Self-action and stimulated scattering effects of sound beams in absorbing liquids—thermal self-focusing

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The mechanisms of self-action of intense sound in viscous homogeneous liquids are described and estimated theoretically. It is shown that dissipative mechanisms are predominant for a broad class of liquids. These mechanisms involve viscous heating and the excitation of "acoustic streaming," whose velocity clearly renormalizes the effective temperature. A number is introduced that distinguishes this class (the thermal analog of the acoustic Mach number). The threshold energy values and the spatial scales of the thermal self-focusing of the sound pulses are calculated. The conditions are found under which the self-action process is more rapid than shock wave formation.

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1. INTRODUCTION

1. Nonlinear acoustics is a thoroughly studied division of the mechanics of continuous media and important results have been obtained by its methods.[1,2] These have considerably deepened our understanding of the structure and properties of condensed matter. At the present time, the phenomenology of the interaction and self-action of sound waves through the elastic mechanism of nonlinearity (the interaction of "sound with sound") has been developed in detail in the researches of Khokhlov and co-workers (see Ref. 2). Nonlinear mechanisms of a nonacoustic nature (see, for example, Refs. 3–7) have been studied with considerably less detail.

The chief obstacle to the generation and transformation of narrow-band acoustic signals (similar to what prevails in nonlinear optics) is the weak dispersion, which does not hinder the rapid enrichment of the spectrum of an intense sound with harmonics. One of the solutions here is the application of methods which introduce artificial sound-velocity dispersion; in liquids, mode dispersion of acoustic waveguides,[8] dispersion of boundary impedances,[9] and impurity resonances, are possible (see, for example, Ref. 10 for gas bubbles in a solution). The application of such methods is based on the fact that nonlinear wave effects are made up of nonlinear distortions that build up with the distance traversed by the wave packet in the medium, and the quantitative measure of them is the spatial scale L over which the distortions reach a definite level. For the process of the multiple generation of harmonics, this is the formation discontinuity length L_{dis}, which is inversely proportional to the amplitude of the sound and to the coefficient of elastic nonlinearity ε. The dispersion that is introduced destroys the phase agreement of the harmonics, extending the scale for all the frequencies except for a limited set singled out by the dispersion curve.

Along with this, there is another logically admissible possibility that consists of assigning to the system a scale L, less than L_{dis}, that is independent of the elastic nonlinearity. It is clear that such a scale can be realized only by including excitations of another physical nature in the interaction with the sound.

2. The mechanisms of such excitations are classified into parametric (motion of the fluid, brought about by adiabatic external action) and nonlinear thermohydrodynamic mechanisms. The number of thermodynamic mechanisms for nonlinear sound effects is exactly equal to the number of pairs of conjugate coordinates and forces that describe the system under consideration. This is first of all the universal thermal (entropy) mechanism, which, together with the elastic mechanism comprises the entire set for classical liquids far from phase transitions. In solutions, a concentration mechanism is added to it—a redistribution of the density of the components, brought about by pressure gradients, and also sound-induced chemical transformations. In conducting liquids, interaction of the sound waves should take place via excitation of magnetohydrodynamic vibrations, and in polarized liquids via low-frequency electrical excitations (in this connection, the anisotropic liquid-crystal mixtures and ferroelectric properties are especially interesting[11]).

The diversity of these effects is sharply narrowed near phase transitions. The anomalous increase in the fluctuations of the order parameter singles out in the pretransition regions only one mechanism that is connected precisely with the excitation of oscillations of the order parameter. Near the stratification point of solutions, for example, the concentration mechanism should predominate, while around the boiling point, the elastic mechanism, and so on. It is essential that in these regions, the discussed effects should not only be emphasized but should also be most informative. The locality, the adjustable duration, and the intensity of the action of the sound open up here possibilities of, say, programmed "motion" over a specified trajectory in the space of the critical variables and, as a consequence, of a detailed measurement of the pretransition values of the parameters over a wide range. A promising development of the idea of Ref. 12, is the realization of a program of acousto-thermodynamic investigations. This program should first include an analysis of the kinetics of phase transitions[13] under the action of in-

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tense sound. Its second part, pertaining to methods of probing phase tansitions, can be established by the study of nonlinear acoustical transformations. The employment of practically all of the phenomenology of nonlinear optics is useful here.[14] Among the investigated effects we should include scattering by excitations of this or that nature—spontaneous and stimulated, including those of acoustical spectroscopy.[15] A number of theoretical and experimental results have already been obtained along these lines; for example, the possibility has been shown of the reversal of the wave front of acoustic beams.[16–19] Studies of the self-action of sound beams have a special value. One aspect here is the establishment of maximum possible transfer of sound energy to a thick layer of liquid, in particular for the purpose of acoustic probing. Another aspect is connected with the kinetics of the destruction of matter; here, as in the similar optical case, new physical effects can appear in principle.

3. It is premature at the present time to analyze in detail the applications of nonlinear thermohydrodynamic phenomena. The analysis of the concentration, magnetohydrodynamic, and other mechanisms is the subject of separate studies. Here we shall investigate a new class of nonlinear acoustic effects connected with the thermal mechanism. It is an important problem by virtue of its universality—sound in principle should increase the temperature of any liquid (a probable exception are mixtures in which endothermic reactions are excited), although, of course, they differ in their susceptibility to sonic heating. The establishment of an explicit quantitative description of the susceptibility is one of the results of our work.

The main content of this paper is a detailed analysis of the conditions of observation of thermal self-focusing of sound (TSFS) in a viscous liquid. This effect was suggested by Askar'yan[3] and is based on the monotonic decrease of sound velocity with temperature usually observed in liquids (exceptions are water at $T \leqslant 74\ ^\circ\mathrm{C}$ and a number of liquid metals—bismuth, antimony, tellurium). The continuous regime of TSFS has been investigated theoretically by Zabolotskaya and Khokhlov.[5] A system of truncated equations that completely describe TSFS in both the continuous and the pulsed regimes is described in Sec. 2. The time of establishment of the continuous regime amounts to several minutes and convective mixing over the entire volume of the liquid can smooth out the effect; it is therefore necessary to resort to a nonstationary regime.

The role of another universal mechanism of self-focusing is also made clear and estimated in Secs. 2 and 3—hydrodynamic flow ("acoustic streaming"), a self-action that takes place through excitation by sound that is absorbed by the medium and is diffracted. The flow always attempts to restore the level of the sound intensity; therefore, it defocusses a beam with limited cross section.[6] In our analysis, this is clearly expressed by the introduction—for the continuous regime—of an effective temperature, which is less than the real by an amount that is proportional to the flow velocity. In the nonstationary regime the flow, it turns out, limits the length of the pulse that can be focused thanks to the thermal nonlinearity.

In addition, our analysis predicts a new effect of stimulated scattering of sound by a temperature wave (it was first considered in more limited form in Ref. 20), and also its analog—scattering from a sound-induced low-frequency hydrodynamic wave.

2. DERIVATION OF THE TRUNCATED EQUATIONS OF SELF-ACTION OF QUASIMONOCHROMATIC SOUND

1. The description of the nonlinear acoustical effects in a homogeneous absorbing liquid is contained in a set of thermodynamical equations, including:

the equation of continuity

$$\partial_t\rho+\partial_b(\rho v_b)=0, \tag{1}$$

the balance-of-forces equation

$$\rho(\partial_t+v_b\partial_b)v_a+\partial_a p=\eta\partial_b{}^2 v_a+(\zeta+\eta/3)\partial_a\partial_b v_b \tag{2}$$

and the equation of entropy production

$$\begin{aligned}&\rho T(\partial_t+v_b\partial_b)s-\varkappa\partial_b{}^2 T\\&=\zeta(\partial_b v_b)^2+\eta\partial_a v_b[\partial_a v_b+\partial_b v_a-{}^2/{}_3\delta_{ab}\partial_c v_c].\end{aligned} \tag{3}$$

Here ρ, p, T and s are the mass density of the liquid, the pressure, the temperature, and the entropy density, respectively; $v_{a,b}$ are the Cartesian components of the velocity; η, ζ, and $\varkappa$ are the coefficients of shear viscosity, bulk viscosity and thermal conductivity. Later on by these same letters (ρ, p, T, s) we shall denote the departures from the equilibrium values labeled by the subscript zero. The equation of state completes the set (1)–(3), i.e., the functions $\rho(p, T)$ and $s(p,T)$; at small departures from equilibrium, it suffices to know the first few thermodynamic derivatives of ρ and s with respect to p and T: $(\partial\rho/\partial p)_T=\rho_p$, $(\partial^2\rho/\partial T^2)_p=\rho_{TT}$, etc.

2. In correspondence with what was said in subsection 1 of Sec. 1, we require that the observation distance L (i.e., all the spatial scales of accumulation of the described effects; in our problem, this is primarily L_f—the focal distance of the nonlinear thermal lens) be shorter than the distance L_{dis} over which the discontinuity is formed:

$$L<L_d=(\rho_0c_0{}^5)^{1/2}/(2I)^{1/2}\omega\varepsilon. \tag{4}$$

Here ω is the frequency, I the sound-wave intensity input,

$$\varepsilon=1+\frac{\rho_0}{2}\left[\left(\frac{\partial c_0}{\partial p}\right)_T+\frac{\alpha T_0}{\rho_0 c_p}\left(\frac{\partial c_0}{\partial T}\right)_p\right]$$

is the coefficient of elastic nonlinearity, α is the coefficient of volume thermal expansion, c_0 is the equilibrium sound velocity, c_p is the isobaric specific heat.

Violation of the inequality (4) does not of course prevent the effect of TSFS; however, the picture becomes so complicated in this case (in particular, the discontinuity in this case should form more rapidly on points of the axis than on the periphery, if self-focusing takes place) that it does not lend itself as yet to an analytic description. The quantitative treatment of the experiment is also made more difficult under these conditions, although determination of the paraxial discontinuity can provide a qualitative indication of the TSFS.

Assuming condition (4) to be satisfied, we can solve the problem in the quasimonochromatic approximation— as the unexcited solution we employ the solution, linearized about the equilibrium point, in conjunction with the linear equation of state:

$$\rho=\left(\frac{1}{c_0^2}+\frac{\alpha^2 T_0}{c_p}\right)p-\rho_0\alpha T, \qquad s=-\frac{\alpha}{\rho_0}p+\frac{c_p}{T_0}T. \tag{5}$$

The dispersion equation has two solutions: $\omega = 0$ and $\omega = c_0k$ (the wave vector $\mathbf{k}$ is directed along the x axis for definiteness), and the high-frequency complex amplitudes are connected by the relations

$$T^\omega=\frac{\alpha T_0}{\rho_0 c_p}p^\omega; \quad v_x^\omega=\frac{1}{\rho_0 c_0}p^\omega. \tag{6}$$

In correspondence with this, we must seek a solution of the nonlinear problem in the form

$$p=p^0+(p^\omega/2)\exp[i\omega(t-x/c_0)]+\text{c.c.} \tag{7}$$

(and similarly for the variables T and v_a), assuming that p^0, p^ω and so forth are slowly varying functions of t, x, and $\mathbf{r}_\perp = (0,y,z)$. $\partial_t \sim \partial_x \sim \mu$, $\nabla_\perp = (0,\partial_y,\partial_z) \sim \mu^{1/2}$ μ is a symbolic smallness parameter. This is the well-known geometrical approximation of diffraction theory.[21]

The correctness of such a quasimonochromatic description in our problem is determined by a whole series of parameters. We first require the satisfaction of the inequality $L/L_{\text{dis}} < 1$; further, changes in the input amplitude of the sound should be slow (in wave scales): the beam width $a > 2\pi c_0/\omega$, the pulse duration $\tau > \omega.^{-1}$ In order to establish all the other limitations, we carry out the natural normalization of the thermodynamic variables to their equilibrium values, and the coordinates to the wave scales; the entropy density is normalized to c_p, the velocity to c_0. In the new variables (we use for them the same notation as before) the set (1)–(3) has the form

$$\partial_t\rho+\partial_b[(1+\mu_v\rho)v_b]=0, \tag{1'}$$

$$(1+\mu_v\rho)(\partial_t+\mu_v v_b\partial_b)v_a+\partial_a p=\mu_\eta[\tilde{\eta}\partial_b^2 v_a+(\tilde{\xi}+\tilde{\eta}/3)\partial_a\partial_b v_b], \tag{2'}$$

$$(1+\mu_v\rho)(1+\mu_\tau T)(\partial_t+\mu_v v_b\partial_b)s=\mu_\varkappa\tilde{\varkappa}\partial_b^2 T+\mu_v^2\mu_\eta\mu_T^{-1}$$
$$\times M_T^{-2}[\tilde{\xi}(\partial_b v_b)^2+\tilde{\eta}\partial_a v_b(\partial_a v_b+\partial_b v_a-{}^2/_3\delta_{ab}\partial_c v_c)] \tag{3'}$$

with the symbolic parameters μ_v, μ_T, μ_η, $\mu_\varkappa$ and the real parameters

$$(\tilde{\xi},\tilde{\eta})=\frac{(\xi,\eta)\omega}{\rho_0 c_0^2}; \quad \tilde{\varkappa}=\frac{\varkappa\omega}{\rho_0 c_0^2 c_p}; \quad M_T^{-2}=\frac{c_0^2}{T_0 c_p}.$$

These terms in (1′)–(3′) which are marked by the symbols μ should be smaller than the terms pertaining to the linear, nondissipative system. This requires first the smallness of the departures from equilibrium which is expressed by the conditions $\mu_v \ll 1$ (it is really governed by the inequality $v \ll c_0$) and $\mu_T \ll 1$ i.e., the inequality $T/T_0 \ll 1$). The smallness of terms with μ_η and $\mu_\varkappa$ is expressed by the inequalities $(\tilde{\xi},\tilde{\eta}) \ll 1$ (the sound absorption coefficient per wavelength is smaller than unity) and $\tilde{\varkappa} < 1$ (the time of diffusion spreading of the temperature excitation in a region with dimensions of the order of a wavelength is much longer than the period of the sound oscillations). The last two inequalities are satisfied in the ultrasonic range by a wide margin: at $T_0 = 20$ °C and at a frequency $\omega/2\pi = 1$ MHz, $\tilde{\eta} = 2.4\times10^{-6}$ in benzene, and $\tilde{\eta} = 2.0\times10^{-3}$; in very viscous glycerine; the corresponding values of $\tilde{\varkappa}$ are[1)] 3.5×10^{-7} and 1.6×10^{-7}.

The parameter M_T^{-2}, as follows from (3), characterizes the susceptibility of the liquid to viscous heating by the sound. It is equal to the ratio of the square of the velocity of sound to the velocity of thermal motion (the thermal analog of the reciprocal of the Mach number) and determines in principle the intensity of the thermohydrodynamic effects in some liquid. Formally, those terms of (3′) which correspond to viscous heating do not vanish in a consistent analysis of the orders of smallness only if $M_T^{-2} \sim \mu^{-1} \gg 1$. In viscous liquids, this is actually the case: under normal conditions, M_T^{-2} (benzene) = 3.5, M_T^{-2} (glycerin) = 5.3. For comparison, M_T^{-2} (water) = 1.7; for an ideal monatomic gas, $M_T^{-2} = 0.4$.

3. The quadratic term in the expansion of ρ and p in the equation of state does not make a contribution to the self-action (it does not have in its spectrum a component with frequency ω), once (4) is assumed; therefore, we carry out the expansion up to cubic terms. The missing connection between the amplitude of the transverse velocity $\mathbf{v}_\perp = (0, v_y, v_z)$ and the amplitude of the pressure is obtained—in the first nonvanishing order—from (2):

$$\mathbf{v}_\perp^\omega=-(i\omega\rho_0)^{-1}\nabla_\perp p^\omega. \tag{8}$$

Now substituting (7) with the constraints (6) and (8) in (1)–(3), eliminating terms with the frequencies $\omega = 0$ and $\omega = c_0k$, we obtain a system of truncated equations that describe the self-action of the sound. The equation for the changes in amplitude of the sound pressure has the form

$$(c_0\partial_x+\partial_t+ic_0^2\nabla_\perp^2/2\omega+c_0\delta)p^\omega=i\omega p^\omega[v_x^0/c_0+(1-a_1)$$
$$\times|p^\omega|^2/8(\rho_0c_0^2)^2-a_2p^0/\rho_0c_0^2-a_3T^0/T_0]. \tag{9}$$

The absorption coefficient (with account of only the viscosity mechanism) is

$$\delta=(\xi+4\eta/3)\omega^2/2(\rho_0c_0^3).$$

The remaining coefficients are expressed in terms of the thermodynamic derivatives of the density and the entropy:

$$a_1=\frac{(\rho_0c_0^3)^2}{16}\left(-q\frac{\partial}{\partial T}+\frac{\partial}{\partial p}\right)^3\rho, \qquad q=\alpha T_0(c_p\rho_0)^{-1},$$

$$a_2=\frac{\rho_0c_0^4}{2}\left(-q\frac{\partial}{\partial T}+\frac{\partial}{\partial p}\right)\frac{\partial\rho}{\partial p},$$

$$a_3=\frac{T_0c_0^2}{2}\left(-q\frac{\partial}{\partial T}+\frac{\partial}{\partial p}\right)\frac{\partial\rho}{\partial T}.$$

The constant q is small in all liquids ($\lesssim 10^{-8}$ deg-s^2-cm-g^{-1}); therefore the coefficients a_i are determined with high accuracy by only the quantity ϵ and by the derivative of the sound velocity with respect to T: $a_1 \approx (\varepsilon-1)^2/2$, $a_2 = \varepsilon - 1$, $a_3 \approx -T_0\gamma_p/2$, where $\gamma_p = (\partial \ln c_0^2/\partial T)_p$.

The terms on the right hand side of (9) correspond to four mechanisms of self-action of the sound: a) excitation of longitudinal flow ("acoustic streaming"), b) noninertial change of state, c) generation of pressure video pulses ("acoustic detection"), d) heating of the liquid by intense sound.

4. The total changes in the sound intensity—both dissipative and diffractive attenuation—bring about the development of streaming.

$$(\partial_t-\nu\nabla_\perp^2)v_x^0+\rho_0^{-1}\partial_x p^0=-\frac{1}{4\rho_0^2c_0^3}(c_0\partial_x+\partial_t)|p^\omega|^2, \tag{10}$$

$\nu = \eta/\rho_0$ is the kinematic viscosity.

Two mechanisms—viscous and adiabatic—also lead to heating:

$$(\partial_t-\chi\nabla_\perp^2)T^0-q\partial_t p^0=\frac{\delta}{\rho_0^2c_0c_p}|p^\omega|^2-\frac{T_0a_4}{4(\rho_0c_0^2)^2}\partial_t|p^\omega|^2. \tag{11}$$

Here $\chi = \varkappa(\rho_0c_p)^{-1}$ is the coefficient of thermal diffusivity,

$$a_4=\frac{(\rho_0c_0^2)^2}{c_p}\left(-q\frac{\partial}{\partial T}+\frac{\partial}{\partial p}\right)^2 s\approx\frac{\alpha c_0^2}{c_p}.$$

Finally, acoustic detection is described by the equations

$$\frac{1+M_T^{-2}(\alpha T_0)^2}{\rho_0c_0^2}\partial_t p^0-\alpha\partial_t T+\partial_a v_a^0=\frac{1}{2\rho_0^2c_0^5}(\partial_t+c_0\delta)|p^\omega|^2$$

$$-\frac{a_2-(qT_0/\rho_0c_0^2)a_3}{2(\rho_0c_0^2)^2}\partial_t|p^\omega|^2, \tag{12}$$

$$\rho_0\partial_t\mathbf{v}_\perp^0+\nabla_\perp p^0=0. \tag{13}$$

The driving terms on the right side of (12) correspond respectively to the combined dissipative and adiabatic (through the nonlinearity of the equation of state) mechanisms.

3. ANALYSIS OF THE CONDITIONS FOR OBSERVATION OF TSFS

1. In order to establish the possibility for observation of thermal self-focusing, we investigate the system (9)–(13), using the results of calculations of its optical analog in the aberration-free approximation.[21,23]

For the inertia-free mechanism (b), the formula

$$L_f=(a/2)(c_0/|\Delta c|)^{1/2}, \tag{14}$$

determines the focusing length. Here $\Delta c\approx\varepsilon^2I(8\rho_0c_0^2)^{-1}$ is the nonlinear change in the sound velocity. Comparing (4) and (14), we obtain the result that the ratio

$$L_f/L_p\approx4\pi a/\lambda>1$$

($\lambda = 2\pi c_0/\omega$ is the wavelength), i.e., this mechanism cannot produce self-focusing. The reason is quite understandable: both processes—the formation of a discontinuity and the narrowing of the beam—are governed here by the same nonlinearity, but the first of them is a plane-wave mechanism, i.e., more rapid, while the second is diffraction origin.

Mechanisms (a), (c), and (d) are generally nonstationary. It is easy to estimate their establishment times from (10)–(12), by eliminating $\mathbf{v}_\perp$ with the help of (13). We get, correspondingly,

$$\tau(v)=a^2/\nu,\quad \tau(p)=a/c_0,\quad \tau(T)=a^2/\chi. \tag{15}$$

The numerical values of these times form a completely determined sequence (we made estimates for benzene and glycerin at $a = 0.5$ cm):

$$\tau(T)=(2.6-2.7)\cdot10^2\ \text{s}\gg\tau(v)$$

$$=(3.5-2.3)\cdot10^{-2}\ \text{s}\gg\tau(p)=(3.8-2.6)\cdot10^{-6}\ \text{s}.$$

The fastest mechanism (c) of acoustic detection, as is seen from this estimate, is always stationary if we do not enter into the range of frequencies above 10–100 MHz. In the steady-state region, it follows from (12) that $p^0\approx\varepsilon I/c_0$, and the estimate according to (14) again gives $L_f/L_{dis}\sim a/\lambda$.

Thus, over the entire real range of τ, the contribution to the self-action of the beam, observed against the background of the developing shock wave, can be made only by the mechanisms (a) and (d)--excitation of streaming and heating.

2. We now compare the intensity of the viscous (T_δ) and adiabatic (T_α) heating. At $\tau\gg\tau(T)$ we have $T_\alpha\approx0$ (this is simply a uniform change of beam temperature). For a nonstationary regime, their estimate from (11) is

$$T_\delta\approx2\delta I\tau(\rho_0c_p)^{-1},\qquad T_\alpha\approx\alpha T_0I(2\rho_0c_0c_p)^{-1}, \tag{16}$$

whence $T_\delta/T_\alpha\gg1$, i.e., only the contribution of viscous heating is achieved in practice if we consider media with $\delta\gtrsim10^{-2}$–10^{-3} cm^{-1} in the case of pulse lengths greater than $(1–10)\,\omega^{-1}$.

There are two reasons for the excitation of streaming. Its maxivelocity, which is due to sound attenuation in the stationary, $\tau>\tau(v)$, and in the nonstationary regimes, is estimated from (10) and (9):

$$v_\delta\approx a^2\delta I/(\eta c_0)^{-1};\ \delta I\tau(\rho_0c_0)^{-1}. \tag{17}$$

The diffraction attenuation alone generates streaming up to the velocity v_d, the maximum value of which is determined by the dependence $I(x)\approx I(0)[1+(x/L_d)^2]^{-1}$, where L_d is the diffraction scale of beam spreading and, is equal, in the absence of self-action, to $-\omega a^2/c_0$. It follows from (10) that

$$v_d\approx a^2I(2\eta c_0L_d)^{-1};\ I\tau(2\rho_0c_0L_d)^{-1}$$

for $\tau>\tau(v)$ and $\tau<\tau(v)$, respectively. The ratio of v_δ to v_d in this and the other case is equal to $2\delta L_d$ and in principle can take on values of the order of 1–10, i.e., the contributions of the dissipative and diffractive sources of streaming can be comparable. Here, however, we encounter the following fine point: in seeking the threshold of self-focusing, we should assume $L_d\to\infty$, since at the threshold (by definition) the diffractive spreading is compensated by the nonlinear self-compression. Thus, in the estimate of the possibilities of self-focusing, we need to take into account only the viscous source of the flow; the exact picture of its development in the self-focusing is complicated and deserves a special analysis.

3. The estimates given in subsections 1 and 2 allow us to simplify the complete system (9)–(13) considerably. In the continuous regime, when $\tau>\tau(T)$, it reduces to only two equations:

$$\begin{aligned}&\left(\partial_x+\frac{ic_0}{2\omega}\nabla_\perp^2+\delta\right)p^\omega=\frac{i\omega\gamma_p}{2c_0}\tilde{T}p^\omega,\\&\nabla_\perp^2\tilde{T}=-\frac{\delta}{\varkappa\rho_0c_0}\left(1+\frac{\varkappa}{\eta\gamma_pc_0^2}\right)|p^\omega|^2,\end{aligned} \tag{18}$$

where the effective "temperature" $\tilde{T}=T^0+2v_x^0(c_0\gamma_p)^{-1}$.

The system (18) is identical to the system that describes (depending on the combination of the signs of the right-hand sides) the self-focusing or self-defocusing.[23]

Self-focusing leads to narrowing to the beam if (aside from the negative nature of γ_p) $\sigma = \varkappa(\eta|\gamma_p|c_0^2)^{-1} < 1$ the loss of heat by streaming does not compensate for the heating. In viscous liquids, the inequality $\sigma < 1$ is well satisfied; σ (benzene) $= 2\times 10^{-2}$, σ(glycerin) $= 3\times 10^{-5}$; the choice of the liquid that is capable in principle of assuring self-focusing is not a difficult one.

The focal length of the thermal lens[23] is equal to $L_f = (\varkappa/\delta|\gamma_p|I)^{1/2}$, and the condition (4) requires the satisfaction of the inequality

$$(\zeta+4\eta/3)c_0^2|\gamma_p|>4\varkappa\varepsilon^2, \tag{19}$$

if the mechanism of absorption is the classical one. At a frequency of 1 MHz, the condition (19) is satisfied in benzene and—with a margin of almost two orders of magnitude—in glycerin. In the first case, however, the damping $\delta \sim \omega^2$ is not exactly satisfied; for similar liquids, (19) is replaced by a more general condition, which limits the frequency range:

$$\omega^2\delta^{-1}(\omega)<\rho_0c_0^5|\gamma_p|(4\varkappa\varepsilon^2)^{-1}.$$

Substitution here of the parameters of benzene and glycerin yields a positive result for $\omega/2\pi \approx 1$–10 MHz.

It remains only to estimate the temperature changes produced by the sound; they should not be too large in order not to destroy the homogeneity of the liquid:

$$T^0\approx 2\delta a^2I/\varkappa=(2/|\gamma_p|)(a/L_f)^2$$

It then follows that the overheating is not too large, $T^0 \sim 1$ °C, even if a shortfocus lens is formed: $L_f \approx 8.5$ cm (benzene) and 16 cm (glycerin). The attenuation at such lengths is also small: $\delta L_f = 8\times 10^{-2}$ (benzene) and 4×10^{-1} (glycerin). The corresponding intensities are $I = \varkappa(2\delta a^2)^{-1} = (0.3$–$0.2)$ W/cm^2.

4. In the pulsed self-focusing regime, the critical parameter is the energy $W = \pi a^2 I\tau$; it also determines the focal length of the lens; therefore the conditions of observation of self-focusing should impose now the lower limit of the duration of the pulse. In the experimentally most convenient range $\tau(v) < \tau < \tau(T)$, this limitation is governed first of all by the requirement $v_x^0 < c_0|\gamma_p|T^0/2$—the thermal converging lens should be stronger than the diverging streaming lens. Substitution of formulas (16) and (17) in the latter inequality yields

$$\tau>\tau_v=\frac{a^2\rho_0c_p}{\eta c_0^2|\gamma_p|}. \tag{20}$$

In glycerin, $\tau_v = 7.6\times 10^{-3}$ s—the heating is stronger than the streaming over the entire range $\tau > \tau(v)$ Benzene defocuses—because of the more developed streaming[η (benzene)/η(glycerin) $= 4\times 10^{-4}$]— pulses of duration less than 4.9 s.

We find the critical self-focusing energy by equating L_f (14) to the diffraction length L_d introduced in subsec. 2, with accuracy to within the numerical factor $8|\Delta c|/c_0 = (0.61\lambda / a)^2$, where the change in velocity Δc is given by the substitution of (16) in (9):

$$W_{cr}=(0.61)^2\pi\rho_0c_p\lambda^2/8|\gamma_p|\delta. \tag{21}$$

For benzene, this quantity amounts to 63 J, for glycerin, 340 J. We recall that up to now we have used in the estimates the well-known tabulated data for the frequency $\omega/2\pi = 1$ MHz. The quantity W_{cr} falls off sharply with increase in frequency; for the classical mechanism of absorption, it is proportional to ω^{-4}; at $\omega/2\pi = 3$ MHz, for example, W_{cr} (glycerin) $= 4.2$ J.

The self-focusing length, in this regime,

$$L_f=a^2(\pi\rho_0c_p)^{1/2}/2(|\gamma_p|\delta W)^{1/2}, \tag{22}$$

in glycerin, is already equal to 3–8 cm at frequencies of 1–3 MHz, when the threshold is exceeded by 1.5 times.

The condition (4), with account of (22), imposes, as pointed out above, a limitation on the pulse length:

$$\tau>\tau_p=c_p(\varepsilon\omega a)^2/|\gamma_p|\delta c_0^5, \tag{23}$$

at $\delta \sim \omega^2$ it does not depend on the frequency; τ_p (benzene) $= 1.8$ s, τ_p (glycerin) $= 0.6$ s. This estimate essentially prohibits the transition to durations $\tau < \tau(v)$.

The absolute increase in the temperature can be calculated by using (16) and (21);

$$T=W(0.61\lambda/2a)^2/|\gamma_p|W_{cr}.$$

For glycerin at $W/W_{cr} = 1.5$ and $a = 0.5$ cm we have at frequencies of 1 and 3 MHz $T = 12$ and 1.5 °C, respectively, i.e., no dangerous overheating of the liquid takes place.

4. CONCLUSION

Observation of continuous TSFS is entirely realistic in terms of the physical parameters a source power below one watt in a beam with a radius of several millimeters at frequencies of 1–3 MHz. It is required, however, by the geometry of the experiment that the development of convection be maintained for a period of time of the order of minutes.

Realization of the pulsed TSFS regime is possible in the same frequency range in the case of pulse durations τ from hundreds of milliseconds and upward; in the opposite case, at above-critical values of the energy, a shock wave is generated. Removal of the defocusing effect of the acoustic streaming is also possible in sufficiently viscous liquids in practically the same range of τ. No dangerous overheating should take place here so long as the energy does not exceed the critical value (which amounts to tenths of a joule in the best case) by not more than 2–3 times. It is essential that even at such energy levels, the observation distance should amount to no more than several (up to ten) centimeters.

[1)] Here and below, we shall use the handbook data.[22]

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