Tunnel states in an amorphous ferromagnet

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A model of an amorphous ferromagnet containing atoms that form two-level systems is considered. The time of relaxation of such systems via interaction with spin waves is calculated, and it is shown that this time can be shorter than the phonon relaxation time. The law governing the damping of the spin waves in such a system is determined and the magnetic contribution to the heat conduction is calculated. In a definite temperature range this contribution can exceed that of the phonons.

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1. INTRODUCTION

The low-temperature properties of amorphous substances are being explained at present on the basis of the concept of structural excitations of a special type, known as two-level systems. The idea of this description is due to Anderson, Halperin, and Varma[1] and to Phillips,[2] and consists of the assumption that the amorphous system contains a definite number of atoms (or groups of atoms) that can be on one of two levels and tunnel between these levels with participation of acoustic phonons. By making certain assumption concerning the statistical distribution function of the tunnel states, Anderson, Halperin, and Varma[1] calculated the contribution linear in the temperature to the heat capacity of glasses and the quadratic temperature dependence of the thermal conductivity, i.e., precisely the temperature dependences of the quantities experimentally observed in glass insulators.

Two-level systems were shortly after observed also in metallic glasses.[3,4]

It is assumed in the present paper that an amorphous ferromagnet also contains a number of atoms that form two-level systems. The atoms can be either magnetic or nonmagnetic. It is clear that the tunnel states due to the magnetic atoms modulate the exchange-interaction energy, the dipole energy, and the magnetic-anisotropy energy. If, however, the tunnel states are formed by nonmagnetic atoms, the latter, participating in the production of an indirect exchange interaction and of the crystal field, also modulate the exchange-interaction and the magnetic-anisotropy energies.[1)] Obviously, tunnel transitions in this amorphous ferromagnet should be produced not only by phonons but also by spin waves. It is these phenomena to which this paper is devoted. In particular, it has turned out that if the dipole-dipole interaction is neglected the spin-wave contribution to the thermal conductivity is proportional to $T^{5/2}$, but contains a large coefficient; A temperature region in which the spin-wave thermal conductivity exceeds that of the phonons can therefore exist. Furthermore, the spin-wave damping is also found to be anomalously strong and to have a rather strong momentum dependence.

A direct experimental check on this prediction of the theory is most important for the study of how the spin-states are made up in magnets. We wish to propose here one variant of such a verification, which we regard as quite attractive.

It is well known (see, e.g., the paper by Laermans *et al.*[7]) that bombardment of crystals by fast electrons and γ rays causes local amorphization and the onset of two-level systems. On the other hand, in ferrite crystals the spin-wave damping is investigated in detail with the aid of parametric-resonance experiments (Gurevich and Anisimov[8]). By irradiating a crystal whose spin-wave damping is well known, it is possible to track the variation of the damping as a function of the irradiation dose, and ascertain the role played in this case by the two-level systems.

It must be noted that the question of the interaction of spin waves with two-level systems in ferromagnets was recently considered by Continentino.[9] He, however, took into account only the modulation of the exchange interaction by two-level systems. But in this case, in view of the law of total-spin conservation, the spin waves can not cause transitions between tunnel states in the homogeneous limit ($k\to 0$). The tunnel modulation of the exchange interaction is therefore inessential in the low-temperature region and it is necessary to study the interaction of spin waves with tunnel states that modulate the magnetic anisotropy and the dipole forces. The smallness connected with the total-spin conservation law was not noted in Ref. 9, so that the energy and temperature dependences obtained there for the physical quantities do not hold in the exchange approximation. Furthermore, the quadratic approximation investigated in Ref. 9 for the spin-wave spectrums is not sufficient at low temperatures, and one must use for the spin-wave energy an exact expression that takes into the dipole forces, as will be done below.

2. CHOICE OF MODEL

We consider (for the sake of argument) a ferromagnet with "easy axis" anisotropy, described by the Hamiltonian

$$\mathscr{H}=-\frac{1}{2}\sum_{ij}J_{ij}\mathbf{S}_i\mathbf{S}_j-D\sum_j(S_j^z)^2$$
$$-\frac{1}{2}(g\mu_0)\sum_{ij}(\mathbf{S}_i\mathbf{S}_jR_{ij}^2-3(\mathbf{S}_i\mathbf{R}_{ij})(\mathbf{S}_j\mathbf{R}_{ij})),\qquad(1)$$

where S_i^α denotes the α-projection of the spin in the i-th lattice site ($\alpha=x, y, z$); J_{ij} is the exchange interac-

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tion of the spins at sites i and j; D is the magnetic anisotropy constant; D is the magnetic anisotropy constant; the z axis is the easy axis.

We write the two-level-system Hamiltonian, as usual,[10] with the aid of a Pauli matrix in the form

$$\mathscr{H}_\tau = {}^1/_2 (\Delta\sigma_z + \Delta_0\sigma_x), \tag{2}$$

where Δ is the difference between the level energies of the levels in two wells, and Δ_0 is the tunneling energy in the usual form[10]:

$$\Delta_0 = \omega_0 e^{-\lambda} \tag{3}$$

(we use a system of units in which $\hbar = k_B = 1$).

It is quite clear that the exchange-interaction energy, the dipole energy, and the magnetic-anisotropy energy are all different, depending on the potential minimum at which the tunneling object is located. Therefore the Hamiltonian of spin interaction with two-level systems can be represented in the form

$$\mathscr{H}' = \mathscr{H}_{ex}' + \mathscr{H}_a' = -\frac{1}{2}\sum_{ija} \delta J_{ij} \mathbf{S}_i \mathbf{S}_j \sigma_z^{(a)} - \sum_{ia} D_i (\mathbf{S}_i \mathbf{n}_i)^2 \sigma_z^{(a)}. \tag{4}$$

The summation here is over all the tunnel states and over the spins that interact with them. The vector $\mathbf{n}_i$ is the direction of the random anisotropy axis. Generally speaking, one should add to (4) the modulated part of the dipole forces, which decreases rapidly with distance (like R^{-4}). This part, however, does not lead to any new phenomena, and can therefore be taken into account effectively by redefining the constant D_i.

In the spin-wave approximation the Hamiltonian (1) takes the form (see, e.g., Ref. 11)

$$\mathscr{H} = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} \alpha_{\mathbf{k}}^+ \alpha_{\mathbf{k}}, \quad \varepsilon_{\mathbf{k}} = (A_{\mathbf{k}}^2 - |B_{\mathbf{k}}|^2)^{1/2}, \tag{5}$$

where ε_k is the spin-wave energy, and for the quantities A_k and B_k we have the expressions

$$A_{\mathbf{k}} = \alpha (ak)^2 + 2\beta\mu_0 M_0 + 2\mu_0 H_i + 4\pi\mu_0 M_0 \sin^2\theta_{\mathbf{k}},$$
$$B_{\mathbf{k}} = 4\pi\mu_0 M_0 \sin^2\theta_{\mathbf{k}} \exp(-2i\varphi_{\mathbf{k}}). \tag{6}$$

Here αa^2 is the spin-wave rigidity coefficient, a is the interatomic distance, α is a quantity of the order of the Curie temperature, M_0 is the saturation magnetization, and H_i is the internal magnetic field. The constant D in (1) was rewritten by us in the form $D = \beta\mu_0 M_0/S$, where now β is the magnetic-anisotropy constant.

The Hamiltonian (4) contains terms that describe transitions that take place between levels of two-level systems and are accompanied by emission and absorption of one or several spin waves, as well as terms describing the elastic and inelastic scattering of the spin waves. The volume part of the Hamiltonian describes in our approximation only the scattering processes. Obviously, the principal role in the low-temperature region is played by the single-quantum processes (the interlevel transitions accompanied by emission of two and more spin waves are small because the phase space is small). That part of the Hamiltonian (4) which corresponds to single-quantum processes is of the form

$$\mathscr{H}_{a1}' = -\frac{1}{4}(2S)^{3/2} \sum_{ia} D_i \sin 2\gamma_i [c_i^+ \exp(i\psi_i) + c_i \exp(-i\psi_i)] \sigma_z^{(a)}. \tag{7}$$

Here c_i and c_i^+ are the spin-deviation Bose operators and are connected with the spin-wave creation and annihilation operators by the known (u, v) transformation (see, e.g., Ref. 11), while γ_i and ψ_i are the azimuthal and polar angles of the vector $\mathbf{n}_i$.

Changing over in (7) to the spin-wave operators α_k^+ and α_k, we represent $\mathscr{H}_{a1}'$ in the form

$$\mathscr{H}_{a1}' = -\sum_{ia} D_i \sum_{\mathbf{k}} \{\alpha_{\mathbf{k}}^+ \Phi_{\mathbf{k}}^* \exp(-i\mathbf{k}\mathbf{R}_i) + \alpha_{\mathbf{k}} \Phi_{\mathbf{k}} \exp(i\mathbf{k}\mathbf{R}_i)\} \sigma_z^{(a)},$$
$$\Phi_k = u_{\mathbf{k}} \exp(-i\psi_i) + v_{\mathbf{k}} \exp(i\psi_i). \tag{8}$$

Here $D_i = 1/4(2S)^{3/2} D_i \sin 2\gamma_i$, and u_k and v_k are transformation coefficients given by[11]

$$u_{\mathbf{k}} = \left(\frac{A_{\mathbf{k}} + \varepsilon_{\mathbf{k}}}{2\varepsilon_{\mathbf{k}}}\right)^{1/2}, \quad v_{\mathbf{k}} = -\frac{B_{\mathbf{k}}}{|B_{\mathbf{k}}|}\left(\frac{A_{\mathbf{k}} - \varepsilon_{\mathbf{k}}}{2\varepsilon_{\mathbf{k}}}\right)^{1/2}. \tag{9}$$

It must be noted that in the case of a noncollinear ferromagnet (asperomagnet) the exchange part can acquire terms responsible for single-quantum transitions, but by virtue of the total-spin conservation in the exchange interaction they must inevitably vanish in the homogeneous limit $\mathbf{k} = 0$. The Hamiltonian exchange part analogous to (8) must therefore include the factor k omitted in Ref. 9. After diagonalizing the tunnel Hamiltonian (2), the interaction (8) takes the form

$$\mathscr{H}_{a1}' = \sum_a (\sigma_z^{(a)} Q_z^{(a)} + \sigma_x^{(a)} Q_x^{(a)}),$$
$$Q_\mu = \sum_{i\mathbf{k}} c_\mu^i [\alpha_{\mathbf{k}}^+ \Phi_{\mathbf{k}}^* \exp(-i\mathbf{k}\mathbf{R}_i) + \alpha_{\mathbf{k}} \Phi_{\mathbf{k}} \exp(i\mathbf{k}\mathbf{R}_i)], \tag{10}$$
$$c_x^i = D_i \Delta_0/E, \quad c_z^i = -D_i \Delta/E, \quad E = (\Delta_0^2 + \Delta^2)^{1/2}.$$

Thus, the final form of the Hamiltonian of our problem is

$$\mathscr{H} = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} \alpha_{\mathbf{k}}^+ \alpha_{\mathbf{k}} + \frac{1}{2}\sum_a E^{(a)} \sigma_z^{(a)} + \sum_a (\sigma_z^{(a)} Q_z^{(a)} + \sigma_x^{(a)} Q_x^{(a)}). \tag{11}$$

It should be noted that this Hamiltonian describes the principal effects of the interaction between spin waves and two-level systems in any ferromagnet, including noncollinear ones. In the latter case, however, the interaction of the two-level systems with the longitudinal fluctuations of the spins can also be significant. At present, however, we know almost nothing about such fluctuations, other than the experimental fact that they exist.

3. RELAXATION TIME OF A TWO LEVEL SYSTEM

The time of relaxation of a two-level system on account of spin-wave absorption and emission can be easily calculated from the Hamiltonian (18) by the same method used in the case of interaction of a two-level system with phonons 1, 2, 3. As a result we obtain from (9)–(11) for the two-level-system reciprocal lifetime governed by the part of the interaction proportional to σ_x

$$\frac{1}{\tau_m} = A\left(\frac{\Delta_0}{E}\right)^2 I(E) \operatorname{cth}\frac{E}{2T}, \tag{12}$$

where

$$A = \frac{1}{4\pi\alpha^{3/2}} (4\pi\mu_0 M_0)^{1/2} \overline{D_i^2},$$

$$I(E) = \frac{1}{2}\int_0^\pi d\theta \sin\theta \left\{\left[\sin^4\theta + \left(\frac{E}{4\pi\mu_0 M_0}\right)^2\right]^{1/2} - \left(\frac{\beta}{2\pi}\right)^2 - \sin^2\theta\right\}^{1/2},$$

where $\overline{D_i^2}$ is the average over both the angle γ_i and the quantity D_i. By regarding in this integral the energy E as being of the order of the temperature T assuming $T \gg \beta\mu_0/M_0$, we can neglect in $I(E)$ the contribution due to the anisotropy. Calculating next the integral for $E \gg 4\pi\mu_0 M_0$ and $E \ll 4\pi\mu_0 M_0$, we obtain for the relaxation time the expressions

$$\frac{1}{\tau_m}=\frac{1}{4\pi\alpha^{3/2}}\overline{D_i^2}\left(\frac{\Delta_0}{E}\right)^2 E^{1/2}\,\mathrm{cth}\frac{E}{2T} \quad (E\gg 4\pi\mu_0 M_0), \tag{13}$$

$$\frac{1}{\tau_m}=\frac{1}{8\alpha^{3/2}\sqrt{2}}\overline{D_i^2}\left(\frac{\Delta_0}{E}\right)^2 (4\pi\mu_0 M_0)^{-1/2} E\,\mathrm{cth}\frac{E}{2T} \quad (E\ll 4\pi\mu_0 M_0). \tag{14}$$

Equation (13) corresponds to the case when the dipole-dipole interaction is neglected, whereas (14) corresponds to the case when the dipole-dipole interaction is significant.[2]

We compare now the relaxation time introduced by the spin waves with the contribution due to processes in which phonons participate. The relaxation time due to the phonons is well known (we use the notation of Ref. 10)

$$\frac{1}{\tau_{ph}}=\left(\frac{\gamma_l^2}{c_l^5}+2\frac{\gamma_t^2}{c_t^5}\right)\frac{E^3}{2\pi\rho}\left(\frac{\Delta_0}{E}\right)^2 \mathrm{cth}\frac{E}{2T}=E\left(\frac{E}{E_0}\right)^2\left(\frac{\Delta_0}{E}\right)^2\mathrm{cth}\left(\frac{E}{2T}\right), \tag{15}$$

where E_0 is of the order of several dozen degrees. The contribution of the spin waves to the relaxation time will obviously prevail over the phonon contribution if the following conditions are satisfied

$$\frac{\overline{D_i^2}}{4\pi\alpha^2}\left(\frac{\alpha}{E_0}\right)^{1/2}\gg\left(\frac{E}{E_0}\right)^{5/2} \quad (E\gg 4\pi\mu_0 M_0), \tag{16}$$

$$\frac{\overline{D_i^2}}{8\alpha^2\sqrt{2}}\left(\frac{\alpha}{4\pi\mu_0 M_0}\right)^{1/2}\gg\left(\frac{E}{E_0}\right)^2 \quad (E\ll 4\pi\mu_0 M_0). \tag{17}$$

It follows from these inequalities that there can exist a temperature region in which the spin-wave relaxation is faster than that of the phonons, i.e., the two-level system "feels" only spin waves.

It must also be noted that in the general case in amorphous magnets the total lifetime of two-level systems is determined by both interaction processes:

$$1/\tau=1/\tau_{ph}+1/\tau_m.$$

4. SPIN-WAVE SPECTRUM

We study now the influence of the tunnel states on the energy spectrum of the spin waves. To this end we consider the dispersion equation for the spin waves

$$\omega-\varepsilon_k-\Sigma(\mathbf{k},\omega)=0,$$

where Σ is the self-energy part of the spin waves and is due to their interaction with the tunnel states.

$\Sigma(k,\omega)$ can be accurately determined by the technique developed in Ref. 13. However, just as in the phonon case (see, e.g., the review in Ref. 14), it is clear that in the lowest order of perturbation theory $\Sigma(k,\omega)$ consists of contributions of two types, resonance and relaxation. These contributions are obtained by averaging the following quantities over all the tunnel-state types:

$$\Sigma_{res}^{(0)}=\overline{D_i^2}\left(\frac{\Delta_0}{E}\right)^2|\Phi_k|^2\,\mathrm{th}\frac{E}{2T}\left(\frac{1}{\omega-E+i\delta}-\frac{1}{\omega+E+i\delta}\right), \tag{18}$$

$$\Sigma_{rel}^{(0)}=-\overline{D_i^2}\left(\frac{\Delta}{E}\right)^2|\Phi_k|^2\chi_{zz}(0)\frac{i/\tau}{\omega+i/\tau},$$
$$\chi_{zz}(0)=\frac{1}{T}\frac{e^{E/T}}{(1+e^{E/T})^2}. \tag{19}$$

These quantities are averaged by the standard method developed in Refs. 1 and 10. This averaging operation can be represented in the form

$$\bar{A}=\bar{P}_m\int_0^{E_m} dE E\int_{\Delta_{0min}}^{E}\frac{d\Delta_0}{\Delta_0(E^2-\Delta_0^2)^{1/2}}A(E,\Delta_0), \tag{20}$$

where $\bar{P}_m$ is the density of the tunnel states that modulate the magnetic anisotropy, E_m is their maximum energy, and Δ_{0min} is the minimum tunneling energy and is determined, for experiments that are fast enough, by their reciprocal times (a detailed discussion of this procedure can be found, e.g., in Ref. 10). In addition, in the calculation of the mean values it is necessary also to average over the polar angle φ_k of the spin wave, in which case the factor $|\Phi_k|^2$ is replaced by A_k/ε_k. As a result of the described procedure, we obtain for the resonance contribution to the self-energy of the spin wave in the case $\omega \ll E_m$

$$\Sigma_{res}(\mathbf{k},\omega)=-2\overline{D_i^2}\frac{A_k}{\varepsilon_k}\bar{P}_m V_0\left\{\ln\frac{E_m}{2\pi T}-\mathrm{Re}\,\psi\left(\frac{1}{2}+i\frac{\omega}{2\pi T}\right)\right\}-i\pi\bar{P}_m V_0\overline{D_i^2}\frac{A_k}{\varepsilon_k}\mathrm{th}\frac{\omega}{2T}, \tag{21}$$

where V_0 is the volume per magnetic atom and ψ is the logarithmic derivative of the Γ function. This expression is obviously valid in the case

$$|\Sigma(\mathbf{k},\varepsilon_k)|\ll\varepsilon_k. \tag{22}$$

For the energy and damping of the spin wave we have then

$$\tilde{\varepsilon}_k=\varepsilon_k-2\bar{P}_m V_0\overline{D_i^2}\frac{A_k}{\varepsilon_k}\left\{\ln\frac{E_m}{2\pi T}-\mathrm{Re}\,\psi\left(\frac{1}{2}+i\frac{\varepsilon_k}{2\pi T}\right)\right\},$$
$$\gamma_{res}^{(k)}=\pi\bar{P}_m V_0\overline{D_i^2}\frac{A_k}{\varepsilon_k}\mathrm{th}\frac{\varepsilon_k}{2T}. \tag{23}$$

The factor A_k/ε_k in this expression stems from allowance from the dipole forces in the unperturbed spin-wave spectrum. If the magnetic anisotropy is neglected, it becomes infinite as $\mathbf{k}\to 0$. The equations in (23) can therefore be correct only at a sufficiently high magnetic anisotropy, when the corrections to the spectrum are small. However, even in this case the correction to the spectrum may not be small because of the large logarithm in (23). The situation here is essentially the same as in the case of resonant interaction of the phonons (see the review in Ref. 14), where a similar large logarithm is present. This indicates that it may be necessary to investigate higher-order approximations in the density of two-level systems, where powers of large logarithms are present.

The relaxation contribution is more complicated. It follows from (13)–(15) that the relaxation time of the two-level system varies in a range from τ_{min} corresponding to $\Delta_0=E$ to τ_{max} at $\Delta_0=\Delta_{0min}$. In the case $\Delta_{0min}\ll T$ which is of greatest interest the relaxation part of Σ can be easily represented in the form

$$\Sigma_{rel}=-\overline{D_i^2}\frac{A_k}{2\varepsilon_k}\bar{P}_mV_0\int\limits_0^{E_m}\frac{dE}{T}\frac{e^{E/T}}{(1+e^{E/T})^2}\left\{\frac{1}{2}\ln\frac{(\omega\tau_{min})^2+1}{(\omega\tau_{max})^2+1}\right.$$

$$+\int\limits_0^1 dx\frac{x(\sqrt{1-x}-x)}{x^2+(\omega\tau_{min})^2}+i\omega\tau_{min}\left[\frac{1}{\omega\tau_{min}}\left(\operatorname{arctg}\frac{1}{\omega\tau_{min}}-\operatorname{arctg}\frac{1}{\omega\tau_{max}}\right)\right.$$

$$\left.\left.-\frac{1}{4}\ln\frac{(\omega\tau_{min})^2+1}{(\omega\tau_{max})^2+1}+\int\limits_0^1 dx\frac{\sqrt{1-x}+(x/2)-1}{x^2+(\omega\tau_{max})^2}\right]\right\}. \quad (24)$$

Comparing this expression with Σ_{res}, we see that owing to the large logarithm (21) the real part of Σ_{rel} is in practice always smaller than $\mathrm{Re}\Sigma_{res}$. At the same time, in the frequency region $\tau_{min}^{-1}(T)\gg\omega\gg\tau_{max}^{-1}$ the combination of arctangents in (24) is equal to $\pi/2$ and the contribution of Σ_{rel} to the spin-wave damping is

$$\gamma_{rel}^{(k)}=\overline{D_i^2}\frac{A_k}{4\varepsilon_k}\pi\bar{P}_mV_0. \quad (25)$$

This quantity is of the same order as $\gamma_{res}^{(k)}$; outside this interval the relaxation damping is negligibly small.

5. THERMAL CONDUCTIVITY AND INTERACTION ENERGY OF TWO-LEVEL SYSTEMS

Obviously the equation for the heat capacity of two-level systems remains unchanged when the interaction with the spin waves is turned on. The situation with the thermal conductivity is here different.

The thermal conductivity due to spin waves can be easily estimated by using the known formula $\varkappa\propto(1/3)C_s l v$, where C_s is the spin heat capacity, v is the spin-wave group velocity, and l is their mean free path. It is convenient here to consider two cases. We first neglect the dipole-dipole interaction; this corresponds to temperatures $T\gg4\pi\mu_0M_0$. In addition, we assume in both cases that the influence of the gap in the spin-wave spectrum is negligibly small, i.e., we consider our system at temperatures higher than the magnetic-anisotropy energy ($T\gg\beta\mu_0M_0$). In this case we obtain for the thermal conductivity the estimate

$$\varkappa\propto\frac{1}{a\bar{P}_mV_0}\frac{\alpha^2}{\overline{D_i^2}}\left(\frac{T}{\alpha}\right)^{5/2}. \quad (26)$$

Comparing this expression with the phonon contribution to the thermal conductivity,

$$\varkappa_{ph}=\frac{\rho}{6\pi\bar{P}_{ph}}\left(\frac{c_l}{\gamma_l^2}+\frac{c_t}{\gamma_t^2}\right)T^2, \quad (27)$$

we see that owing to the large coefficient in (26) the spin-wave contribution to the thermal conductivity, at temperatures

$$T>\alpha\left(\frac{\bar{P}_m}{\bar{P}_{ph}}\right)^2\left(\overline{D_i^2}V_0\frac{\rho ca}{\gamma^2}\right)^2$$

exceeds the phonon contribution.

In the other limiting case, when the dipole-dipole interaction is significant, i.e., at $\beta\mu_0M_0\ll T\ll4\pi\mu_0M_0$, an estimate of the thermal conductivity due to the spin waves yields

$$\varkappa\propto\frac{1}{a\bar{P}_mV_0}\frac{\alpha^2}{\overline{D_i^2}}\left(\frac{T}{4\pi\mu_0M_0}\right)\left(\frac{T}{\alpha}\right)^{5/2}. \quad (28)$$

In this case a comparison with the phonon contribution (26) shows that the temperatures at which $\varkappa>\varkappa_{ph}$ must satisfy the inequality

$$T>\alpha\left(\frac{4\pi\mu_0M_0}{T}\right)^2\left(\frac{\bar{P}_m}{\bar{P}_{ph}}\right)^2\left(\overline{D_i^2}V_0\frac{\rho ca}{\gamma^2}\right)^2.$$

At lower temperatures $T\sim\beta\mu_0M_0$ we can no longer neglect the gap in the spin-wave spectrum, and the spin-wave contribution to the thermal contribution is exponentially small in this temperature region.

The resultant picture for the thermal conductivity of an amorphous ferromagnet is thus the following. At low temperatures the thermal conductivity is governed by the phonons and has a quadratic temperature dependence. With rising temperature, a large contribution is made to the thermal conductivity by the term due to the spin waves, and at sufficiently high temperatures the thermal conductivity is proportional to $T^{5/2}$.

We consider, finally, the question of the interaction of tunnel centers on account of spin-wave exchange, using the standard perturbation-theory method. From Eqs. (5), (6), and (8)–(10) we obtain the equation

$$\mathscr{H}_{int}=-\frac{1}{2}\sum_{a\neq b}\sum_{\mu\nu}J_{\mu\nu}^{ab}\sigma_\mu^{(a)}\sigma_\nu^{(b)},$$

$$J_{\mu\nu}^{ab}=c_\mu^a c_\nu^b\cos(\psi_a-\psi_b)\frac{v_0}{(2\pi)^3}\int dk\exp(ik\mathbf{R}_{ab})$$

$$\times\frac{1}{\alpha(ak)^2+2\beta\mu_0M_0+2\mu_0H_i}\left[1-\frac{4\pi\mu_0M_0\sin^2\theta_k}{\alpha(ak)^2+2\beta\mu_0M_0+2\mu_0H_i+8\pi\mu_0M_0\sin^2\theta_k}\right]. \quad (29)$$

The second term in the square brackets of this expression is smaller than 1/2, and allowance for it does not change the qualitative picture of the interaction between the centers. This interaction takes the same form as the known Suhl-Nakamura interaction, i.e., at large distances it is proportional to $R_{ab}^{-1}\exp(-q_0R_{ab})$, where

$$q_0=\frac{1}{a}[(2\beta\mu_0M_0+2\mu_0H_i)\alpha^{-1}]^{1/2},$$

but the presence of the factor $D_aD_b\cos(\psi_a-\psi_b)$ makes the sign of the interaction uncertain. We note also that such a simple formula for $\mathscr{H}_{int}$ can be used at $\mu\neq\nu$ only when the distances between centers are not too large; the corresponding criterion can be written in the form

$$|E_a-E_b|\ll\varepsilon_{k=R_{ab}^{-1}}. \quad (30)$$

In conclusion one of the authors, Yu. N. Skryabin of the Metal Physics Institute of the Urals Scientific Center of the USSR Academy of Sciences, is grateful to the Leningrad Institute of Nuclear Physics for hospitality.

[1]It is well known (see, e.g., Refs. 5 and 6) that random anisotropy destroys the long-range magnetic order. A constant magnetic field or else a constant magnetic anisotropy, however, ensure stability of the ferromagnetic ground state. The existence of this stabilization is in fact assumed hereafter.

[2]An expression similar to (13), with replacement (in our notation) of $\overline{D_i^2}$ by $\overline{(\delta J)^2}$, was obtained in Ref. 9 in the exchange approximation. As already noted, however, in this approximation the interaction between spin waves and two-level systems in weak at low energies; in the corresponding expression it is then necessary to replace $\overline{D_i^2}$ by $\overline{(\delta J)^2}E/\alpha$. At high energies the exchange approximation becomes decisive.

Corresponding to this case are the equations obtained in this paper, provided $\overline{D_i^2}$ is replaced in them by $\overline{(\delta J)^2}E/\alpha$ or by $\overline{(\delta J)^2}T/\alpha$.

[1]P. W. Anderson, B. I. Halperin, and C. M. Varma, Phil. Mag. **25**, 1 (1972).

[2]W. A. Phillips, J. Low Temp. Phys. **7**, 351 (1972).

[3]S. Hunklinger, J. Physique **39**, C6, 1444 (1978).

[4]J. R. Matey and A. C. Anderson, Phys. Rev. **B16**, 3406 (1977).

[5]Y. Imry and S. K. Ma, Phys. Rev. Lett. **35**, 1399 (1975).

[6]R. A. Pelcovits, E. Pytte, and J. Rudnick, *ibid*. **40**, 476 (1978).

[7]C. Laermans, A. M. de Goer, and M. Locatelli, Phys. Lett. **80A**, 331 (1980).

[8]A. G. Gurevich and A. N. Anisimov, Zh. Eksp. Teor. Fiz. **68**, 677 (1975) [Sov. Phys. JETP **41**, 336 (1975)].

[9]M. A. Continentino, J. Phys. **C14**, 3527 (1981).

[10]J. L. Black and B. I. Halperin, Phys. Rev. **B16**, 2879 (1977).

[11]A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskii, Spin Waves, Wiley, 1968.

[12]Y. Ishikawa, S. Onodera, and K. Tajima, Sol. St. Commun. **38**, 561 (1981).

[13]S. V. Maleev, Zh. Eksp. Teor. Fiz. **79**, 1995 (1980) [Sov. Phys. JETP **52**, 1008 (1980)].

[14]S. Hunklinger and W. Arnold, in: Physical Acoustics, R. N. Thurston and W. P. Mason, eds., Academic, 1976, Vol. 12, p. 155.

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