Influence of nonlinear effects on the surface impedance of a conductor

A. P. Kopasov and A. G. Skoryukov
Gor'kiĭ Physicotechnical Research Institute

(Submitted 3 July 1981; resubmitted 31 March 1982)
Zh. Eksp. Teor. Fiz. **83**, 607–619 (August 1982)

The influence of the nonlinear effects on the surface impedance of a conductor located in a constant magnetic field **H** oriented parallel to its surface is investigated in the case when the skin effect has the anomalous character. It is assumed that the time taken by an electron to traverse the skin layer is much shorter than the mean free time and the period of the electromagnetic wave. The reflection of the electrons by the surface is considered to be nearly diffuse scattering. The corrections to the surface impedance that stem from the following nonlinear currents are estimated: the current that is cubic in the field of the fundamental wave (the first harmonic), the current produced by the fields of the first and second harmonics together, and the current produced by the electric field of the first harmonic and the magnetic field of the rectified radioelectric current. Normally, all the three corrections are of the same order of magnitude. In certain cases the influence of the homogeneous magnetic field produced in the interior of the sample by the radioelectric current predominates. The nonlinearity can be significant even when the magnetic field of the wave is much weaker than the field **H**.

PACS numbers:73.25. + i

A finite-amplitude wave incident on a conductor changes the effective conductivity of the conductor at the wave frequency. This makes the reflection and absorption coefficients dependent on the wave amplitude. Such self-action effects have been experimentally observed in metals and semimetals in a number of investigations under the conditions of both the anomalous[1,2] and the normal[3] skin effect. In Ref. 3 theoretical calculations of the fundamental-wave-amplitude dependence of the absorption and the amplitudes of the higher harmonics under normal-skin-effect and $\omega\tau \ll 1$ (where ω is the wave frequency and τ is the relaxation time) conditions are also performed which are in good agreement with experiment. But no theoretical analysis of the various mechanisms leading to the dependence of the surface impedance on the wave amplitude under anomalous-skin-effect conditions has so far been performed.

In the present paper we study the influence of the nonlinear effects on the surface impedance of a conductor located in a constant magnetic field **H** oriented parallel to its surface under anomalous skin effect conditions, when the following inequalities are satisfied:

$$\frac{\delta}{v_F\tau} \ll 1, \quad \frac{\delta\omega}{v_F} \ll 1, \quad \frac{\delta}{r_H} \ll 1, \tag{1}$$

where δ is the skin depth, v_F is the Fermi velocity, and r_H is the Larmor radius. We consider the magnetic field **H** to be nonquantizing. Furthermore, we shall assume that the time of transit of the electrons through the skin layer is much shorter than the mean free time and the period of the electromagnetic wave, i.e., that

$$\left|\frac{\omega+i\tau^{-1}}{\Omega}\right| \left(\frac{\delta}{r_H}\right)^{1/2} \ll 1, \tag{2}$$

where Ω is the cyclotron frequency. The reflection of the electrons by the sample surface is considered to be nearly diffuse scattering.

In our situation the self-action effects turn out to be due to the change that occurs in the effective conductivity as a result of the influence on the electron motion of the magnetic fields produced in the conductor by the incident wave. In this case not only the magnetic field of the fundamental wave, but also the magnetic field of the second harmonic, as well as the magnetic field of the rectified (radioelectric) current, is important.

NONLINEAR CONDUCTIVITY IN A MAGNETIC FIELD

Let the electromagnetic field have the form

$$\begin{aligned} \mathbf{E}(\mathbf{r},t) &= \mathbf{E}(\mathbf{k}_1,\omega_1)\exp(i\mathbf{k}_1\mathbf{r}-i\omega_1 t)+\ldots \\ &\quad +\mathbf{E}(\mathbf{k}_N,\omega_N)\exp(i\mathbf{k}_N\mathbf{r}-i\omega_N t), \\ \mathbf{H}(\mathbf{r},t) &= \mathbf{H}(\mathbf{k}_1,\omega_1)\exp(i\mathbf{k}_1\mathbf{r}-i\omega_1 t)+\ldots \\ &\quad +\mathbf{H}(\mathbf{k}_N,\omega_N)\exp(i\mathbf{k}_N\mathbf{r}-i\omega_N t), \end{aligned} \tag{3}$$

where $\mathbf{k}_1 \| \mathbf{k}_2 \| \ldots \mathbf{k}_N \| y$.

Let us find the nonlinear current $j_\alpha^{(N)}(\mathbf{r},t)$ produced at the frequency $\omega = \omega_1+\omega_2+\ldots+\omega_N$, by the electromagnetic field (3) in an unbounded conductor:

$$\begin{aligned} j_\alpha^{(N)}(\mathbf{r},t) &= j_\alpha^{(N)}(\mathbf{k},\omega)e^{i\mathbf{k}\mathbf{r}-i\omega t}, \\ \mathbf{k} &= \mathbf{k}_1+\mathbf{k}_2+\ldots+\mathbf{k}_N. \end{aligned} \tag{4}$$

Let us write the kinetic equation for the electron distribution function f in the form

$$\begin{aligned} &\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}+\frac{dt_1}{dt}\frac{\partial f}{\partial t_1}+e\mathbf{E}(\mathbf{r},t)\mathbf{v}\frac{\partial f}{\partial \varepsilon} \\ &+e\left(E_z(\mathbf{r},t)+\frac{1}{c}[\mathbf{v}\times\mathbf{H}(\mathbf{r},t)]_z\right)\frac{\partial f}{\partial p_z} = -\frac{f-f_0}{\tau}, \end{aligned} \tag{5}$$

where t_1 is the time of the motion along the trajectory in the constant magnetic field **H** (the field **H** is directed along the z

 0038-5646/82/080334-08$04.00

axis), ε is the energy, p_z is the component of the momentum along the z axis, and f_0 is the equilibrium distribution function.

For the derivative dt_1/dt we have

$$\frac{dt_1}{dt}=1+\frac{\partial t_1}{\partial \mathbf{p}}\left(e\mathbf{E}(\mathbf{r},t)+\frac{e}{c}[\mathbf{v}\times\mathbf{H}(\mathbf{r},t)]\right). \qquad (6)$$

The nonlinear current can be computed by finding from the kinetic equation (5) the corresponding correction $f^{(N)}(t_1)e^{i\mathbf{kr}-i\omega t}$ to the distribution function:

$$j_\alpha^{(N)}(\mathbf{k},\omega)=\frac{2e}{(2\pi)^3}\left|\frac{eH}{c}\right|\int d\varepsilon dp_z dt_1 v_\alpha(t_1)f^{(N)}(t_1). \qquad (7)$$

Let us define the nonlinear conductivity tensor

$$\sigma^{(N)}_{\alpha\alpha_1\alpha_2\ldots\alpha_N}(\mathbf{k}_1,\omega_1;\mathbf{k}_2,\omega_2;\ldots\mathbf{k}_N,\omega_N)$$

by the following relation:

$$j_\alpha^{(N)}(\mathbf{k},\omega)=\hat{P}(\mathbf{k}_1,\omega_1,\alpha_1;\mathbf{k}_2,\omega_2,\alpha_2;\ldots\mathbf{k}_N,\omega_N,\alpha_N)$$
$$\times\,\sigma^{(N)}_{\alpha\alpha_1\alpha_2\ldots\alpha_N}(\mathbf{k}_1,\omega_1;\mathbf{k}_2,\omega_2;\ldots\mathbf{k}_N,\omega_N) \qquad (8)$$
$$\times E_{\alpha_1}(\mathbf{k}_1,\omega_1)E_{\alpha_2}(\mathbf{k}_2,\omega_2)\ldots E_{\alpha_N}(\mathbf{k}_N,\omega_N),$$

where $\hat{P}(\mathbf{k}_1,\omega_1,\alpha_1;\mathbf{k}_2,\omega_2,\alpha_2;\ldots\mathbf{k}_N,\omega_N,\alpha_N)$ is the operator of symmetrization with respect to the frequencies, the wave vectors, and the vector indices (sum over all transpositions).

Let us introduce the tensor $A^{(N)}_{\alpha\alpha_1\ldots\alpha_N}$, such that

$$\sigma^{(N)}_{\alpha\alpha_1\alpha_2\ldots\alpha_N}(\mathbf{k}_1,\omega_1;\mathbf{k}_2,\omega_2;\ldots\mathbf{k}_N,\omega_N)$$
$$=\left(\frac{ck_2}{\omega_2 H}\right)\left(\frac{ck_3}{\omega_3 H}\right)\cdots\left(\frac{ck_N}{\omega_N H}\right) \qquad (9)$$
$$\times A^{(N)}_{\alpha\alpha_1\alpha_2\ldots\alpha_N}(k_1,k_2,\ldots k_N;\omega_1,\omega_2,\ldots,\omega_N),$$
$$k_l=k_{yl},\quad l=1,2,3\ldots$$

By solving the kinetic equation (5) by the iterative procedure, we can establish the following recursion formulas, which allow us to compute the components of the tensor $\sigma^{(N)}$ in the leading approximation in the parameter δ/r_H characterizing the anomalousness (we assume that $|k_l|\sim\delta^{-1}$):

$$A^{(l)}_{\alpha\alpha_1\alpha_2\ldots\alpha_N}(k_1,k_2,\ldots,k_l;\omega_1,\omega_2,\ldots,\omega_l)$$
$$=\left(1+\frac{k_1}{k_2}\right)A^{(l-1)}_{\alpha\alpha_1\alpha_2\ldots\alpha_N}(k_1+k_2,k_3,\ldots,k_l;\omega_1+\omega_2,\omega_3,\ldots,\omega_l)$$
$$-\frac{k_1}{k_2}A^{(l-1)}_{\alpha\alpha_1\alpha_2\ldots\alpha_N}(k_1,k_2+k_3,k_4,\ldots,k_l;\omega_1,\omega_2+\omega_3,\omega_4,\ldots,\omega_l), \qquad (10)$$

for $N\geqslant l\geqslant 3$, and

$$A^{(2)}_{\alpha\alpha_1\alpha_2\ldots\alpha_N}(k_1,k_2;\omega_1,\omega_2)=\left(1+\frac{k_1}{k_2}\right)$$
$$-A^{(1)}_{\alpha\alpha_1\alpha_2\ldots\alpha_N}(k_1+k_2;\omega_1+\omega_2)$$
$$-\frac{k_1}{k_2}A^{(1)}_{\alpha\alpha_1\alpha_2\ldots\alpha_N}(k_1;\omega_1). \qquad (11)$$

The function $A^{(1)}_{\alpha\alpha_1\alpha_2\ldots\alpha_N}(k_1;\omega_1)$ has the form

$$A^{(1)}_{\alpha\alpha_1\alpha_2\ldots\alpha_N}(k_1;\omega_1)=\frac{e^2}{4\pi^2|k_1|}$$
$$\times\sum_s\int_{\varepsilon=\varepsilon_F}dp_z\frac{\rho^{(s)}v^{(s)}_\alpha v^{(s)}_{\alpha_1}\ldots v^{(s)}_{\alpha_N}}{(v^{(s)}_x)^{N+1}}\,\mathrm{cth}\left\{\left(-i\frac{T}{2}(\omega_1+i\tau^{-1})\right\}\right.,$$
$$T=\frac{2\pi}{\Omega}, \qquad (12)$$

where ε_F is the Fermi energy and ρ is the radius of curvature of the trajectory in $\mathbf{p}$ space. All the quantities in the integrand in (12) are taken at the $v_y=0$ stationary-phase points, which are numbered by the index s. We shall assume that there are only two such points on each trajectory (i.e., that $s=1,2$). For a convex Fermi surface the formula (12) can be transformed into the form

$$A^{(1)}_{\alpha\alpha_1\ldots\alpha_N}(k_1;\omega_1)=\frac{e^2}{4\pi^2|k_1|}$$
$$\times\int_0^{2\pi}\frac{d\varphi}{K(\varphi)}\frac{n_\alpha n_{\alpha_1}\ldots n_{\alpha_N}}{n_x^{N-1}}\,\mathrm{cth}\left\{-i\frac{T}{2}(\omega_1+i\tau^{-1})\right\}, \qquad (13)$$

where n_α is the unit vector along the normal to the Fermi surface, $K(\varphi)=K(\varphi,\theta=\pi/2)$ is the Gaussian curvature of the Fermi surface, and φ and θ are the azimuthal and polar angles of the vector n_α (the polar axis coincides with the y axis).

Let us note that the nonlinearity in the present situation is due to the Lorentz force (this is due to the fact that the force eE is $v_F/\omega\delta$ times weaker than the Lorentz force).

The formulas (11) and (13) are generalizations of the formulas (21) and (22) in Ref. 4, in which the nonlinear conductivity $\sigma^{(2)}_{\alpha\beta\gamma}(k_1,\omega;k_2,\omega)$ at the frequency of the second harmonic is obtained. From the formulas (12) and (13) it follows that, as in the linear theory, the contribution of the longitudinal fields to the transverse current is insignificant.

Here we neglect that part of the nonlinear conductivity $\sigma^{(N)}$ which undergoes geometrical-resonance oscillations, and describes the anomalous penetration of the field, since we can, in the electromagnetic-wave-reflection problem, considered in the leading approximation in the parameter δ/r_H, neglect the anomalous penetration effects (except in the situation when the Fermi surface has sections the diameter of whose orbit does not depend on p_z (Ref. 5) and in the case of resonance at the extreme frequencies $\omega=l\Omega_{\rm extr}$ (where l is a whole number) when $\omega\tau\gtrsim r_H/\delta$ (Ref. 6).

If the Fermi surface is closed, then the integrand in (13) can have a singularity when $n_x=0$. Analysis shows that this singularity should be integrated in the sense of the principal value. But when $N\geqslant 3$, for certain components of the tensor $A^{(1)}_{\alpha\alpha_1\ldots\alpha_N}$, e.g., for the component $A^{(1)}_{zzzz}$, this singularity cannot be integrated even in the sense of the principal value. Naturally, all the components of $\sigma^{(N)}_{\alpha\alpha_1\ldots\alpha_N}$ are in fact finite. In the present case it is necessary to correctly take into account the contribution to the nonlinearity of the electrons near the elliptic reference points with orbit diameter

$$2r(p_z)\leqslant\delta(|(\omega+i\tau^{-1})/\Omega|^2+1),$$

to which the stationary-phase method is inapplicable. We shall not dwell here on the investigation of this quite complicated problem, since below we shall need only the components $\sigma^{(2)}_{xxx}$ and $\sigma^{(3)}_{xxxx}$, for which the formulas (9)–(13) give asymptotically exact expressions in the closed-Fermi-surface case as well [the integrand in (13) does not have a singularity at all].

The explicit expressions for the components $\sigma^{(2)}_{xxx}$ and $\sigma^{(3)}_{xxxx}$ have the form

$$\sigma^{(2)}_{xxx}(\mathbf{k}_1,\omega_1;\ \mathbf{k}_2,\omega_2)=\frac{c}{\omega_2 H}\left(\frac{k_1+k_2}{|k_1+k_2|}B(\omega_1+\omega_2)-\frac{k_1}{|k_1|}B(\omega_1)\right),\tag{14}$$

$$\sigma^{(3)}_{xxxx}(\mathbf{k}_1,\omega_1;\ \mathbf{k}_2,\omega_2;\ \mathbf{k}_3,\omega_3)$$

$$=\frac{c^2}{\omega_2\omega_3 H^2}\left[\left(k_1+k_2-\frac{k_1k_3}{k_2+k_3}\right)\frac{k_1+k_2+k_3}{|k_1+k_2+k_3|}\right.$$
$$\left.\times B(\omega_1+\omega_2+\omega_3)-|k_1+k_2|B(\omega_1+\omega_2)+\frac{|k_1|k_3}{k_2+k_3}B(\omega_1)\right],\tag{15}$$

where

$$B(\omega)=\frac{e^2}{4\pi^2}\int_0^{2\pi}\frac{d\varphi}{K(\varphi)}n_x^2\,\mathrm{cth}\left\{-i\frac{T}{2}(\omega+i\tau^{-1})\right\}.\tag{16}$$

If, on the other hand, the Fermi surface is open, but the trajectories are closed and the inequality

$$2r(p_z)\gg\delta(|(\omega+i\tau^{-1})/\Omega|^2+1)$$

is satisfied for all p_z, then the relations (9)–(13) give for all the components of the nonlinear conductivity tensor $\sigma^{(N)}_{\alpha\alpha_1\ldots\alpha_N}$ asymptotically exact expressions that are valid in the leading approximation in the parameter δ/r_H.

THE NONLINEAR CORRECTIONS TO THE SURFACE IMPEDANCE

Let an electromagnetic wave of frequency ω, whose electric field vector is perpendicular to the constant magnetic field $\mathbf{H}\|z$, be incident normally at the surface of a conductor occupying the half-space $y>0$. We shall assume that the Fermi surface (like any other constant-energy surface) has a symmetry plane perpendicular to $\mathbf{H}$ (for example, this can be a sphere or a corrugated cylinder with axis parallel to $\mathbf{H}$). Let, moreover, $\Omega(p_z)=\mathrm{const}$.

On account of the symmetry of the Fermi surface, the linear-conductivity component $\sigma_{zx}=0$. Therefore, in the present case the electric field of the linear approximation in the conductor will have only an x component (we have in mind only the transverse components, since the longitudinal components can be neglected). As can be seen from (12) and (13), the nonlinear-conductivity components $\sigma^{(N)}_{zxx\ldots x}$ also vanish. Consequently, the electric fields and the currents, both linear and nonlinear, will be perpendicular to $\mathbf{H}$. (It is clear that the vanishing of the components $\sigma^{(N)}_{zxx\ldots x}$, which has been proved by us for an unbounded medium, is connected only with the symmetry of the Fermi surface, and, consequently, it will occur also in the case of the semifinite conductor, at any rate, when the scattering of the electrons by the sample surface is isotropic.)

In the linear theory the surface impedance $\zeta_L(\omega)$ is defined by the relation

$$\zeta_L(\omega)=-E_x^{(1)}(0,\omega)/H_z^{(1)}(0,\omega)=-E^{(1)}(0,\omega)/H^{(1)}(0,\omega),\tag{17}$$

where $E^{(1)}(0,\omega)$ and $H^{(1)}(0,\omega)$ are the amplitudes of the electric and magnetic fields of frequency ω at the surface of the conductor, as computed in the linear approximation. In third-order perturbation theory, there arises at the frequency of the incident wave a nonlinear current $j^{(3)}(y,\omega)$ that emits some additional electromagnetic field at the same frequency. Let $\Delta E(0,\omega)$ be the amplitude of the electric field, emitted by the current $j^{(3)}(y,\omega)$, at the surface of the conductor; $\Delta H(0,\omega)$, the amplitude of the magnetic field.

In the nonlinear theory the surface impedance $\zeta_{NL}(\omega)$ can be defined similarly to (17):

$$\zeta_{NL}(\omega)=-E(0,\omega)/H(0,\omega),\tag{18}$$

where $E(0,\omega)$ and $H(0,\omega)$ are the amplitudes, computed with allowance for the nonlinear effects, of the electric and magnetic fields of fundamental frequency at the surface of the conductor.

The surface impedance $\zeta_{NL}(\omega)$ introduced in accordance with (18) will describe in the usual fashion the electromagnetic-wave reflection at the fundamental frequency (i.e., will determine the amplitude and phase of the reflected wave at the frequency ω).

Setting

$$E(0,\omega)=E^{(1)}(0,\omega)+\Delta E(0,\omega),$$

$$H(0,\omega)=H^{(1)}(0,\omega)+\Delta H(0,\omega)$$

and taking account of the fact that

$$|\Delta E(0,\omega)|=|\Delta H(0,\omega)|\ll|E^{(1)}(0,\omega)|\ll|H^{(1)}(0,\omega)|,$$

we obtain from (18) for the nonlinear correction to the impedance the relations

$$\frac{\Delta\zeta_{NL}(\omega)}{\zeta_L(\omega)}=\frac{\zeta_{NL}(\omega)-\zeta_L(\omega)}{\zeta_L(\omega)}=\frac{\Delta E(0,\omega)}{E^{(1)}(0,\omega)}.\tag{19}$$

The strength $\Delta E(0,\omega)$ of the secondary field emitted by the nonlinear current $j^{(3)}(y,\omega)$ can be found, using the reciprocity theorem. As a result, we shall have

$$\frac{\Delta\zeta_{NL}(\omega)}{\zeta_L(\omega)}=-\frac{4\pi}{c}\frac{\zeta_L(\omega)}{(E^{(1)}(0,\omega))^2}\int_0^\infty dy\,j^{(3)}(y,\omega)E^{(1)}(y,\omega),\tag{20}$$

where $E^{(1)}(y,\omega)$ is the electric field in the conductor as computed in the linear approximation.

The nonlinear current $j^{(3)}(y,\omega)$ can be represented in the form of a sum of three terms

$$j^{(3)}(y,\omega)=j_1^{(3)}(y,\omega)+j_2^{(3)}(y,\omega)+j_3^{(3)}(y,\omega),\tag{21}$$

where $j_1^{(3)}(y,\omega)$ is the current that is cubic in the field of the first harmonic, $j_2^{(3)}(y,\omega)$ is the current produced by both the field of the first harmonic and the field of the second harmonic, and $j_3^{(3)}(y,\omega)$ is the current produced by the electric field of the first harmonic and the magnetic field of the rectified radioelectric current.

Below we estimate and compare the contributions of the individual terms in (21) to the surface impedance. We shall assume that the surface scattering of the electrons is close to diffuse scattering, or, more exactly, that the reflectivity factor p, which depends on the glancing angle α, differs significantly from unity when $\alpha\sim(\delta/r_H)^{1/2}$, but that $p(\alpha=0)=1$, $p(\alpha)$ decreasing smoothly from 0 to values $\sim(\delta/r_H)^{1/2}$ as α is varied. (The going to unity of the reflectivity parameter $p(\alpha=0)$ normally occurs in the case of electron reflection by a metal surface.[7]) It is known from the linear theory[8,9] that, in the present case, we can actually consider the problem in infinite space if we are not interested in factors of the order of unity. But the linear-conductivity tensor $\sigma_{\alpha\beta}$ of the unbounded medium should then be replaced by a modified tensor $\tilde{\sigma}_{\alpha\beta}$ (see, for example, Ref. 9). The difference between $\tilde{\sigma}_{\alpha\beta}$ and $\sigma_{\alpha\beta}$ lies in the fact that in the tensor $\tilde{\sigma}_{\alpha\beta}$ the factor describing the cyclotron resonance is taken into account only at those $v_y=0$ stationary-phase points for which the orbit in real space can be a resonance orbit.

Here we also shall, in computing the nonlinear current $j^{(3)}(y,\omega)$, consider infinite space, assuming that the fields are established by a current sheet lying in the $y=0$ plane, and modifying in the indicated manner the resonance factors in the linear and nonlinear conductivities. Let us express the nonlinear current $j^{(3)}(y,\omega)$ in terms of the value of the amplitude of the electric field of the linear approximation on the current sheet, which field we identify with the field $E^{(1)}(0,\omega)$ at the metal surface. We must note here that, as follows from the results of Refs. 4 and 10, there are additional limitations on the applicability of such an approach in the nonlinear theory. It is shown in Ref. 10 that the nonlinear conductivity of a semifinite conductor can differ significantly from the conductivity of the infinite medium even when the surface reflection of the electrons with glancing angles $\sim(\delta/r_H)^{1/2}$ is close to diffuse reflection (the case of nearly diffuse reflection considered in Ref. 10). Since $p(\alpha=0)=1$, the distribution function $f^{(1)}$ of the first approximation in the field is a continuous function, i.e., there is realized in it a continuous transition from the volume electrons (i.e., the electrons that do not collide with the surface) to the electrons that are diffusely reflected from the surface. For $\Omega\tau\gg1$ in the vicinity of the resonance $\omega=l\Omega$, where l is a whole number, the gradients of the distribution function $f^{(1)}$ are large in the region of the variables t_1, p_z, ε, and y that separates the volume electrons from the electrons that are diffusely scattered. As a result the region in question makes a significant contribution to the nonlinear conductivity at the frequency of the second harmonic, since the second-order—in the field—distribution function can be expressed in terms of the derivatives of the function $f^{(1)}$. It turns out that this contribution to the $\omega=l\Omega$ resonance increases more rapidly, as the parameter $\Omega\tau$ increases, than the nonlinear conductivity of the unbounded medium if $\omega\tau\gg(r_H/\delta)^{1/2}$ (Ref. 10). Thus, near the $\omega=l\Omega$ resonance the condition of applicability of the approach based on the reduction of the problem to one in infinite space should be the satisfaction of the inequality

$$\omega\tau\lesssim(r_H/\delta)^{1/2}. \tag{22}$$

If it is valid, then the character of the cyclotron-resonance singularities of the nonlinear response of the semifinite conductor (under our assumption about the character of the surface reflection) is the same as in the case of the unbounded medium.

Furthermore, it follows from the results of Ref. 11 that we must, in computing the current $j_2^{(3)}(y,\omega)$ produced by the fields of the first and second harmonics together, correctly allow for the reflection of the electrons by the surface in the vicinity of the odd resonance $\omega=(l+\frac{1}{2})\Omega$ if $\Omega\tau\gg1$. This is due to the fact that there are in the present situation two characteristic lengths: δ_1, the skin depth at the frequency ω, and δ_2, the skin depth at the frequency 2ω, with $\delta_2\ll\delta_1$.

Let us now proceed to estimate the nonlinear corrections to the surface impedance. Let us represent the Fourier transform $E^{(1)}(k,\omega)$ of the field of the first harmonic in the form

$$E^{(1)}(k,\omega)=i\frac{\omega}{c}\frac{E^{(1)}(0,\omega)}{\zeta_L(\omega)}\delta_1^2e_1(k\delta_1). \tag{23}$$

Let us define the skin depth δ_1 at the frequency ω by the relation

$$\delta_1=\left|\frac{\omega e^2}{\pi c^2}\int_0^{2\pi}\frac{d\varphi}{K(\varphi)}n_x^2R(\omega)\right|^{-1/3}, \tag{24}$$

$$R(\omega)=(1-\exp\{iT(\omega+i\tau^{-1})\})^{-1}.$$

It follows from (23) that the function $e_1(k)$ satisfies the normalization condition

$$\int_{-\infty}^{\infty}dke_1(k)=-i\frac{\zeta_L(\omega)c}{\delta_1\omega}\sim1. \tag{25}$$

On reducing the problem to one in infinite space, we obtain for $e_1(k)$ with allowance for the modification of the linear-conductivity tensor the expression

$$e_1(k)=-\frac{1}{\pi}\left\{k^2-\frac{i}{|k|}\tilde{R}(\omega)\right\}^{-1},\quad \tilde{R}(\omega)=\frac{R(\omega)}{|R(\omega)|}. \tag{26}$$

Let us first consider the contribution of the current $j_1^{(3)}(y,\omega)$. Expressing it in terms of the tensor element $\sigma_{xxxx}^{(3)}$, and taking (15) and (23) into account, we obtain for the corresponding nonlinear correction $(\Delta\zeta_{NL}(\omega)/\zeta_L(\omega))$, after some transformations the expression

$$\left(\frac{\Delta\zeta_{NL}(\omega)}{\zeta_L(\omega)}\right)_1=\frac{\delta_1\omega}{c\zeta_L(\omega)}\left[I_1\tilde{R}(\omega)+I_2\tilde{R}(-\omega)+I_3\frac{R(2\omega)}{|R(\omega)|}\right]\frac{|H^{(1)}(0,\omega)|^2}{H^2}, \tag{27}$$

where

$$I_m=\pi\int_{-\infty}^{\infty}dk\,dk_1\,dk_2\,dk_3\delta(k-k_1-k_2-k_3)\times e_1(k)e_1(k_1)e_1(k_2)e_1^*(k_3)F_m,\qquad m=1,2,3, \tag{28}$$

$$F_1=-\frac{|k|}{2}-|k_1|,\quad F_2=\frac{|k_3|}{2},\quad F_3=|k_1+k_2|.$$

It follows from (28) and (26) that the quantities I_m depend only on the parameters $\omega\tau$ and Ω/ω, i.e.,

$$I_m = I_m(\omega\tau, \Omega/\omega). \tag{29}$$

It is not difficult to verify that the coefficients I_m remain finite at all values of the magnetic field, even when $\omega\tau \to \infty$. We should, in estimating them, take the following circumstance into account. As is well known, weakly damped short cyclotron waves, whose existence was predicted by Kaner and Skobov,[12] can be intensely excited in a metal. Mathematically, this is manifested in the fact that the function $e_1(k)$ (more exactly, the functions that are analytic continuations of $e_1(k)$ into the entire complex plane from the real positive and real negative semiaxes) can have in a definite range of magnetic-field strengths poles lying close to the real axis. The dispersion equation for the cyclotron waves in our case has the form

$$k^3 \mp i\tilde{R}(\omega) = 0, \tag{30}$$

where the sign minus (plus) corresponds to the root with $\operatorname{Re} k > 0$ ($\operatorname{Re} k < 0$).

It follows from (30) that slowly decaying cyclotron waves exist when the following inequalities are satisfied:

$$\tau^{-1} \ll l\Omega - \omega \ll \Omega. \tag{31}$$

For the wave vector $\operatorname{Re} k(\omega)$ of the cyclotron wave and the damping constant $\operatorname{Im} k(\omega)$ we obtain

$$|\operatorname{Re} k(\omega)| \approx (\lambda_1)^{1/3} \approx 1,$$
$$|\operatorname{Im} k(\omega)| \approx \frac{1}{3}\lambda \approx \frac{1}{3}\left[\frac{1}{(l\Omega-\omega)\tau} + \frac{\pi(l\Omega-\omega)}{\Omega}\right], \tag{32}$$

where

$$\lambda = \operatorname{Re}\tilde{R}(\omega), \quad \lambda_1 = \operatorname{Im}\tilde{R}(\omega).$$

(In the dimensional variables $|\operatorname{Re} k(\omega)| = \delta_1^{-1}$, $|\operatorname{Im} k(\omega)| = \lambda/3\delta_1$.)

As can be seen from (32), the cyclotron-wave damping constant is nonzero even at $\tau = \infty$, which is due to the surface scattering of the electrons. In the absence of slowly decaying cyclotron waves $e_1(k)$ is a smooth function of the wave vector k.

Taking the foregoing into account, we can easily estimate the integrals I_m in (27). As a result, we obtain

$$\left(\frac{\Delta\zeta_{NL}(\omega)}{\zeta_L(\omega)}\right)_1 = c_1\frac{|H^{(1)}(0,\omega)|^2}{H^2}. \tag{33}$$

When the inequalities (31) are satisfied, the dominant contribution to the correction $(\Delta\zeta_{NL}(\omega)/\zeta_L(\omega))_1$ to the surface impedance is made by the cyclotron waves, and the coefficient $c_1 \sim \lambda^{-1} \gg 1$. At the odd resonance $\omega = (l+\frac{1}{2})\Omega$, the coefficient c_1 increases in proportion to the parameter $\Omega\tau$ as the latter increases. At points far from the resonance and also at the even resonance $\omega = l\Omega$, we have $|c_1| \sim 1$.

Let us now estimate the contribution $(\Delta\zeta_{NL}(\omega)/\zeta_L(\omega))_2$ of the electromagnetic field of the second harmonic to the correction to the surface impedance. The Fourier transform $E^{(2)}(k, 2\omega)$ of the field of the second harmonic can be found from the equation

$$\left[k^2 - \frac{8\pi i\omega}{c^2}\tilde{\sigma}(k,2\omega)\right]E^{(2)}(k,2\omega) = \frac{8\pi i\omega}{c^2}j^{(2)}(k,2\omega), \tag{34}$$

where

$$j^{(2)}(k,2\omega) = \int_{-\infty}^{\infty} dk_1\, dk_2\, \delta(k-k_1-k_2)\,\tilde{\sigma}^{(2)}(k_1,\omega;k_2,\omega) \times E^{(1)}(k_1,\omega)E^{(1)}(k_2,\omega), \tag{35}$$

$\tilde{\sigma}(k, 2\omega)$ and $\tilde{\sigma}^{(2)}(k_1, \omega; k_2, \omega)$ are the modified linear and nonlinear conductivities at the frequency 2ω ($\tilde{\sigma}$ and $\tilde{\sigma}^{(2)}$ differ from the corresponding quantities for the unbounded medium in that the resonance factors $\operatorname{cth}\{-\frac{1}{2}iT(s\omega + i\tau^{-1})\}$ are replaced by $R(s\omega)$, $s = 1, 2$).

It is not difficult to verify that, if the inequalities (31) are satisfied, then the Fourier transform $E^{(2)}(k, 2\omega)$ has spikes at $k = \pm\operatorname{Re} k(2\omega)$ and $k = \pm 2\operatorname{Re} k(\omega)$ [$\operatorname{Re} k(2\omega)$ is the wave vector of the cyclotron wave at the frequency 2ω, and in the dimensional variables $|\operatorname{Re} k(2\omega)| = \delta_2^{-1}$]. The spikes at $k = \pm\operatorname{Re} k(2\omega)$ correspond to the excitation of a free wave; those at $k = \pm 2\operatorname{Re} k(\omega)$, of an induced wave, which stems from the cyclotron waves of fundamental frequency. These spikes do not overlap. In other words, the synchronism condition $\operatorname{Re} k(2\omega) = 2\operatorname{Re} k(\omega)$ is not fulfilled for the cyclotron waves, since, in fact, $\operatorname{Re} k(2\omega) = \operatorname{Re} k(\omega)$.

Finding $E^{(2)}(k, 2\omega)$ from (34), we can easily compute the nonlinear current $j_2^{(3)}(y, \omega)$ and estimate the corresponding correction to the surface impedance. (We shall, in computing the contribution of the current $j_2^{(3)}(y, \omega)$ to the impedance, assume that $\delta_1 \sim \delta_2$, an assumption which is valid at points far from the resonance $\omega = (l+\frac{1}{2})\Omega$ and at the resonance $\omega = l\Omega$. But if $\delta_2 \ll \delta_1$, then, as has already been noted above, we must correctly allow for the reflection of the electrons by the sample surface.) We find in the present case that the correction $(\Delta\zeta_{NL}(\omega)/\zeta_L(\omega))_2$ is of the same order of magnitude as $(\Delta\zeta_{NL}(\omega)/\zeta_L(\omega))_1$.

In the second approximation in the amplitude of the incident wave there arises, besides the current at the frequency 2ω, a rectified radioelectric current $\mathbf{j}^{(2)}(y, 0)$ flowing in the skin layer in the direction parallel to its surface. In our situation the radioelectric current $\mathbf{j}^{(2)}(y, 0)$ will have only an x component, for which we easily obtain from (14) the expression

$$j^{(2)}(y,0) = \frac{e^2}{4\pi^2}\frac{c}{\omega H}\int_0^{2\pi}\frac{d\varphi}{K(\varphi)}n_x^2\int_{-\infty}^{\infty} dk_1\, dk_2\, e^{i(k_1+k_2)y}\frac{k_1}{|k_1|} \times\{R(\omega)E^{(1)}(k_1,\omega)E^{(1)*}(k_2,\omega) - R(-\omega) \times E^{(1)*}(k_1,\omega)E^{(1)}(k_2,\omega)\}. \tag{36}$$

The radioelectric current $j^{(2)}(y, 0)$ produces an additional constant magnetic field $H^{(2)}(y, 0)$. The magnetic field $H^{(2)}(y, 0)$ depends on how the radioelectric-current circuit is closed (actually, the sample is bounded). We shall assume that the radioelectric-current circuit is closed on the half-space $y > 0$ (generally speaking, this is determined by the experimental conditions). Consequently,

$$H^{(2)}(y,0)=\frac{4\pi}{c}\int_0 dy'j^{(2)}(y',0). \quad (37)$$

Let us represent the magnetic field $H^{(2)}(y,0)$ in the form

$$H^{(2)}(y,0)=\tilde{H}^{(2)}(y,0)+\Delta H^{(2)}, \quad (38)$$

where $\Delta H^{(2)}=H^{(2)}(\infty,0)$. The characteristic variation scale of the function $\tilde{H}^{(2)}(y,0)$ is δ_1. Substituting (36) into (37), we obtain for the strength $\Delta H^{(2)}$ of the additional magnetic field in the interior of the sample the expression

$$\Delta H^{(2)}=[I_0\tilde{R}+\text{c.c.}]\frac{|H^{(1)}(0,\omega)|^2}{H}, \quad (39)$$

where

$$I_0=i\int_{-\infty}^{\infty}dk_1\,dk_2\frac{k_1}{|k_1|(k_1+k_2)}e_1(k_1)e_1^*(k_2). \quad (40)$$

The $(k_1+k_2)^{-1}$ singularity in (40) is integrated in the sense of the principal value.

Substituting (26) into (40) and (40) into (39), we find after some transformations that

$$\Delta H^{(2)}=S\left(\omega\tau,\frac{\Omega}{\omega}\right)\frac{|H^{(1)}(0,\omega)|^2}{H}, \quad (41)$$

where

$$S\left(\omega\tau,\frac{\Omega}{\omega}\right)$$

$$=\frac{4}{\pi^2}\left\{\lambda^2\int_0^{\infty}dk_1\,dk_2\frac{k_1k_2(k_1^2+k_2^2+k_1k_2)}{[(k_1^3+\lambda_1)^2+\lambda^2][(k_2^3+\lambda_1)^2+\lambda^2]}\right.$$

$$\left.-\lambda_1\int_0^{\infty}dk_1\,dk_2\frac{k_1k_2}{k_1+k_2}\frac{(k_1^3+\lambda_1)(k_2^3+\lambda_1)+\lambda^2}{[(k_1^3+\lambda_1)^2+\lambda^2][(k_2^3+\lambda_1)^2+\lambda^2]}\right\}. \quad (42)$$

The dimensionless function $S(\omega\tau,\Omega/\omega)$ depends resonantly on the magnetic field, its characteristic value being of the order of unity. It follows from (42) that $S(\omega\tau,\Omega/\omega=2/l)=11/27$ (for $\Omega/\omega=2/l$ the coefficient $\lambda_1=0$). It is also not difficult to show that the function $S(\omega\tau,\Omega/\omega)$ can change its sign near the $\omega=l\Omega$ resonance if the parameter $\Omega\tau$ is sufficiently large. Indeed, the coefficient $\lambda_1\gg\lambda>0$ when the inequalities $\Omega\gg\omega-l\Omega\gg\tau^{-1}$ are satisifed. Consequently, in the present case the second term in the curly brackets in the formula (42) can become dominant, i.e., the function $S(\omega\tau,\Omega/\omega)$ can become negative, whereas at resonance $S(\omega\tau,\Omega/\omega=1/l)=11/27>0$. The expressions (41) and (42) are formally obtained in the case of the cylindrical Fermi surface as well when the effects of the anomalous penetration are neglected, but in the present case we must, generally speaking, take the effects into consideration. As can be seen from (42), the slowly decaying cyclotron waves (if they exist) make to the field a contribution of the same order of magnitude as the skin component $E^{(1)}(y,\omega)$, $H^{(1)}(y,\omega)$ of the electromagnetic field.

The magnetic field $\Delta H^{(2)}$ produced in the interior of the sample by the radioelectric current has been experimentally observed in bismuth in the radio-frequency region where $\omega\tau\ll1$ by Babkin and Dolgopolov.[13] Its existence is, as stated in Ref. 13, indicated by the observed alternate-field-amplitude-dependent shift, along the magnetic-field axis, of the radioelectric-current-related size-effect lines relative to the radio-frequency size-effect lines. In Ref. 13, besides describing the experimental investigation, Babkin and Dolgopolov also derive expressions for the radioelectric current and the current at the second-harmonic frequency under conditions of anomalous skin effect (i.e., with allowance for the effects of the anomalous penetration) and the simultaneous satisfaction of the inequalities $\omega\tau\ll1$ (low frequencies) and $\Omega\tau\gg1$ (strong magnetic fields). They, moreover, assume that the Fermi surface is a cylinder whose axis is parallel to the magnetic fields $\mathbf{H}$ and $\mathbf{H}^{(1)}(y,\omega)$.

It is not difficult to compare our results with the results obtained in Ref. 13. Taking into consideration the formula (12), we can verify that, in the situation considered in Ref. 13, the expresions derived by us for the smooth part of the nonlinear conductivity $\sigma^{(2)}_{xxx}$ at the frequency 2ω and at zero frequency essentially coincide with the expressions obtained in Ref. 13. (There are, however, discrepancies in the numerical coefficients. The formulas (7) and (9) in Ref. 13 evidently contain some errors, which are partially corrected in Ref. 14.) Originally, it was asserted in Ref. 13 that, according to calculations, the secondary magnetic field $\Delta\mathbf{H}^{(2)}$ coincides in direction with $\mathbf{H}$. But in Ref. 14 it is asserted that, according to calculations, $\Delta\mathbf{H}^{(2)}$ and $\mathbf{H}$ are oppositely directed.

As can be seen from the expressions (41) and (42) obtained by us, for $\omega\tau\ll1$ and $\Omega\tau\gg1$ the function $S(\omega\tau,\Omega/\omega)>0$ (since $\lambda\simeq1$, $|\lambda_1|\ll1$) and, consequently, the field $\Delta\mathbf{H}^{(2)}$ coincides in direction with $\mathbf{H}$. This is in accord with the experimental data obtained by Babkin and Dolgopolov,[13,14] since they observed a shift of the size-effect lines for the radioelectric current relative to the radio-frequency size-effect lines and toward the region of lower intensities of the magnetic field $\mathbf{H}$. (Although the anomalous-penetration effects, which we neglected in the derivation of (41) and (42), are important in the case of the cylindrical Fermi surface, it can be shown that allowance for them does not change the sign of $\Delta\mathbf{H}^{(2)}$ when $\omega\tau\ll1$ and $\Omega\tau\gg1$.)

Let us now estimate the effect of the magnetic field $H^{(2)}(y,0)$ on the surface impedance. Let us represent the magnetic field $H^{(2)}(y,0)$ in the form (38), and consider first the contribution due to the field $\tilde{H}^{(2)}(y,0)$. As can be verified, the corresponding correction to the impedance has the same order of magnitude as the correction given in (33). The effect of the homogeneous magnetic field $\Delta H^{(2)}$ can be taken into account by replacing H by $H+\Delta H^{(2)}$. Finally, we obtain

$$\frac{\Delta\zeta_{NL}(\omega,H)}{\zeta_L(\omega,H)}=\tilde{c}\frac{|H^{(1)}(0,\omega)|^2}{H^2}+\frac{\zeta_L(\omega,H+\Delta H^{(2)})-\zeta_L(\omega,H)}{\zeta_L(\omega,H)}. \quad (43)$$

When the inequalities (31) are satisfied, the coefficient $\tilde{c}\sim\lambda^{-1}\gg1$; in the remaining case $|\tilde{c}|\sim1$ (as has already been noted, we exclude from consideration the range of magnetic-field strengths near the odd resonance $\omega=(l+\frac{1}{2})\Omega$ when

$\Omega\tau \gg 1$). Expanding the second term in (43) in a series in powers of $\Delta H^{(2)}$, we obtain

$$\frac{\Delta\zeta_{NL}(\omega,H)}{\zeta_L(\omega,H)}=\tilde{c}\frac{|H^{(1)}(0,\omega)|^2}{H^2}+\frac{\Delta H^{(2)}}{\zeta_L(\omega,H)}\frac{d\zeta_L(\omega,H)}{dH}. \tag{44}$$

For $\Omega\tau \gg 1$ we find that at frequencies far from the resonance

$$\frac{d\zeta_L(\omega,H)}{dH}\sim\frac{\omega+i\tau^{-1}}{\Omega}\frac{1}{H}\zeta_L(\omega,H). \tag{45}$$

If $\omega\sim\Omega$ and $\omega\tau\gg 1$, then, as can be seen from (44) and (45), all the three terms in the nonlinear current $j^{(3)}$, namely, the term cubic in the field of the first harmonic, the term due to both the field of the first, and the field of the second, harmonic, and the term produced by the electric field of the first harmonic and the magnetic field of the radioelectric current, make contributions to the surface impedance that are of the same order of magnitude, with

$$\Delta\zeta_{NL}(\omega, H)/\zeta_L(\omega, H)\sim|H^{(1)}(0, \omega)|^2/H^2. \tag{46}$$

It is not difficult to see that a similar situation will obtain in the $\omega\tau \ll 1$, $\Omega\tau \gg 1$ case.

For $\omega\tau \gg \Omega\tau \gg 1$ the dominant contribution of the nonlinear correction to the surface impedance at frequencies far from resonance is connected with the influence of the homogeneous magnetic field $\Delta H^{(2)}$:

$$\frac{\Delta\zeta_{NL}(\omega,H)}{\zeta_L(\omega,H)}=\frac{\Delta H^{(2)}}{\zeta_L(\omega,H)}\frac{d\zeta_L(\omega,H)}{dH}\sim\frac{\omega}{\Omega}\frac{|H^{(1)}(0,\omega)|^2}{H^2}. \tag{47}$$

For $\Omega\tau \gg 1$ we obtain for the correction to the surface impedance at frequencies close to the even resonance $\omega = l\Omega$ the following estimate:

$$\frac{\Delta\zeta_{NL}(\omega,H)}{\zeta_L(\omega,H)}\sim\frac{\omega}{\omega-l\Omega+i\tau^{-1}}\frac{|H^{(1)}(0,\omega)|^2}{H^2}, \tag{48}$$

this correction being, as can easily be seen, due to the field $\Delta H^{(2)}$. An exception is the situation in which weakly damped cyclotron waves are excited, i.e., in which the inequalities (31) are satisfied, if, moreover, $l\sim 1$. In the present case the corrections to the impedance that stem from the nonlinear currents $j_1^{(3)}$, $j_2^{(3)}$, and $j_3^{(3)}$ are of the same order of magnitude.

We should note the following quite interesting circumstance. If weakly damped cyclotron waves are not excited, then the formula (43) for the nonlinear impedance is, as is easy to see, valid when the inequality

$$|H^{(1)}(0, \omega)|/H \ll 1, \tag{49}$$

which guarantees the applicability of the perturbation theory in terms of the highly inhomogeneous fields, which change over distances $\sim\delta_{1,2}$, is satisfied. As follows from (47) and (48), the satisfaction of the inequality (49) is not a sufficient condition for the expansion of the expression (43) in a series in powers of $\Delta H^{(2)}$, i.e., for the transition from (43) to (44), to be justified. The condition of applicability of the perturbation theory in terms of the homogeneous field $\Delta H^{(2)}$ is, generally speaking, more rigid than (49), but its fulfillment is not required, since the field $\Delta H^{(2)}$ is taken into account exactly. In other words, situations are possible in which the formula (43) describes a sharp change in the surface impedance (according to the foregoing, this can occur at frequencies far from a resonance when $\omega\tau \gg \Omega\tau \gg 1$ and at frequencies close to the even resonance $\omega = l\Omega$ if $\Omega\tau \gg 1$ and the inequalities (31) are not satisfied). In such cases

$$\zeta_{NL}(\omega, H)=\zeta_L(\omega, H+\Delta H^{(2)}).$$

The nonlinear impedance $\zeta_{NL}(\omega, H)$ introduced by the relation (18) characterizes the reflection of the electromagnetic wave at the frequency of the fundamental harmonic. Since $|\zeta_{NL}| \ll 1$, $|\zeta_L| \ll 1$ and the amplitude of the reflected sound harmonic[4,11]

$$E^{(2)}(0, 2\omega)\sim\zeta_L(H^{(1)}(0, \omega))^2/H,$$

we can verify that the nonlinear impedance $\zeta_{NL}(\omega, H)$ also characterizes the energy dissipation in the sample (the energy that can be carried away by the second and higher harmonics is negligibly small). This was demonstrated earlier by Dubovskiĭ[15] for the normal skin effect at low frequencies, i.e., at frequencies satisfying the condition $\omega\tau \ll 1$.

Above we assumed that the sample is acted upon by an electromagnetic field of constant amplitude for an infinitely long time. In practice, to prevent strong overheating of the sample, fairly short pulses are often used. It should be noted that the results obtained by us are not always applicable when the pulsed regime is employed, since in this instance the radioelectric current will depend on the time, and its screening should, generally speaking, be taken into account. Let, for definiteness, the pulse have a rectangular shape. Let us denote its duration by t_p. Clearly, the formula (43) for the nonlinear impedance will remain valid if $t_p \gg \tau$ and $t_p \geqslant t_0$, where t_0 is a time interval such that the normal skin effect obtains at frequencies $\sim t_0^{-1}$ (in other words, over the time period t_0 the magnetic field of the rectified current penetrates into the metal to a depth much greater than r_H). If the pulse is short, i.e., if $\tilde{t}_0 \geqslant t_p \gg \tau$, where $\tilde{t}_0$ is a time interval such that the anomalous skin effect obtains at frequencies $\sim\tilde{t}_0^{-1}$, then a homogeneous field does not exist in the interior of the sample (at distances $\sim r_H$ from the surface). Consequently, in such a situation the second term in the formula (43) should actually be discarded.

It follows from the estimates obtained by us for the nonlinear corrections to the impedance that the nonlinearity can be quite substantial in weak magnetic fields (it was precisely in weak magnetic fields that the dependence of the surface impedance of the wave amplitude was observed by Cochran and Shiffman[1] and Gantmakher[2]). We must, however, bear in mind that these estimates are applicable when $H \gtrsim H_c$, where the field H_c is determined from the condition

$$\left|\frac{\omega+i\tau^{-1}}{\Omega}\right|\left(\frac{\delta}{r_H}\right)^{1/2}\sim 1.$$

In metals with skin depth $\delta\sim 10^{-5}$ cm at frequencies $\omega/2\pi\sim 10$ GHz, for $\omega\tau \gtrsim 1$ the field $H_c\sim 10$ Oe. In bismuth, $H_c\sim 0.01$–0.1 Oe in the radio-frequency region.[2,14] As a rule, in such fields $\Omega\tau < 1$. Consequently, we see that, as the magnetic field H decreases and approaches the region of fields $H\sim H_c$ from the right, the nonlinear corrections (for a fixed value of $H^{(1)}(0, \omega)$) increase. It can also be shown that, as H decreases further and crosses into the region $H<H_c$, the nonlinear corrections begin to decrease rapidly (if, of course, the condition $|H^{(1)}(0, \omega)|<H_c$ is fulfilled; otherwise the sample may go over into the current state[14,16]), i.e., the nonlinearity manifests itself most strongly at $H\sim H_c$.

We have not considered here the nonlinearity connected with the warming up of the sample. Estimates similar to those presented in Refs. 14 and 17 show that even at frequencies $\omega \sim 10^{10}$–10^{11} we can, by using short pulses, realize in the typical metals, as well as in the bismuth-type semimetals, at temperatures 1–4 K, situations in which the nonlinearity due to the Lorentz force is quite strong, i.e., $|H^{(1)}(0, \omega)| \sim H > H_c$, but the warm-up is negligible. The effect of the warm-up decreases further with decreasing frequency.

[1]J. F. Cochran and C. A. Shiffman, Bull. Am. Phys. Soc. **10**, 110 (1965).
[2]V. F. Gantmakher, Pis'ma Zh. Eksp. Teor. Fiz. **2**, 557 (1965) [JETP Lett. **2**, 346 (1965)].
[3]V. V. Vas'kin, I. F. Voloshin, V. Ya. Demikhovskiĭ, and L. M. Fisher, Fiz. Nizk. Temp. **5**, 605 (1979) [Sov. J. Low Temp. Phys. **5**, 289 (1979)]; Tezisy dokladov na 20 Vsesoyuznom soveshchanii po fizike nizkikh temperatur (Abstracts of Papers Presented at the Twentieth All-Union Conf. on Low Temperature Physics), Moscow, 1979, p. 8.
[4]A. P. Konasov, Zh. Eksp. Teor. Fiz. **72**, 191 (1977) [Sov. Phys. JETP **45**, 100 (1977)].
[5]É. A. Kaner and V. F. Gantmakher, Usp. Fiz. Nauk **94**, 193 (1968) [Sov. Phys. Usp. **11**, 81 (1968)].
[6]M. Ya. Azbel', Zh. Eksp. Teor. Fiz. **39**, 400 (1960) [Sov. Phys. JETP **12**, 283 (1961)].
[7]A. F. Andreev, Usp. Fiz. Nauk **105**, 113 (1971) [Sov. Phys. Usp. **14**, 609 (1972)]; V. I. Okulov and V. N. Ustinov, Fiz. Nizk. Temp. **5**, 213 (1979) [Sov. J. Low Temp. Phys. **5**, 101 (1979)].
[8]M. Ya. Azbel' and É. A. Kaner, Zh. Eksp. Teor. Fiz. **32**, 896 (1957) [Sov. Phys. JETP **5**, 730 (1957)].
[9]R. Chambers, in: The Physics of Metals, Vol. 1: Electrons (ed. by J. Ziman), Camb. U. Press, Cambridge, 1969 (Russ. Transl., Mir, M., 1972).
[10]A. P. Kopasov and I. N. Mol'kov, Phys. Status Solidi **111**, No. 2, 1982.
[11]A. P. Kopasov, Zh. Eksp. Teor. Fiz. **78**, 1408 (1980) [Sov. Phys. JETP **51**, 709 (1980)].
[12]É. A. Kaner and V. G. Skobov, Fiz. Tverd. Tela (Leningrad) **6**, 1104 (1964) [Sov. Phys. Solid State **6**, 851 (1964)].
[13]G. I. Babkin and V. T. Dolgopolov, Zh. Eksp. Teor. Fiz. **66**, 1461 (1974) [Sov. Phys. JETP **39**, 717 (1974)].
[14]V. T. Dolgopolov, Doctoral Dissertation, Chernogolovka, 1979.
[15]L. B. Dubovskiĭ, Zh. Eksp. Teor. Fiz. **58**, 2110 (1970) [Sov. Phys. JETP **31**, 1138 (1970)].
[16]V. T. Dolgopolov, Zh. Eksp. Teor. Fiz. **68**, 335 (1975) [Sov. Phys. JETP **41**, 173 (1975)].
[17]V. T. Dolgopolov, Usp. Fiz. Nauk **130**, 241 (1980) [Sov. Phys. Usp. **23**, 134 (1980)].

Translated by A. K. Agyei