Resonant cooling of the spin system of a superconductor lattice nuclei following optical orientation of the electrons

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Cooling of the spin system of lattice nuclei as a result of hyperfine interaction with polarized electrons is considered when the mean electron spin (**S**) and the external alternating magnetic field ($\mathbf{H}_1$) oscillate at a frequency close to that of the nuclear magnetic resonance. The equations obtained for the Zeeman and dipole-dipole reservoir temperatures are an extension of the Provotorov equations to the case of interaction between nuclei and polarized electrons. Solutions of the equations are presented for strong and weak magnetic fields. It is shown that the degree of cooling of the nuclear spin system has a resonant dependence on the modulation frequency of **S** and of $\mathbf{H}_1$. In the case of an extremely weak alternating field the dependence is determined by the correlators of the total spin of the lattice nuclei. The solution for a saturating field $\mathbf{H}_1$ does not depend on the form of the spin correlators.

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1. INTRODUCTION

Optical orientation of the electrons in a semiconductor in a magnetic field causes polarization of the crystal-lattice nuclei.[1–5] As shown in Refs. 2 and 3, this effect is the result of lowering the temperature of the nuclear spin system by the contact with the nonequilibrium spins of the photoelectrons.

The usefulness of the concept of nuclear spin temperature becomes most clearly manifest when the nuclei are cooled by electrons whose polarization is modulated in time:

$$\mathbf{S}(t)=\mathbf{S}_0\cos\omega t.$$

In this case the average spin introduced into the nuclear spin system is zero. If, however, the external magnetic field is also modulated:

$$H_\alpha(t)=H_{0\alpha}+H_{1\alpha}\cos(\omega t+\varphi_\alpha),$$

the average energy change of the nuclei is generally speaking different from zero, and this leads to cooling.[4–6]

At low values of the constant magnetic field ($H_0\lesssim H_L$, where H_L is the characteristic field produced at the nucleus by the neighboring nuclei) the theory of cooling of a nuclear spin system when **S** and **H** are modulated was developed earlier.[6] In this paper we investigate theoretically the cooling process in the case of a strong constant magnetic field ($H_0\gg H_L$).

The cases of weak and strong fields H_0 differ in that at $H_0\gg H_L$ the state of the nuclear spin system is described by two parameters, the temperatures of the Zeeman and of the spin-spin reservoirs, whereas at $H_0\lesssim H_L$ these temperatures are equal. Thus, in the case of a weak constant field it suffices to determine the change produced in the total nuclear-spin energy by the contact with the polarized electrons, but at $H_0\gg H_L$ we must know the partition of this energy between the Zeeman and the spin-spin reservoirs. (The problem is easiest to solve for a sufficiently strong alternating field $\mathbf{H}_1$ perpendicular to $\mathbf{H}_0$ and oscillating at a frequency ω close to the NMR frequency.[5] In this case the Zeeman and the spin-spin temperatures are equal in a rotating coordinate frame.)

In Sec. 2, starting from simple phenomenological consideration we find how the energy input to the lattice-nuclei spin system is partitioned by the Zeeman and the spin-spin reservoirs, and in Sec. 3 we present a rigorous derivation of equations for their temperatures. These equations are similar in many respects to the Provotorov equations.[7]

2. PHENOMENOLOGICAL DERIVATION OF THE EQUATIONS FOR THE TEMPERATURES OF THE ZEEMAN AND THE SPIN-SPIN RESERVOIRS

If the spin system of the lattice nuclei is acted upon by a strong constant ($H_0\gg H_L$) and a weak alternating ($H_1\ll H_L$) magnetic field, the reciprocal temperatures of the Zeeman (α) and of the spin-spin (β) reservoirs are determined, as is well known, by the system of Provotorov equations[7]:

$$\frac{d\alpha}{at}=-W\left(\alpha-\frac{\Delta}{\omega_0}\beta\right)-\frac{\alpha}{T_{1Z}}+J_\alpha=0,$$
$$\frac{d\beta}{dt}=W\frac{\Delta}{\omega_L'^2}(\omega_0\alpha-\Delta\beta)-\frac{\beta}{T_{1D}}+J_\beta=0, \quad (1)$$

where $\Delta=\omega_0-\omega$; ω_0 is the NMR frequency in the constant field H_0, ω is the frequency of the oscillations of the alternating field H_1, T_{1Z} and T_{1D} are the characteristic relaxation times of the Zeeman and of the spin-spin temperatures to the lattice temperature; $W\sim H_1^2$ is the Δ-dependent rate of change of α and β under the influence of the alternating field, and ω'_L is the nuclear-spin precession frequency in the local field H'_L. We have added in Eqs. (1) the terms J_α and J_β, which give the rates of change of the Zeeman and of the spin-spin temperatures on account of the contact with the polarized electrons. We must find the form of these correction terms.

It can be easily seen that J_α and J_β are connected with the energy fluxes into the Zeeman (q_α) and into the spin-spin (q_β) reservoirs by the following relations:

$$J_\alpha=q_\alpha\left(\frac{dE_Z}{d\alpha}\right)^{-1}=-\left(\frac{I}{\mu_I}\right)^2\frac{q_\alpha}{H_0^2},$$
$$J_\beta=q_\beta\left(\frac{dE_D}{d\beta}\right)^{-1}=-\left(\frac{I}{\mu_I}\right)^2\frac{q_\beta}{H_L'^2}, \quad (2)$$

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where μ_I and I are the magnetic moment and the spin of the lattice nucleus and the derivatives with respect to α and β were calculated in the high-temperature approximation, when

$$E_Z=-(\mu_I/I)^2H_0^2\alpha,\quad E_D=-(\mu_I/I)^2H_L'^2\beta.$$

To solve our problem it suffices to determine the fluxes q_α and q_β connected with the polarization of the nuclear spins by the oriented electrons.

We obtain first the total energy flux $q_\alpha+q_\beta$. The hyperfine interaction leads to transfer of the spin of the oriented electrons to the lattice nuclei. The spin flux into the nuclear system is

$$\mathbf{j}=\mathbf{S}/T_{1l}',$$

where $\mathbf{S}$ is the average electron spin and $T_{1l}'\gtrsim 1$ sec is the time of longitudinal relaxation of the nuclear spins on the electrons. If the nuclei are in a magnetic field $\mathbf{H}(t)$, the spin flux $\mathbf{j}$ gives rise to an energy flux

$$q_S(t)=-\left(\frac{\mu_I}{I}\right)\mathbf{H}(t)\mathbf{j}(t)=-\left(\frac{\mu_I}{I}\right)\frac{1}{T_{1l}'}\mathbf{H}(t)\mathbf{S}(t). \qquad (3)$$

This flux is the consequence of the work performed on the nuclear spins by the polarized electrons.

The alternating magnetic field also performs work. The corresponding energy flux is[8]

$$q_H(t)=-\left(\frac{\mu_I}{I}\right)\frac{d\mathbf{H}(t)}{dt}\mathbf{I}(t), \qquad (4)$$

where $\mathbf{I}(t)$ is the value of the spin of the lattice nuclei at the instant of time t. Since we are interested in that part of the flux q_H' which is connected with the polarization of the nuclei by the oriented electrons, we must determine the time variation of the equilibrium spin added by the electrons to the nuclear spin system:

$$\mathbf{I}'(t)=\int_{-\infty}^{t}\hat{G}(t-\tau)\mathbf{j}(\tau)\,d\tau=\frac{1}{T_{1l}'}\int_{-\infty}^{t}\hat{G}(t-\tau)\mathbf{S}(\tau)\,d\tau. \qquad (5)$$

The time-dependent tensor $\hat{G}$ describes here the time variation of the nonequilibrium spin introduced into the lattice nuclear system by the oriented electrons. In other words, $\hat{G}$ is the Green's function for the nonequilibrium spin of the nuclei. (If the strong constant magnetic field is directed along the z axis, we have $G_{zx}=G_{xz}=G_{zy}=G_{yz}=0$, while the components $G_{xx}=G_{yy}$ and $G_{xy}=-G_{yx}$ decrease to zero within a time on the order of the nuclear-spin transverse-relaxation time $T_2\sim 10^{-4}$ sec, while the component $G_{zz}(\tau)$ decreases slowly with increasing τ, within a time on the order of the longitudinal relaxation time T_1.)

Using (3)–(5) we obtain an expression for the total energy flux into the nuclear spin system as a result of the simultaneous action of the alternating field and of the polarized electrons:

$$\langle q_\alpha+q_\beta\rangle=\langle q_S+q_H'\rangle=-\frac{1}{T_{1l}'}\left(\frac{\mu_I}{I}\right)\left\langle \mathbf{H}(t)\mathbf{S}(t)+\frac{d\mathbf{H}}{dt}\int_{-\infty}^{t}\hat{G}(t-\tau)\mathbf{S}(\tau)\,d\tau\right\rangle. \qquad (6)$$

The angle brackets denote here averaging over the period of the modulation of $\mathbf{S}$ and $\mathbf{H}_1$.

It can be seen even from this expression that if $\mathbf{S}$ and $\mathbf{H}_1$ are parallel to the field $\mathbf{H}_0$ and oscillate at frequency $\omega\gg T_1^{-1}$, then $\langle q_\alpha+q_\beta\rangle=0$. Indeed, in this case the component G_{zz} in the integrand of (6) can be regarded with high accuracy as equal to unity. Then

$$\langle q_\alpha+q_\beta\rangle=\langle q_S+q_H'\rangle\sim\mathbf{SH}_1-\mathbf{SH}_1=0.$$

The change of the nuclear spin-system energy can thus be due only to the $\mathbf{S}$ and $\mathbf{H}_1$ components that are perpendicular to $\mathbf{H}_0$. We shall therefore analyze hereafter the situation $\mathbf{S}\perp\mathbf{H}_0\perp\mathbf{H}_1$.

We examine now the division of the flux q_S+q_H' among the Zeeman and the spin-spin reservoirs. To this end we obtain the energy flux q_α into the Zeeman reservoir (q_β is connected with q_α and with q_S+q_H' by the obvious relation $q_\beta=q_S+q_H'-q_\alpha$).

Since the energy of the Zeeman subsystem is

$$E_Z=-(\mu_I/I)H_0I_z$$

and H_0 is independent of the time, the energy flux q_α is expressed in terms of the z component of the spin flux:

$$q_\alpha=-(\mu_I/I)H_0j_z.$$

Assuming that $\mathbf{S}$ lies in a plane perpendicular to $\mathbf{H}_0$, and recognizing that $G_{zx}=G_{zy}=0$, we easily find that j_z (and hence q_α) is zero in the absence of an alternating field. If $H_1\neq 0$, the precession of the nonequilibrium spin about $\mathbf{H}_1$ leads to the onset of

$$j_z=-(\mu_I/I)[\mathbf{H}_1(t)\times\mathbf{I}'(t)]_z.$$

The energy flux into the Zeeman reservoir is then

$$q_\alpha=\left(\frac{\mu_I}{I}\right)^2\frac{1}{T'}\left\langle \mathbf{H}_0\left[\mathbf{H}_1(t)\int_{-\infty}^{t}\hat{G}(t-\tau)\mathbf{S}(\tau)\,d\tau\right]\right\rangle. \qquad (7)$$

The expressions obtained can be rewritten in an unusually simple form by using the complex representation for the oscillating $\mathbf{S}$ and $\mathbf{H}_1$:

$$\mathbf{S}(t)=\mathrm{Re}\,\mathbf{S}e^{i\omega t},\quad \mathbf{H}_1(t)=\mathrm{Re}\,\mathbf{H}_1e^{i\omega t}.$$

Then

$$\langle q_S+q_H'\rangle=-\frac{1}{2T_{1l}'}\left(\frac{\mu_I}{I}\right)\mathrm{Re}\{\mathbf{H}_1^*\mathbf{S}-i\omega\mathbf{H}_1^*(\hat{g}(\omega)\mathbf{S})\}, \qquad (8)$$

$$q_\alpha=\frac{1}{2T_{1l}'}\left(\frac{\mu_I}{I}\right)^2\mathrm{Re}(\mathbf{H}_0[\mathbf{H}_1^*\times\hat{g}(\omega)\mathbf{S}]) \qquad (9)$$

and the energy flux into the spin-spin reservoir is

$$q_\beta=-\frac{1}{2T_{1l}}\left(\frac{\mu_I}{I}\right)\times\mathrm{Re}\left\{\mathbf{H}_1^*\mathbf{S}-i\omega\mathbf{H}_1^*(\hat{g}(\omega)\mathbf{S})+\frac{\mu_I}{I\hbar}\mathbf{H}_0[\mathbf{H}_1^*\times\hat{g}(\omega)\mathbf{S}]\right\}. \qquad (10)$$

Here

$$g(\omega)=\int_{-\infty}G(\tau)e^{i\omega\tau}\,d\tau \qquad (11)$$

is the Fourier transform of the tensor $\hat{G}(\tau)$.

Substituting the relations (9) and (10) in (1) and (2) we obtain a generalization of the Provotorov equations to the case of interaction between nuclear spins and oriented electrons. In the next section we present a rigorous microscopic derivation of these equations, on the basis of the transport equation for the spin-system density matrix.[2,6] This derivation makes it possible to determine the connection between the coefficients of the system (1) and the microscopic parameters of the problem.

3. MICROSCOPIC DERIVATION OF THE EQUATIONS FOR THE ZEEMAN AND SPIN-SPIN TEMPERATURES

Let the lattice nuclei be located in a strong magnetic field ($H_0 \gg H_L$) and let them interact with the polarized electron and with the weak alternating field $\mathbf{H}_1$. Following Ref. 6 we assume that the frequency of the modulation of $\mathbf{S}$ and $\mathbf{H}_1$ is high enough ($\omega T_1 \gg 1$). In this case the spin density matrix of the lattice nuclei can be represented in the form

$$\Phi(t) = \Phi_0 + \Phi_1(t),$$

where Φ_0 is independent of the time and $\Phi_1(t)$ oscillates at the frequency ω. As shown in Ref. 6, the system of equations for Φ_0 and Φ_1 is of the form

$$\frac{i}{\hbar}\frac{d\Phi_0}{dt} = [\mathcal{H}_{N0}, \Phi_0] + \overline{\hat{L}}(t, \Phi_1) - \frac{i}{\hbar}\hat{F}(\Phi_0) = 0, \tag{12}$$

$$\frac{i}{\hbar}\frac{d\Phi_1}{dt} = [\mathcal{H}_{N0}, \Phi_1] + \hat{L}(t, \Phi_1). \tag{13}$$

Here $\mathcal{H}_{N0} = \mathcal{H}_z + \mathcal{H}'_d$ is the nuclear spin Hamiltonian part which does not depend explicitly on the time, with

$$\mathcal{H}_z = -\sum_n (\mathbf{H}_0, \hat{\mathbf{I}}^n)\mu_I/I$$

the energy opertor of the nuclear spins in the constant magnetic field $\mathbf{H}_0$, and $\mathcal{H}'_d$ is the (truncated) part, which commutes with $\mathcal{H}_z$, of the operator of the spin-spin interactions between the nuclei.[1)] The operator $\hat{L}$ describes the interaction of the nuclear spins with the alternating magnetic field $\mathbf{H}_1$ and the polarization of the nuclei by oriented electrons, while the operator $\hat{F}$ describes the relaxation of the nuclei on the electrons in the conduction band:

$$\hat{L}(t, \Phi) = -\left(\frac{\mu_I}{I}\right)\sum_n [(\mathbf{H}_1(t), \hat{\mathbf{I}}^n), \Phi] + \sum_{\substack{mn\\ \alpha\beta\gamma}} \frac{2a_{mn}}{\hbar^2}\varepsilon_{\alpha\beta\gamma}S_\gamma(t)[\hat{I}_\alpha^n\{\hat{I}_\beta^m\Phi\}],$$

$$\hat{F}(\Phi_0) = -\frac{1}{4}\sum_{\alpha mn} a_{mn}[\hat{I}_\alpha^n[\hat{I}_\alpha^m\Phi_0]]. \tag{14}$$

Here $S_\gamma(t)$ is the time-dependent γ projection of the average spin of the oriented electrons; $\varepsilon_{\alpha\beta\gamma}$ is a unit antisymmetric tensor of third rank, and $\{\hat{I}_\beta\Phi\} = \hat{I}_\beta\Phi + \Phi\hat{I}_\beta$. The coefficients a_{mn} in (14) depend on the hyperfine interaction and on the time of the correlations of the orbital motion of the electrons.[2–6] The bar over the operator $\hat{L}$ in Eq. (12) denotes averaging over the modulation period $T = 2\pi/\omega$ of the electron spin and of the alternating magnetic field.[2)] (Here and elsewhere we assume for simplicity that the magnetic field has the same value for all the lattice nuclei.)

Since the operators $\mathcal{H}_z$ and $\mathcal{H}'_d$ commute with each other, the density matrix can be described within the spin-temperature concept with the aid of the two temperatures α and β of the Zeeman and the dipole-dipole reservoirs, respectively. We shall seek the equations for these temperatures in the high-temperature approximation, when

$$\Phi_0 = 1 - \alpha\mathcal{H}_z - \beta\mathcal{H}_d'. \tag{15}$$

Substituting (15) in (12), multiplying the right- and left-hand sides of the resultant equation by $\mathcal{H}_z$, and calculating the trace over the quantum numbers of the nuclei, we obtain an expression for the derivative $d\alpha/dt$ in terms of α and β. An analogous operation with $\mathcal{H}'_d$ yields an expression for $d\beta/dt$. The behavior of the temperatures α and β is then described by the system

$$\frac{d\alpha}{dt} = \left(\frac{I}{\mu_I}\right)^2\frac{i}{\hbar}\frac{\operatorname{Sp}\mathcal{H}_z\overline{\hat{L}}(t, \Phi_1)}{H_0^2\operatorname{Sp}\hat{I}_z^2} - \frac{\alpha}{2\hbar^2 N}\sum_m a_{mm},$$

$$\frac{d\beta}{dt} = \left(\frac{I}{\mu_I}\right)^2\frac{i}{\hbar}\frac{\operatorname{Sp}\mathcal{H}_d'\overline{\hat{L}}(t, \Phi_1)}{H_L'^2\operatorname{Sp}\hat{I}_z^2} - \frac{\beta}{2\hbar^2 N}\frac{\tilde{H}_L^2}{H_L'^2}\sum_m a_{mm}, \tag{16}$$

where $\hat{I}_z$ is the operator of the z projection of the total spin of the lattice nuclei; N is the total number of the nuclei;

$$H_L'^2 = (I/\mu_I)^2\operatorname{Sp}\mathcal{H}_d'^2/\operatorname{Sp}\hat{I}_z^2 N;$$

$$\tilde{H}_L^2 = \left(\frac{I}{\mu_I}\right)^2\left(\sum_{mn\alpha} a_{mn}\operatorname{Sp}[\mathcal{H}_d', \hat{I}_\alpha^n][\hat{I}_\alpha^m\mathcal{H}_d']\right)\Big/ 2\operatorname{Sp}\hat{I}_z^2\sum_m a_{mm}.$$

The second terms in the right-hand sides of the system (16) are proportional to the traces of $(\mathcal{H}_z\hat{F})$ and $(\mathcal{H}'_d\hat{F})$. They correspond to the heating of the Zeeman and of the dipole-dipole reservoirs by relaxation on the electrons. The first terms in the right-hand sides of these equations describe the rates of change of the Zeeman and dipole-dipole temperatures because of the interaction with the alternating field and the polarized electrons, with

$$\operatorname{Sp}\mathcal{H}_d'\overline{\hat{L}}(t, \Phi_1) = \operatorname{Sp}\mathcal{H}_{N0}\overline{\hat{L}}(t, \Phi_1) - \operatorname{Sp}\mathcal{H}_z\overline{\hat{L}}(t, \Phi_1).$$

Calculations similar to those in Ref. 6 lead to the set of Eqs. (1) with the following parameters:

$$\frac{1}{T_{1Z}} = \frac{1}{2\hbar^2 N}\sum_m a_{mm}, \quad \frac{1}{T_{1D}} = \frac{1}{T_{1Z}}\frac{\tilde{H}_L^2}{H_L'^2},$$

$$W = \frac{1}{2}\left(\frac{\mu_I}{I}\right)^2\operatorname{Re}(\mathbf{H}_1^*, \hat{g}\mathbf{H}_1), \tag{17}$$

where

$$\hat{g}_{\alpha\beta}(\omega) = \int_{-\infty}^{0}\frac{\operatorname{Sp}(\hat{I}_\alpha(0)\hat{I}_\beta(\tau))}{\operatorname{Sp}\hat{I}_z^2}e^{i\omega\tau}d\tau \tag{18}$$

is the Fourier transform of the correlators of the projections of the total spin of the lattice nuclei, and the dependence of $\hat{I}_\beta$ on τ is given by the usual expression

$$\tilde{I}_\beta(\tau)=\exp(i\mathscr{H}_{N0}\tau/\hbar)\tilde{I}_\beta\exp(-i\mathscr{H}_{N0}\tau/\hbar).$$

In the considered case of a strong constant magnetic field, the essential components of the tensor[3] $\hat{g}$ [Eq. (18)] are connected with one another by the simple relations[8]

$$g_{xx}=g_{yy}=ig_{yx}=-ig_{xy}.$$

This makes is possible to rewrite the generalized Provotorov equations (2), (10), and (11) in a simple and compact form:

$$\frac{d\alpha}{dt}=\frac{1}{T_{1Z}}\left\{\frac{4\,\mathrm{Re}(iH_{1+}{}^{*}S_{+}g_{xx})}{\hbar H_0}-\alpha\right\}$$
$$-|H_{1+}|^2\left(\frac{\mu_I}{I}\right)\mathrm{Re}\,g_{xx}\left(\alpha-\frac{\Delta}{\omega_0}\beta\right),$$
$$\frac{d\beta}{dt}=\frac{1}{T_{1D}}\left\{\frac{I}{\mu_I}\frac{\mathrm{Re}[H_{1+}{}^{*}S_{+}(1+2i\Delta g_{xx})]}{\tilde{H}_L{}^2}-\beta\right\}$$
$$+|H_{1+}|^2\left(\frac{\mu_I}{I}\right)\mathrm{Re}\,g_{xx}\frac{\omega_0\Delta}{\omega_L'^2}\left(\alpha-\frac{\Delta}{\omega_0}\beta\right). \qquad (19)$$

Here $\Delta=\omega_0-\omega$; ω_0 is the NMR frequency; ω_L' is the fequency of the nuclear-spin precession in the field H_L', and

$$S_+=(S_x+iS_y)/\sqrt{2},\qquad H_{1+}=(H_{1x}+iH_{1y})/\sqrt{2}$$

are the complex amplitudes of the components of $\mathbf{S}(t)$ and $\mathbf{H}_1(t)$, which rotate around $\mathbf{H}_0$ in the same direction as the precession of the nuclear spins.

To conclude this section we shall show that the cooling of the Zeeman and spin-spin reservoirs has a resonant character. Indeed, the rate of cooling of the Zeeman reservoir is proportional to $g_{xx}(\omega)$, and consequently decreases rapidly with increasing difference $\Delta=\omega_0-\omega$ (at large values of Δ we have $2g_{xx}\approx i/\Delta$). The rate of cooling of the spin-spin reservoirs contains terms Δg_{xx} $(\mathbf{H}_0\cdot[H_1^*\times\mathbf{S}])$ and can become appreciable far from the resonant frequency. However, the relaxation of the spin-spin temperature under the influence of the alternating field is determined by the large parameter $\omega_0\Delta/\omega_L'^2$, which causes the spin-spin reservoir temperature to increase just as rapidly with increasing deviation from resonance. The most intense cooling of the nuclear spin-spin system takes place thus when $\mathbf{S}$ and $\mathbf{H}_1$ oscillate at frequencies close to the NMR frequency.

4. ANALYSIS OF RESULTS

Although it is easy to obtain a general stationary solution of the system (19), the expressions obtained for α and β are unwieldy and not very perspicuous. We confine ourselves here therefore to an analysis of two very simple situations, when 1) the alternating magnetic field is weak, so that $WT_{1I}\ll 1$, and 2) H_1 is so strong that $WT_{1I}\gg 1$.

Let $WT_{1I}\ll 1$. Then

$$\alpha=-\frac{4}{\hbar H_0}\mathrm{Im}(H_{1+}{}^{*}S_{+}g_{xx}),$$
$$\beta=\frac{2I}{\mu_I}\frac{\mathrm{Re}[H_{1+}{}^{*}S_{+}(1+2i\Delta g_{xx})]}{\tilde{H}_L{}^2}. \qquad (20)$$

It can be seen that the temperatures of the Zeeman and dipole-dipole reservoirs are different. Their values are determined by the correlators of the total spin components of the lattice nuclei. As a rule there are no known explicit expressions for these correlators. Equations (20) make it possible to find their values from an analysis of experimental data.

We consider now the second case, when WT_{1I}. Then

$$\alpha=\frac{\Delta}{\omega_0}\beta, \qquad (21)$$

$$\beta=\frac{2\mu_I}{I\hbar^2}\frac{\mathrm{Re}(H_{1+}{}^{*}S_{+})}{\tilde{\omega}_L{}^2+(\omega_0-\omega)^2}, \qquad (22)$$

where $\tilde{\omega}_L$ is the frequency of the Larmor precession of the nuclear spin in the field $\tilde{H}_L$.

In this case the nuclear-spin correlators do not enter in the final answer. The dependence of the dipole-dipole reservoir temperature on the modulation frequency of $\mathbf{S}$ and $\mathbf{H}_1$ has a clearly pronounced resonant character. The maximum cooling is reached when the modulation frequency equals the NMR frequency. The half-width of the resonance curve is determined by the value of the local field H_L. The temperatures of the Zeeman and of the dipole-dipole reservoirs, just as in the case of saturation of the nuclear magnetic resonance, differ by the factor $\Delta/\omega_0=(\omega_0-\omega)/\omega_0$. Thus, for cases when the modulation frequency $\omega<\omega_0$ or $\omega>\omega_0$ the signs of the Zeeman-reservoir temperatures are opposite. At exact resonance ($\omega=\omega_0$) this temperature becomes infinite ($\alpha=0$).

We now compare the results of the theory developed above with the equations used in Ref. 5 to describe the cooling of a nuclear spin system in a rotating coordinate frame (RCF). It is known[7] that on going from the lab to the RCF the dipole-dipole reservoir temperature remains unchanged, but the Zeeman temperature changes by $\omega_0/(\omega_0-\omega)$ times ($\alpha_{\mathrm{RCF}}=\alpha\omega_0/\Delta$). Thus, in the case of a sufficiently strong alternating magnetic field we have $\alpha_{\mathrm{RCF}}=\beta_{\mathrm{RCF}}=\beta$, and the temperature of the nuclear spin system in the RCF is given by (22). The only difference between this equation and the result of Ref. 5 is that the denominator of (22) does not contain the squared amplitude of the alternating field. (We recall that all the calculations were carried out in the approximation $H_1\ll H_L$, and this is why the denominator of (22) does not contain the term H_1^2, which is small compared with H_L^2).

If, however, the equalization of the Zeeman and of the dipole-dipole temperatures is rapid enough, the simple formula of Ref. 5 can no longer be used to describe the cooling of the nuclear spins in the RCF. In this case the cooling must be calculated with the aid of the system (19) or its RCF analog. The simple relations (20) are valid for extremely weak H_1.

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[1] Since we seek a solution of the problem for the case of strong magnetic fields ($H_0\gg H_L$), we need retain in the operator $\mathscr{H}_{N0}$, in contrast to Ref. 6, only the truncated part $\bar{\mathscr{H}}_d'$ (Ref. 7).

[2] We note that we can neglect in the operator $\hat{L}$ the terms containing the electron spin.[6]

[3] We recall that the cooling of the nuclear spin system at $H_0\gg H_L$ is due only to those components of $\mathbf{H}_1$ and $\mathbf{S}$ that are perpendicular to $\mathbf{H}_0$. We are therefore considering the case when $\mathbf{H}_1\perp\mathbf{H}_0\perp\mathbf{S}$.

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