temperature dependence of β_s and determined numerical values of this quantity.

Near the λ-point (at least in the range $|\tau| \lesssim 3 \times 10^{-2}$) the values of β_s, β_p, and C_p are connected by the simple thermodynamic relations[4,24]

$$\beta_p = \beta_s - \left(\rho^{-1}\frac{\partial\rho}{\partial P}\right)_T \left(\frac{dP}{dT}\right)_s, \tag{10}$$

$$C_p = T\left(\frac{\partial S}{\partial T}\right)_\lambda + TV\left(\frac{\partial P}{\partial T}\right)_\lambda \beta_p, \tag{11}$$

where $(\rho^{-1}\partial\rho/\partial P)_T$ is the isothermal compressibility; $(\partial S/\partial T)_\lambda$ is the temperature derivative of the entropy along the λ-line; $(\partial P/\partial T)_s$ and $(\partial P/\partial T)_\lambda$ are the derivatives of pressure along the saturated vapor pressure curve and along the λ-line respectively. The features of the behavior of β_p and C_p near the λ-point can be determined from the results obtained and Eqs. (10) and (11), and experimental values of the critical exponents α and α' and the coefficients in Eqs. (4) to (7) can be calculated.

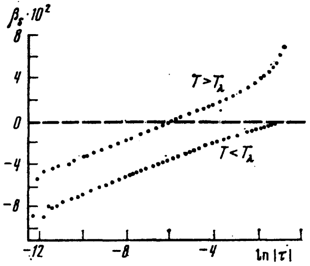

FIG. 3. Temperature dependence of the coefficient of thermal expansion of ^{4}He under the saturated vapor pressure.

The expansion coefficient at saturated vapor pressure was determined by measuring the shift in resonance frequency of the superconducting resonator filled with ^{4}He, using the method described above. Numerical values of β_s were determined from a formula derived from Eq. (1), connecting the temperature derivative of the permittivity at saturated vapor pressure $(d\varepsilon/dT)_s$ with β_s:

$$\beta_s=\left(\frac{d\varepsilon}{dT}\right)_s\frac{3}{(\varepsilon-1)(\varepsilon+2)}\frac{1+\alpha_1/\alpha_0V}{1+2\alpha_1/\alpha_0V}. \tag{12}$$

The experimental values of β_s over the whole temperature range are shown in Fig. 3. Only those values of β_s which were used to determine the form of the divergence of the thermodynamic quantities near the λ-point are shown in Table III. Some values of β_s in the range $|T-T_\lambda|<5\times10^{-5}$ K (separated by the horizontal line) were not used in the analysis since an inaccuracy in determining β_s at these temperatures can be produced by the simultaneous participation of the two phases of ^{4}He in each measurement. The experimental values obtained for β_s agree with measurements by other authors[7,14,27] within a few percent.

Treatment of the results by the least squares method showed that at a logarithmic temperature dependence β_s in the range $|T-T_\lambda|<4\times10^{-2}$ K it can be well represented by

$$10^3\beta_s=(8.21\pm0.02)\ln|\tau|+(47.75\pm0.15),\quad T>T_\lambda, \tag{13}$$

$$10^3\beta_s=(7.98\pm0.02)\ln|\tau|+(11.83\pm0.12),\quad T<T_\lambda. \tag{14}$$

TABLE III.

$\tau<0$	$10^2\beta_s$	$\tau>0$	$10^2\beta_s$
$-4.6\cdot10^{-6}$	−8.390	$5.5\cdot10^{-6}$	−5.363
$-7.3\cdot10^{-6}$	−7.942	$7.4\cdot10^{-6}$	−4.693
$-9.2\cdot10^{-6}$	−7.856	$1.2\cdot10^{-5}$	−4.373
$-1.2\cdot10^{-5}$	−8.145	$1.5\cdot10^{-5}$	−4.206
$-1.4\cdot10^{-5}$	−7.919	$2.5\cdot10^{-5}$	−3.970
$-1.8\cdot10^{-5}$	−7.475	$3.2\cdot10^{-5}$	−3.682
$-2.7\cdot10^{-5}$	−7.186	$5.06\cdot10^{-5}$	−3.334
$-3.73\cdot10^{-5}$	−6.989	$5.62\cdot10^{-5}$	−3.283
$-4.70\cdot10^{-5}$	−6.751	$7.60\cdot10^{-5}$	−3.022
$-6.91\cdot10^{-5}$	−6.483	$1.11\cdot10^{-4}$	−2.719
$-9.67\cdot10^{-5}$	−6.206	$1.62\cdot10^{-4}$	−2.376
$-1.34\cdot10^{-4}$	−5.921	$2.35\cdot10^{-4}$	−2.072
$-1.75\cdot10^{-4}$	−5.709	$3.23\cdot10^{-4}$	−1.800
$-2.49\cdot10^{-4}$	−5.439	$4.24\cdot10^{-4}$	−1.574
$-3.27\cdot10^{-4}$	−5.206	$6.58\cdot10^{-4}$	−1.227
$-4.10\cdot10^{-4}$	−5.050	$9.669\cdot10^{-4}$	−0.913
$-4.60\cdot10^{-4}$	−4.947	$1.331\cdot10^{-3}$	−0.630
$-5.43\cdot10^{-4}$	−4.831	$1.796\cdot10^{-3}$	−0.399
$-6.77\cdot10^{-4}$	−4.630	$2.233\cdot10^{-3}$	−0.230
$-8.747\cdot10^{-4}$	−4.416	$2.670\cdot10^{-3}$	−0.099
$-1.202\cdot10^{-3}$	−4.190	$3.513\cdot10^{-3}$	0.142
$-1.565\cdot10^{-3}$	−3.976	$4.098\cdot10^{-3}$	0.274
$-1.763\cdot10^{-3}$	−3.875	$6.031\cdot10^{-3}$	0.583
$-2.311\cdot10^{-3}$	−3.670	$9.162\cdot10^{-3}$	0.946
$-2.951\cdot10^{-3}$	−3.451	$1.151\cdot10^{-2}$	1.099
$-3.729\cdot10^{-3}$	−3.250	$1.483\cdot10^{-2}$	1.318
$-5.548\cdot10^{-3}$	−2.963	$1.851\cdot10^{-2}$	1.467
$-6.455\cdot10^{-3}$	−2.848		
$-7.827\cdot10^{-3}$	−2.671		
$-9.779\cdot10^{-3}$	−2.506		
$-1.303\cdot10^{-2}$	−2.293		
$-1.791\cdot10^{-2}$	−2.036		

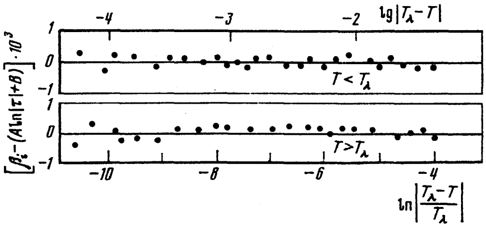

FIG. 4. Deviation of the experimental values of the isobaric thermal expansion coefficient from a logarithmic dependence near the λ-transition.

We used Eq. (10) which relates β_p and β_s in order to examine the nature of the divergence of the thermodynamic properties near the λ-point and to calculate the critical exponents α and α' and the coefficients A and A' and B and B' from the experimental results. The values of compressibility $(\rho^{-1}\partial\rho/\partial P)_T$ and of $(dP/dT)_s$ used in Eq. 10 were taken from other work.[8,28,29]

The temperature dependence of the isobaric expansion coefficient was determined by comparing values of β_p calculated from experiment with the analytical relations (4), (5) and (8), (9). We obtained the following values for the coefficients by comparing the experimental results with the logarithmic dependence of Eqs. (8) and (9) (see Fig. 4):

$$A=8.24\cdot10^{-3}\pm2\cdot10^{-5},\quad A'=8.03\cdot10^{-3}\pm2\cdot10^{-5},$$

$$B=4.60\cdot10^{-2}\pm1.4\cdot10^{-4},\quad B'=1.02\cdot10^{-2}\pm1.2\cdot10^{-4}.$$

It can be seen from this that a small but noticeable difference exists between the coefficients A and A' ($A/A'=1.026$), which is not exceeded by the sum of the systematic and random errors (taking account of systematic errors, $A/A'=1.026\pm0.013$). Such a difference between A and A' could be produced either by a small systematic error which was not taken into account or by a weak departure of β_p from a logarithmic temperature dependence as a result of the critical exponents α and α' not being zero.

A detailed analysis was therefore carried out of the agreement of the experimental results with the analytical relation $\beta_p\sim A\alpha^{-1}|\tau|^{-\alpha}+B$, and the critical exponents α, α' and the coefficients A, A' and B, B' in Eqs. (4) and (5) were evaluated. It follows from this analysis that at $T>T_\lambda$

$$\alpha=-0.006\pm0.0025,\quad A=(7.9\pm0.6)\cdot10^{-3},\quad B=1.35\pm0.5;$$

and at $T<T_\lambda$

$$\alpha'=0.000\pm0.0025,\quad A'=(7.95\pm0.5)\cdot10^{-3},\quad B'=4.86\pm0.3.$$

The values obtained for the critical exponents show that with high accuracy, the divergence of the thermodynamic properties of ^{4}He in the region of the λ-transition is, in all probability, of logarithmic form. The small departure of α from $\alpha=0$ at $T>T_\lambda$ in our work could be produced by a small systematic error in measuring β_s in the region near the maximum density, where the relative accuracy in determining the expansion coefficient was less than in the rest of the range over which β_s was measured.

We should note that critical exponents $\{\alpha, \alpha'\} \neq 0$ were obtained in a number of measurements[8,9,19] of the temperature dependences of C_p, C_s, and β_p. In order to reconcile α and α' with the theoretical values α_{th}, these authors used Eqs. (6) and (7) with additional correction terms. By choosing parameters A, A'; B, B' and D, D' for individual experiments, values of the critical exponents close to $\alpha_{th} = -0.026$ could be obtained within the experimental accuracy. In our view, however, such a procedure for treating the experimental results contains too many adjustable parameters and can give a non-zero value just because of the statistical scatter of experimental points.

In conclusion we should remark that the statistical method of treating the experimental results plays a not unimportant part in evaluating the critical exponents. Several adequate methods for doing this have been developed (see, for example, Refs. 8, 30, 31). In the present work an analysis similar to that of Ahlers[8] has been used. At the outset we set out goal at a confidence level (0.95) in order to obtain the most probable values of the critical exponents and for testing the hypotheses, and then determined the values of α, A, and B and their scatter.

5. CONCLUDING REMARKS

The measurements made of molar volume and expansion coefficient and studies of critical phenomena near the λ-transition of liquid helium are based on the application of the Clausius-Mosotti relation, from which information on density and its variations can be obtained from the temperature dependence of the permittivity. Both the instant of the λ-transition and small variations in density near T_λ can be recorded with high accuracy using the break in the $\varepsilon(T)$ curve. It can be seen that a similar method can be used successfully to study the properties of He-II so as to verify some predictions of the phenomenological theory of superfluidity near the λ-point[6] and their quantitative comparison with experiment. One of the most important problems in this field is the measurement of the shift in the temperature of the λ-transition in narrow gaps (films, capillaries etc) and the determination of the numerical value of the free parameter M which occurs in Ginzburg and Sobyanin's theory.[6] For example, the magnitude of the shift in T_λ as a function of the parameter M for a plane gap if thickness d is[6]

$$\Delta T_\lambda[\mathrm{K}] = T_\lambda - T_\lambda(d) = 2.53\cdot 10^{-11}\frac{[(3+M)/3]^{3/4}}{d^{3/2}}. \qquad (15)$$

Results available for porous media qualitatively confirm the existence of a $\Delta T_\lambda \sim d^{-3/2}$ relation for such a shift in T_λ. However, it is practically impossible to compare them quantitatively with the theoretical relations because of the uncertainty in the size of the pores. It would also be interesting to measure the difference in the densities of ^{4}He in a gap and in the bulk in order to study the influence of size effects on the change in helium density near the λ-transition.

By using the method described, direct measurement of ΔT_λ and of the change in density of liquid ^{4}He can be carried out in a single or in several gaps in the form of parallel-plate capacitors with accurate control of the width d [down to $\sim(1-2)\times 10^{-5}$ cm], so that quantitative comparison can then be made between theory and experiment.

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