Relaxation in ferromagnetic metals

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The interaction between nuclear spin waves and conduction electrons is considered. It is shown that these processes are very important at infralow temperatures. They lead to large scattering probabilities and determine the establishment of the thermodynamic equilibrium in the subsystems. A theory of relaxation in ferromagnetic metals at infralow temperatures is developed.

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Research into the region of infralow temperatures has become quite vital of late. Cryogenic technology makes now possible experiments at temperatures $T \lesssim \hbar\omega_n$, where $\omega_n = AS/\hbar$ is the nuclear-spin precession frequency, A is the hyperfine interaction constant, and S is the spin of the outer electron shell of the atom), i.e., of the order of 10^{-2} K (see Refs. 1–3). At these temperatures, not only electron but also nuclear spins are ordered systems. The properties of nuclear-magnetically ordered substances are substantially different than at ordinary temperatures. Owing to the smallness of the thermal energy ($\sim T$) compared with the gap in the magnon spectrum, these excitation are practically completely "frozen out." The pricipal role in the heat transfer is then assumed by the nuclear spins—by nuclear spin waves (NSW).

As shown in Refs. 4 and 5, dissipation processes with participation of NSW play a very important role in relaxation at infralow frequencies. The last stage of relaxation in ferro- and antiferromagnetic dielectrics is determined by the interaction of the NSW with phonons. However, owing to small phase space of the phonons the corresponding scattering probability is very low, meaning that the time required to established thermodynamic equilibrium in gases of NSW, magnons, and phonons becomes long. This is slow relaxation.

In ferromagnetic metals (FMM), as will be shown below, the situation changes. The presence of conduction electrons alters greatly the course of relaxation in FMM compared with dielectrics,[4,5] and influences strongly the time of establishment of thermal equilibrium in the subsystems.

The NSW spectrum produced via hyperfine interaction has the following structure[1)]:

$$\omega_n(\mathbf{k}) = \omega_n - \Delta\omega_n(\mathbf{k}), \tag{1}$$

where $\hbar\Delta\omega_n(\mathbf{k}) = A^2 IS/\hbar\omega_{e\mathbf{k}}$ describes the dispersion of the NSW, $\omega_{e\mathbf{k}} = \omega_e + \omega_E(ak)^2$ is the magnon dispersion law, $\hbar\omega_E \sim \Theta_c$ are Curie temperatures, $\omega_e = \mu_e(H + H_a)/\hbar$ is the precession frequency of the magnetic moment, H is the external magnetic field and is directed along the easy magnetization axis z, H_a is the magnetic-anisotropy field, and I is the nuclear spin. Since $\Delta\omega_n(\mathbf{k}) \ll \omega_n$, we can neglect the NSW dispersion when the scattering probability is determined.

The parameters that enter in our problem are subject to the following chain of inequalities:

$$\begin{aligned} &\Delta\omega_n(\mathbf{k}) \ll \omega_n \ll \omega_e \ll \omega_E, \\ &\Theta_D \ll J_{sd} \ll \varepsilon_F, \end{aligned} \tag{2}$$

where Θ_D is the Debye temperature, ε_F is the Fermi energy, and J_{sd} is the characteristic s–d exchange energy according to the Vonsovskiĭ model[7] (see Sec. 1).

In contrast to Refs. 4 and 5, in the present paper the creation (annihilation) operators will be designated by the subscripts m, n, e, and ph for the magnons, NSW, electrons, and phonons, respectively. The same corresponding symbols will label also the damping coefficients.

1. INTERACTION OF NSW WITH CONDUCTION ELECTRONS

We choose the interaction of the conduction electrons with the spin waves (magnons) in the form given in Ref. 7:

$$\hat{\mathcal{H}}_{em} = \frac{J_{sd}(2S)^{1/2}}{2\mathfrak{N}^{3/2}} \sum_{\mathbf{k}_{1,2,3}} \hat{a}^+_{e\mathbf{k}_1\uparrow}\, \hat{a}_{e\mathbf{k}_2\downarrow}\, \hat{a}^+_{m\mathbf{k}_3} \Delta(1-2+3) + \text{c.c.}, \tag{3}$$

where $\mathfrak{N}$ is the number of lattice sites, and $\hat{a}^+_{e\mathbf{k}\sigma}$ is the creation operator for electrons with spin projection $\boldsymbol{\sigma}$.

Changing from the creation (annihilation) operators $\hat{a}^+_{m\mathbf{k}}$ ($\hat{a}_{m\mathbf{k}}$) of the spin deviations to the creation (annihilation) operators of the spin waves $\hat{\alpha}^+_{m\mathbf{k}}$ ($\hat{\alpha}_{m\mathbf{k}}$) via the canonical transformation[8]

$$\hat{a}_{m\mathbf{k}} \approx \hat{\alpha}_{m\mathbf{k}} + v_{\mathbf{k}}\hat{\alpha}^+_{n-\mathbf{k}}, \tag{4}$$

where $v_{\mathbf{k}} = -A(IS)^{1/2}/\hbar\omega_{e\mathbf{k}}$ is the small (in absolute value) amplitude of this transformation and connects the spin-deviation operators with the NSW ($\hat{\alpha}^+_{n\mathbf{k}}$ is the operator of creation of NSW with momentum $\mathbf{k}$), we obtain for the Hamiltonian of the interaction between the conduction electrons and the NSW the expression

$$\hat{\mathcal{H}}_{en} = \frac{J_{sd}(2S)^{1/2}}{2\mathfrak{N}^{3/2}} \sum_{\mathbf{k}_{1,2,3}} v_{\mathbf{k}_3}\hat{a}^+_{e\mathbf{k}_1\downarrow}\, \hat{a}_{e\mathbf{k}_2\uparrow}\, \hat{\alpha}^+_{n\mathbf{k}_3} \Delta(1-2+3) + \text{c.c.} \tag{5}$$

A. To determine the NSW and electron damping coefficients we can use a diagram technique. To this end we introduce the NSW Green's function

$$Q_{\mathbf{k}n}(\tau_1 - \tau_2) = \langle T_\tau | \tilde{\alpha}_{n\mathbf{k}}(\tau_1)\, \tilde{\alpha}^+_{n\mathbf{k}}(\tau_2) | \rangle, \tag{6}$$

where $\tilde{\alpha}^+_{n\mathbf{k}}(\tau) = \exp(\hat{\mathcal{H}}\tau)\hat{\alpha}^+_{n\mathbf{k}}\exp(-\hat{\mathcal{H}}\tau)$ is the Heisenberg representation of the NSW creation (annihilation) operators,

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{en}, \quad \hat{\mathcal{H}}_0 = \frac{1}{\mathfrak{N}}\sum_{\mathbf{k}} \omega_n(\mathbf{k})\, \hat{\alpha}^+_{n\mathbf{k}}\hat{\alpha}_{n\mathbf{k}}.$$

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In the absence of the interaction (5) we have

$$Q_{\mathbf{kn}}^{(0)}(\omega_s)=1/(\omega_{n\mathbf{k}}-i\omega_s); \tag{7}$$

where $\omega_s = 2\pi sT$ and s is an integer.

The fact that the temperature region considered by us is determined by the condition $T \lesssim h\omega_n \ll \varepsilon_F$ enables us to use for the conduction-electron Green's functions an expression that is valid at absolute zero temperature (see, e.g., Ref. 9). As shown repeatedly in a number of papers (in particular, in Ref. 10), the largest contribution to the damping coefficents (and accordingly to the dispersion law) is made by the momentum-space region near the Fermi surface. We can therefore write for the electron Green's functions the following approximate expression:

$$G_\sigma(\varepsilon, \mathbf{p}) = \frac{1}{\varepsilon_\sigma - \varepsilon_F - v_{F\sigma}(p - p_{F\sigma})}. \tag{8}$$

Since $\varepsilon_\sigma = \varepsilon(p) - \sigma J_{sd}$, this leads naturally to a dependence of p_F and v_F on σ. Owing to the s–d interaction, the Fermi surface splits into two separated by a distance $\Delta p \sim J_{sd}/v_F$.

The lifetime of the NSW excitations is determined by the imaginary part of a self-energy functions characterized by an increment to the NSW Green's function on account of the interaction (5):

$$-\frac{\delta Q_{\mathbf{kn}}^{(0)}(\omega_s)}{[Q_{\mathbf{kn}}^{(0)}(\omega_s)]^2} = \delta Q_{\mathbf{kn}}^{(0)-1}(\omega_s) = -J_{sd}^2 v_{\mathbf{k}}^2 \int \frac{d^3p}{(2\pi\hbar)^3} \cdot \frac{\theta(\varepsilon_\uparrow(\mathbf{p}+\hbar\mathbf{k})-\varepsilon_F)-\theta(\varepsilon_\downarrow(\mathbf{p})-\varepsilon_F)}{\hbar\omega_{n\mathbf{k}}+\varepsilon_\downarrow(\mathbf{p})-\varepsilon_\uparrow(\mathbf{p}+\hbar\mathbf{k})-i0}, \tag{9}$$

$$\theta(x)=\begin{cases}1 & x\geqslant 0\\ 0 & x<0.\end{cases}$$

Separating the imaginary part and performing simple operations, we obtain for the NSW damping

$$\gamma_{n\mathbf{k}} = \frac{1}{\tau_{n\mathbf{k}}} = \frac{J_{sd}^2 v_{\mathbf{k}}^2 \omega_n}{8\pi^2 p_F^3} \times \oint_{\varepsilon=\varepsilon_F - J_{sd}/2} \frac{dS}{|\nabla_{\mathbf{p}}\varepsilon|}\delta\left(\varepsilon_F + \frac{J_{sd}}{2} - \varepsilon(\mathbf{p}+\hbar\mathbf{k})\right). \tag{10}$$

To determine $\gamma_{n\mathbf{k}}$ in analytic form we use the quadratic and isotropic electron dispersion law in the effective-Mass approximation: $\varepsilon_{\mathbf{p}} = p^2/2m^*$. The integral in (10) can then be easily obtained, and as a result we get

$$\gamma_{n\mathbf{k}} = \frac{J_{sd}^2 v_{\mathbf{k}}^2 \omega_n}{16\pi\varepsilon_F^2}\frac{p_F}{\hbar k}, \tag{11}$$

if $J_{sd}/2v_F\hbar \leqslant k \leqslant 2p_F/\hbar$. This means that both short-wave and long-wave NSW take part in the scattering processes. When calculating the average probability (this reduces simply to integration of (11) with respect to k) the upper limit of the integration can be set equal to infinity.[2)]

As a result

$$\gamma_{ne} = \bar{\gamma}_{n\mathbf{k}} = \overline{\frac{1}{\tau_{n\mathbf{k}}}} = \frac{1}{(4\pi)^3}\frac{\omega_n}{\omega_e}A^2 IS/\hbar\varepsilon_F\left(1+\frac{\omega_E}{4\omega_e}\frac{\hbar\omega_E}{\varepsilon_F}\right) \tag{12}$$

(we have used here the relation $J_{sd}^2 = \hbar\omega_E\varepsilon_F$).

B. The damping coefficient of the electronic states can also be easily obtained. To this end we determine the correction to the electron spectrum on account of the interaction (5). According to the standard procedure,[9]

$$\delta\varepsilon_\sigma = -\frac{J_{sd}^2 T}{2}\sum_{s=-\infty}^{\infty}\int\frac{d^3\mathbf{k}}{(2\pi)^3}v_{\mathbf{k}}^2 Q_{\mathbf{kn}}^{(0)}(\omega_s) \times\{G(\varepsilon-\varepsilon_{F\sigma}-i\omega_s, \mathbf{p}-\hbar\mathbf{k})-G(-i\omega_s, p-\hbar\mathbf{k})\},$$

where we have introduced $\varepsilon_{F\sigma} = \varepsilon_F - \sigma J_{sd}$. If we introduce the variable $p_1 = |\mathbf{p} - \hbar\mathbf{k}|$ and use the fact that

$$d^3k = 2\pi k^2 dk\sin\theta d\theta \approx 2\pi k dk dp_1,$$

we can obtain after simple calculations (the dispersion of the NSW can be neglected in this case because $\Delta\omega_{n\mathbf{k}} \ll \omega_n$.)

$$\delta\varepsilon_\sigma = -\frac{J_{sd}^2}{8\pi^2 v_{F\sigma}p_{F\sigma}}\int_0^\infty v_x^2 x dx \times\int_{-\infty}^{\infty}\operatorname{ctg}\frac{\omega}{2T}\frac{d\omega}{\omega-\omega_n+i0}\theta(|\varepsilon-\varepsilon_F|-\hbar\omega), \tag{13}$$

$$x=ak, \qquad v_{F\sigma}=\partial\varepsilon/\partial p|_{p=p_{F\sigma}}.$$

Since it must be taken into account for the NSW that $T \neq 0$, the summation over s was replaced in the derivation of (13) by integration over the frequencies, with introduction of $\cot(\omega/2T)$, which has poles at the points $\omega = \omega_s = 2\pi sT$. Next, as is customary in analytic continuation, the integration contour is rotated through $\pi/2$ and aligned with the line $\omega = \omega_n$. Thus, separating the imaginary part in $\delta\varepsilon_\sigma$ we obtain

$$\gamma_{ene}^{(\sigma)} = \frac{1}{\tau_{ene}^{(\sigma)}} = \frac{J_{sd}^2}{8\pi v_{F\sigma}p_{F\sigma}\hbar}\int_0^\infty v_x^2 x dx \times\int_{-\infty}^{\infty}\operatorname{ctg}\frac{\omega}{2T}\delta(\omega-\omega_n)\theta(|\varepsilon-\varepsilon_F|-\hbar\omega).$$

The remaining elementary calculations yield readily for the damping the following expression, which is valid in the absence of NSW dispersion:

$$\gamma_{ene}^{(\sigma)} = \frac{J_{sd}^2}{8\pi v_{F\sigma}p_{F\sigma}\hbar}\operatorname{ctg}\frac{\hbar\omega_n}{2T}\frac{A^2 IS}{\hbar^2\omega_e\omega_E}\theta(|\varepsilon-\varepsilon_F|-\hbar\omega_n). \tag{14}$$

Comparison of γ_{en} with the frequency γ_{eph} calculated in Ref. 10 shows that at in the temperature region $T \ll T_0$, where

$$T_0 \cong (I/S)^{1/3}(\Theta_D^2\hbar\omega_n/\omega_e)^{1/3},$$

the scattering of electrons by NSW is most substantial (T_0 is of the order of liquid-helium temperature).

At typical parameter values $J_{sd} \approx 10^3$K, $\varepsilon_F \approx 10^4$K, $\hbar\omega_n = T \approx 10^{-2}$K, $\hbar\omega_E = 10^2$K, $\hbar\omega_e \sim 0.1$–10^2K we find that γ_{en} ranges from 10^8 to 10^5 sec^{-1}.

The probability of electron scattering by magnons at infralow temperatures, as shown in Ref. 11, is exponentially small. It is clear that this process is unimportant when relaxation is considered in our case.

2. RELAXATION IN A SYSTEM OF MAGNONS, NSW, ELECTRONS, AND PHONONS

The equations obtained in the preceding section for the damping coefficients and describing the interaction of electrons with NSW, as well as the results of Ref. 4, allow us to

proceed to a description of relaxation in ferromagnetic metals at infralow frequencies.

Before we proceed to analyze the transport equation, we write down the inequalities that govern the scattering probabilities that enter in the problem:

$$\begin{aligned} 1/\tau_{mn} &\gg 1/\tau_{mnph} - \text{for magnons,} \\ 1/\tau_{nn} &\gg 1/\tau_{ne} - \text{for NSW,} \\ 1/\tau_{en} &\gg 1/\tau_{ee} \gg 1/\tau_{eph} - \text{for electrons.} \end{aligned} \tag{15}$$

If we use the results of Ref. 4 and the values of τ_{ne} and τ_{en}, we easily obtain all these relations. For example,

$$\frac{1}{\tau_{ne}} \approx \frac{\hbar\omega_E}{\varepsilon_F}\frac{\omega_n}{\omega_e}\frac{1}{\tau_{nn}} \ll \frac{1}{\tau_{nn}},$$
$$\frac{1}{\tau_{en}} \approx \frac{\varepsilon_F}{\hbar\omega_e}\frac{1}{\tau_{ee}} \gg \frac{1}{\tau_{ee}},$$

and we have used in the last relation the fact that the temperatures considered by us are defined by the inequality $T \lesssim \hbar\omega_n$, and $J_{sd}{}^2 = \hbar\omega_E\varepsilon_F$.

In the inequalities (15), $1/\tau_{nn}$ is the probability of the four-particle scattering of NSW by one another, $1/\tau_{mn}$ is the damping of the magnons by the NSW, $1/\tau_{mnph}$ is the damping of a magnon by an NSW with emission (absorption) of a phonon, $1/\tau_{ee}$ and $1/\tau_{eph}$ corresponds to the processes of electron-electron and electron-phonon interaction, cited, e.g., in Ref. 9. By summing we can establish the following the time hierarchy

$$\tau_{en} < \tau_{nn} < \tau_{phe} < \tau_{mn} < \tau_{mnph}.$$

Thus, according to (15), the system of transport equations takes the form

$$\begin{aligned} \dot{N}_{\mathbf{k}} &= L_{nn}(N_{\mathbf{k}}) + L_{ne}(N_{\mathbf{k}}, n_{e\mathbf{k}}), \\ \dot{n}_{\mathbf{k}} &= L_{mn}(n_{\mathbf{k}}, N_{\mathbf{k}}) + L_{mnph}(n_{\mathbf{k}}, N_{\mathbf{k}}, f_{\mathbf{k}}), \\ \dot{n}_{e\mathbf{k}} &= L_{en}(n_{e\mathbf{k}}, N_{\mathbf{k}}), \\ \dot{f}_{\mathbf{k}} &= L_{ph\mathbf{k}}(f_{\mathbf{k}}, n_{e\mathbf{k}}). \end{aligned} \tag{16}$$

The longest time in the subsystems considered is τ_{mnph}. Relative to this time we can set all the collision integrals equal to zero, i.e., L_{en}, L_{nn}, L_{phe}, $L_{mn} = 0$.

It follows from the vanishing of the first collision integral that the electron system keeps up with the nuclear system, i.e., after fast "bosonization" of the NSW gas

$$N_{\mathbf{k}}(t) = \left[\exp\left(\frac{\hbar\omega_n - \xi_n(t)}{T_n}\right) - 1\right]^{-1}$$

with a chemical potential $\xi_n(t) \neq 0$ and a temperature T_n, the electron temperature T_e becomes equalized practically instantaneously with the nuclear temperature T_n. The characteristic time of this equalization is $\sim\tau_{en}$, and during this time the chemical potential $\xi_n(t)$ of the NSW also tends to zero.

The next stage is $T_{ph} \to T_n$ (with the electrons as intermediaries). The characteristic time of this equalization is $\sim\tau_{phe}$. Next, owing to the interaction of the magnons with the NSW, the magnon temperature $T_m \to T_n$ within a time of the order of τ_{mn}. Their chemical potential in this case is $\xi_m(t) \neq 0$.

The approach $\xi_m(t) \to 0$ is described by the equation

$$\dot{\bar{n}}_{\mathbf{k}} = L_{mnph}(\bar{n}_{\mathbf{k}}, \bar{N}_{\mathbf{k}}, \bar{f}_{\mathbf{k}}); \tag{17}$$

here

$$\bar{n}_{\mathbf{k}} = \left[\exp\left(\frac{\hbar\omega_{e\mathbf{k}} - \xi_m(t)}{T_n}\right) - 1\right]^{-1},$$
$$\bar{N}_{\mathbf{k}} = \left[\exp\left(\frac{\hbar\omega_n}{T_n}\right) - 1\right]^{-1}, \quad \bar{f}_{\mathbf{k}} = \left[\exp\left(\frac{\hbar\omega_{p\mathbf{k}}}{T_n}\right) - 1\right]^{-1}.$$

If (17) is integrated over all $\mathbf{k}$, straightforward but rather cumbersome calculations yield the equation

$$\dot{\xi}_m = \frac{T_n}{\tau}\left[\exp\left(-\frac{\xi_m}{T_n}\right) - 1\right], \tag{18}$$

where $1/\tau \sim 1/\tau_{mnph}$; its solution is

$$\xi_m(t) = T_n \ln\,(1 + Ce^{-t/\tau}); \tag{19}$$

here C is the integration constant and is determined from the initial conditions. In the derivations of (18), which describes the final stage of the establishment of the thermodynamic equilibrium, we have assumed that $\hbar\omega_e \gg \xi_m$, and this made it possible to obtain so simple a differential equation. As seen from its solution, the characteristic time of the final establishment of the equilibrium is $\sim\tau_{mnph}$. According to Ref. 4,

$$\tau_{mnph} \sim \left(\frac{\Theta_D}{\gamma\mu_e M_0}\right)^2 \frac{\rho a^4 c_s}{\hbar}\frac{\Theta_D{}^2}{A^2 IS}\frac{1}{\omega_e},$$

where c_s is the speed of sound, γ is the magnetostriction constant, and ρ is the density. It can fluctuate in a very wide range. The most typical values of τ_{mnph} are of the order of minutes.

Thus, thermodynamic equilibrium is established in the system with the temperature of all the subsystems, which is equal to the temperature of the nuclei.

The cause of the relaxation described by us here is, naturally, the presence of the conduction electrons. In Refs. 4 and 5 was considered relaxation in ferro- and antiferromagnetic dielectrics. The time of the ultimate establishment of thermodynamic equilibrium in them was characterized by the interaction of the NSW with phonons. This time is very long. Depending on the various constants contained in the answer, it can vary in a very wide range, from hundreds of hours to dozens of years.

In the case considered here we were interested only in the establishment of the internal thermal equilibrium in the sample, and were not interested in its connection with the helium bath (it is understood that the sample was cooled in a helium bath). This case is of interest because the role of the thermostat is assumed not by the phonons, as is usually the case, but by the NSW gas with its own temperature T_n.

Thus, the conduction electrons interacting with the NSW contribute to a rapid relaxation of the magnon and phonon subsystem temperatures to the temperature of the nuclei. This is the fast relaxation.

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[1)] It may be questioned why no account is taken in the NSW dispersion of the contribution from the indirect Ruderman-Kittel interaction of the

nuclear spins with one another. The point is, as shown in Ref. 6 (Chap. 4, Sec. 1) that the contribution from the Suhl-Nakamura interaction greatly exceeds the Ruderman-Kittel interaction. It cannot be neglected in the calculation of the electron kinetic characteristics, for it becomes substantial.

[2)] Since the integral with respect to k converges well on the upper limit, the integration region can be extended to infinity when calculating the corresponding probability.

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