Vacuum energy and large-scale structure of the Universe

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Small longitudinal quantum fluctuations of the metric in cosmological models with metastable vacuum are considered. A Hamiltonian formalism is constructed and the system of small perturbations quantized. The single-loop corrections are taken into account in the Einstein equations. The stability of the models with respect to perturbations of the metric is investigated. The spectrum of fluctuations of the metric is calculated. Under certain conditions, the amplitude of the spectrum is sufficient for the formation of galaxies during the hydrodynamic stage after the decay of the polarized vacuum.

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1. INTRODUCTION

The significant progress in the construction of a unified theory of the electromagnetic, weak, and strong interactions achieved following the clearer understanding of the part played by the gauge properties of these interactions[1] has led to new and rather unusual ideas concerning the early stages in the evolution of our Universe. New possibilities have appeared for solving a number of important cosmological problems associated with the baryon asymmetry of the Universe[2] and its homogeneity and isotropy.[3] In addition, we have got nearer to solving two other problems: How and from what perturbations were the galaxies and clusters of galaxies formed?

In the present paper, we consider the problem of the initial inhomogeneity spectrum and investigate the connection between this spectrum and the observed large-scale structure of the Universe. It should be noted that at the present time the problem of the initial fluctuation spectrum is solved in a rather pragmatic manner[4] in theories of galaxy formation. In fact, the perturbation spectrum and its amplitude are chosen to ensure the observed parameters of the galaxies and clusters. However, it would be more satisfactory to avoid introducing arbitrary additional assumptions into the theory and instead obtain the initial spectrum from fundamental principles, restricting oneself to fluctuations that are unvoidably present on the background of a homogeneous and isotropic Universe. What is the possible nature of such perturbations? The most natural would appear to be quantum-mechanical fluctuations of the metric. They are necessarily present in any system possessing a finite energy density, and their existence is intimately related to the uncertainty principle.[5] One also cannot rule out the possibility of thermal fluctuations. However, their existence on the scale of galaxies raises serious doubts, since at the Planck time the co-moving galactic scales greatly exceed the characteristic sizes of the causally connected regions.[6] Therefore, in what follows we shall restrict ourselves to considering only quantum-mechanical perturbations.

In our earlier paper[7] we showed that in a Universe filled with hydrodynamic matter, with equation of state $p = p(\varepsilon)$ satisfying the condition $p + \varepsilon \sim \varepsilon$, quantum fluctuations of the metric are inadequate for the formation of galaxies. Gauge theories of interactions open up new possibilities. These theories predict the existence of "quasivacuum" stages in the early evolution of the Universe.[8,9] During these stages, the dynamical evolution of the Universe is determined by the vacuum energy with effective equation of state $p_v = -\varepsilon_v$. Under the assumption that hydrodynamics is applicable, we demonstrated earlier the fundamental possibility of obtaining fairly large amplitudes of inhomogeneities in models with a quasivacuum stage.[7] However, due to the limitations of the hydrodynamic approximation, we did not create a complete theory capable of describing as well small quantum-mechanical perturbations in the quasivacuum stage in an empty Universe (without particles), in which hydrodynamic fluctuations do not exist. In the present paper, operating in the framework of purely field models, we attempt to construct such a theory.

Our paper is arranged as follows. In Sec. 2, we consider the example of a "typical" gauge theory, which leads to the presence of a quasivacuum stage in the evolution of the Universe. In Sec. 3, we describe the initial background model of the Universe. In Sec. 4, we identify the true physical degree of freedom characterizing the system of small longitudinal perturbations of the metric, describe the evolution of classical perturbations, and derive a variational principle. In Sec. 5, we present the procedure of canonical quantization. In Sec. 6, we consider the stability of the original cosmological model with respect to vacuum fluctuations of the metric. The spectrum of pregalactic inhomogeneities due to the occurrence of zero-point vibrations is calculated in Sec. 7.

2. VACUUM ENERGY IN GAUGE THEORIES

Gauge theories of elementary interactions have the consequence that during the very early stages in the evolution of our Universe the vacuum could have had a nonvanishing energy density. Such possibilities were analyzed in the framework of unified theories by Kirzhnits, Linde, and others (see the review of Ref. 8). Let us consider the simplest Lagrangian with local Abelian $U(1)$ symmetry:

$$\mathscr{L} = |D_\nu \varphi|^2 - {}^1/_2 \lambda^2 (|\varphi|^2 - {}^1/_2 \sigma^2)^2 - {}^1/_4 F^{\mu\nu} F_{\mu\nu}, \tag{2.1}$$

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where φ is a complex scalar field, $D_\nu = \partial_\nu - ieA_\nu, F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, where A_ν is the potential of the vector field. The scalar Higgs field φ in the Lagrangian (2.1) (or set of fields performing the same functions) is a necessary attribute of all renormalizable gauge theories.[1,8] Because of the specific self-interaction potential of this field,

$$V(\varphi) = {}^1/_2\lambda^2(|\varphi|^2 - {}^1/_2\sigma^2)^2 \tag{2.2}$$

the minimum of the potential corresponds to the existence of a homogeneous nonvanishing expectation value $|\varphi| = \sigma/\sqrt{2}$ of the field.

When allowance is made for the single-loop corrections, the state of the scalar field will be determined by an "effective" potential, which is the sum of the "classical" potential (2.2) and the σ-dependent shift of the vacuum energy density due to the zero-point (or thermal) vibrations of the field φ. In this case, if the coupling constants satisfy certain relations, the "effective" potential can have an additional local minimum at $\varphi = 0$ (see Fig. 1).[8] If the depth of this minimum is sufficiently small, the state with $\varphi = 0$ is metastable and has nonvanishing vacuum energy density:

$$\varepsilon_{\varphi v} = V(\varphi = 0) = m_\varphi^2 m_{A_\nu}^2 / 8e^2. \tag{2.3}$$

The vacuum energy-momentum tensor $T^i_{k\varphi v} = \varepsilon_{\varphi v}\delta^i_k$ corresponds to the Λ term in Einstein's equations. As is readily seen in (2.3), the vacuum energy density is determined by the characteristic masses and coupling constants. Therefore, in principle, there could exist in the Universe condensates corresponding to the different types of interaction and possessing different energy densities. In particular, the vacuum energy density in a grand unification theory could be comparable with the Planck density ($\varepsilon_{\rm Pl} \sim 10^{93}$ g/cm^3), since the masses that occur in the various forms of the theory may be comparable with the Planck mass.[1,9,10]

Suppose the potential of some scalar field φ has a local minimum as shown in Fig. 1. As an example, let us consider a Universe that begins its expansion from the state with $\varphi = 0$. This could be due, for example, to the restoration of symmetry at high temperatures (for a detailed discussion of the various possibilities, see the review of Ref. 8). At the start of expansion, ordinary hydrodynamic matter could also be present in the Universe. However, its density falls during the process of expansion, and after a fairly short period of time this matter can be ignored and only the purely vacuum stage considered.[7]

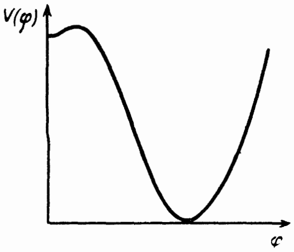

FIG. 1. Characteristic form of the effective potential $V(\varphi)$ of the scalar field φ in gauge theories.

Because the state with $\varphi = 0$ is metastable, this vacuum stage lasts for only a finite time, after which the Universe must make a phase transition and go over to the state with $|\varphi| \sim \sigma\sqrt{2}$. Then the vacuum energy is transformed into the energy of ordinary matter, and the Universe enters the hydrodynamic (Friedmann) expansion regime.

We now consider the question of the fluctuations of the metric in the vacuum stage associated with the perturbation of the condensate field φ. In the first order in $\delta\varphi$, the perturbations of the vacuum energy-momentum tensor are equal to zero:

$$\delta T^{\,i}_{k\,\varphi v} = \frac{\partial V}{\partial \varphi}(\varphi = 0)\,\delta\varphi\delta_k{}^i = 0 \tag{2.4}$$

and, therefore, the corresponding longitudinal adiabatic perturbations of the metric will also have the value zero. At the first glance, it might therefore appear that there will be no longitudinal fluctuations of the metric in the linear approximation. However, it can be shown that this is not the case. As was demonstrated in Ref. 7, the hope of obtaining fairly large fluctuations can be justified only when the density of the vacuum condensate does not differ strongly from the Planck density. And it is well known that at such densities the single-loop polarization corrections to Einstein's equations are important. It is allowance for these corrections that gives rise to nonvanishing quantum longitudinal fluctuations of the metric already in the linear approximation.

To conclude this section, we emphasize that the main results of the paper relating to the spectrum of pregalactic inhomogeneities will be independent of the concrete assumptions made concerning the gauge theories. Their validity rests solely on the existence in the Universe of a quasivacuum stage with a definite energy density of the vacuum condensate that is transformed into the energy of relativistic particles at a certain time as a result of a phase transition.

3. BACKGROUND COSMOLOGICAL MODEL

We consider a homogeneous isotropic Universe—for simplicity of zero spatial curvature[1)]—with metric

$$ds^2 = g_{ik}dx^i dx^k = a^2(\eta)\left(d\eta^2 - \sum_{\alpha=1}^{3}(dx^\alpha)^2\right), \tag{3.1}$$

where η is the conformal time.

As was noted in Sec. 2, the contribution of the energy-momentum tensor of the ordinary matter can be ignored in the quasivacuum stage. Therefore, in the Einstein equations describing this stage we take into account only the energy of the vacuum condensate and the contribution of the polarized conformal fields in the strong gravitational field:

$$R_k{}^i - {}^1/_2\delta_k{}^i R = 3l_{\rm Pl}^2(\varepsilon_{\varphi v}\delta_k{}^i + \langle T_k{}^i\rangle_v), \tag{3.2}$$

where $h = c = 1$, $l_{\rm Pl} = (8\varkappa G/3)^{1/2}$ is the Planck length, and $\varepsilon_{\varphi v} = $ const is the vacuum energy, which remains constant during the expansion process. In the conformally flat metric (3.1), there is no production of massless particles, and the energy-momentum tensor of the polarized quantum fields, $\langle T^i_k\rangle_v = \langle T^i_k\rangle_{v1} + \langle T^i_k\rangle_{v2}$ consists of solely of local terms that arise in the regularization process[11]:

$$\langle T_k^{\ i}\rangle_{v1}=\frac{1}{3l_{Pl}^2H^2}\left(R_l^{\ i}R_k^{\ l}-\frac{2}{3}RR_k^{\ i}-\frac{1}{2}\delta_k^{\ i}R_m^{\ l}R_l^{\ m}+\frac{1}{4}\delta_k^{\ i}R^2\right),$$

$$\langle T_k^{\ i}\rangle_{v2}=-\frac{1}{18l_{Pl}^2M^2}\left(2R^{;i}_{\ ;k}-2\delta_k^{\ i}R^{;l}_{\ ;l}-2RR_k^{\ i}+\frac{1}{2}\delta_k^{\ i}R^2\right),$$

(3.3)

where $H^2=360\pi/Gk_2$, $M^2=-360\pi/Gk_3$, and the constants k_2 and k_3 are determined by the contribution of the quantum fields with different spins. The solutions of Eqs. (3.2)–(3.3) are meaningful only when

$$|R^{ik}R_{ik}|,\ |R^{iklm}R_{iklm}|,\ldots \ll l_{Pl}^{-4},$$

since the polarization terms in (3.3) are written down in the single-loop approximation. To justify the applicability of the single-loop approximation, we assume that $(Hl_{Pl})^2$, $(Ml_{Pl})^2 \ll 1$. These conditions will be satisfied if the number of elementary fields is sufficiently large and there is no special canceling of the contributions of the different fields to the polarization tensor $\langle T^i_{\ k}\rangle_v$.

For the stability of Minkowski space, which is obvious, is necessary for k_3 to be negative ($k_3<0, M^2>0$). If $\varepsilon_{\varphi v}<\frac{1}{4}(Hl_{Pl})^2\varepsilon_{Pl}$ ($\varepsilon_{Pl}=l_{Pl}^{-4}$ is the Planck density) and $H^2>0$, then it can be shown that the system of equations (3.2)–(3.3) has solutions of de Sitter type:

$$a(\eta)=-\frac{1}{\varkappa\eta},\qquad -\infty<\eta<0,$$

$$\varkappa_{1,2}=\frac{H}{\sqrt{2}}\left(1\pm\left[1-\frac{4\varepsilon_{\varphi v}l_{Pl}^2}{H^2}\right]^{1/2}\right)^{1/2}, \qquad (3.4)$$

$$R_\beta^{\ \alpha}=R_\alpha^{\ \beta}=-3\varkappa^2\delta_\beta^{\ \alpha},\quad R=-12\varkappa^2.$$

In the most interesting case, $\varepsilon_{\varphi v}\ll(Hl_{Pl})^2\varepsilon_{Pl}$, the solution with $\varkappa_1\approx H$, which was proposed by Starobinsky in Ref. 12, describes a de Sitter stage in which the energy of the polarized vacuum of the conformal fields is dominant ($\langle T^i_{\ k}\rangle_v\sim(Hl_{Pl})^2\varepsilon_{Pl}\gg\varepsilon_{\varphi v}$). Note that the homogeneity and isotropy of the Universe does not make this stage unavoidable. Its existence must be assumed by choosing appropriate initial conditions.

The other solution with $\varkappa_2\approx l_{Pl}\varepsilon_{\varphi v}^{1/2}$ corresponds to a Universe whose dynamical evolution is determined by vacuum energy that arises in gauge theories ($\varepsilon_{\varphi v}\gg\langle T^i_{\ k}\rangle_v\sim(\varepsilon_{\varphi v}/(Hl_{Pl})^2\varepsilon_{Pl})\varepsilon_{\varphi v}$). If the relations between the parameters of the gauge theories are such that the formation of a condensate with nonvanishing energy density is unavoidable, then a homogeneous isotropic Universe will necessarily pass through a de Sitter stage of this type.

In the present paper, we concentrate mainly on the question of the fluctuations of the metric in the de Sitter stage whose existence is due to the scalar fields ($\varkappa\approx l_{Pl}(\varepsilon_{\varphi v})^{1/2}$). The results associated with the quantum fluctuations in the model proposed by Starobinsky[12] ($\varkappa\approx H$) have been given in our preceding paper Ref. 13.

We note that the metric (3.1) with scale factor (3.4) can be reduced to a static form by a coordinate transformation.[14] The de Sitter Universe with the metric (3.4) is characterized by a constant negative 4-curvature and the absence of matter, and formally it does not have a singularity. However, the assumption that there is ordinary matter in the Universe or allowance for the finite time during which the metastable vacuum exists ensures a singular nature of the cosmological solutions.[7,13]

4. EVOLUTION OF CLASSICAL PERTURBATIONS

We consider the small perturbations of the metric (3.1), (3.4) satisfying Eqs. (3.2). We shall make our analysis in a synchronous frame with $\delta g_{i0}=0$, $\delta^g_{\alpha\beta}=-h_{\alpha\beta}$.

In the general case, the polarization tensor $\langle T^i_{\ k}\rangle_v$ contains additional terms compared with (3.3), including perturbations of the Weyl tensor.[15] However, for the de Sitter Universe (3.4) these corrections to $\langle T^i_{\ k}\rangle_v$ are zero in the linear approximation. Therefore, to investigate the perturbations in the given concrete case we can use the expressions (3.3) for $\langle T^i_{\ k}\rangle_v$. Using (2.4) and linearizing Eqs. (3.2) and (3.3), we obtain the following equations for small perturbations on the background of the metric (3.4):

$$\delta R_0^{\ 0}=\frac{1}{CM^2a^2}\left(\frac{a'}{a}\delta R'-\frac{1}{3}\delta R_{,\alpha\alpha}\right)+\frac{\varkappa^2}{C}\left(\frac{1}{M^2}-\frac{1}{H^2}+\frac{1}{2\varkappa^2}\right)\delta R, \qquad (4.1a)$$

$$\delta R_0^{\ \alpha}=\frac{1}{3CM^2a}\left(\frac{1}{a}\delta R_{,\alpha}\right)', \qquad (4.1b)$$

$$\delta R_\beta^{\ \alpha}-\frac{1}{2}\delta_\beta^{\ \alpha}\delta R=\frac{1}{3CM^2a^2}\left[\delta_\beta^{\ \alpha}\left(\delta R''+\frac{a'}{a}\delta R'-\delta R_{,\gamma\gamma}-3\varkappa^2a^2\delta R\right)+\delta R_{,\alpha\beta}\right], \qquad (4.1c)$$

where $C=1+4\varkappa^2/M^2-2\varkappa^2/H^2, C\neq0$, the prime denotes differentiation with respect to η, and a comma followed by an index the partial derivative with respect to the corresponding x^α.

For longitudinal (scalar) perturbations, δR does not vanish.[2)] In the general case, the linearized system of equations (4.1), written down for such perturbations of the metric, is a system of two equations, each of the fourth order. However, as we shall see subsequently, in the case of the de Sitter metric (3.4) (and in Minkowski space) the solution of the system (4.1) reduces ultimately to solution of a second-order equation for δR. It is obtained by linearizing the contracted equations (3.2):

$$\delta R''+2\frac{a'}{a}\delta R'-\Delta\delta R+M^2a^2\left(1-2\frac{\varkappa^2}{H^2}\right)\delta R=0. \qquad (4.2)$$

In Minkowski space, for which $a=1$, the coefficient in front of δR is M^2. In this case, the equation analogous to (4.2) describes scalar particles with mass M.

The system of equations (4.1c) is complete for determining the evolution of perturbations $h_\alpha^{\ \beta}=a^{-2}h_{\alpha\beta}$ of the metric. For the quantization, we must find the action for the considered system of perturbations. The polarization tensor $\langle T^i_{\ k}\rangle_{v2}$ is obtained by varying terms of the type $\int R^2\sqrt{-g}\,d^4x$ and is conserved: $\langle T^i_{\ k}\rangle_{v2;i}=0$.[17] The tensor

$\langle T^i_k\rangle_{v1}$ is conserved only in the case of a homogeneous, isotropic Universe. However, it is easy to see that in the first order in the perturbations $\langle T^i_k\rangle_{v1}$ is conserved in the Universe we are considering: $\langle T^{i(0)}_k + \delta T^i_k\rangle_{v1;i} = 0$. Therefore, for $\langle \delta T^i_k\rangle_{v1}$ there must also exist a variational principle.

We find the action for small perturbations in the same way as in our previous paper Ref. 7, expanding the total action to second order in $h^\alpha_\beta, h^{\alpha\prime}_\beta$, etc:

$$\delta S_b = -\frac{1}{6l_{Pl}^2}\Big[C\int d^4x\, a^2\Big\{\frac{1}{2}(h^{,\pi}h_{\pi,\gamma} - h^{\gamma}_{\varkappa,\delta}h^{\delta,\varkappa}_{\gamma}) + \frac{1}{4}(h^{\gamma}_{\delta,\varkappa}h^{\delta,\varkappa}_{\gamma} - h^{,\pi}h_{,\pi}) + \frac{1}{4}(h'^2 - h_\beta{}^{\alpha\prime}h_\alpha{}^{\beta\prime})\Big\}$$
$$-\frac{1}{6M^2}\int d^4x\Big\{h'' + 3\frac{a'}{a}h' + h^{\gamma,\pi}_{\pi,\gamma} - h^{,\pi}_{,\pi}\Big\}^2 + \int d^4x(*)\Big], \quad (4.3)$$

where the symbol (*) stands for divergent terms, and $h^\alpha_{\beta,\gamma} = h^{\alpha,\gamma}_\beta = \partial h^\alpha_\beta/\partial x^\gamma$. Varying (4.3) with respect to h^α_β, we obtain Eqs. (4.1c).

We consider perturbations h^α_β of potential type:

$$h_\beta{}^\alpha(\mathbf{x},\eta) = (\nabla^\alpha\nabla_\beta - {}^1/_3\delta_\beta{}^\alpha\Delta)\lambda(\mathbf{x},\eta) - {}^1/_3\delta_\beta{}^\alpha\Delta\mu(x,\eta), \quad (4.4)$$

where Δ is the Laplacian, and $\nabla^\alpha = \nabla_\alpha = \partial/\partial x^\alpha$. In the general case, the solutions of the equations for the adiabatic fluctuations contain not only the physical modes but also fictitious modes associated with the arbitrariness in the choice of the synchronous frame of reference.[18] Because the de Sitter cosmology is translationally invariant with respect to the time (the unperturbed $R = \text{const}$), $\delta R = 0$ on the fictitious perturbations. For this reason, δR characterizes only the true perturbations and is an invariant variable.

We express the action (4.4) in terms of δR. For this, we first of all write out the necessary components δR^i_k:

$$\delta R_0{}^0 = \frac{1}{2a^2}\Delta\Big(\mu'' + \frac{a'}{a}\mu'\Big), \quad \delta R_0{}^\alpha = -\frac{1}{3a^2}\nabla^\alpha\Delta(\lambda+\mu)', \quad (4.5\text{a,b})$$

$$\delta R_\beta{}^\alpha = -\frac{1}{2a^2}\nabla^\alpha\nabla_\beta\Big(\lambda'' + 2\frac{a'}{a}\lambda' + \frac{1}{3}\Delta(\lambda+\mu)\Big) \quad \text{for } \alpha\neq\beta, \quad (4.5\text{c})$$

$$\delta R = \frac{1}{a^2}\Delta\Big(\mu'' + 3\frac{a'}{a}\mu' - \frac{2}{3}\Delta(\lambda+\mu)\Big). \quad (4.5\text{d})$$

We introduce the functions

$$V = a\lambda', \quad \Phi = (\lambda+\mu)',$$
$$\Psi = \frac{a'}{a}\mu' - \frac{1}{3}\Delta(\lambda+\mu), \quad \delta R = \Delta\tilde{\phi}. \quad (4.6)$$

Using (4.1) and (4.5), we find that these functions satisfy the equations

$$\Phi = -\frac{1}{CM^2}a\Big(\frac{1}{a}\tilde{\Phi}\Big)', \quad V' = -\frac{a}{3CM^2}\Delta\tilde{\phi},$$
$$\Psi = -\frac{1}{CM^2}\Big(\frac{a'}{a}\tilde{\phi}' - \frac{1}{3}\Delta\tilde{\phi} - \varkappa^2a^2\tilde{\phi}\Big). \quad (4.7)$$

Substituting (4.4) in (4.3) and expressing λ and μ in terms of V by means of (4.7), and making fairly long calculations, we reduce the action for the considered perturbations to the form

$$\delta S_b = -\frac{1}{6l_{Pl}^2}\Big\{C\int d^4x\Big[\frac{3}{2}a^4\Big(\frac{1}{a^2}V'\Big)'^2 - \frac{3}{2}\nabla_\alpha V'\nabla^\alpha V' - \frac{3}{2}CM^2a^2V'^2 + \frac{\partial}{\partial\eta}\Big(\frac{1}{2\eta}\nabla_\alpha V\nabla^\alpha V - \Delta VV'\Big) + \frac{\partial}{\partial x^\varkappa}\Big(\frac{1}{a^2}V'(a^2\nabla^\varkappa V)'\Big) + (*)\Big]. \quad (4.8)$$

The variation of this action with respect to V gives an equation for the variable V. We add to the action (4.8) a number of divergent counter terms and express V' in terms of δR [see (4.7)]. Then we finally obtain

$$\delta S_b = \frac{1}{36CM^4l_{Pl}^2}\times\int d^4x a^2\Big(\delta R'^2 - \nabla_\alpha\delta R\nabla^\alpha\delta R - M^2a^2\Big(1 - 2\frac{\varkappa^2}{H^2}\Big)\delta R^2\Big). \quad (4.9)$$

Varying (4.9) with respect to δR, we obviously obtain the field equation (4.2).

5. QUANTIZATION

To quantize the longitudinal perturbations, we introduce the new variable

$$\varphi = (18C)^{-1/2}a\delta R/l_{Pl}M^2, \quad (5.1)$$

which characterizes the physical degrees of freedom of the considered field. For it, the action can be written as follows:

$$\delta S_b = \frac{1}{2}\int d^4x\Big[\varphi'^2 - \nabla_\alpha\varphi\nabla^\alpha\varphi + \Big(\frac{a''}{a} + M^2a^2\Big(2\frac{\varkappa^2}{H^2} - 1\Big)\Big)\varphi^2\Big]. \quad (5.2)$$

It differs from the action (4.9) by a divergent term.[3)] Varying (5.2) with respect to φ, we obtain the equation

$$\varphi'' - \Delta\varphi - \Big(\frac{a''}{a} + M^2a^2\Big(2\frac{\varkappa^2}{H^2} - 1\Big)\Big)\varphi = 0. \quad (5.3)$$

We quantize the real scalar field φ in the standard manner in the framework of the canonical Hamiltonian formalism. We define the generalized momentum conjugate to the field variable φ by

$$\pi = \partial\mathscr{L}/\partial\varphi' = \varphi'.$$

Then the corresponding Hamiltonian is

$$\mathscr{H} = \mathscr{H}_n + \mathscr{H}_{\rm int} = \int d^3x(\pi\varphi' - \mathscr{L})$$
$$= \frac{1}{2}\int d^3x(\pi^2 + \nabla_\alpha\varphi\nabla^\alpha\varphi + M^2a^2\varphi^2) + \frac{1}{2}\int d^3x\Big(-\frac{a''}{a} - 2\frac{M^2a^2\varkappa^2}{H^2}\Big)\varphi^2. \quad (5.4)$$

Here, we distinguish two terms: $\mathscr{H}_n$ is the Hamiltonian of the system in the absence of an external gravitational field, and $\mathscr{H}_{\rm int}$ is the energy of the interaction of the perturbations with the external gravitational field.

Going over from the field variables φ and π to the corresponding operators, we determine for them commutation relations on the hypersurface $\eta_0 = \text{const}$:

$$[\hat{\varphi}(\eta_0, \mathbf{x})\hat{\varphi}(\eta_0, \mathbf{x}')] = [\hat{\pi}(\eta_0, \mathbf{x})\hat{\pi}(\eta_0, \mathbf{x}')] = 0, \quad [\hat{\varphi}(\eta_0, \mathbf{x})\hat{\pi}(\eta_0, \mathbf{x}')] = i\delta(\mathbf{x}-\mathbf{x}'). \tag{5.5}$$

The field equation (5.3) is equivalent to the Heisenberg equations

$$[\hat{\varphi}\hat{\mathscr{H}}] = i\hat{\varphi}', \quad [\hat{\pi}\hat{\mathscr{H}}] = i\hat{\pi}'.$$

The creation and annihilation operators $\hat{b}_k^+$ and $\hat{b}_k^-$ for the considered perturbations (scalarons) are defined by expanding the field operator $\hat{\varphi}$ with respect to a complete system of orthonormalized solutions of Eq. (5.3):

$$\hat{\varphi} = \frac{1}{\sqrt{2}}\int \frac{d^3k}{(2\pi)^{3/2}}\{e^{i\mathbf{kx}}u_k^*(\eta)\hat{b}_k^- + e^{-i\mathbf{kx}}u_k(\eta)\hat{b}_k^+\} \tag{5.6}$$

(the functions $e^{i\mathbf{kx}}/(2\pi)^{3/2}$ are orthonormalized eigenfunctions of the operator Δ). The operators $\hat{b}_k^+$ and $\hat{b}_k^-$ satisfy standard commutation relations for bosons:

$$[b_k^- b_{k'}^+] = \delta_{kk'}. \tag{5.7}$$

It follows from the definition (5.6) and Eq. (5.3) that the complex amplitude $u_k(\eta)$ must satisfy the equation

$$u_k'' + \left[k^2 - \frac{2}{\eta^2}\left(1 + \frac{M^2}{H^2} - \frac{M^2}{2\varkappa^2}\right)\right]u_k = 0 \tag{5.8}$$

and the normalization condition

$$u_k' u_k^* - u_k^{*\prime} u_k = 2i, \tag{5.9}$$

which is necessary for the consistency of the commutation relations (5.5) and (5.7).

Turning to the definition of the Hilbert state space, we assume the existence of an eigenvector $|0\rangle$ of the operators b_k^- corresponding to zero eigenvalues: $\hat{b}_k^- |0\rangle = 0$. The remaining states can be obtained by successive application of the operators $\hat{b}_k^+$. We define the complete system of eigenfunctions $u_k(\eta)$ as the solutions of Eq. (5.8) with the initial conditions

$$u_k(\eta_0) = \Omega_0^{-1/2}, \quad u_k'(\eta_0) = i\Omega_0^{1/2}, \quad \Omega_0^2 = k^2 + M^2 a_0^2. \tag{5.10}$$

These initial conditions are consistent with the normalization (5.9) of the basis funcitons and correspond to the minimum of the Hamiltonian $\mathscr{H}_n$ averaged over the state $|0\rangle$.

We note that the spectrum of the considered fluctuations in the region of wave numbers of greatest interest to us, namely, $-k\eta_0 \gg 1$ (η_0 is the hypersurface on which the initial quantum state is specified), does not depend in the employed order of perturbation theory on the principle used to choose the initial vacuum, since for such fluctuations

$$k^2 \gg U = \frac{2}{\eta_0^2}\left(1 + \frac{M^2}{H^2} - \frac{M^2}{2\varkappa^2}\right).$$

For this reason, the possible changes in sign of the Hamiltonian (5.4) due to the negative interaction energy $\mathscr{H}_{\text{int}}$ are not important for us.

To conclude this section, we write down the solution of Eq. (5.8) with the initial conditions (5.10). It can be expressed in terms of Bessel functions and has the form

$$u_k(\eta) = \frac{\pi}{4}(-\eta)^{1/2}\left[\frac{\varkappa^2}{M^2 + \varkappa^2(k\eta_0)^2}\right]^{1/4} \times [(1 + 2i((k\eta_0)^2 + M^2/H^2)^{1/2}) \times (Y_{3\nu/2}(-k\eta_0)J_{3\nu/2}(-k\eta) - J_{3\nu/2}(-k\eta_0)Y_{3\nu/2}(-k\eta)) + 2\eta_0(Y'_{3\nu/2}(-k\eta_0)J_{3\nu/2}(-k\eta) - J'_{3\nu/2}(-k\eta_0)Y_{3\nu/2}(-k\eta))], \tag{5.11}$$

where $J_{3\nu/2}$ and $Y_{3\nu/2}$ are Bessel functions of order $3\nu/2$,

$$\nu = (1 + 8M^2/9H^2 - 4M^2/9\varkappa^2)^{1/2}.$$

We emphasize once more that the nonvanishing initial conditions (5.10) correspond to minimal quantum fluctuations and are necessarily present in the considered Universe.

6. QUANTUM FLUCTUATIONS AND BACKGROUND COSMOLOGICAL MODEL

To characterize the level of the fluctuations in the considered model, we calculate the correlation function of the perturbations of the curvature scalar. The fluctuations due to the single-loop corrections to the Einstein equations can be appreciable on fairly large scales only when the vacuum energy density, which determines the expansion rate of the Universe in the de Sitter stage, does not differ too strongly from the Planck value. To avoid unnecessary complication of the final expressions, we restrict ourselves to calculations of perturbations for a Universe with $\varkappa \gg M$.[4] In this case, using (5.1) and (5.6) and asymptotic expansions of the solution (5.11), we find the following expression for the correlation function of the curvature fluctuations at times $|\eta| \ll |\eta_0|$:

$$\langle 0|\delta R(\mathbf{x})\delta R(\mathbf{x}+\mathbf{r})|0\rangle = \int \frac{d^3k_p}{(2\pi)^3}\exp(i\mathbf{k}_p\mathbf{r}_p)|\delta R_{k_p}|^2$$

$$\approx 9Cl_{\text{Pl}}^2 M^4\left\{\frac{1}{16}\left(2 + \frac{2}{3\nu}\right)\left(\frac{\eta_0}{\eta}\right)^{3\nu} \times \int_0^{\varkappa(\eta/\eta_0)} \frac{d^3k_p}{(2\pi)^3}\exp(i\mathbf{k}_p\mathbf{r}_p)\left(\frac{\eta^2}{(k_p\eta_0)^2 + M^2\eta^2}\right)^{1/2} + \frac{2^{3\nu-1}}{\pi}\Gamma^2\left(\frac{3\nu}{2}\right)\int_{\varkappa(\eta/\eta_0)}^{\varkappa} \frac{d^3k_p}{(2\pi)^3}\exp(i\mathbf{k}_p\mathbf{r}_p)\frac{1}{k_p^3}\left(\frac{\varkappa}{k_p}\right)^{3(\nu-1)} + \int_{\varkappa}^{\infty}\frac{d^3k_p}{(2\pi)^3}\exp(i\mathbf{k}_p\mathbf{r}_p)\frac{1}{k_p}\right\}. \tag{6.1}$$

Here, $k_p = k/a$ and $r_p = ra$ are, respectively, the physical wave vector and the physical scale (measured in cm^{-1} and cm); It is also assumed that $M \lesssim H$ and $H > \varkappa > 2MH/(8M^2 + 9H^2)$ so that ν is a real number.

The correlation function is conveniently characterized by its spatial spectrum $P^2(k_p)$:

$$\langle 0|\delta\hat{R}(\mathbf{x})\delta\hat{R}(\mathbf{x}+\mathbf{r})|0\rangle = \frac{1}{2\pi^2}\int P^2(k_p)\frac{\sin k_p r_p}{k_p r_p}\frac{dk_p}{k_p}.$$

By virtue of the isotropy, $P(k_p)$ depends only on the modulus of k_p (and the time) and characterizes the amplitude of the

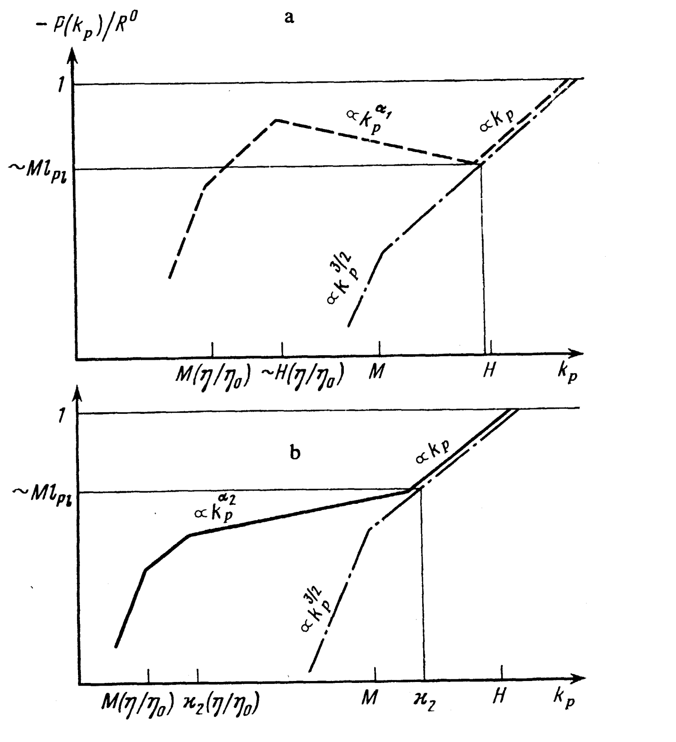

FIG. 2. Perturbation spectrum $\delta R/R^{(0)}$ of the curvature scalar in the de Sitter stage of expansion of the Universe due: a) to polarization of the vacuum of the conformal physical fields in the gravitational field, $\varkappa_1 \approx H$ ($\alpha_1 \approx M^2/H^2$ for $M^2 \ll H^2$) and b) nonvanishing vacuum energy density in gauge thories, $\varkappa_2 \approx l_{Pl}\epsilon_{\varphi v}^{1/2}$ ($\alpha_2 \approx M^2/3l_{Pl}^2\epsilon_{\varphi v}$ for $M^2 \ll 3l_{Pl}^2\epsilon_{\varphi v}$).

perturbations of the curvature scalar on scales $r_p \sim 1/k_p$. The dependence of $P(k_p)$ and k_p given by (6.1) at the time $|\eta| \ll |\eta_0|$ (η_0 is the initial hypersurface) is shown in Fig. 2. The broken line corresponds to the case $H \geqslant \varkappa > H/\sqrt{2}$, when the vacuum energy density is determined by the polarization of the conformal fields. The fluctuations in a Universe in which the vacuum energy associated with Higgs fields is dominant are shown for the case $H \gg \varkappa \gg M$ by the continuous line.

In Sec. 7, it will be shown that the characteristic perturbations of the metric correspond to perturbations of the curvature scalar. Therefore, these last give a fairly good characterization of the level of the fluctuations. As can be seen from Fig. 2, the graphs of $P(k_p)$ against k_p have three inflections. The first, at $k_p \sim M(\eta/\eta_0)$, is due to the inflection in the initial vacuum spectrum, whereas the other two, at $k_p \sim \varkappa(\eta/\eta_0)$ and $k_p \sim \varkappa$, are due to the evolution of the perturbations. For the intermediate scales $\varkappa > k_p > \varkappa(\eta/\eta_0)$, the amplitude of the fluctuations remains constant, and the length of the intermediate region increases with the passage of time in the direction of small k_p. In the case $H \geqslant \varkappa > H\sqrt{2}$, the spectrum has a maximum, which is situated at $k_p \sim \varkappa(\eta/\eta_0)$. The amplitude of the perturbations in the region of the maximum increases and at a certain time $\eta = \eta_f$ the perturbations $P(k_{pf})$ of the curvature scalar at scales $k_{pf} \sim \varkappa(\eta_f/\eta_0)$ becomes of the same order as the curvature scalar $R^{(0)} = -12\varkappa^2$, which characterizes the unperturbed model. It is clear that from then on the back reaction of multiple production of excitations (scalarons) on the evolution of the "background" model of the Universe becomes important. Thus, the de Sitter Universe with $H \geqslant \varkappa > H/\sqrt{2}$ is unstable with respect to perturbations of the metric of scalar type and due to the presence of the vacuum fluctuations of the perturbation field it has a finite lifetime (for more detail, see Ref. 13).

The situation becomes qualitatively different in the case of a de Sitter stage due to a delayed cosmological phase transition ($\varkappa < H/\sqrt{2}$). It can be seen from Fig. 2 that in this case the amplitude of the fluctuations decreases monotonically in the region of small k_p and the perturbations do not increase to the magnitudes that characterize the background model. Therefore, this de Sitter Universe will be stable with respect to the production of longitudinal perturbations of the metric ("scalarons"). Nevertheless, the time of its existence will be finite. This is due to the metastability of the vacuum condensate itself, which has a nonzero energy density. The decay of the vacuum is a phase transition of the first or second kind (the type of phase transition is determined by the parameters of the gauge theories).[8,9]

We emphasize once more that, depending on $\varkappa$, the de Sitter stage is finite for qualitatively different reasons. For $H \geqslant \varkappa > H/\sqrt{2}$, it is finite because of the instability of the polarized vacuum with respect to the production of longitudinal perturbations of the metric, whereas for $H/\sqrt{2} > \varkappa$ the cause of the finite lifetime of the de Sitter state is the rearrangement of the scalar fields.

As a result of the instability of the de Sitter models of the considered types, matter is produced in the Universe, and it goes over to an ordinary Friedmann expansion regime. The de Sitter stages described above can be realized only as finite (in time) intermediate stages in the evolution of our Universe.

7. FORMATION OF GALAXIES

In modern theories of the formation of galaxies, one of the most important questions is the origin of the initial spectrum of perturbations. It was shown earlier in Ref. 7 that in a singular "hydrodynamic" Universe with $p + \varepsilon \sim \varepsilon$ (p is the pressure and ε is the energy density) the quantum-mechanical initial perturbations are insufficient for the formation of the observed large-scale structure. Therefore, the question of the natural occurrence of an initial inhomogeneity spectrum capable of leading subsequently to the formation of galaxies and clusters of galaxies in a Universe with a vacuum (de Sitter) stage is of particular interest.

To compare the obtained perturbation spectra with modern theories of galaxy formation, we need expressions for the correlation functions of the perturbations of the metric. From Eq. (4.5a) for δR_0^0, integrating, we find

$$h=h_\alpha{}^\alpha=-\Delta\mu=\int\frac{d\eta}{a}\int a^3\,\delta R_0{}^0\,d\eta. \qquad (7.1)$$

Using (4.6) and (4.7), we obtain the corresponding expression for λ:

$$\Delta\lambda=-\frac{1}{CM^2}\int a\left(\frac{1}{a}\delta R\right)' d\eta-\Delta\mu. \qquad (7.2)$$

Knowing λ and μ from (4.4), we can find the perturbations of the metric h^{α}_{β}. Besides physical modes, they include fictitious modes corresponding to transformations of the coordinate system that leave it synchronous.[8]

The existence of the fictitious modes unfortunately renders the synchronous frame inconvenient for investigation of perturbations of the metric. To establish how strongly the metric of the perturbed de Sitter model differs from (3.4), we go over from the synchronous coordinates η, x^{α} to new coordinates $\tilde{\eta},\tilde{x}^{\alpha}$ by means of[19]

$$\tilde{\eta}=\eta+\frac{1}{2}\frac{\partial\lambda}{\partial\eta},\quad \tilde{x}^{\alpha}=x^{\alpha}+\frac{1}{2}\frac{\partial\lambda}{\partial x^{\alpha}}. \qquad (7.3)$$

In the coordinates $\tilde{\eta},\tilde{x}^{\alpha}$ the interval ds^2 in the linear order in the perturbations of the metric has the form

$$ds^2=a^2(\tilde{\eta})\left[(1+2\phi)d\tilde{\eta}^2-(1-2\psi)\sum_{\alpha=1}^{3}(d\tilde{x}^{\alpha})^2\right], \qquad (7.4)$$

where

$$\phi=-\frac{1}{2a}(a\lambda')',\quad \psi=\frac{1}{2}\left(\frac{a'}{a}\lambda'+\frac{1}{3}\Delta(\lambda+\mu)\right). \qquad (7.5)$$

The perturbations $\delta R(\tilde{\eta},\tilde{x}^{\alpha})$ of the curvature scalar in the new coordinates in the de Sitter Universe are equal to $\delta R(\eta, x^{\alpha})$ in the synchronous coordinates, since $R^{(0)}_{,i}=0$.

Substituting the expressions (7.1) and (7.2) for λ and μ in (7.5) and using Eqs. (4.1) and (4.2), we find

$$ds^2=a^2(\tilde{\eta})\left(1+\frac{1}{3CM^2}\delta R(\tilde{\eta},\tilde{x}^{\alpha})\right)\left(d\tilde{\eta}^2-\sum_{\alpha=1}^{3}(d\tilde{x}^{\alpha})^2\right), \qquad (7.6)$$

i.e., the scalar perturbations of the metric in the de Sitter Universe are conformally flat. For $H>\varkappa\gg M$, their amplitude is exactly equal to that of the relative perturbations $\delta R/R^{(0)}$ of the curvature scalar, whose correlation function is given in (6.1). In a fairly wide range of scales, the amplitude of the perturbations of the metric is proportional to $(Ml_{\rm Pl})$. The spectrum of the fluctuations deviates only slightly from a flat spectrum. In an ordinary hydrodynamic Universe, the metric perturbations on scales greater than the horizon have two physical modes, one decreasing and one constant. The constant mode subsequently leads to the formation of galaxies and clusters of galaxies. The contemporary horizon (10^{28} cm) corresponded to the scale[5)] $(10^{28}-10^{40})l_{\rm Pl}$ at the time of transition of the Universe to the Friedmann stage. For example, if the duration of the de Sitter stage is $>10^2\varkappa^{-1}$ (Ref. 9), $(Ml_{\rm Pl})\sim10^{-3}$, $(Hl_{\rm Pl})\sim10^{-1}$ (Ref. 12), and $\varepsilon_{\varphi v}\sim10^{-8}\varepsilon_{\rm Pl}$ (Ref. 10), then, as is readily seen from (6.1) (see also Fig. 2), the spectrum of the perturbations on scales of clusters of galaxies will be almost flat with amplitude $\delta g\sim10^{-4}$. Thus, if the Universe did evolve through a de Sitter stage with a vacuum energy of sufficiently high density the resulting perturbations of the metric (for a reasonable choice of the values of $M, H, \varepsilon_{\varphi v}$) were quite sufficient for the formation of the galaxies and clusters of them. At the same time, the perturbation spectrum agrees with the spectra allowed by modern theories of galaxy formation.[4]

In conclusion, we note that all quantum fields contribute to M and H, and their values can be calculated only in a future unified field theory. The necessary condition on the vacuum energy density $\varepsilon_{\varphi v}$ can be satisfied in a grand unification theory.[10]

8. CONCLUSIONS

During its evolution, the Universe could certainly have passed through a de Sitter stage with finite duration. The possible existence of such stages arises for two qualitatively different reasons. One is that under certain conditions the dynamics of the Universe is determined by the energy density of the gravitationally polarized vacuum.[12] In this case, the energy density of the polarized vacuum does not differ strongly from the Planck density and during the de Sitter stage fluctuations could have arisen sufficient for the formation of galaxies.[13] Nevertheless, the existence of a de Sitter stage of this kind does not appear to us to be sufficiently well justified, since homogeneity and isotropy of the metric in no way mean that such a stage must have occurred in the past.

The other possibility of realization of a de Sitter stage is associated with the fact that during the early stages in the evolution of our Universe the vacuum in gauge theories could have had a nonvanishing energy density. For certain relations between the parameters of the gauge theories, the Universe must necessarily have passed through such de Sitter stages.[8] For this reason, de Sitter stages associated with delayed cosmological phase transitions appear to us more realistic. In the present paper, we have shown that at vacuum energy densities not differing too strongly from Planck densities the perturbations of the metric generated during the de Sitter stage are quite sufficient for the formation of the large-scale structure of the Universe.

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[1)] For a Universe with nonvanishing spatial curvature all the main conclusions of our paper remain valid.

[2)] For gravitational waves, $\delta R=0$. And, as is readily seen from (4.1), the equations that describe the evolution of the gravitational waves are identical to the linearized Einstein equations with vanishing right-hand side. They were quantized in Starobinsky's paper Ref. 16.

[3)] In quantum theory, the addition of divergent counter terms to the action means that there is a certain renormalization.

[4)] If the de Sitter stage is associated with scalar fields with energy density $\varepsilon_{\varphi v}\ll(Hl_{\rm Pl})^2\varepsilon_{\rm Pl}$, then the condition $\varkappa\gg M$ takes the form $\varepsilon_{\varphi v}\gg(Ml_{\rm Pl})^2\varepsilon_{\rm Pl}$.

[5)] The density at the start of the Friedmann expansion is $\sim\varepsilon_{\varphi v}$.

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