Spatial spin resonance in amplitude-modulated magnetic fields

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Spatial spin resonance of polarized neutrons is investigated in a system with an amplitude-modulated alternating-sign magnetic field. Cases of exponential and Gaussian modulation are considered. It is shown that sideband maxima of the resonant spectrum are suppressed by these choices of modulation.

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1. INTRODUCTION

By spatial spin resonance (SSR) is usually meant the selective spin flip of polarized neutrons in a system of two mutually perpendicular magnetic fields that are constant in time. By selectivity is meant the fact that SSR takes place only for neutrons having a definite energy corresponding to resonance condition, in contrast to the resonance case, in which this dependences is much more weakly pronounced.

Spatial spin resonance was predicted in 1962 (Ref. 1) and was first confirmed experimentally in 1968 (Ref. 2). This phenomenon was later incorporated in a magnetic monochromator for polarized thermal neutrons.[3,4] Drabkin, Ruban, and Sbitnev[5,6] considered in detail the SSR and the methods of its realization, based on regarding this phenomenon as a Rabi resonance[7] in the coordinate frame of the moving neutron. This phenomenon was analyzed in Refs. 5 and 6 also on the basis of a group-theoretical approach. In the cited papers the SSR was considered in a system consisting of a homogeneous magnetic field and a perpendicular to a constant-amplitude periodic magnetic field. In this paper we examine the behavior of a neutron in a system with an amplitude-modulated alternating-sign magnetic field.

Let the neutron move in the system of the uniform magnetic field **H** perpendicular to a spatially oscillating field **h**. In the inertial coordinate frame connected with the moving neutron, the time-alternating field **h** can be regarded as a superposition of two fields rotating in opposite directions at a frequency

$$\omega=\pi V/\delta,$$

where V is the neutron velocity and δ is the spatial half-period of the field **h**. At $\omega=\Omega$ (Ω is the Larmor frequency of the neutron in the field **H**), a resonant spin flip takes place. And if $h\ll H$, the resonance is due mainly to the **h** component that rotates in the precession direction.[5,6] The Pauli equation for the spin amplitudes leads in this case to the system of equations[8]

$$\begin{aligned} i\hbar\dot{C}_\uparrow&=-\mu_n(HC_\uparrow+h_0e^{i\omega t}C_\downarrow),\\ i\hbar\dot{C}_\downarrow&=\mu_n(HC_\downarrow-h_0e^{-i\omega t}C_\uparrow), \end{aligned} \tag{1}$$

where $C_\uparrow$ and $C_\downarrow$ are the amplitudes of the states with spins parallel and antiparallel to the field H, respectively, μ_n is the magnetic moment of the neutron, and h_0 is the amplitude of the rotating component of the field **h** and is generally speaking time-dependent.

The exact solution of the system (1) for an arbitrary $h_0(t)$ dependence and for arbitrary ω has not been found. It was shown in Ref. 9, however, that there exist exact solution for an arbitrary amplitude envelope of the field **h**, provided the latter varies "slowly" compared with the rotation period, if the resonance condition $\omega=\Omega$ is satisfied. Kaplan[9] obtained also approximate solutions at arbitrary ω for two-level systems of quite general type.

We consider the case of exponential modulation:

$$h_0(t)=r_0\exp\{-a|t|\}\quad(0\ll a\ll\Omega),$$

which admits of an exact analytic solution.[11] Solving the system (1) under the initial conditions

$$C_\downarrow(-t_0)=0,\quad |C_\uparrow(-t_0)|=1\quad(t_0\geqslant0),$$

we obtain the spin-flip probability at $\omega=\Omega$ in the form

$$W(t)=|C_\downarrow(t)|^2=\sin^2\left[\frac{\Gamma}{a}-\frac{\Gamma}{2a}(e^{-at_0}+e^{-at})\right], \tag{2}$$

where $\Gamma=2\mu_n r_0/\hbar$. Let $t=t_0$. At

$$\frac{\Gamma}{a}(1-e^{-at_0})=\frac{\pi}{2}(2k+1),\quad k=0,1,\ldots \tag{3}$$

complete spin flip takes place.

In the absence of modulation we have[5,6]

$$W(t)=\frac{\Gamma^2}{\Gamma^2+(\omega-\Omega)^2}\sin^2\left\{\frac{t-t_0}{2}[\Gamma^2+(\omega-\Omega)^2]^{1/2}\right\}.$$

At $\omega=\Omega$ this expression goes over into (2) as $a\to0$, and the condition for total spin flip is

$$\Gamma t=\pi(2k+1),\quad k=0,1,\ldots$$

Let now $h_0(t)=r_0e^{-a^2t^2}$. It is impossible to solve exactly the system (1) in this case. At $\omega=\Omega$, however, the polarization can be obtained by using the result of Ref. 9 for a two-level system without relaxation, in the form

$$P=|C_\uparrow|^2-|C_\downarrow|^2=\cos\left(\int_{-t_0}^{t}r\,dt\right),$$

where r is the Larmor-precession frequency in a field of amplitude equal to that of the rotating component of the field **h**. Indeed, in the inertial frame of a moving neutron rotating at a frequency $\mathbf{\Omega}=(2\mu_n/\hbar)\mathbf{H}$, the magnetic field $H=0$. For resonant neutrons ($\omega=\Omega$), furthermore, the resonance component of the field **h** has a constant direction and the spin-flip process constitutes only precession in the field **h** through an angle

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$$\alpha = \int_{-t_0}^{t} r(t)\,dt.$$

If

$$\alpha = \pi(2k+1), \quad k=0, 1, \ldots, \tag{4}$$

total spin slip of the neutron takes place. For exponential modulation, the condition (4) leads to (3). For Gaussian modulation $r(t) = \Gamma \exp(-a^2t^2)$, and we obtain the condition for total spin flip in the form ($t = t_0$)

$$\frac{\Gamma}{a}\Phi(at_0) = \sqrt{\pi}(2k+1), \quad k=0, 1, \ldots, \tag{5}$$

where Φ is the Gaussian integral.

For an alternating-sign "square-wave" field **h** of the form

$$h(x) = (-1)^k h_0, \quad {}^1/_2\delta(2k-1) < x < {}^1/_2\delta(2k+1), \quad k=0, \pm 1, \ldots \tag{6}$$

The Fourier series is

$$h(x) = \frac{4}{\pi} h_0 (\cos Ux - {}^1/_3 \cos 3Ux + {}^1/_5 \cos 5Ux - \ldots).$$

The first harmonic enters in $h(x)$ with a coefficient $4/\pi$. Taking this into account, as well as the fact that at $h \ll H$ and $\omega = \Omega$ the spin rotation angle per period is proportional to the amplitude of the field **h** (Refs. 5 and 6), and changing from (3) and (5) for the rotating components of the field h to the lab frame, we obtain the conditions of total spin flip for the first harmonic in the form

$$\frac{\tau}{\mu}(1-e^{-\mu N/2}) = \frac{\pi}{4}(2k+1), \tag{7}$$

$$\frac{\tau}{\mu}\Phi\left(\mu\frac{N}{2}\right) = \frac{\sqrt{\pi}}{2}(2k+1). \tag{8}$$

Here $\tau = r_0/H \ll 1$ and μ is the coefficient of the change of the amplitude of **h** and satisfies the condition $\mu N/2 = at_0$. Here $(N+1)$ is the total number of the spatial half-periods of the field **h**. In particular at $\mu = 0$ we have, in accord with Ref. 5, $\tau N = \pi/2$. We note that at $\tau \ll 1$ the resonance condition ($\omega = \Omega$)

$$\pi V/\delta = 2\mu_n H/\hbar$$

remains unchanged when slow modulation ($\mu \ll 1$) is introduced.

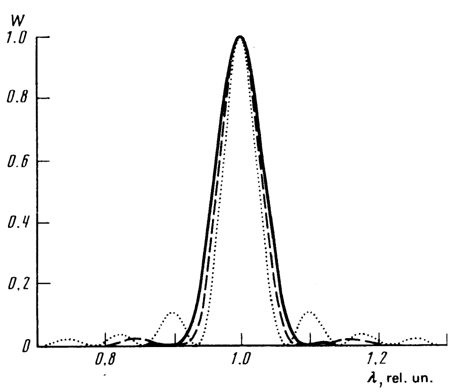

FIG. 1. Theoretical plots of the neutron spin-flip probability vs the relative wavelength in the absence of modulation (dotted line), for exponential modulation (dashed) and for Gaussian modulation (solid).

Figure 1 shows plots of the theoretically calculated dependences of the spin flip probabilities in SSR against the neutron relative wavelength for a square-wave field **h** of the type (6) without modulation and with parameters $\mu = 0$, $N = 27$, $\tau = 0.057$ (dotted line), with exponential modulation $\mu = 0.1$, $N = 27$, $\tau = 0.104$ (dashed line), and $\mu = 0.1$, $N = 27$, $\tau = 0.095$ with Gaussian modulation (solid line). The parameters μ, N, and τ are connected by the total spin-flip conditions (7) and (8). When modulation is introduced, the sidebands of the maxima decrease, and the half-width of the principal maximum increases while its amplitude remains constant. Given N, these changes manifest themselves to a greater degree in the case of Gaussian modulation. The calculations have shown also that with increasing N and at constant μ the sidebands vanish completely.

2. EXPERIMENTAL RESULTS

The resonator was constructed by the method described in Ref. 2: aluminum foil was placed between dielectric liners of equal thickness δ in fanfold form, the only exception being that the foil had initially a special profile [Fig. 2(a)] (the dashed lines show the folds). Figure 2(b) shows the file as folded, and when direct current flow through it an alternating-sign amplitude-modulated field is produced [Fig. 2(c)]. This magnetic field agrees qualitatively with the results of the theoretical calculation of the field along the longitudinal axis of the fanfold, under the condition that $N \gg 1$ and the liner thickness is small compared with the height and width of the fanfold and is large compared with the foil thickness.

The jump of the field h on the longitudinal axis on going through the foil between the k-th and $(k+1)$-st layers is

$$\Delta h_k = 4\pi I/cb_k, \tag{9}$$

where I is the current and b_k is the foil thickness at this

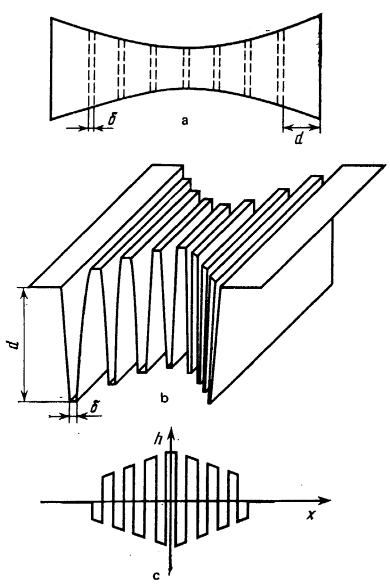

FIG. 2. a) Profiled foil (the dashed lines show the fold lines); b) fanfolded foil; c) distribution of magnetic field along the longitudinal axis of the fanfold.

point. Under the indicated assumption, and also under the condition $\mu \ll 1$, the magnetic field h changes insignificantly over the half-period and the profile of the fanfold can be calculated for the specific form of the modulation. The experiments were performed for Gaussian modulation, for which

$$h_0=r_0e^{-\mu^2k^2},\quad k=0,\pm1,\ldots\pm N/2.$$

From (9) we obtain

$$b_k=\frac{4\pi}{c}\frac{I}{r_0}[1+e^{-\mu^2(2k+1)}]^{-1}e^{\mu^2k^2}.$$

The resonant value of the field is obtained in the form

$$I_p=b_0r_0\frac{c}{4\pi}[1+e^{-\mu^2}],$$

where b_0 is the foil thickness at $k = 0$ and r_0 is determined from the total flip condition (8).

To investigate the spectral dependence of the spin-flip probability, an essembly whose setup is described in detail in Ref. 3 was placed in a horizontal channel of the VVR-M reactor. A spin-flipper and an SSR resonator were placed between two magnetized Fe–Co mirrors that acted as a polarizer and an analyzer. The spectral measurements were made by a time-of-flight procedure with a resolution $\Delta\lambda = 3\cdot10^{-2}$ Å. Four spectra were recorded:

1) $S_1(\lambda)$—the spectrum formed by the polarizer and analyzer when the spin-flipper is connected in the "antiparallel" position, at which adiabatic spin flip takes place in the entire wavelength range; the resonator was turned off ($h = 0$);

2) $S_2(\lambda)$—spin-flipper in the "parallel" position (there is no spin flip), the resonator is turned off;

3) $S_3(\lambda)$—resonance spectrum (spin-flipper in antiparallel position, resonator turned on);

4) $S_4(\lambda)$—spin flipper in parallel position, resonator turned on.

The resonator activating field was in all cases $H = 212$ Oe, corresponding to a resonant wavelength $\lambda = 2.9$ Å (at $\delta = 1.86$ mm); the current in the fanfold was $I_r = 19$ A.

The probability of spin-flip in the resonator as a function of the neutron wavelength was calculated from the formula

$$W(\lambda)=\frac{1}{2}\left[1+\frac{S_3(\lambda)-S_4(\lambda)}{S_2(\lambda)-S_1(\lambda)}\right].$$

We note that this equation does not depend on the probability of spin flipping by the flipper, nor on the polarizing ability of the polarizer and analyzer.

Figure 3 shows the experimental dependence of the spin-flip probability on the wavelength and the calculated curve (dashed) for a resonator with Gaussian modulation of the field **h**.

3. CONCLUSION

1. We have proposed and experimentally realized a procedure for obtaining alternating-sign amplitude-modulated magnetic fields with the aid of a profiled fanfolded current-carrying foil. By varying the profile it is possible to obtain an alternating-sign field modulated in accordance with various functions that change little within the period of the alternating-sign field, and thus influence strongly the shaper of the spectrum of the spatial spin resonance.

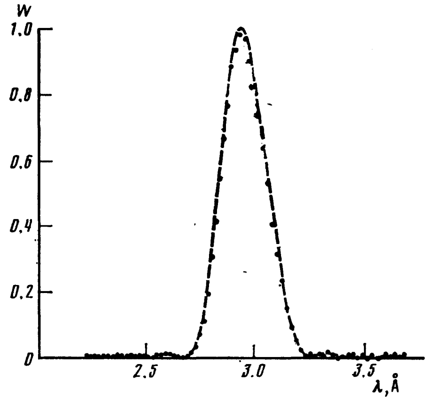

FIG. 3. Measured neutron spin-flip probability vs wavelength for Gaussian modulation of the field h. Dashed curve—theory.

2. We have shown that exponential or Gaussian modulation of the alternating-sign field of the SSR resonator suppresses the sidebands of the maxima of the resonance spectrum at a sufficiently large number of half-periods. Thus, for 27 half-periods and $\mu = 0.1$ the sideband amplitude is only 1.5% percent of the amplitude of the principal maximum. With increasing number of half-periods, their amplitude tends asymptotically to zero, and the half-width of the principal maximum tends to a constant value determined only by the field-variation coefficient μ. This makes it possible to increase the transmission and the spectral resolution of the magnetic monochromator proposed in Ref. 3 for polarized neutrons, without introducing additional elements into the system. However, owing to the background "pedestal,"[3,4] the minimum possible dispersion of the light is reached for resonance lines with half-width $\Delta\lambda/\lambda \approx 0.1$.

We point out the possibility of obtaining, in the coordinate frame of the neutron, of an amplitude-modulated magnetic field h. Such a field is produced by a system of two fanfolds described above [Fig. 2(b)], one imbedded in the other and shifted by one-quarter of the period and rotated 90° relative to each other. The activating field H needed to reproduce the SSR in this case should be directed along the neutron-beam axis.

In conclusion, we consider it our pleasant duty to thank G. M. Drabkin for suggesting the examination of SSR in amplitude-modulated magnetic fields. We are indebted also to V. A. Ruban for help with the work and for useful remarks, and to G. S. Golubev, T. I. Krivshich, G. A. Panev, D. N. Orlova, and V. Yu. Karasanidze for help with the experiment.

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