# Influence of a current and of a magnetic field on the properties of magnetic superconductors of the $HoMo_6S_8$ type

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We investigate the properties of the inhomogeneous domain-type magnetic structure (*DS* phase) produced in magnetic superconductors such as $HoMo_6S_8$ in the region where superconductivity and magnetism coexist. We show that the superconducting critical current in the *DS* phase decreases to zero when the temperature is lowered to $T_{c2}^{(c)}$ (the supercooling temperature of the *DS* phase). The wave vector of the magnetic structure decreases at the same time with increasing current flowing through the sample. We investigate the behavior of the *DS* in a magnetic field, obtain the phase diagram in the $(H,T)$ plane. In the region where the field penetrates, the *DS* phase is substantially altered: new peaks $2nQ$ ($n$ is an integer) appear in the neutron scattering, and the wave vector $Q$ decreases with increasing field.

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## I. INTRODUCTION

The great interest in the coexistence of superconductivity and magnetism is due to the fact that the interaction of these two competing order parameters leads to the apperance of new states, in which the magnetic ordering and the superconducting pairing are transformed in such a way that they can coexist. From this point of view, greatest interest attaches to the situation first considered by Anderson and Suhl,[1] wherein in the absence of Cooper pairing ferromagnetic ordering should appear at the point $\theta_c \ll T_c$, where $T_c$ is the critical temperature of the superconducting transition. Anderson and Suhl have assumed that $\theta_c$ is a second-order phase transition point for the magnetic subsystem. Under these assumptions, they have shown[1] that supeconductivity transforms the magnetic interactions in the system in such a way that what appears at the second-order phase transition point $T_M \approx \theta_c$ is not ferromagnetic ordering but an inhomogeneous magnetic state with large wave vectors $Q \gg \xi_0^{-1}$, where $\xi_0$ is the superconducting correlation length ($\xi_0 = hv_F/\pi\Delta_0$, $\Delta_0$ is the superconducting gap that would exist in the superconductor at $T=0$ in the absence of localized moments (LM), and $\Delta_0 \approx 1.76\ T_c$, and $v_F$ is the Fermi velocity of the electrons). In the inhomogeneous state, the directions of the magnetic moment in space change so rapidly that the exchange field of the LM is effectively averaged out over the superconducting correlation length, and the superconductivity is capable of surviving under these conditions even at a relatively large absolute value of the exchange field.

Anderson and Suhl took into account only exchange interaction (XI) of conduction electrons and of localized moments. Blount and Varma, as well as a few others (see Ref. 2 and the literature cited there) have shown that an inhomogeneous magnetic structure appears in a superconductor also in a model in which only the electromagnetic interaction (EI) of the electrons and of the LM is taken into account.

To describe real compounds such as $HoMo_6S_8$ and $ErRh_4B_4$ it is necessary to take into account both mechanisms whereby the electrons and the LM interact, and also consider the magnetic anisotropy (MA), which restricts greatly the possibilities of changing the directions of the moments in space. The problem of determining the structure of a superconducting phase with inhomogeneous magnetic ordering and the regions in which it exists, with allowance for the XI, EI, and MA, assuming a second-order phase transition at the point $\theta_c$ in the absence of superconductivity was solved in Ref. 3.

The quantitative results of Ref. 3 were obtained for a sufficiently dirty superconductors with electron mean free path $l$ satisfying the condition $Q^{-1} \ll l \ll \xi_0$, $(\theta_{ex} v_F^2/N(0))^{1/2}$, where $\theta_{ex}$ is the contribution of the long-range part of the exchange interaction to the energy of the ferromagnetic state of the system per LM, and $N(0)$ is the density of the electronic states per LM.

The main results of Ref. 3 reduce to the following statements.

(a) Below the point $T_M$ there appears in the superconducting phase an inhomogeneous magnetic structure with wave vector **Q**. The value of $Q$ is the result of a compromise between the energy of the inhomogeneity of the magnetic subsystem, on the one hand, and the energy of the interaction of the superconducting and magnetic subsystems on the other. The former reaches a minimum at a small value of $Q$, while the latter, which includes the XI and the EI, tends to increase $Q$. If $\theta_{ex}$ is not very small compared with the analogous contribution $\theta_m$ of the electromagnetic interaction, the interaction between the superconducting and magnetic subsystems is determined mainly by the exchange term. More accurately, the electromagnetic contribution can be neglected if the condition $\theta_{ex} \gg \theta_m (\lambda_L\ Q)^{-2}$ is satisfied, where $\lambda_L$ is the London penetration depth. The XI, however, of necessity governs substantially the characteristics of the magnetic subsystem, inasmuch as in real compounds $\theta_m$ is of the order of or somewhat larger than $\theta_{ex}$. The small role of the EI in the interaction between the superconducting and magnetic

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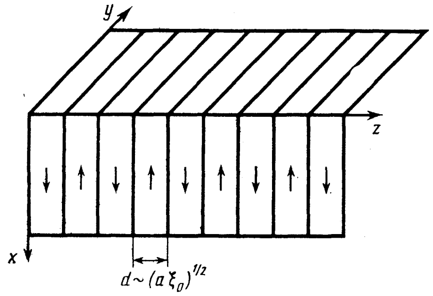


FIG. 1. Magnetic structure of the main type in superconductors of the type $ErRh_4B_4$ and $HoMo_6S_8$.

subsystems is due to the small value of $(\lambda_L Q)^{-2} \approx a\xi_0/\lambda_L^2$, where $a$ is a parameter that characterizes the rigidity of the magnetic system and is of the order of the interatomic distance.

(b) The magnetic anisotropy localizes the change of the direction of the moments inside the domain walls. As a result, the inhomogeneous magnetic structure in the superconducting phase takes below the $T_M$ point the form of a one-dimensional plane-parallel domain structure (Fig. 1). The structure wave vector $Q = \pi/d$, where $d$ is the domain thickness, decreases with decreasing temperature. This change is small and its order of magnitude is $Q \approx (a\xi_0)^{-1/2}$. $(2n+1)Q$ peaks where $n$ is an integer, should appear in the case of neutron scattering. In an ideal domain structure, the intensity of the peaks is proportional to $(2n+1)^{-2}$ in a single crystal and to $(2n+1)^{-4}$ in a polycrystal.

(c) The domain structure is transverse because of the electromagnetic part of the energy of the magnetic subsystem. The direction of **Q** in the plane perpendicular to the direction of the magnetic moment inside the domains depends on the anisotropy of the parameters $a$ and $v_F$. The domain has 180-degree walls only if the vector **Q** is not perpendicular to the easy plane of the crystal.

(d) The magnetic moment inside the domains is smaller than the value that would be present in a ferromagnet in the absence of a superconducting pair, but this difference is very small and is of the order of $(a/\xi_0)^{1/2}$. The supeconductivity has likewise practically no influence on the structure of the domain walls.

(e) At $\theta_{ex} < \theta_{ex}^{(c)} \approx T_{c1}^3 v_F N^2(0)/aT_M$ the superconducting phase with the domain structure (*DS* phase) remains stable down to zero temperature. If $\theta_{ex} > \theta_{ex}^{(c)}$, a first-order phase transition takes place from the *DS* phase into the normal ferromagnetic phase *FN* with decreasing temperature. In Ref. 3 we determined the points $T_{c2}$ of the first-order transition, the supercooling point $T_{c2}^{(c)}$ of the *DS* phase, and the superheating point $T_{c2}^{(\omega)}$ of the *FN* phase, the point $T_{c2}^{(\omega)}$ practically coinciding with $T_M$.

(f) The influence of the magnetic structure of the *DS* phase on the superconducting-condensation is similar to the influence of magnetic impurities, for which the magnetic-scattering time $\tau_m$ is defined by

$$\tau_m^{-1} = 7\zeta(3)\theta_{ex}N(0)s^2(T)/\pi v_F Q(T),$$

where $\zeta(x)$ is the Riemann function and $s(T)$ is the relative average localized moment chosen such that $s(0) = 1$ (see Ref. 9).

(g) Above the $T_M$ point the magnetic-moment fluctuations have a ferromagnetic character everywhere with the exception of a very narrow region near $T_M$. In crystals with easy-axis anisotropy the magnetic fluctuations are suppressed by the long-range part of the dipole interaction. Therefore the self-consistent field approximation for the magnetic subsystem yields in this case not only qualitative but also quantitative results.

The results (a)–(g) agree with the experimental data for $HoMo_6S_8$ (Ref. 4–6). The parameters obtained for this compound are $T_{c1} = 1.8$ K, $T_M = 0.7$ K and $T_{c2} \approx 0.65$ K. Neutron-scattering investigations of $HoMo_6S_8$ polycrystals have shown that when they are heated from the low-temperature region only the ferromagnetic phase is observed, in accord with the conclusion (e). The intensity of the ferromagnetic peak decreases with rising temperature, as should be the case for a second-order transition, and the assumption that the transition at the point $T_M$ is of second order is fully applicable to this compound. The behavior of an inhomogeneous magnetic structure to which a magentic field is applied was investigated in Refs. 4–6. We shall therefore consider below theoretically, within the framework of the same assumptions as in Ref. 3, the influence of a magnetic field and of a superconducting current on the *DS* phase, and compare the theoretical conclusions with the experimental data.

The results of the investigation of neutron scattering in polycrystals and single crystals of $ErRh_4B_4$ (Refs. 7 and 8) contradict the conclusions (b) and (c), and indicate that in the absence of superconductivity the transition into the ferromagnetic phase is apparently of first order (of the singlet ferromagnetic transition type). Therefore the theory of Ref. 3 and the results cited below are not applicable directly to $ErRh_4B_4$.

In Sec. II below we investigate the properties of the *DS* phase under conditions when a superconducting current of density $j$ flows through the sample. This current should lead to a decrease in the superconducting order parameter $\Delta$. Therefore, by passing current through $HoMo_6S_8$ samples in the region of the existence of the *DS* phase one can observe a decrease of the wave vector $Q$ with increasing superconducting current. We shall calculate the function $Q(j)$ for thin films, in which the action of the magnetic field of the current on the *DS* phase can be neglected. Under the same conditions we shall obtain the dependence of the critical superconducting current $j_c(T)$ in the *DS* phase and show that $j$ drops to zero when the temperature decreases from $T_M$ to the *DS*-phase supercooling point $T_{c2}^{(c)}$.

In Secs. III and IV we shall investigate the effect of a magnetic field on the *DS* phase in thin films of thickness $L$ small compared with the London penetration depth $\lambda_L$. We shall obtain for the films the region of existence of the *DS* phase in the $(T,H)$ plane and consider the change induced in the magnetic structure by the field.

In Secs. V and VI we shall find the region of existence of the *DS* phase in the $(T,H)$ plane for bulky samples and calculate the lower critical field $H_{c1}$, as well as the depth of penetration of the field in the $DS$ plane. In all our calculations we consider dirty superconductors with a mean free path satisfying the condition $(\theta_{ex} v_F^2/N(0))^{1/2} \gg l \gg Q^{-1}$. The condition $l \gg Q^{-1}$ means that we exclude from consideration dirty superconductors with mean free path approaching the interatomic distance.

## II. EFFECT OF A CURRENT ON THE SUPERCONDUCTING PHASE WITH INHOMOGENEOUS MAGNETIC ORDERING (*DS* PHASE)

### A. Eulenberg equations and their solution

We consider a system of electrons within the framework of the BCS model. The localized magnetic moments, which are regularly located at the crystal-lattice sites will be described within the framework of the mean-field approximation. In accordance with conclusion (a) it suffices to take into account the action of only the exchange field on the conduction electrons. Within the framework of these approximations, the Hamiltonian of the electron system is

$$\mathscr{H}=\int d^3\mathbf{r}\left[\psi^+(\mathbf{r})\frac{\mathbf{p}^2}{2m}\psi(\mathbf{r})+\Delta(\mathbf{r})\psi^+(\mathbf{r})\psi^+(\mathbf{r})+\Delta^*(\mathbf{r})\psi(\mathbf{r})\psi(\mathbf{r}) + \frac{|\Delta(\mathbf{r})|^2}{g}+\sum_i I(\mathbf{r}-\mathbf{r}_i)\psi^+(\mathbf{r})\sigma\psi(\mathbf{r})\mathbf{s}_i\right]+\mathscr{H}_{sc}, \quad (1)$$

where $\psi(\mathbf{r})$ are spinor operators, $\boldsymbol{\sigma}$ are Pauli matrices, $\Delta(\mathbf{r})$ is the superconducting order parameter, $g$ is the effective-interaction parameter of the electrons and phonons, $\mathbf{s}_i$ describes the mean value of the localized moment at the site $i$, $0 \leqslant s_i \leqslant 1$ the term $\mathscr{H}_{sc}$ describes electron scattering by nonmagnetic impurities, and we assume the scattering potential to be pointlike. In (1) we have neglected the anisotropy of the electron velocity on the Fermi surface, assuming this anisotropy to be small.

We consider next a magnetic structure with wave vectors $Q \ll 2\pi n^{1/3}$, where $n$ is the LM density. According to (1), the electrons are acted upon by the components of the exchange field with wave vectors of order $Q$ and wave vectors of order $2\pi n^{1/3}$, since the LM lattice is discrete. The action of the LM on the superconductivity is neglected (see Ref. 3), so that the exchange field $\mathbf{h}$ depends on the continuous variable $\mathbf{r}$ and $\mathbf{h}(\mathbf{r})' = h_0\,\mathbf{s}(\mathbf{r})$, where $h_0 = n\,I(0)$, $I(0) = \int \tilde{I}(\mathbf{r})d^3\mathbf{r}$ and $\theta_{ex} = h_0^2 N(0)$.

Our problem is to find the self-consistency equations for $\Delta(\mathbf{r})$ in the presence of a superconducting current. Knowledge of this equation yields the functional of the free energy of the system, minimization of which enables us to determine all the characteristics of the $DS$ phase (see Ref. 3).

The Green's functions for superconducting electrons satisfy a system of equations of the Eulenberg type[2]

$$\begin{gathered}\left[\tilde{\omega}(\mathbf{r})+ih(\mathbf{r})\pm\frac{1}{2}(\mathbf{v}\nabla)\right]f^{\mp}(\mathbf{v},\mathbf{r})=\tilde{\Delta}^{\mp}(\mathbf{r})g(\mathbf{v},\mathbf{r}),\\ g^2(\mathbf{v},\mathbf{r})+f^+(\mathbf{v},\mathbf{r})f^-(\mathbf{v},\mathbf{r})=1,\quad l=v_F\tau,\\ \tilde{\omega}(\mathbf{r})=\omega+\frac{1}{2\tau}\bar{g}(\mathbf{r}),\quad \tilde{\Delta}^{\pm}(\mathbf{r})=\Delta^{\pm}(\mathbf{r})+\frac{1}{2\tau}\bar{f}^{\pm}(\mathbf{r}),\\ \Delta^{\pm}(\mathbf{r})=gN(0)\sum_{\omega}\bar{f}^{\pm}(\mathbf{r}),\quad \bar{g}(\mathbf{r})=\int\frac{d\Omega}{4\pi}g(\mathbf{v},\mathbf{r}),\\ \bar{f}^{\pm}_{(\mathbf{r})}=\int\frac{d\Omega}{4\pi}f^{\pm}(\mathbf{v},\mathbf{r}),\quad g(\mathbf{v},\mathbf{r})=i\int\frac{d\xi}{2\pi}G_{++}(\mathbf{p},\mathbf{r}),\\ f^{\sigma}(\mathbf{v},\mathbf{r})=\sigma\int\frac{d\xi}{2\pi}\mathscr{F}_{-\sigma\sigma}(\mathbf{p},\mathbf{r}),\end{gathered}\quad (2)$$

where the symbol $\sigma = \pm$ characterizes the direction of the spin of the functions $G$ and $\mathscr{F}$, where $\mathbf{v}$ is the velocity on the Fermi surface, and averaging over the angles $\Omega$ denotes averaging over the direction of the velocity $\mathbf{v}$ on the Fermi surface. Equations (2) were written for the case when the moments inside the domains are directed along one axis ($z$), i.e., within the domains we have $s_x = s_y = 0$ and $s_z(\mathbf{r}) = s_z(\mathbf{r}+2\mathbf{d}) = s$ and $s_z(\mathbf{r}+\mathbf{d}) = -s_z(\mathbf{r}) = -s$, where $2d$ is the period of the domain structure.

In the presence of a superconducting current, the pairs have a c.m.s. momentum $2q$, and the solution of Eq. (2) must be sought in the form $\Delta^{\pm}(\mathbf{r}) = \Delta(\mathbf{r})e^{\pm 2i\mathbf{qr}}$, where $\mathbf{q}$ is determined by the given current densty $\mathbf{j}$:

$$\mathbf{j}=-\frac{2ie}{m}N(0)\sum_{\omega}\int\frac{d\Omega}{4\pi}\left(\mathbf{v}-\frac{i\nabla}{m}\right)g(\mathbf{v},\mathbf{r}). \quad (3)$$

Actually at $j \neq 0$ we can replace $\omega$ in (2) by $\omega + i(\mathbf{v}\cdot\mathbf{q})$ and seek the solution of (2) in the form $\Delta^+(\mathbf{r}) = \Delta^-(\mathbf{r}) = \Delta(\mathbf{r})$.

Just as in Ref. 3, we obtain the solution for the case of a dirty superconductor when the conditions $(h\tau)^2 \ll 1$, $\Delta_0\tau \ll 1$, $Ql \gg 1$ and $v_F q\tau \ll 1$ are satisfied. In Ref. 3 it is shown that under these conditions we can neglect the coordinate dependence of $\Delta$ at $j = 0$, and Eqs. (2) can be easily solved by expanding $h(\mathbf{r})$, $g(\mathbf{v},\mathbf{r})$, and $f^{\pm}(\mathbf{v},\mathbf{r})$ in Fourier series.

$$g(\mathbf{v},\mathbf{r})=\sum_k g_k(\mathbf{v})e^{i\mathbf{Q}\mathbf{r}k},\ h(r)=\sum_k h_k e^{i\mathbf{Q}\mathbf{r}k}=\frac{4h_0 s}{\pi}\sum_{h=0}^{\infty}\frac{\sin(\mathbf{Q}\mathbf{r}n)}{2n+1} \quad (4)$$

and analogously for $f^{\pm}(\mathbf{v},\mathbf{r})$ and $\Delta(\mathbf{r})$. The condition $(h\tau)^2 \ll 1$ ensures smallness of the harmonics $f_k^{\pm}$, $g_k$, and $\Delta_k$ with $k \neq 0$, and the use of perturbation theory in terms of these harmonics, makes it possible to write down equations for the quantities $g_0(\mathbf{v})$ and $f_0^{\pm}(\mathbf{v})$, and for the component $\Delta_k$ with $k = 0$ which will hereafter be designated $\Delta$. This solution method can be used also in the presence of current, and as a result we obtain from (2)

$$(\tilde{\omega}_0+i\mathbf{q}\mathbf{v})f_0(\mathbf{v})-\tilde{\Delta}g_0(\mathbf{v})=-f_0(\mathbf{v})g_0(\mathbf{v})\sum_k\frac{h_k h_{-k}\tau}{1+k^2\tau^2(\mathbf{v}\mathbf{Q})^2} \quad (5a)$$

$$f_0^2(\mathbf{v})+g_0^2(\mathbf{v})=1,\quad \tilde{\omega}_0=\omega+\frac{1}{2\tau}\bar{g}_0,\quad \tilde{\Delta}=\Delta+\frac{1}{2\tau}\bar{f}_0, \quad (5b)$$

where in the derivation of (5) we used also the conditions $Ql \gg 1$ and $\Delta\tau \ll 1$. From (5) it follows that at $(h\tau)^2 \ll 1$ and $qv_F\tau \ll 1$ we can replace $f_0(\mathbf{v})$ and $g_0(\mathbf{v})$ in the right-hand side of (5a) by $\bar{f}_0$ and $\bar{g}_0$, respectively. After this replacement and after introducing the parameter $u = \tilde{\omega}_0/\tilde{\Delta}$ we obtain equations for $\bar{f}_0$, $\bar{g}_0$, or for $u$ in the form

$$\omega \bar{f}_0 - \Delta \bar{g}_0 = -\bar{f}_0 \bar{g}_0/\tau_m, \quad \bar{f}_0^2 + \bar{g}_0^2 = 1, \tag{6a}$$

$$\frac{\omega}{\Delta} = u\left[1 - \frac{1}{\tau_m \Delta (1+u^2)^{1/2}}\right], \quad \tau_m^{-1} = \frac{2\tau v_F^2 q^2}{3} + \frac{7\zeta(3) h_0^2 s^2}{\pi v_F Q}. \tag{6b}$$

Relation (6) which connects $\omega$ with $\Delta$, is similar to the corresponding relation for a superconductor with magnetic impurities (see Ref. 8), for which the scattering time is equal to $\tau_m$. Thus, at $(h\tau)^2 \ll 1$, $\Delta\tau \ll 1$, and $(Ql)^{-1} \ll 1$ we obtain the self-consistency equation for the parameter $\Delta$:

$$\ln \frac{\Delta_0}{\Delta} - f(x) = 0, \tag{7a}$$

$$f(x) = \begin{cases} \dfrac{\pi x}{4} & x = (\tau_m \Delta)^{-1} \leqslant 1 \\ \operatorname{arch} x + \dfrac{1}{2}\left(x \arcsin \dfrac{1}{x} - \sqrt{1-x^{-2}}\right), & x > 1 \end{cases}. \tag{7b}$$

The superconducting part of interest to us of the free-energy functional of the system is obtained by multiplying the left-hand side of (7a) by $\Delta N(0)$ and integrating from 0 to $\Delta$. As a result we obtain for the complete functional of the system $\mathcal{F}(s,Q,\Delta,q,T)$ at $T \leqslant T_M \ll T_{c1}$ and at a specified current density $j$ the expression

$$\mathcal{F}(s, Q, \Delta, q, T) = \mathcal{F}_m(s, Q, T) + \mathcal{F}_s(\Delta) + \mathcal{F}_{int}(s, Q, \Delta, q) - 2eqj, \tag{8}$$

$$\mathcal{F}_s(\Delta) = -\frac{1}{2} N(0) \Delta^2 \ln \frac{e\Delta_0^2}{\Delta^2}, \quad e = 2.718,$$

$$\mathcal{F}_{int}(\Delta, s, Q, q) = N(0)\left(\frac{\pi\Delta}{2\tau_m} - \frac{1}{3\tau_m^2}\right), \quad \tau_m \Delta \geqslant 1, \tag{9}$$

$$\mathcal{F}_m(Q, s, T) = -\theta s^2 - T\sigma(s) + \frac{1}{\pi} Q\eta(s, T),$$

where the functional $\mathcal{F}_m(Q,s,T)$ was obtained in Ref. 3, $\theta$ is the energy of the ferromagnetic state per LM, $\sigma(s)$ is the LM entropy, and $\eta$ is the surface energy of the domain wall (see Ref. 3). It is easy to verify from (3) and (7) that $j = 2e\partial\mathcal{F}_{int}/\partial q$. Therefore the equilibrium values of the parameters $\Delta$, $Q$, $q$ and $s$ are obtained from the condition that the total functional (8) be a minimum at the given value of the current $j$.

### B. Dependence of the wave vector of the magnetic structure on the current, and the critical current in the *DS* phase

Minimization of the free-energy functional (9) with respect to $q$, $\Delta$, and $Q$ yields the dependences of $\Delta$ and $Q$ on the $j$ in the implicit form

$$Q(x) = Q_0(T) g^{1/2}(x) \varphi^{1/2}(x), \quad \Delta(x) = \Delta_0 g^2(x),$$

$$g(x) = e^{-\pi x/8}, \quad \varphi(x) = 1 - \frac{4x}{3\pi}, \quad Q_0^2(T) = \frac{7\zeta(3)\pi\Delta_0\theta_{ex}s^2(T)}{2v_F\eta(T)}, \tag{10}$$

$$j(x) = 2.98 j_{c0}[x - \alpha g^{-3}(x)\varphi^{-1/2}(x)]^{1/2}\varphi(x) g^{-3}(x),$$

$$\alpha(T) = 7\zeta(3) h_0^2 s^2(T)/\pi v_F Q_0(T)\Delta_0,$$

$$j_{c0} = 1.71 eN(0)\Delta_0^{3/2}\tau^{1/2} v_F,$$

where $j_{c0}$ is the critical current that would flow in the system at $T = 0$ in the absence of the LM. It is equal in practice to the critical current in a nonmagnetic superconducting phase at temperatures $T_{c1} \gg T > T_M$.

A plot of the function $j(x,\alpha)/j_{c0}$ for several values of $\alpha$ is shown in Fig. 2. The value $\alpha_{c2} = 0.168$ corresponds to the temperature $T_{c2}$, and $\alpha \approx 0.26$ to the temperature $T_{c2}^{(c)}$. The dashed curve in this figure shows the function $Q(x)/Q_0$. In the temperature interval from $T_M$ to $T_{c2}$, the value of $Q(j)$ decreases by approximately 15% when the current is increased from zero to the critical value $j_c(\alpha)$. With further decrease of the temperature from $T_{c2}$ to $T_{c2}^{(c)}$ the total change $Q$ with increasing current decreases and vanishes as $T_{c2}^{(c)}$. Therefore the optimum conditions for observing the $Q(j)$ dependence are realized near the temperature $T_{c2}$, where the change of $Q$ is still large enough and the current $j_c$ is approximately half the value of $j_{c0}$. At small $j/j_c$ the wave vector decreases quadratically with $j$, and a noticeable decrease of $Q$ takes place only in the current interval from $0.5j_c$ to $j_c$.

The dependence of the critical current $j_c$ on the temperature, i.e., on the parameter $\alpha$, is shown in Fig. 3. In the considered case of a dirty superconductor with $(h\tau)^2 \ll 1$, the value of the critical current does not depend on the direction of $\mathbf{j}$ relative to the direction of the magnetic moment inside the domains or relative to the direction of the wave vector of the magnetic structure.

To decrease the action of the magnetic field of the current on the magnetic structure it is necessary to choose a sample in the form of a thin film and direct the current such that the influence of its magnetic field be a minimum. The influence of the magnetic field on the *DS* phase is considered below, and it follows from this consideration that the smallest effect is produced by a field perpendicular to the magnetization inside the domains and parallel to the hardest-magne-

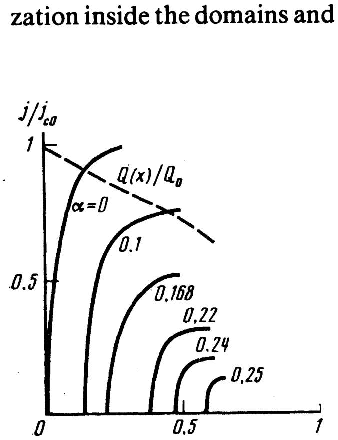


FIG. 2. Dependence of superconducting current and of the magnetic-structure wave vector $Q$ on the parameter $x$ (see the text).

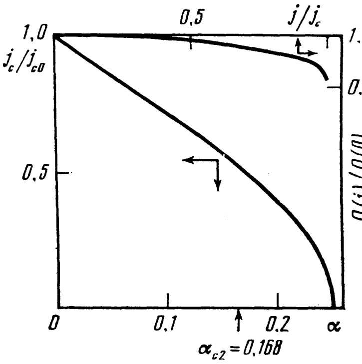


FIG. 3. Dependence of the superconducting critical current on the parameter $\alpha$ (see the text and Eq. (10)).

tization axis. In this geometry the value of the field $H$ should be less than $2(D+\theta_{ex})\cdot\Delta_0/h_0\mu_\perp$, where $D$ is the anisotropy parameter and $\mu_\perp$ is the magnetic moment at the site for the hard magnetization direction. This condition leads to a restriction on the plate thickness $L$ at a given current density $j$, i.e., to the condition

$$L<0.186h_0c(\Delta_0\tau)^{-1/2}(v_Fe\mu_\perp n)^{-1}(j_c/j)\,(D+\theta_{ex})/\theta_{ex},$$

where $c$ is the speed of light and $e$ is the electron charge.

## III. INFLUENCE OF MAGNETIC FIELD PARALLEL TO THE MOMENTS INSIDE THE DOMAINS ON THE STRUCTURE OF THE *DS* PHASE IN A THIN PLATE

### A. Region of existence of the *DS* phase in the $(H,T)$ plane

The most interesting and strongest influences of a magnetic field on the *DS* phase manifest themselves under conditions when the magnetic field is directed along the magnetic moment inside the *DS*-phase domains and if the magnetic field penetrates fully into the sample. We therefore investigate first the behavior of a sample in the form of a plate whose thickness $L$ is small compared with the London penetration depth $\lambda_L$ in the *DS* phase (the value of $\lambda_L$ in the *DS* phase will be obtained below). We consider a plate cut in such a way that the moment inside the domains lies in the plane of the plate and is directed along the $x$ axis (the [111] axis in $HoMo_6S_8$). We shall disregard the influence of the boundary conditions on the magnetic structure of the *DS* phase. To this end, the structure wave vector $\mathbf{Q}$ (which is parallel to the $z$ axis) should lie in the plane of the plate, or the plate thickness $L$ should be much larger than the domain thickness $d=\pi/Q$, if the vector $\mathbf{Q}$ does not lie in the plane of the plate.

We obtain now the region of the existence of the *DS* phase for such a plate in the case of a field parallel to the direction of the moment inside the domains, i.e., a field directed along the $x$ axis. Under the influence of the field, the thickness of the domains with magnetization along the field $(+s)$ increases, and opposite to the field $(-s)$ decreases. We denote these thicknesses of such domains by $d(1\pm\delta)$. The Fourier expansion of the average value of the moment $s_x(z)$ is then

$$s_x(z)=s\delta+\sum_{n=1}^{\infty}\frac{2s}{\pi n}\{[1-(-1)^n\cos(\pi n\delta)]\sin nQz+(-1)^n\sin(\pi n\delta)\cos(nQz)\}. \qquad (11)$$

Expression (11) does not take into account the finite thickness of the domain wall. It is shown in Ref. 3 that the influence of the domain walls on the superconductivity can be neglected, and we are at present likewise not interested in the contribution of the walls to the neutron scattering. Therefore expression (11) is sufficient for our purposes. We neglect the orbital effect of the field on the superconductivity by virtue of the condition $L\ll\lambda_L$, and we must take into account only the action of the exchange field $h_0s_x(z)$.

In part I we solved the problem for a rapidly alternating exchange field in the presence of a current. We now consider the problem without a current, but in the presence of a constant exchange field $\bar h=h_0s\delta$. Allowance for this field reduces to replacing in (2)–(6) the real variable $\omega$ by the complex variable $z=\omega+i\bar h$. In the self-consistency equation for $\Delta$, instead of integrating $\bar f_0(\omega)$ along the real axis $\omega$ we must calculate the integral of the functions $\bar f_0(z)$ along a path parallel to the real axis and shifted above it by an amount $\bar h$. The result of the integration does not depend on $\bar h$ (and $\Delta$ does not depend on $\bar h$) so long as this integration path does not cross the singular points of the function $f(z)$. The singular points of $f(z)$ correspond to zeros of $dz/d\bar f_0$, while the function $\bar f_0(z)$ is determined by Eqs. (6a), i.e., by the equation

$$z^2=\left(\frac{1}{f^2}-1\right)\left(\Delta-\frac{f}{\tau_m}\right)^2,\quad \tau_m^{-1}=\frac{7\zeta(3)h_0^2s^2}{\pi v_FQ}G(\delta),$$
$$G(\delta)=\frac{4}{7\zeta(3)}\sum_{n=1}^{\infty}\frac{1-(-1)^n\cos\pi n\delta}{n^3}. \qquad (12)$$

At $\Delta\tau_m>1$, the singular point of $f(z)$ closest to the real axis is located at $z=i\Delta[1-(\Delta\tau_m)^{-2/3}]^{3/2}$, and so long as $\bar h<h_c=\Delta[1-(\Delta\tau_m)^{-2/3}]^{2/3}$ the quantity $\bar h$ does not enter in the self-consistency equation for $\Delta$. This means that the paramagnetic susceptibility of the electron system is zero in weak fields, and the functional of the free energy of the system takes the form

$$\mathcal{F}(s,Q,\Delta,\delta,T)=-\theta s^2-T\sigma(s)+\eta(s,T)Q/\pi-\frac{H^2}{8\pi}-\mu_\parallel Hs\delta$$
$$+\theta_{ex}s^2\delta^2-\frac{1}{2}N(0)\Delta^2\ln\frac{e\Delta_0^2}{\Delta^2}+N(0)\left(\frac{\pi\Delta}{2\tau_m}-\frac{1}{3\tau_m^2}\right),$$
$$\Delta\tau_m>1,\quad h_0\delta s<h_c=\Delta[1-(\Delta\tau_m)^{-2/3}]^{3/2}. \qquad (13)$$

There is no superconducting solution at $\bar h>h_c$.

We have noted above that the magnetic structure of the *DS* phase acts on the superconductivity in analogy with magnetic impurities for which the magnetic-scattering time is equal to $\tau_m$. We see now that this analogy is not complete, since in our system the parallel paramagnetic susceptibility $\chi_{e,\parallel}$ is zero, and in a superconductor with magnetic impurities it differs from zero at any value of the parameter $\Delta\tau_m$. The reason why these two systems differ is that a superconductor with magnetic impurities is completely isotropic in spin space, whereas in the *DS* phase there is a preferred direction parallel to the moment inside the domains. Therefore the paramagnetic responses of these two electron systems to the constant field are different. In a superconductor with magnetic impurities, the ferromagnetic susceptibility of the electrons is isotropic, while in the *DS* phase it is anisotropic. We shall see below that the paramagnetic electronic response of the *DS* to a perpendicular field differs from zero (just as the response of a superconductor with magnetic impurities), i.e., in the *DS* phase we have $\chi_{e,\parallel} = 0$, but $\chi_{e,\perp} > 0$.

Minimizing the functional (13) with respect to the variables $s$, $Q$, $\Delta$ and $\delta$ we can determine the equilibrium characteristics of the *DS* phase and the region of the existence of this phase, as functions of the magnetic field and of the temperature.

Minimization with respect to $\delta$ can be effected without taking into account the last term in (13), since it is small compared with the sixth term relative to the parameter $\Delta / v_F\ Q \ll 1$. Therefore $\delta = \mu_\parallel\ H/2\theta_{ex}$. The magnetic susceptibility of the localized moments in the *DS* phase in a thin plate is $\chi_\parallel = \mu_\parallel^2\ n/2\theta_{ex}$ and the electronic contribution to the susceptibility is negligible small compared with the LM contribution. Thus, the susceptibility $\chi_\parallel$ in the *DS* phase does not depend on the temperature in the entire region of existence of the *DS* phase.

On minimizing with respect to $s$, the third and last terms of (13) can also be neglected, and the parameter $s$ is determined by the first two terms of the functional (13). These same two terms determine the equilibrium value of $s(T)$ in the ferromagnet in the absence of a field and of superconductivity, and we arrive at the conclusion that in the *DS* phase the value of $s(T)$ hardly differs from the moment in the ferromagnetic phase in the absence of superconductivity and of a magnetic field.

Minimization with respect to $\Delta$ and $Q$ leads to equations for the equilibrium values of $\Delta\ (T,H)$, $Q\ (T,H)$ and of the free energy $\mathscr{F}(T,H)$ in the form

$$\Delta(T,H) = \Delta_0 e^{-\pi x/4}, \quad Q(T,H) = Q_0 e^{-\pi x/8}\left(1 - \frac{4x}{3\pi}\right)^{1/2} G^{1/2}(\delta),$$
$$\mathscr{F}(T,H) = -\theta s^2 - T\sigma(s) - \frac{H^2}{8\pi} - \frac{(\mu_\parallel H)^2}{4\theta_{ex}} - \frac{N(0)}{2}\Delta_0^2 e^{-\pi x/2}\left(1 - \frac{3\pi x}{2} + 2x^2\right), \tag{14}$$

where $x = (\tau_m\ \Delta)^{-1}$ is defined by the equation

$$F(x) = xe^{-3\pi x/8}\left(1 - \frac{4x}{3\pi}\right)^{1/2} = \alpha(T)G(\delta), \quad \delta = \frac{\mu_\parallel H}{2\theta_{ex} s} < 1, \ x < 1. \tag{15}$$

(*a*) *DS-phase supercooling line*. The function $F(x)$ has a maximum at $x = 0.68$. Therefore the region of existence of the *DS* phase is determined, as a function of $T$ and $H$, by the conditions

$$s^2(T,H) \leqslant s^2(T_{c2}^{(c)}, 0)\, Q_0(T)\, Q_0^{-1}(T_{c2}^{(c)})\, G^{-1}(\delta),$$
$$\mu_\parallel H \leqslant 2\theta_{ex}(\Delta_0/h_0)\, e^{-\pi x/4}(1 - x^{2/3})^{3/2}. \tag{16}$$

According to (16), the *DS*-phase supercooling line in the $H,T$

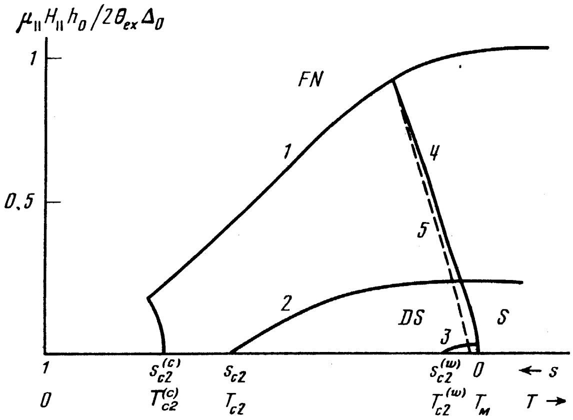


FIG. 4. Region of existence of the *DS* phase in the $(H,T)$ plane in the case of a field parallel to the moment direction in the domains. The temperature scale is the normalized mean value $s(T)$ of the moment in the ferromagnetic phase in the absence of superconductivity and of a magnetic field. Curve 1—*DS*-phase supercooling line, 2—line of $DS \to FN$, first-order transition curve 3—*FN*-phase superheat line, curve 4 separates the superconducting phase with homogeneous magnetic ordering from the phase with inhomogeneous magnetic order (in the mean-field approximation). Curve 5 shows the region of establishment of a clearly pronounced domain structure (11).

plane consists of two segments defined by the relations

$$\mu_\parallel H = 2\theta_{ex}(\Delta_0/h_0)\, e^{-\pi x/4}(1 - x^{2/3})^{3/2},$$
$$F(x) = \alpha(T)\, G(\mu_\parallel H/2\theta_{ex} s(T)), \tag{17a}$$

$$s^2(T) = s^2(T_{c2}^{(c)}) + \frac{\pi^2 \ln 2}{28\zeta(3)}\left(\frac{\mu_\parallel H}{\theta_{ex}}\right)^2. \tag{17b}$$

They intersect at the point with coordinates

$$s^2(T) = s^2(T_{c2}^{(c)}) + 0.152(\Delta_0/h_0)^2, \quad \mu_\parallel H = 0.108\theta_{ex}\Delta_0/h_0.$$

The resultant *DS*-phase supercooling curve in the $(H,T)$ plane is shown schematically in Fig. 4, curve 1. In the same figure, the temperature scale is taken to be the equilibrium value of $s(T)$ in the ferromagnetic phase without superconductivity or an external field, while $s$ is reckoned to the left from the point $T_M$, where $s = 0$, to the point $s = 1$, which corresponds to $T = 0$. Figure 4 corresponds to the situation $\Delta_0/h_0 \ll s_{c2}$.

(*b*) *First-order* $DS \to FN$ *transition line*. To obtain the first-order transition line it is necessary to equate the equilibrium free energy (14) of the *DS* phase to the energy of the normal ferromagnetic phase, which corresponds to the minimum of the functional

$$\mathscr{F}(s,\ T) = -\theta s^2 - T\sigma(s) - \mu_\parallel H s - H^2/8\pi \tag{18}$$

with respect to the variable $s$. At low temperatures, where $s \gg \Delta_0/h_0$, the first-order transition line is determined by the relation

$$\mu_\parallel H s(T) = {}^1/_2 N(0)\, \Delta_0^2 e^{-\pi x/2}\left(1 - \frac{3\pi x}{2} + 2x^2\right), \tag{19a}$$
$$F(x) = 0.066 s^2(T)/s^2(T_{c2}), \tag{19b}$$

and in the region of small $s(T) \ll (N(0)\Delta_0^2/b\theta)^{1/4}$ we obtain

$$\mu_{\parallel}H=2\theta_{ex}\frac{\Delta_0}{h_0}\cdot 0.738\left(\frac{b\theta\Delta_0^2}{\theta_{ex}h_0^2}\right)^{1/4},\quad b=\frac{4(I^2+I+{}^1/_2)}{5I(I+1)},\qquad (20)$$

where $I$ is the effective localized moment at low temperatures $T\lesssim T_M$. The first-order transition line is shown by curve 2 of Fig. 4. The latent heat of the $DS\rightarrow FN$ transition in the region $s(T)\gg\Delta_0/h_0$ amounts to

$$q=\left(0.709-\frac{3\pi x}{8}+\frac{1}{2}x^2\right)\Delta_0^2N(0)\left(\frac{d\ln s^2}{d\ln T}\right),\qquad (21)$$

and the parameter $x$ is given by (19b).

(c) *Superheat line of normal ferromagnetic phase FN.* This line is determined by the equation $h_0\,s(T,H)=\Delta_0/2$, where $s(T,H)$ is the equilibrium value of the moment in the $FN$ phase in the absence of a field, i.e., the value of $s$ that minimizes the functional (18). This line passes through the point $H=0$, $s(T)=\Delta_0/2h_0$; at small $H$ the value of $s(T)$ increases linearly with increasing $H$ and takes the asymptotic value $\mu_{\parallel}\ H=b\theta(\Delta_0/2h_0)^3$ at $s\ll\Delta_0/2h_0$. The $FN$-phase superheat line is shown by curve 3 of Fig. 4.

(d) *Line separating the superconducting phases with homogeneous and inhomogeneous magnetic ordering (the phases S and DS).* We determine now the line on which a domain structure appears. To the right of this line in Fig. 4 is located the $s$-phase with homogeneous magnetic ordering $s_0\neq 0$ in a magnetic field $H>0$, and on the line itself there appear Fourier components $s_q\neq 0$. The functional of the free energy takes, accurate to terms of second order in $s_{xq}$, the form (see Ref. 3)

$$\mathcal{F}\{s_{xq},s_0,T\}=-(\theta-\theta_{ex})s_0^2-\mu_{\parallel}HS_0-T\sigma(s_0)$$
$$+\sum_q\left(\frac{3}{2}bs_0^2-\theta+T\frac{3I}{2(I+1)}+\theta a^2q^2+\frac{\pi^2\Delta_0\theta_{ex}}{2v_Fq}\right)s_qs_{-q}.\ (22)$$

From (22) we obtain $s_0=\mu_{\parallel}H/2\theta_{ex}$, and the line of transition into the inhomogeneous magnetic state takes the form $s(T)=3^{1/2}s_0$, accurate to terms of higher order in $s_0\ll 1$. The line at which the inhomogeneous magnetic structure appears (the second-order transition line) is shown by the line 4 in Fig. 4. Near this line, the magnetic structure takes the form $s_x(z)\sim\sin Q_M z$ with $Q_M^3\approx(\pi^2\Delta_0\theta_{ex}/4a^2v_F\,\theta)$ (Ref. 1), and this structure goes over into a domain structure when the temperature or the field is decreased. In the domain structure, the domain width $\pi(1-\delta)/Q$ should be large compared with the domain-wall width $a/s(T)$ (see Ref. 3 at $T_M-T\ll T_M$). There exists therefore a transition region between the sinusoidal solution and the domain structure. This transition region is given by the relation $\mu_{\parallel}\ H=2\theta_{ex}[s(T)-aQ(T)/\pi]$ and is shown by the dashed curve 5 of Fig. 4. At $H=0$ it passes through the point $s(T)\approx(a/\xi_0)^{1/3}$. In the transition region, the parameter $Q$ varies quite rapidly from the value $Q_M$ to the value given by (14).

Thus, Fig. 4 represents the region of the existence of a superconducting phase with an inhomogeneous magnetic-order parameter in the $(H,T)$ plane for a parallel field. The forms of the curves 1–4 are determined by the parameters $\Delta_0/h_0$ and $s_{c2}=0.208(\Delta_0/h_0)(v_F\,Q_0/\Delta_0)^{1/2}$, and Fig. 4 shows the situation $1>s_{c2}^{(c)}\gg\Delta_0/h_0$. In principle the $DS$ phase can remain stable down to $T=0$ (if $s_{c2}>1$) or remain metastable down to $T=0$ ($s_{c2}<1$, but $s_{c2}^{(c)}=1.26s_{c2}>1$). In any case, however, the $DS$ phase exists only in fields $\mu_{\parallel}\ H<2\theta_{ex}\,\Delta_0/h_0$.

**B. Dependence of magnetic structure of the *DS* phase on the parallel field**

It follows from (14) that the wave vector $Q$ of the domain structure decreases with increasing field at a given temperature. According to Fig. 4, the maximum changes in the field from zero to values of the order of $2\theta_{ex}\,\Delta_0/h_0\,\mu_{\parallel}$ are possible at temperatures determined by the condition $s(T)\approx\Delta_0/h_0$. The parameter $\delta$ increases in this case from zero to a value on the order of unity when the $DS$-phase stability limit is reached. The quantity $G^{1/2}(\delta)$, which describes the dependence of $Q$ on $H$ at a given temperature ($\delta=\mu_{\parallel}H/\theta_{ex}\,s(T)$), is shown in Fig. 5, from which it is seen that when the parameter $\delta$ increases from zero to 0.5 the change of $Q$ is approximately 20%. Such a change can be easily discerned in experiment.

When $\delta$ is changed, according to (11), the relative intensities of the component $nQ$ in the neutron scattering also change. In particular, in the presence of a magentic field there appear peaks $2nQ$, which are missing at $H=0$. For the intensity of a peak $nQ$ in a single crystal we obtain from (11) the expression

$$\frac{I_n(H)}{I_1(0)}=\frac{1-(-1)^n\cos\pi n\delta}{2n^2},\quad \delta=\frac{\mu_{\parallel}H}{2\theta_{ex}s(T)}.\qquad (23)$$

The dependence of the relative intensity of the peaks on the value of the parallel fields can also be detected in experiment by neutron scattering. We note that the value of $s(T)$ can be obtained from data on the temperature dependence of the neutron-scattering amplitude. The parameter $\theta_{ex}$ can be calculated if the susceptibility of the plate in the parallel magnetic field is known (see III A above).

**IV. EFFECT OF FIELD PERPENDICULAR TO THE MAGNETIZATION INSIDE THE DOMAINS ON THE *DS* PHASE IN A THIN PLATE**

In this section we investigate the behavior of the $DS$ phase in a field perpendicular to the magnetization inside the domains. We consider as before the case when the field is parallel to the surface of the plate and disregard orbital effects, assuming that the plate thickness is $L\ll\lambda_L$ and considering fields $H\ll H_{c2}^*(0)$, where $H_{c2}^*(0)$ is the upper orbital critical field.

In the system considered by us, the electrons are acted upon by an exchange field $h_x(\mathbf{r})=h_0\,s_x(\mathbf{r})$ that varies in space, and by a constant exchange field $h_z=h_0s_z$ perpendicular to it (the magnetic field is directed along the $z$ axis). We shall show below that the component $s_z$ is small, and will disregard the change of the $s_x$ component inside the domains on account of the appearance of the $s_z$ component. Thus, the moment inside the domains is characterized by the quantities $s_x=\pm s$ and $s_z$.

We start with the Gor'kov equation for the Green's function

$$G(\mathbf{r},\tau,\mathbf{r}',0)=-i\langle T_\tau\psi(\mathbf{r},\tau)\psi^+(\mathbf{r}',0)\rangle,\psi=(\psi_\uparrow,\psi_\downarrow^+,\psi_\downarrow,\psi_\uparrow^+).$$

We must take into account the action exerted on the superconductivity by the exchange fields $h_z$ and $h_x(\mathbf{r})$ and the scattering of the electrons by nonmagnetic impurities. The field $h_x(\mathbf{r})$ contains the Fourier components $(2n+1)Q$ with $n=0, 1, 2, \ldots$. The equations for the function $G$ are of the form

$$\hat{G}^{-1}G(\mathbf{r},\tau;\tau',0)=1,\qquad \hat{G}^{-1}=\hat{G}_0^{-1}+\hat{h},$$

$$G=\begin{pmatrix} G_+ & G_1\\ G_2 & G_-\end{pmatrix},\quad G_0^{-1}=\begin{pmatrix} G_+^{-1} & 0\\ 0 & G_-^{-1}\end{pmatrix},\quad \hat{h}=\begin{pmatrix} 0 & h_1\\ h_2 & 0\end{pmatrix}. \tag{24}$$

We change over to the momentum representation in terms of the difference $\mathbf{r}-\mathbf{r}'$ and introduce the Green's function

$$G(\mathbf{p},\mathbf{r})=\int d\mathbf{r}'\,e^{i\mathbf{p}\mathbf{r}'}G(\mathbf{r},\tau,\mathbf{r}+\mathbf{r}',0).$$

The operator $\hat{G}^{-1}$ takes then the form

$$G_\pm^{-1}(\mathbf{p},\mathbf{r})=\begin{pmatrix} i\omega\mp h_z+\xi(\mathbf{p}) & \mp\Delta\\ \pm\Delta & -i\omega\pm h_z+\xi(\mathbf{p})\end{pmatrix}-\frac{1}{2\tau}\int\frac{d\mathbf{p}}{(2\pi)^3}G_\pm(\mathbf{p},\mathbf{r}), \tag{25a}$$

$$\hat{h}_{1,2}(\mathbf{r})=\begin{pmatrix} h_x(\mathbf{r}) & 0\\ 0 & h_x(\mathbf{r})\end{pmatrix}-\frac{1}{2\tau}\int\frac{d\mathbf{p}}{(2\pi)^3}G_{1,2}(\mathbf{p},\mathbf{r}), \tag{25b}$$

where $\xi(\mathbf{p})=\varepsilon(\mathbf{p})-\varepsilon_F$ and $\varepsilon(\mathbf{p})$ is the electron energy.

The procedure for solving Eqs. (24) and (25) is similar to that used in Refs. 3 and 10. We expand the Green's function $G(\mathbf{p},\mathbf{r})$ in a Fourier series in the variable $\mathbf{r}$, with Fourier coefficients $\tilde{G}(\mathbf{p},n)$. The Fourier component $n$ of the Green's function is small in the parameter $(h_x\tau)^n$ and we confine ourselves to terms of second order in $h_x$ in the equation for the mass operator of the zeroth component of the Green's function. In this case the discarded terms are small in the parameter $(h_x\tau)^2$. The equation for the zeroth component of the Green's function $\tilde{G}(\mathbf{p},0)$ is

$$\tilde{G}(\mathbf{p},0)=\tilde{G}_0(\mathbf{p},0)+\sum_{\mathbf{q}}\tilde{G}_0(\mathbf{p},0)\hat{h}_{\mathbf{q}}\tilde{G}(\mathbf{p}+\mathbf{q},0)\hat{h}_{-\mathbf{q}}\tilde{G}(\mathbf{p},0), \tag{26a}$$

$$\mathbf{q}=n\mathbf{Q},\ \hat{G}_0^{-1}\tilde{G}_0(\mathbf{p},0)=1, \tag{26b}$$

with only the zeroth component of the Green's function retained in the last term of the right-hand side of (25a) (the higher components are small in terms of the parameter $|h_x|/v_FQ$), while in the right-hand side of (25b) we leave out the last term, since it is small in the parameter $(Ql)^{-1}$. We write the solution for the function $\tilde{G}(\mathbf{p},0)$ in the form

$$\tilde{G}(\mathbf{p},0)=\begin{pmatrix}\tilde{G}_+(\mathbf{p}) & 0\\ 0 & \tilde{G}_-(\mathbf{p})\end{pmatrix},\ \tilde{G}_\pm^{-1}(\mathbf{p})=\begin{pmatrix} i\omega_{1,2}+\xi & \mp\Delta_{1,2}\\ \pm\Delta_{1,2} & -i\omega_{1,2}+\xi\end{pmatrix} \tag{27a}$$

and then Eq. (26b) takes the form

$$\tilde{G}_0^{-1}(\mathbf{p})=\begin{pmatrix}\tilde{G}_{0+}^{-1}(\mathbf{p}) & 0\\ 0 & \tilde{G}_{0-}^{-1}(\mathbf{p})\end{pmatrix}$$

$$\tilde{G}_{0\pm}^{-1}(\mathbf{p})=\begin{pmatrix} i\omega\pm h_z+\xi & \mp\Delta\\ \pm\Delta & -i\omega\mp h_z+\xi\end{pmatrix}-\frac{1}{2\tau}\int\frac{d\mathbf{p}}{(2\pi)^3}\tilde{G}_\pm(\mathbf{p}). \tag{27b}$$

Substituting (27) in (26a) and integrating first with respect to the angle variable of the momentum, and then with respect to energy, we obtain equations for $\omega_{1,2}$ and $\Delta_{1,2}$:

$$\omega_1=\omega-ih_z+\frac{\omega_1}{2\tau(\omega_1^2+\Delta_1^2)^{1/2}}-\frac{\pi}{v_FQ}\sum_n\frac{|h_n|^2\Delta_1(\omega_1\Delta_2+\omega_2\Delta_1)}{2n\tau(\omega_2^2+\Delta_2^2)^{1/2}(\omega_1^2+\Delta_1^2)^{3/2}} \tag{28}$$

$$\Delta_1=\Delta-\frac{\Delta_1}{2\tau(\omega_1^2+\Delta_1^2)^{1/2}}+\frac{\pi}{v_FQ}\sum_n\frac{|h_n|^2\omega_1(\omega_1\Delta_2+\omega_2\Delta_1)}{2n\tau(\omega_2^2+\Delta_2^2)^{1/2}(\omega_1^2+\Delta_1^2)^{3/2}}$$

and analogous expressions with the substitutions $\Delta_1\leftrightarrow\Delta_2$, $\omega_1\leftrightarrow\omega_2$, $h_z\leftrightarrow -h_z$. We note that for our problem the solution of the Gor'kov-Nambu equations is simpler than the solution of equations of the Eulenberg type because in (27b) it is easier to carry out first integration with respect to the angles and then with respect to energy.

We introduce the variables $u_{1,2}=\omega_{1,2}/\Delta_{1,2}$ and, using the small parameter $\tau\Delta\ll 1$, we obtain

$$\frac{\omega-ih_z}{\Delta}=u_1-\frac{x(u_1+u_2)}{2(1+u_2^2)^{1/2}},\quad \frac{\omega+ih_z}{\Delta}=u_2-\frac{x(u_1+u_2)}{2(1+u_2^2)^{1/2}},$$
$$x=\frac{7\zeta(3)h_0^2s^2}{\pi v_FQ\Delta},\quad \Delta=\lambda\sum_\omega\frac{1}{(1+u_1^2)^{1/2}}. \tag{29}$$

The solution of (29) is of the form $u_{1,2}=u\pm iv$, where $u$ and $v$ are real numbes. At small $h_z\ll\Delta$ the self-consistency equation for $\Delta$ takes the form

$$\ln\frac{\Delta_0}{\Delta}=f(x)+x\frac{h_z^2}{2\Delta^2}F(0,x), \tag{30}$$

where the function $f(x)$ is defined in (7), while $F(0,x)$ at $x\leqslant 1$ is given by the integral

$$F(0,x)=\int_0^\infty\frac{(1-u^2)\,du}{(1+u^2)^3}\left[1-\frac{u^2x}{(1+u^2)^{3/2}}\right]^{-2}, \tag{31}$$

and $F(0,x)=\pi/8+4x/105+O(x^2)$. Integrating (30) with respect to $\Delta$ we obtain the functional of the free energy of the system at $x\leqslant 1$:

$$\mathcal{F}(s_z,s,\Delta,Q)=-\theta s^2-T\sigma(s)+\theta_{ex}s_z^2[1-C(0,x)]-\mu Hs_z+Ds_z^2+\frac{\eta(s,T)Q}{\pi}-\frac{1}{2}N(0)\Delta^2\left[\ln\frac{e\Delta_0^2}{\Delta^2}+\pi x-\frac{2x^2}{3}\right]-\frac{H^2}{8\pi}, \tag{32}$$

$$C(0,x)=\int_0^x F(0,x)\,dx,$$

where $D$ is the anisotropy parameter. It is seen from (32) that the paramagnetic susceptibility of the electrons in a perpendicular field is different from zero and is proportional to $C(0,x)$ in full analogy with the behavior of a superconductor with magnetic impurities.[9]

In the case of small $x \ll 1$ and arbitrary fields $h_z \leqslant \Delta$ we obtain accurate to terms of order $x$, the self-consistency equation for $\Delta$:

$$\ln\frac{\Delta_0}{\Delta}=\frac{x}{2}K\left(\frac{h_z}{\Delta}\right), \qquad (33)$$

which corresponds to the free-energy functional

$$\mathcal{F}(s_z, s, \Delta, Q)=-\theta s^2-T\sigma(s)+\theta_{ex}s_z^2+\frac{1}{2}N(0)\Delta^2 xE\left(\frac{h_0 s_z}{\Delta}\right)$$
$$-\mu_\perp H s_z+Ds_z^2+\frac{\eta(s,T)Q}{\pi}-\frac{1}{2}N(0)\Delta^2\ln\frac{e\Delta_0^2}{\Delta^2}-\frac{H^2}{8\pi}, \qquad (34)$$

where $K(y)$ and $E(y)$ are elliptic functions.

Minimization of (34) with respect to $s_z$, $s$, $Q$, and $\Delta$ yields their equilibrium values. For the susceptibility of this system we obtain $\chi_\perp = \mu_\perp^2 n/(\theta_{ex} + D)$, it is less than the parallel susceptibility in a ratio $(1 + D/\theta_{ex})^{-1}\mu_\perp^2/\mu_\parallel^2$. The region of existence of the $DS$ phase in the $(H,T)$ plane at small $x$ (in practice from $T_{c2}$ to $T_M$) is determined by the same relations as in the parallel field, if $\theta_{ex}$ is replaced by $\theta_{ex} + D$ and the slope of the line separating the phases $DS$ and $S$ is decreased by a factor $\sqrt{3}$.

The quantity $Q$ decreases with increasing field, and at small $x$ this function takes the form

$$Q(H)=Q(0)\left[\frac{2}{\pi}E(\delta_\perp)\right]^{1/2}, \quad \delta_\perp=\frac{\mu_\perp H h_0}{2\Delta_0(\theta_{ex}+D)}. \qquad (35)$$

A plot of the function $Q(\delta_\perp)/Q(0)$ is shown in Fig. 5. In the region of the existence of the $DS$ phase, the quantity $\delta_\perp$ varies from zero to a value on the order of unity, and the maximum decrease of $Q$ with increasing field is approximately 15–20% in the temperature interval from $T_{c2}$ to $T_M$. Thus, the influence of the perpendicular field on the wave vector of the $DS$ phase is much weaker than that of the parallel field even at $D = 0$. The magnetic-field interval of the existence of the $DS$ phase is also anisotropic, and for a perpendicular field at small $x$ it is larger in the ratio $(1 + D/\theta_{ex})^{-1}\mu_\parallel/\mu_\perp$.

We have neglected above the orbital effects, as can be done if $H_{c1}^{(u)} \ll H_{c2}^{*}(0)$.

Neglecting the action of the perpendicular magnetic field we can calculate now the field dependence of the mean value $\bar{Q}(H)$ for polycrystalline samples. As a result we obtain

$$\bar{Q}(H)/Q(0)=\delta_p^{-1}\int_0^{\delta_p} G^{1/2}(x)\,dx, \text{ where } \delta_p=\mu_\parallel H/2\theta_{ex}s(T).$$

The function $\bar{Q}(\delta_p)/Q(0)$ is plotted in Fig. 5.

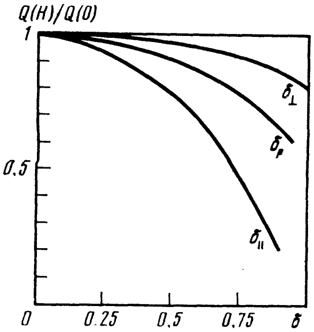


FIG. 5. Dependence of the magnetic-structure vector $Q$ on the magnetic field at a given temperature.

## V. LOWER CRITICAL MAGNETIC FIELD $H_{c1}$ IN THE $DS$ PHASE

We consider now a bulky sample and obtain for it the lower critical magnetic field $H_{c1}$, which determines the appearance of a vortex structure in the $DS$ phase. We confine ourselves to an investigation of superconductors with $\lambda_L \gg \xi$ where $\lambda_L$ is the London penetration depth of the field in the nonmagnetic phase $S$ at temperatures $T_M < T \ll T_{c1}$, and $\xi$ is a correlation length or order $(\xi_0 l)^{1/2}$. We shall show below that magnetic fields with Fourier components (vectors) of the order $\lambda_L^{-1}$ take part in the formation of the vortex structure of the $DS$ phase. If $\lambda_L \gg \xi$, the response of the $DS$ phase to these fields is approximately the same as the response to a constant field. Let the constant magnetic field be directed along the $z$ axis. The electrons are then acted upon by a constant exchange field $h_2$, a magnetic field $\mathbf{B} = \operatorname{curl} \mathbf{A}$ directed along the $z$ axis, and a rapidly oscillating exchange field $\mathbf{h}(\mathbf{r}) = (h_x(\mathbf{r}), h_y(\mathbf{r}), h_z(\mathbf{r}))$ of the domain structure. To determine the field $H_{c1}$ it is necessary to find the functional of the free energy of the superconductor in the presence of such fields.

Calculations similar to those performed in Sec. IV yield the equations

$$\frac{\omega-ih_z}{\Delta}=u_1\left(1-\frac{x_1}{(1+u_1^2)^{1/2}}\right)-\frac{x_2(u_1+u_2)}{2(1+u_2^2)^{1/2}}$$

$$\frac{\omega+ih_z}{\Delta}=u_2\left(1-\frac{x_1}{(1+u_1^2)^{1/2}}\right)-\frac{x_2(u_1+u_2)}{2(1+u_1^2)^{1/2}}, \qquad (36)$$

$$x_1=\frac{\pi}{2v_F Q\Delta}\sum_n\frac{|h_{zn}|^2}{n}, \quad x_2=\frac{\pi}{2v_F Q\Delta}\sum_n\frac{|h_{xn}|^2+|h_{yn}|^2}{n},$$

$$1=\lambda\sum_\omega(1+u_1^2)^{-1/2}.$$

These equations are generalizations of Eqs. (6), (12), and (29), and have been obtained under the same assumptions. At $h_z \ll \Delta$ the self-consistent field equation takes the form

$$\ln\frac{\Delta_0}{\Delta}=f(x_1+x_2)+x_2\frac{h_z^2}{2\Delta^2}F(x_1,x_2),$$

$$F(x_1,x_2)=\int_0^\infty\frac{(1-u^2)\,du}{(1+u^2)^3}\left[1-\frac{x_1+u^2x_2}{(1+u^2)^{3/2}}\right]^{-2} \qquad (37)$$

$$=\frac{\pi}{8}+\frac{16x_1}{21}+\frac{4x_2}{105}+o(x^2),$$

where the notation is the same as in (30). Integrating with respect to $\Delta$, we obtain from (37) the functional (32), in which $x = x_1 + x_2$, while $C(0,x)$ must be replaced by

$$C(x_1, x_2)=x_2\int_0^1 F(\alpha x_1, \alpha x_2)\,d\alpha$$

$$=x_2\int_0^\infty \frac{(1-u^2)\,du}{(1+u^2)^3}\left[1-\frac{x_1+u^2x_2}{(1+u^2)^{3/2}}\right]^{-1}. \qquad (38)$$

The paramagnetic susceptibility of the electrons is now proportional to the function $C(x_1,x_2)$, and is different from zero in the presence of the perpendicular field $\mathbf{h}_\perp(\mathbf{r}) = (h_x(\mathbf{r}), h_y(\mathbf{r}),0)$, that intermixes the states with different values of the spin projection on the direction of the field $h_z$.

At slow spatial variations of the fields $h_z$ and $B(\mathbf{r})$, the free-energy functional (32), (38) remains valid for volumes in which this field be regarded as constant. Summing over the space occupied by the superconductor, we arrive at the density of the total functional of the Gibbs free energy

$$\mathscr{F}(s_z, s, Q, \Delta, \mathbf{A}(\mathbf{r}))=-\theta s^2(\mathbf{r})+\{\theta_{ex}[1-C(x_1,x_2)]+D\}s_z^2(\mathbf{r})$$
$$-T\sigma(s(\mathbf{r}))+\eta(s(\mathbf{r}),T)Q/\pi-\mu B(\mathbf{r})s_z(\mathbf{r})$$
$$+\frac{B^2(\mathbf{r})}{8\pi}-\frac{N(0)\Delta^2}{2}\left[\ln\frac{e\Delta_0^2}{\Delta^2}-\pi x+\frac{2}{3}x^2\right]$$
$$+\frac{1-4x/3\pi}{8\pi\lambda_L^2}\left[\mathbf{A}(\mathbf{r})-\frac{c}{e}\nabla\varphi(\mathbf{r})\right]^2-\frac{B(\mathbf{r})H}{4\pi}, \quad x=x_1+x_2, \qquad (39)$$

calculated accurate to terms $\mathbf{A}^2$ at an arbitrary phase $\mathbf{A}$. In (39) the quantity $\Delta$ and the function $\varphi(\mathbf{r})$ are the modulus and the phase of the above parameter, i.e., $\Delta(\mathbf{r}) = \Delta\exp(i\varphi(\mathbf{r}))$, and the parameter $D$ is the anisotropy energy when the direction of the moment is along the $z$ axis.

If the vortex filament is located at the origin along the $z$ axis, then the phase $\varphi(\mathbf{r})$ satisfies the equation

$$[\mathrm{rot}\,\nabla\varphi(\mathbf{r})]_z=\pi\delta(\mathbf{r}). \qquad (40)$$

Minimizing (39) with respect to $s_z(\mathbf{r})$, we obtain the equilibrium value of $s_z$ in the form

$$s_z(\mathbf{r})=\mu B(\mathbf{r})/\{2\theta_{ex}[1-C(x_1, x_2)]+2\pi\mu^2 n+D\}. \qquad (41)$$

Minimization of the functional (39) with respect to $\mathbf{A}(\mathbf{r})$ and $\varphi(\mathbf{r})$ yields the Maxwell equation

$$\left[\mathbf{q}^2+\lambda_L^{-2}\left(1-\frac{4x}{3\pi}\right)\right]B_\mathbf{q}=4\pi\mu\mathbf{q}^2 s_{z\mathbf{q}}+\frac{\Phi_0}{\lambda_L^2}\left(1-\frac{4x}{3\pi}\right),$$
$$\mathbf{q}=(q_x, q_y, 0). \qquad (42)$$

Substituting (41) in (42) we get

$$B_\mathbf{q}=\Phi_0\left[1+\lambda_L^2\mathbf{q}^2\left(1-\frac{4x}{3\pi}\right)^{-1}\times\left(1-\frac{2\pi\mu^2 n}{\theta_{ex}[1-C(x_1,x_2)]+D+2\pi\mu^2 n}\right)\right]^{-1}. \qquad (43)$$

This expression shows that the magnetic flux is quantized in the usual manner

$$\int d^2\mathbf{r}\,B(\mathbf{r})=B_{q=0}=\Phi_0.$$

Using Maxwell's equation (42) and (43), we obtain for the energy difference between a superconductor *with and* without a vortex the expression

$$\mathscr{F}_{vort}-\mathscr{F}_0=\frac{B(\mathbf{r}=0)-4\pi\mu S_z(\mathbf{r}=0)}{8\pi}-\frac{\Phi_0 H}{4\pi}. \qquad (44)$$

By setting (44) equal to zero we obtain

$$H_{c1}=\frac{\Phi_0(1-4x/3\pi)}{4\pi\lambda_L^2}\ln\frac{\tilde\lambda_L}{\xi};$$
$$\tilde\lambda_L=\lambda_L\left(1-\frac{4x}{3\pi}\right)^{-1/2}\left[1+\frac{2\pi\mu^2 n}{\theta_{ex}[1-C(x_1,x_2)]+D}\right]^{-1/2}. \qquad (45)$$

The quantity $\tilde\lambda_L$ determines also the penetration depth for a field parallel to the surface in the case of specular reflection of the electrons from the surface.

It is seen from (45) that the field $H_{c1}$ decreases when the superconductor goes over into *DS* phase, and that for a field parallel to the easy plane this decrease near the point $T_M$ is approximately $\sqrt{3}$.

## VI. REGION OF EXISTENCE OF THE *DS* PHASE IN A BULKY SAMPLE

We have obtained earlier the region of existence of the *DS* phase in a plate of thickness $L \ll \lambda_L$ with the magnetic field parallel to the plane of the plate. We now consider the behavior of a bulky sample of thickness $L \gg \lambda_L$ in a magnetic field parallel to the surface.

In the bulky sample, the magnetic field penetrates into a surface layer having a thickness on the order of $\tilde\lambda_L$. We consider superconductors in which $\tilde\lambda_L \gg \xi$ and the condition $[H_c^{(u)}/H_{c2}^*(0)]^2 \ll 1$ for the smallness of the orbital effect is satisfied.

If $\tilde\lambda_L \gg \xi$, the magnetic field destroys the superconductivity in the surface layer when the exchange field exceeds here the critical value. Since the destruction of the superconductivity in the surface layer is accompanied by destruction of the superconductivity in the entire sample, curve 1 of Fig. 5 is the boundary of the coexistence of the *DS* phase with respect to a parallel field both in a plate and in a bulky sample (for a perpendicular field, just as in the case of a plate, the corresponding results are obtained by replacing $\theta_{ex}$ by $\theta_{ex}+D$).

It is easily seen that the thermodynamic first-order transition field $H_c$, obtained for the plate, describes also the first-order transition $DS \to FN$ in the surface layer, since the energy of the superconducting currents in the *DS* phase is negligibly small compared with the energies that determine the transition. Indeed, the term $H^2/8\pi$ in (18) is small compared with the term $\mu Hs$ in the energy of the *FN* phase on the entire line of the $DS \to FN$ first-order transition. It is obvious also that curve 3 of Fig. 4 for the superheat field of the *FN* phase is valid also for the bulky sample.

The line 4 of Fig. 4, which separates the *DS* and *S* phases, retains its form in the surface layer of the bulky sample, but inside the superconductor, in the Meissner state, it becomes vertical.

The magnetic structure changes under the influence of the field in those sample regions in which the magnetic field penetrates. At $H < H_{c1}$ this change will be observed only in a surface layer of thickness $\tilde{\lambda}_L$, while in the vortical state the magnetic structure changes also in those regions where the vortices are located. The character of the change of the structure, just as in the case of a plate, depends on the direction of the field relative to the direction of a moment inside the domains.

## VII. MAIN CONCLUSIONS AND COMPARISON WITH THE EXPERIMENTAL RESULTS

1. In a thin plate of thickness $L \ll \tilde{\lambda}_L$, for a field parallel to the surface, the magnetic susceptibility of a superconductor in the $DS$ phase is $\chi = \mu^2 n/(\theta_{ex} + D)$, where $\mu$ is the moment and $D$ is the magnetic-anisotropy parameter for the given field direction. Therefore measurements of the magnetic susceptibility in the $DS$ phase yield the exchange parameter $\theta_{ex}$ that determines the most significant characteristics of magnetic superconductors.

2. In a thin plate, the wave vector of the magnetic structure decreases appreciable under the influence of a magnetic field parallel to the surface and to the direction of the moment inside the domains (see Fig. 5), and peaks $2nQ$ appear in the neutron scattering, where $n$ is an integer (see Eq. (23)). In the case of a field perpendicular to the direction of the moments inside the domains, the wave vector of the magnetic structure $Q$ decreases insignificantly with increasing field, and the distribution of the intensities of the neutron-scattering peaks is independent of the field.

3. In a thin plate the supercooling magnetic field of the phase $H_c^{(u)}$ is equal to approximately $2(\theta_{ex} + D)\Delta_0/h_0\mu$ near $T_M$, and decreases with decreasing temperature (Fig. 4).

4. In a bulky sample, the changes of the magnetic structure are observed where the magnetic field penetrates. The region of existence of the $DS$ phase under the condition $\tilde{\lambda}_L \gg \xi$ takes the same form as in a thin plate.

5. The field $H_{c1}$ in which vortices appear in the $DS$ field near $T_M$ remains approximately the same as in the nonmagnetic $S$ phase, and with decreasing temperature the field $H_{c1}$ decreases insignificantly. The depth of penetration of the field decreases on going into the $DS$ phase by a factor $(1 + 4\pi\chi)^{1/2}$.

6. The superconducting critical current $j_c$ in the $DS$ phase decreases to zero with decreasing temperature from $T_M$ to $T_{c2}^{(c)}$ (Fig. 2). The wave vector of the magnetic structure decreases with increasing superconducting current. This decrease is most noticeable near the point $T_{c2}$. The $Q(j)$ dependence is such that in weak currents the change of $Q$ is negligible and becomes observable in practice only when $j$ approaches the value $j_c$.

The action of the magnetic field on the superconducting phase with inhomogeneous magnetic order was investigated so far only in polycrystalline samples of $HoMo_6S_8$. Lynn *et al.* investigated neutron scattering as a function of the magnetic field in the region of existence of an inhomogeneous magnetic structure.[4,6] In $HoMo_6S_8$ the inhomogeneous magnetic structure vanishes in fields above 0.5 kOe. In the same compound, a decrease of the wave vector of the magnetic structure was observed upon application of the field, and this change amounted to approximately 20% when the field increased from zero to 0.2 kOe at 0.67 K. This $Q(H)$ dependence agrees with our conclusions too. For the curve of Fig. 5 for the polycrystalline sample we determine $\delta_p = 0.8$ and, taking $s(T) \approx 0.5$ from Fig. 8 of Ref. 6, we obtain the estimate $\theta_{ex} \approx 0.2$ K. Ishikawa *et al.*[11] estimated the suppression of $T_c$ due to exchange scattering in $HoMo_6S_8$. These data yield the close estimate $\theta_{ex} \approx 0.15$ K. Taking $N(0) \approx 3$ eV$^{-1}$ (Ref. 11), we obtain $h_0 \approx 20$ K. In a parallel field the inhomogeneous structure vanishes then if

$$H_\parallel < H_\parallel^{(u)} = 2\theta_{ex}\Delta_0/\mu_\parallel h_0 \approx 0{,}14 \text{ kOe}$$

($\mu_\parallel \approx 9\mu_B$). This value agrees with the previously given field value (0.5 kOe) above which the peaks of the inhomogeneous structure are no longer observable.

In $HoMo_6S_8$ the upper critical magnetic field $H_{c2}^*(0)$ is approximately 3 kOe (Ref. 12), and the condition for neglecting the orbital effects, used by us in the calculation of $H_c^{(u)}$, is satisfied.

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