Novel class of nonlinear surface waves: asymmetric modes in a symmetric layered structure

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Wave propagation is considered in a symmetric layered structure consisting of a layer having a linear dielectric constant between two layers of a medium whose dielectric constant depends quadratically on the amplitude of the wave electric field. It is shown that besides symmetric and antisymmetric modes there can exist in the structure asymmetric modes at energy fluxes exceeding a certain minimum value. Plots of the energy flux vs the propagation constant are obtained for each of the modes considered. It is shown that asymmetric-modes branch out in these plots from the symmetric and antisymmetric modes upon increase of the energy flux in the wave. The plots themselves are N-shaped in a certain range of layer thicknesses; this may lead to bistable states of the surface waves if the energy flux in the wave is the external parameter.

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1. INTRODUCTION

Interest in the optics of layered media having nonlinear dielectric properties has increased of late. In particular, in Refs. 1–4 is considered a new class of surface waves on the interface between two media, one of which[1–3] or both[4] have a dielectric constant with a quadratic dependence on the wave field:

$$\varepsilon=\varepsilon_0+\alpha|E|^2. \tag{1}$$

A feature of these waves is that the dispersion relation contains as a parameter, besides the frequency and the wave vector, also the square of the field of the electromagnetic wave. This means that at a given frequency it is possible to control the propagation constant of the wave by varying its energy flux, thereby adding substantially to the number of phenomena that can occur in the linear optics of surface waves. Moreover, as shown in Refs. 1–4, when account is taken of the nonlinearity, wave solutions are obtained having no analog whatever in ordinary optics. One of the now investigated new properties of nonlinear surface waves is the possibility of their direct excitation by a bounded light beam incident on the interface,[5] without the use of prisms or periodic structures.

This paper deals with nonlinear surface waves in a certain model layered structure consisting of a layer of thickness $2d$ and dielectric constant ε_1, placed between two layers of a medium with a dielectric constant of the form (1). Such a system reduces in two limiting cases to those already investigated: at $2d\gg\lambda$, waves of the type considered in Refs. 2 and 3 can exist in it on the two interfaces, and in the case $\alpha\to 0$ the system is a symmetric dielectric waveguide in which symmetric and antisymmetric modes can propagate. In the general case, as will be shown below, the system has a number of new interesting properties, the most pronounced of which is that there can exist in it, besides the symmetric and antisymmetric modes, also asymmetric types of waves at energy fluxes above a certain threshold. A second interesting property of the structure is that the dependence of the energy flux in the wave propagating in the layer on the propagation constant turns out to be N-shaped, and this can lead to bistable states of these waves.

2. DISPERSION RELATIONS FOR A NONLINEAR LAYERED STRUCTURE

Let a plane-parallel layer of thickness $2d$ occupy a strip $-d<z<d$. We assume that $\varepsilon_1>\varepsilon_0$ and that the nonlinear medium outside the layer is self-focusing, i.e., $\alpha>0$. The problem is to find the solution of Maxwell's equations in the form of surface waves for a medium with a dielectric constant

$$\varepsilon(z,|E|^2)=\begin{cases}\varepsilon_1 & \text{at} \quad |z|<d\\ \varepsilon_0+\alpha|E|^2 & \text{at} \quad |z|>d\end{cases}. \tag{2}$$

The necessary solutions for the electric field in all of space will be sought in the form of a wave polarized along the y axis and propagating along the x axis:

$$E_y(x,z,t)=E(z)\exp i(k_0n_xx-\omega t), \tag{3}$$

where ω is the frequency of the wave, $k_0=\omega/c$, and n_x is the propagation constant. The variables in (3) can be separated and then the function $E(z)$ should satisfy the equation

$$d^2E/dz^2-k_0^2[n_x^2-\varepsilon(z,|E|^2)]E=0. \tag{4}$$

In a nonlinear medium, Eq. (4) has a solution that decreases at infinity and is of the form

$$E(z)=\left(\frac{2}{\alpha}\right)^{1/2}\frac{q}{\operatorname{ch}k_0q(z-z_i)};\quad z_i=\begin{cases}z_1 & \text{at} \quad z<-d\\ z_2 & \text{at} \quad z>d\end{cases}, \tag{5}$$

where $q=(n_x^2-n_0^2)^{1/2}$, $n_0=\varepsilon_0^{1/2}$, and z_i are constants determined from the boundary conditions. The form of the solution in the layer $-d<z<d$ depends on n_x. At $n_x>n_1$

$$E(z)=\begin{cases}A\operatorname{ch}k_0\Gamma(z-d_0), & \text{(6a)}\\ A\operatorname{sh}k_0\Gamma(z-d_0), & \text{(6b)}\end{cases}$$

where $\Gamma=(n_x^2-n_1^2)^{1/2}$, $n_1=\varepsilon_1^{1/2}$. At $n_x<n_1$

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$$E(z)=\begin{cases}A\cos k_0\gamma(z-d_0), & (7a)\\ A\sin k_0\gamma(z-d_0), & (7b)\end{cases}$$

where $\gamma=(n_1^2-n_x^2)^{1/2}$. The constants A and d_0 are also determined from the boundary conditions. Although (7b) can be obtained from (7a) by renormalizing the constant d_0, we shall consider both solutions, since they can be obtained by analytic continuation from (6a) and (6b).

Consider the dispersion relations that result from solutions such as (6). Equating the fields and their derivatives on the boundaries $z=-d$ and $z=d$ we obtain a system of four equations with the unknowns A, d_0, z_1, z_2 and n_x:

$$\left(\frac{2}{\alpha}\right)^{1/2}\frac{q}{\operatorname{ch}k_0q(d+z_1)}=\begin{cases}A\operatorname{ch}k_0\Gamma(d+d_0), & (8a)\\ -A\operatorname{sh}k_0\Gamma(d+d_0), & (8b)\end{cases}$$

$$\left(\frac{2}{\alpha}\right)^{1/2}q^2\frac{\operatorname{sh}k_0q(d+z_1)}{\operatorname{ch}^2k_0q(d+z_1)}=\begin{cases}-A\Gamma\operatorname{sh}k_0\Gamma(d+d_0), & (8c)\\ A\Gamma\operatorname{ch}k_0\Gamma(d+d_0), & (8d)\end{cases}$$

$$\left(\frac{2}{\alpha}\right)^{1/2}\frac{q}{\operatorname{ch}k_0q(d-z_2)}=\begin{cases}A\operatorname{ch}k_0\Gamma(d-d_0), & (9a)\\ A\operatorname{sh}k_0\Gamma(d-d_0), & (9b)\end{cases}$$

$$\left(\frac{2}{\alpha}\right)^{1/2}q^2\frac{\operatorname{sh}k_0q(d-z_2)}{\operatorname{ch}^2k_0q(d-z_2)}=\begin{cases}-A\Gamma\operatorname{sh}k_0\Gamma(d-d_0), & (9c)\\ -A\Gamma\operatorname{ch}k_0\Gamma(d-d_0). & (9d)\end{cases}$$

The system (8) (the upper rows in the right-hand sides of the equations) was obtained with the aid of (6a), while the system (9) (the lower rows) with the aid of (6b). In the case $n_x<n_1$ the system of boundary conditions can be obtained by putting $\Gamma=i\gamma$ in (8) and (9). We consider first the system (8). Eliminating the variables in succession, we reduce the system to a single equation with unknown d_0 and n_x:

$$\frac{q^2-\Gamma^2\operatorname{th}^2k_0\Gamma(d-d_0)}{q^2-\Gamma^2\operatorname{th}^2k_0\Gamma(d+d_0)}=\frac{\operatorname{ch}^2k_0\Gamma(d-d_0)}{\operatorname{ch}^2k_0\Gamma(d+d_0)}. \tag{10}$$

It is easy to show, for example by a graphic method, that Eq. (10) has a unique solution $d_0=0$ at arbitrary $n_x>n_1$. This solution corresponds to the symmetric modes of the layered structure. If $d_0=0$ is substituted in the system (8), obviously $z_1=-z_2$ and the second pair of equations is identical to the first. Dividing one equation by the other we obtain the dispersion relation in terms of the variables ω, n_x, and z_1:

$$\operatorname{th}k_0q(z_1+d)=\begin{cases}-\dfrac{\Gamma}{q}\operatorname{th}k_0\Gamma d & \text{at}\quad n_x>n_1, \quad (11a)\\[2mm] \dfrac{\gamma}{q}\operatorname{tg}k_0\gamma d & \text{at}\quad n_x<n_1. \quad (11b)\end{cases}$$

The square of the amplitude of the field A inside the layer is for the symmetric mode,

$$A^2=\begin{cases}\dfrac{2}{\alpha}\dfrac{q^2}{\operatorname{ch}^2k_0\Gamma d}\left(1-\dfrac{\Gamma^2}{q^2}\operatorname{th}^2k_0\Gamma d\right) & \text{at}\quad n_x>n_1, \quad (12a)\\[2mm] \dfrac{2}{\alpha}\dfrac{q^2}{\cos^2k_0\gamma d}\left(1-\dfrac{\gamma^2}{q^2}\operatorname{tg}^2k_0\gamma d\right) & \text{at}\quad n_x<n_1. \quad (12b)\end{cases}$$

If we start from the system of boundary conditions (9), the equations for d_0 can be rewritten in the form

$$\frac{q^2-\Gamma^2\operatorname{cth}^2k_0\Gamma(d-d_0)}{q^2-\Gamma^2\operatorname{cth}^2k_0\Gamma(d+d_0)}=\frac{\operatorname{sh}^2k_0\Gamma(d-d_0)}{\operatorname{sh}^2k_0\Gamma(d+d_0)}. \tag{13}$$

This equation also has a solution $d_0=0$, as can be easily verified by direct substitution. There are also other solutions, which we shall consider below. Substituting the value $d_0=0$ in the system of boundary conditions (9), just as in the preceding case, we obtain the dispersion relation in terms of the variables n_x, ω, and z_1:

$$\operatorname{th}k_0q(d+z_1)=\begin{cases}-\dfrac{\Gamma}{q}\operatorname{cth}k_0\Gamma d & \text{at}\quad n_x>n_1, \quad (14a)\\[2mm] -\dfrac{\gamma}{q}\operatorname{ctg}k_0\gamma d & \text{at}\quad n_x<n_1. \quad (14b)\end{cases}$$

The square of the amplitude of the field of the antisymmetric solution is

$$A^2=\begin{cases}\dfrac{2}{\alpha}\dfrac{q^2-\Gamma^2\operatorname{cth}^2k_0\Gamma d}{\operatorname{sh}^2k_0\Gamma d} & \text{at}\quad n_x>n_1, \quad (15a)\\[2mm] \dfrac{2}{\alpha}\dfrac{q^2-\gamma^2\operatorname{ctg}^2k_0\gamma d}{\sin^2k_0\gamma d} & \text{at}\quad n_x<n_1. \quad (15b)\end{cases}$$

The dispersion relations (11) and (14) depend on the parameter z_1, which is connected with the energy flux in the system. The limit $z_1\to+\infty$ corresponds to the case of extremely small energy fluxes, and in this case formulas (11b) and (14b) coincide with the dispersion relations for the symmetric and antisymmetric modes of the dielectric waveguide, which have one or several solutions for n_x in the interval $n_0<n_x<n_1$, depending on the thickness of the carrying layer. In the case of a nonlinear structure, the dispersion relations have solutions also at $n_x>n_1$. In this case $z_1<-d$ and the function $E(z)$ is such that the two maximum values of the field turn out to be beyond the limits of the field. It is natural to name the corresponding solutions nonlinear surface waves.

We obtain now the nonzero solutions of (13). To this end we reduce (13) to a common denominator and obtain after simple transformations

$$[\operatorname{sh}^2k_0\Gamma(d+d_0)-\operatorname{sh}^2k_0\Gamma(d-d_0)]\times\Big[\operatorname{sh}^2k_0\Gamma(d+d_0)\operatorname{sh}^2k_0\Gamma(d-d_0)-\frac{\Gamma^2}{q^2}(\operatorname{ch}^2k_0\Gamma(d+d_0)\operatorname{ch}^2k_0\Gamma(d-d_0)-1)\Big]=0. \tag{16}$$

Equating to zero the first square bracket in (16) yields the already known solution $d_0=0$. We transform the second bracket in (16) by using the formulas for the sum and difference of hyperbolic functions. We then obtain the following biquadratic equation in $\sinh k_0\Gamma d$:

$$(a_1^2-1)\operatorname{sh}^4k_0\Gamma d_0+2[b_1^2(a_1^2+1)+a_1^2]\times\operatorname{sh}^2k_0\Gamma d_0+b_1^2(2a_1^2+a_1^2b_1^2-b_1^2)=0, \tag{17}$$

where $a_1=\Gamma/q$ and $b_1=\sinh k_0\Gamma d$. It has two solutions for $\sinh^2k_0\Gamma d$:

$$\operatorname{sh}^2k_0\Gamma d_0^{(1,2)}=\frac{(1+a_1^2)b_1^2+a_1^2\pm a_1(4b_1^4+4b_1^2+a_1^2)^{1/2}}{1-a_1^2}. \tag{18}$$

In the region $n_x<n_1$ these solutions can be continued by means of the formulas

$$\sin^2k_0\gamma d_0^{(1,2)}=\frac{(1-a_2^2)b_2^2+a_2^2\pm a_2(-4b_2^4+4b_2^2+a_2^2)^{1/2}}{1+a_2^2}, \tag{19}$$

where $a_2 = \gamma/q$, $b_2 = \sin k_0\gamma d$. Equations (18) and (19) determine four different solutions for d_0. The two positive roots $d_0^{(1)}$ and $d_0^{(2)}$ satisfy, for all n_x from the region in which they are defined, the inequalities $d_0^{(1)} > d$ and $d_0^{(2)} < d$, correspond to two qualitatively different distributions of the field $E(z)$. At $d_0 = d_0^{(1)}$ the field has in the region $z < -d$ one maximum from which it decreases on both sides, whereas at $d = d_0^{(2)}$ the field has a maximum in the region $z < -d$, reverses sign at the point $z = d_0^{(2)}$, and has a minimum in the region $z > d$. The two negative roots $-d_0^{(1)}$ and $-d_0^{(2)}$ correspond to the solutions obtained for $E(z)$ from the foregoing by the transformation $z \to -z$.

The roots $d_0^{(1)}$ and $d_0^{(2)}$ are functions of the thickness $2d$ and of the propagation constant n_x. We consider now the regions where the roots are defined as functions of these variables. We investigate first the root $d^{(1)}$ (with a plus sign in front of the radical). The root is defined for all $n_x > n_1$, since the expression in the right-hand side of (18) is always positive. In the region $n_x < n_1$, the expression in the right-hand side of (19) is also positive, but for a solution to exist it is necessary that the right-hand side of (19) not exceed unity. This condition is equivalent to the inequality

$$\sin^2 k_0\gamma d \leqslant \frac{2n_x - (n_1^2 + n_0^2)}{n_1^2 - n_0^2}. \tag{20}$$

All the solutions of this inequality are located in the region $[(n_1^2 + n_0^2)/2]^{1/2} < n_x < n_1$. At all d there exists a solution $n_x^0 < n_x < n_1$ such that at $n_x = n_x^0$ the right-hand side of (19) is equal to unity, $k_0\gamma d_0 = \pi/2$, and the solution $E(z) = A \sin k_0\gamma(z - d_0)$ is transformed into the symmetrical solution $E(z) = A \cos k_0\gamma z$. At $2d/\lambda > [2/(n_0^2 + n_1^2)]^{1/2}$, besides the indicated interval, there can exist additional intervals in which an asymmetric mode exists, and at the end points of these intervals the solution also degenerates to a symmetric one of higher order. Thus, the root d_0^1 corresponds to asymmetric modes, which at definite n_x become separated from the symmetric modes. To be definite, we call the lowest of these modes A.

We consider now the root $d_0^{(2)}$ (with a minus sign in front of the radical). It is easy to show that the right-hand side of (19) does not exceed unity in this case at all admissible $a_2^2 > 0$ and $0 < b_2^2 < 1$, and we must find the intervals of n_x in which the right-hand sides of (18) and (19) are positive. The corresponding criteria for the two equations are respectively $b_1^2 > 2a_1^2/(1 - a_1^2)$ and $b_2^2 > 2a_2^2/(1 + a_2^2)$. These conditions reduce to the inequalities

$$\left(\frac{\operatorname{sh} k_0\Gamma d}{k_0\Gamma d}\right)^2 > \frac{2}{(k_0 d)^2 (n_1^2 - n_0^2)} \quad (n_x > n_1), \tag{21a}$$

$$\left(\frac{\sin k_0\gamma d}{k_0\gamma d}\right)^2 > \frac{2}{(k_0 d)^2 (n_1^2 - n_0^2)} \quad (n_x < n_1). \tag{21b}$$

All the solutions of (21) are also located in the region $[(n_1^2 + n_0^2)/2]^{1/2} < n_x < \infty$. If $2d/\lambda < \pi^{-1}[2/(n_1^2 - n_0^2)]^{1/2}$, then the first of these inequalities determines the interval $n_x' < n_x < \infty$ in which $d_0^{(2)}$ exists, and at $n_x = n_x'$ we have $d_0^{(2)} = 0$ and the solution degenerates into an antisymmetric one. The second inequality has in this case no solutions. On the other hand, if $2d/\lambda > \pi^{-1}[2/(n_1^2 - n_0^2)]^{1/2}$, the inequality (21a) is valid for all $n_x > n_1$, while (21b) has a solution $n_x' < n_x \leqslant n_1$. Thus, the interval in which $d_0^{(2)}$ exists increases with increasing d. In addition, as seen from (21b), at $2d/\lambda > [2/(n_1^2 - n_0^2)]^{1/2}$ there arise additional intervals of n_x, in which $d_0^{(2)}$ exists, and at the end points of these intervals $d_0^{(2)} = 0$. Thus, $d_0^{(2)}$ correspond to an asymmetric mode, which becomes separated from the antisymmetric mode at definite n_x. We call it mode B.

The square of the amplitude A for both asymmetric modes at $n_x > n_1$ is

$$A^2 = \frac{2}{\alpha} \frac{q^2 - \Gamma^2 \operatorname{cth}^2 k_0\Gamma(d \pm d_0)}{\operatorname{sh}^2 k_0\Gamma(d \pm d_0)}. \tag{22a}$$

In the case $n_x < n_1$

$$A^2 = \frac{2}{\alpha} \frac{q^2 - \gamma^2 \operatorname{ctg}^2 k_0\gamma(d \pm d_0)}{\sin^2 k_0\gamma(d \pm d_0)}. \tag{22b}$$

In (22) both signs in the arguments of the hyperbolic and trigonometric functions yield, following the substitution $d_0 = d_0^{(1,2)}$, the same value for A^2. Substituting the expressions for $d_0^{(1,2)}$ in the system of boundary conditions (8) and (9), we can find the dispersion relations for the modes A and B, expressed in terms of the variables ω, n_x, z_1, or z_2. In this case, however, we choose as the third independent variable the energy flux in the mode.

3. ENERGY FLUX IN THE WAVE

It is more convenient to transform in all the dispersion relations from the variable z_1 to the energy flux in the structure, since the energy flux, alongside the frequency and the wave vector, can serve as an external parameter specified in the experiment. The energy flux is calculated by integrating the averaged Poynting vector with respect to the variable z between infinite limits

$$S = \frac{cn_x}{8\pi} \int_{-\infty}^{\infty} E^2(z)\, dz. \tag{23}$$

This formula gives the energy flux per unit width (along the y axis) of the layer. For symmetric modes, a simple integration using Eqs. (5)–(7), (11), and (12) leads to the following expressions the function $S(n_x, \omega)$. At $n_x > n_1$ we have

$$S = S_0 n_x (q + \Gamma \operatorname{th} k_0\Gamma d) \times \left[\frac{2}{k_0 d} + \frac{q - \Gamma \operatorname{th} k_0\Gamma d}{\operatorname{ch}^2 k_0\Gamma d}\left(1 + \frac{\operatorname{sh} 2k_0\Gamma d}{2k_0\Gamma d}\right)\right]. \tag{24a}$$

At $n_x < n_1$

$$S = S_0 n_x (q - \gamma \operatorname{tg} k_0\gamma d) \left[\frac{2}{k_0 d} + \frac{q + \gamma \operatorname{tg} k_0\gamma d}{\cos^2 k_0\gamma d}\left(1 + \frac{\sin 2k_0\gamma d}{2k_0\gamma d}\right)\right], \tag{24b}$$

where $S_0 = cd/4\pi\alpha$. The equations obtained are the dispersion relations for the symmetrical modes in terms of the variables ω, n_x, and S. At each fixed S these relations give the connection between the frequency and the wave vector. It is also to easy to verify that at $S = 0$ Eq. (24b) reduces to the dispersion relation for the symmetric modes of a linear waveguide, while as $d \to \infty$ Eq. (24) yields double the energy flux of the nonlinear surface waves considered in Refs. 2 and 3.

For the antisymmetrical modes, integration using Eqs. (5)–(7) (14), and (15) leads to the following expressions for the energy flux. At $n_x > n_1$

$$S = S_0 n_x (q + \Gamma \operatorname{cth} k_0 \Gamma d) \times \left[\frac{2}{k_0 d} + \frac{q - \Gamma \operatorname{cth} k_0 \Gamma d}{\operatorname{sh}^2 k_0 \Gamma d} \left(\frac{\operatorname{sh} 2k_0 \Gamma d}{2k_0 \Gamma d} - 1 \right) \right]. \tag{25a}$$

In the case $n_x < n_1$

$$S = S_0 n_x (q + \gamma \operatorname{ctg} k_0 \gamma d) \times \left[\frac{2}{k_0 d} + \frac{q - \gamma \operatorname{ctg} k_0 \gamma d}{\sin^2 k_0 \gamma d} \left(1 - \frac{\sin 2k_0 \gamma d}{2k_0 \gamma d} \right) \right]. \tag{25b}$$

Here, too, it is easily seen that at $S = 0$ Eq. (25b) reduces to a dispersion relation for odd modes of the linear waveguide, and as $d \to \infty$ Eq. (25a) gives double the energy flux of the nonlinear surfaces waves.

For the asymmetric modes A and B, the formulas for the energy flux are the same, but in place of d_0 it is necessary to substitute in them different values : $d_0^{(1)}$ or $d_0^{(2)}$. At $n_x > n_1$ we have

$$S = S_0 n_x \left\{ \frac{1}{k_0 d} [2q + \Gamma \operatorname{cth} k_0 \Gamma (d + d_0) + \Gamma \operatorname{cth} k_0 \Gamma (d - d_0)] + \frac{q^2 - \Gamma^2 \operatorname{cth}^2 k_0 \Gamma (d \pm d_0)}{\operatorname{sh}^2 k_0 \Gamma (d \pm d_0)} \left(\frac{\operatorname{sh} 2k_0 \Gamma d}{2k_0 \Gamma d} \operatorname{ch} (2k_0 \Gamma d_0) - 1 \right) \right\}. \tag{26a}$$

In the case $n_x < n_1$,

$$S = S_0 n_x \left\{ \frac{1}{k_0 d} [2q + \gamma \operatorname{ctg} k_0 \gamma (d + d_0) + \gamma \operatorname{ctg} k_0 \gamma (d - d_0)] + \frac{q^2 - \gamma^2 \operatorname{ctg}^2 k_0 \gamma (d \pm d_0)}{\sin^2 k_0 \gamma (d \pm d_0)} \left(1 - \frac{\sin 2k_0 \gamma d}{2k_0 \gamma d} \cos 2k_0 \gamma d_0 \right) \right\}. \tag{26b}$$

For positive and negative d_0 these formulas give identical values of S. At $k_0 \gamma d_0 = \pi/2$ and at $d_0 = 0$ Eqs. (26) reduce respectively to (24) and (25), as expected.

4. NUMERICAL CALCULATIONS AND DISCUSSION OF RESULTS

We used the formulas derived in the preceding section to calculate numerically the functions $S(n_x, \omega)$ for several values of the system parameters. The results of these calculations are shown in Fig. 1. In place of the frequency we chose the equivalent parameter $2d/\lambda = \omega d/\pi c$. At low energy fluxes, as can be seen from the figure, the layered structure has a discrete set, which depends on the parameter $2d/\lambda$, of values of n_x. At small $2d/\lambda$, the propagation constants of the symmetric and antisymmetric modes increase monotonically with increasing energy flux (Fig. 1a). However, at layer

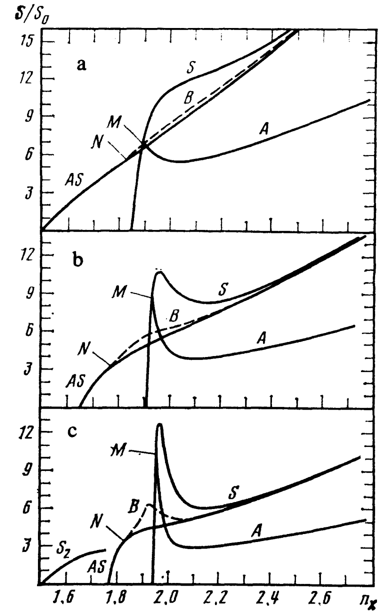

FIG. 1. Dependence of the dimensionless energy flux S/S_0 on the propagation constant n_x for a structure with parameters $n_0 = 1.5$, $n_1 = 2.0$; $2d/\lambda = 0.4$ (a), 0.6 (b), 0.8 (c). The curves are marked as follows: S—plot for symmetric mode; AS—for antisymmetric mode; A—for asymmetric mode A; B—for asymmetric mode B; S_2—for a symmetric mode of second order.

thicknesses exceeding a certain critical value, the plot of the symmetric mode assumes an N-shape (Figs. 1b, 1c). On the decreasing section of the curve, the symmetric mode is unstable. This means that if the external parameter is the energy flux in the structure, the state of the nonlinear surface waves is bistable: the same value of the energy flux corresponds to two stable values of the propagation constant.

Before the maximum S is reached, an asymmetric-mode curve A branches away at the point M from the curve of the symmetric mode. This branch corresponds to two different solutions, which are made equal by the transformation $z \to -z$. The transition from the symmetric to the asymmetric mode at the point M is continuous. For each layer thickness there is a certain minimum energy flux needed for the existence of the mode A. To the right of the point M, the branch of the mode A has a descending section, which is apparently also unstable. In order to determine the behavior of the system when the point M is reached, however, we must solve the problem of the stability to transformation into other modes. We did not do this in the framework of the present study. The energy flux in the mode A is smaller than in the symmetric mode in the entire interval of variation of n_x.

The curves for the antisymmetric mode can start out at small $2d/\lambda$ from finite energy fluxes (at $2d/\lambda < 0.4$), in which case the $S(n_x)$ curve is also monotonic. However, when $2d/\lambda$ increases the antisymmetric-mode branch also assumes an N-shape (Fig. 1c). The assymetric mode B is separated from the antisymmetric-mode branch at the point N, and the transition from the antisymmetric mode to the mode B is also continuous in the parameters n_x and S. The mode B, in analogy with the mode A, can exist only if it has a certain minimum energy flux.

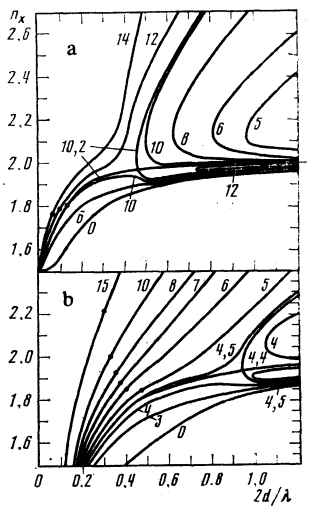

FIG. 2. Dispersion curves for the symmetric (a) and antisymmetric (b) modes at a constant energy flux. The numbers on the curves denote the dimensionless energy flux S/S_0. The thick points on the curves denote the points M (a) and N (b).

Calculations by formulas (24)–(26) at other values of the parameters show that the characteristic features of the $S(n_x)$ curve do not depend on the concrete values of n_0 and n_1. With decreasing difference $n_1 - n_0$, however, these features manifest themselves at smaller energy fluxes S and at larger thicknesses $2d/\lambda$.

Figures 2 and 3 show the dispersion curves $n_x(2d/\lambda)$ for symmetric and antisymmetric modes, obtained from the relation $S(n_x, 2d/\lambda) = \text{const}$. At a zero energy flux these lines reduce to ordinary dispersion curves of the discrete modes of the waveguide. With increasing energy flux, the dispersion curves are shifted. In particular, a shift takes place in the cutoff frequency of the antisymmetric mode.

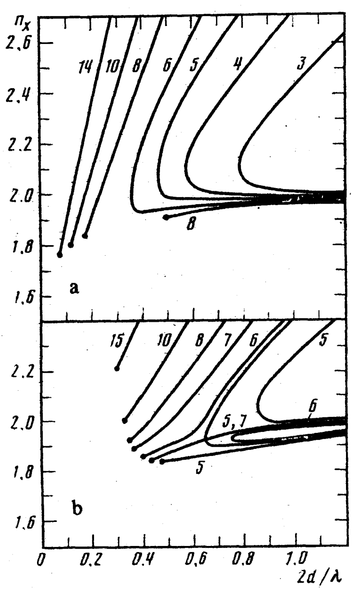

FIG. 3. Dispersion curves for the asymmetric mode A (a) and for the asymmetric mode B (b) at a constant energy flux. The notation is the same as in Fig. 2.

This effect can be used, for example, to produce energy flux stabilizers in the waveguide. At a certain energy flux ($S/S_0 = 10.2$ in Fig. 2a, $S/S_0 = 4.4$ in Fig. 2b) the curves branch out at a certain point. For the values $2d/\lambda$, which are located to the right of the branching point, the $S(n_x)$ curves are N-shaped. It is seen from Fig. 2 that the critical value $2d/\lambda$ for the antisymmetric modes is larger than the corresponding value of $2d/\lambda$ for the symmetric mode.

The curves of Fig. 3 can be regarded as continuations of the curves corresponding to the same energy fluxes shown in Fig. 2. They branch away from the curves of Fig. 2 at the points M and N, which are shown thicker in the figure. Thus, the dispersion curves for the symmetric and antisymmetric modes are also split; the dispersion curves of the modes A and B branch out from them at certain points M and N.

5. CONCLUSION

Our analysis of the nonlinear symmetrical layered structure has made it possible to draw a number of interesting conclusions:

1. Allowance for the nonlinearity in the layered system modifies substantially the set of natural modes of the linear waveguide—besides the symmetric and antisymmetric mode, asymmetric modes of type A and B can propagate in the system.

2. For the existence of this mode, certain minimum energy fluxes are needed.

3. Branches of these modes are split off at certain characteristic points from the symmetric and antisymmetric mode branches, respectively, and these modes can be transformed into symmetric and antisymmetric by continuous variation of the external parameters—the energy flux and the wave vector.

4. In a certain range of values of the carrying-layer thickness or of the wavelength, the nonlinear surface waves are bistable: one value of the energy flux in the wave corresponds to two values of the wave vector.

These features of nonlinear surface waves can apparently find application in integrated-optics instruments and deserve further theoretical and experimental study.

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