Statistics of the levels in small metallic particles

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The problem of the computation of the two-level correlation function in a disordered metal is considered for the case in which the sample is of dimension much smaller than the diffusion length. The supersymmetry method, which allows the problem to be reduced to a problem of the zero-dimensional nonlinear σ model, is used. The symmetry of the collective variable is determined by the presence or absence of magnetic and spin-orbit interactions. It is shown that the correlation function depends on these interactions, and coincides in the various limits with the corresponding correlation functions obtained with the aid of Dyson's hypothesis for orthogonal, unitary, and symplectic ensembles. The computed correlation function determines the response to an external electromagnetic field.

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1. INTRODUCTION

The principal result of quantum mechanics is the deduction that the levels of a finite system are discrete, their positions and spacing being dependent upon the boundary conditions for, and the interactions in, the system. In the simplest cases the problem of the determination of these quantities can be solved either exactly or on the basis of some approximation. But in many cases the interactions are so complicated that it is impossible to calculate the levels. At the same time, the complexity and diversity of the interactions lead to the idea of a statistical description, in which we lose information about the individual levels and go over to mean quantities. Such quantities can be, for example, the density of states, the level correlations, etc. A similar approach is used in statistical physics, where information about the individual particles is given up, and averages over a large number of particles are computed.

The idea that the levels in a bounded volume can be considered statistically was first put forward by Wigner[1] for the description of the levels in complex nuclei. In such nuclei a large number of particles interact in unknown fashion. The resulting problem consists in the exact mathematical determination of the ensemble of systems in which all the possible particle interactions are equally probable. The most convenient of the ensembles determined by Wigner is the Gaussian ensemble. According to the hypothesis on which the Gaussian statistics is based, each physical system possessing N quantum states appears in the ensemble with statistical weight $D(H)$:

$$D(H)=A\exp\left[-\frac{1}{a^2}\sum_{m,n}^{N}H_{mn}^2\right],$$

where the H_{mn} are the matrix elements of the Hamiltonian.

The basic assumption used is the assumption that the various elements H_{mn} are statistically independent. This assumption is quite arbitrary. Furthermore, it is not possible to determine in terms of the H_{mn} an ensemble in which all the interactions would be equally probable.

The next step on the road to the construction of a statistical theory of levels was made by Dyson,[2] who rejected the description with the aid of the Hamiltonian matrix elements. In the Dyson theory the major role is played by an $N\times N$ unitary matrix S, where N is the number of levels. The eigenvalues of the unitary matrices S are N complex numbers $\exp(i\theta_n)$ located on the unit circle. The exact correspondence between S and H is not established. According to Dyson, the behavior of n ($\ll N$) successive levels of a real system is statistically equivalent to the behavior of n successive angles θ_i on the unit circle. It is assumed that all the possible unitary matrices S appear with equal probabilities. This assumption leads to nontrivial correlation properties of the levels.

To describe centrosymmetric systems invariant under time reversal, Dyson used symmetric unitary matrices connected with an orthogonal group. Systems that are not invariant under time reversal were described by arbitrary unitary matrices. Noncentrosymmetric systems that are invariant under time reversal were described by symplectic matrices. The corresponding ensembles were called orthogonal, unitary, and symplectic. It should be emphasized that the statistical description proposed by Dyson, as well as Wigner's description, is a hypothesis, and not a rigorously substantiated theory. Apparently, other mathematical ensembles satisfying the same physical assumptions can be constructed.

Subsequently, such important quantities as level-correlation functions were computed on the basis of Dyson's statistical description.[2,3] The calculations required considerable mathematical proficiency. The correlation functions for all the three ensembles were found in these calculations. In all the three cases these quantities were found to be slowly decreasing oscillating functions with period corresponding to the mean level spacing. At the same time no oscillations were observed in the density of states.

In 1965 Gor'kov and Éliashberg[4] applied the Dyson statistics to systems of entirely different nature. These authors considered small metallic particles. They assumed that the electrons interact not with each other, but with the impurities present in the volume. Thus, in contrast to the case

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of nuclei, it was not the various interparticle interactions that were random, but the external potential. The computation of the responses to an external electromagnetic field requires knowledge of the level correlation function. In Ref. 4 correlation functions computed on the basis of the Dyson hypothesis are used for this purpose, the orthogonal ensemble being identified with the case in which only scattering on ordinary nonmagnetic impurities occurs. For systems containing magnetic impurities, or located in a magnetic field, the unitary ensemble is used; for systems with spin-orbit interactions, the symplectic ensemble.

On the other hand, the statistical properties of the levels in metallic particles can be investigated by solving the Schrödinger equation in the given impurity potential and averaging over the positions of the impurities. Such an approach would allow us to verify the basic principles used in the phenomenological description of the levels in a bounded volume. Thus far, it has not been possible to solve this problem, since the existing traditional methods, which are based on perturbation-theory-series calculations,[5] lead to unsurmountable mathematical difficulties. At the same time, it is quite easy to formulate this problem within the framework of the supersymmetry method proposed by the present author.[6] Below we show that the problem of the computation of the level correlation function reduces to the problem of the investigation of the zero-dimensional variant of the supersymmetric σ model introduced in Ref. 6. The symmetry group of the σ model depends on the presence of magnetic and spin-orbit interactions, as a result of which three different types of symmetry are possible. The zero-dimensional σ-model calculations allow us to obtain the explicit form of the level correlation functions.

2. REDUCTION OF THE PROBLEM TO ONE IN THE ZERO-DIMENSIONAL SUPERSYMMETRIC σ MODEL

Let us consider a metallic particle with finite dimensions a. Let us assume that the dimensions a are much greater than the interatomic distances. The quantity a can be arbitrarily large if we are considering the case of zero temperature and are investigating the lowest-lying excitations. At finite temperatures the dimension a should not exceed the critical diffusion length $L_c = (D\tau_\varepsilon)^{1/2}$, where τ_ε is the inelastic-scattering time and D is the diffusion coefficient. The volume can contain randomly disposed impurities. The allowed energy values ε_k are the eigenvalues for the Schrödinger equation:

$$H\varphi_k=\varepsilon_k\varphi_k, \quad H=\tilde{H}+H_1, \quad \langle H_1\rangle=0, \tag{1}$$

where the φ_k are the eigenfunctions corresponding to the energies ε_k.

The angle brackets in (1) denote averaging over the positions of the impurities, $\tilde{H}$ is the regular part of the Hamiltonian (the kinetic energy and the energy of interaction with the external magnetic field), and H_1 is the energy of interaction with the impurities. It is assumed that the Hamiltonian H_1 in (1) can cover both the scattering on the ordinary impurities and the scattering on the magnetic and spin-orbit impurities.

Let us define the level correlation function $R(\omega)$ with the aid of the following formula:

$$R(\omega) = \left\langle \frac{1}{2\omega\nu^2} \sum_{m_1,m_2} \left(\operatorname{th}\frac{\varepsilon_{m_1}-\varepsilon_0}{2T} - \operatorname{th}\frac{\varepsilon_{m_2}-\varepsilon_0}{2T} \right) \delta(\omega-\varepsilon_{m_1}+\varepsilon_{m_2}) \right\rangle. \tag{2}$$

In (2) summation is performed over all the levels of the system and ε_0 is the Fermi energy. Below we shall consider the limit of low temperatures $T \ll \Delta$, where Δ is the mean level spacing, equal to

$$\Delta=(\nu V)^{-1},$$

$\nu = mp_0/2\pi^2$ is the density of states at the Fermi surface and V is the volume.

The function $R(\omega)$ is equal to the probability that two levels will be a distance ω apart. It is precisely this function that determines the response to an external electromagnetic field in metallic particles. The expression (2) can be rewritten in the computationally more convenient form

$$R(\omega) = \frac{1}{2\pi^2\omega\nu^2} \left\langle \operatorname{Re} \int_{-\infty}^{\infty} \sum_{m_1,m_2} \left(\operatorname{th}\frac{\varepsilon+\omega}{2T} - \operatorname{th}\frac{\varepsilon}{2T} \right) (G_{m_1}^R(\varepsilon+\omega) G_{m_2}^A(\varepsilon) - G_{m_1}^R(\varepsilon+\omega) G_{m_2}^R(\varepsilon)) \, d\varepsilon \right\rangle. \tag{3}$$

In the formula (3) $G_m^R(\varepsilon)$ and $G_m^A(\varepsilon)$ are the retarded and advanced Green functions of the Schrödinger equation (1), which are given by the expression

$$G_m^{R,A}(\varepsilon) = \frac{1}{\varepsilon-\varepsilon_m+\varepsilon_0 \pm i\delta}. \tag{4}$$

The problem of the computation of the correlation function $R(\omega)$ consists in the determination of the Green function of the Schrödinger equation (1), the substitution of the function into (3), and the averaging over the impurities.

The averaging of the product $G_{m1}^R(\varepsilon+\omega)G_{m2}^R(\varepsilon)$ is simple if the mean free path in the metal is much greater than the interatomic distance. It is precisely this limit of sufficiently pure metals that will be considered. Limiting ourselves to the simplest perturbation-theory diagrams,[5] we find that for $\tau\varepsilon_0 \gg 1, \Delta \ll \tau^{-1}$ (τ is the mean free time)

$$\sum_{m_1,m_2} G_{m_1}^R(\varepsilon+\omega) G_{m_2}^R(\varepsilon) = -\pi^2\nu^2. \tag{5}$$

We cannot compute the average of $G_{m1}^R(\varepsilon+\omega)G_{m2}^A(\varepsilon)$ at low frequencies by limiting ourselves to the consideration of a few orders of perturbation theory because of the appearance of divergent terms as $\omega \to 0$ (Ref. 7). For the purpose of carrying out the averaging of $G_{m1}^R(\varepsilon+\omega)G_{m2}^A(\varepsilon)$ over the impurities, we find it convenient to write the Green functions in terms of an integral over commuting and anticommuting variables.[6,8,9] Repeating the computations carried out in Ref. 6 for the density correlator, we obtain

$$\langle G_{m_1}^{R}(\varepsilon+\omega)G_{m_2}^{A}(\varepsilon)\rangle$$
$$=\frac{1}{64}\sum_{\substack{\alpha,\beta\\ m_1,m_2}}\int\langle\psi_{\alpha m_1}(1+\Lambda_{\alpha\alpha})\bar{\psi}_{\alpha m_1}\psi_{\beta m_2}(1-\Lambda_{\beta\beta})\bar{\psi}_{\beta m_2}e^{-L}\rangle d\psi, \tag{6}$$

where the Lagrangian L has the form

$$L=i\int\bar{\psi}(X)\,(-\hat{H}_0+{}^1/_2\omega\Lambda+H_1)\,\psi(X)\,dX. \tag{7}$$

In (6) and (7) $X=\{r,\alpha\}$ denotes the particle coordinate and spin and ψ is a supervector having the classical Fermi and Bose variables as its components. In the case of spinless particles ψ has 8 components, which can be written in the form

$$\psi^a=\begin{pmatrix}u^a\\ v^a\end{pmatrix},\quad u^a=\frac{1}{\sqrt{2}}\begin{pmatrix}\chi^{a*}\\ \chi^a\end{pmatrix},\quad v^a=\frac{1}{\sqrt{2}}\begin{pmatrix}s^{a*}\\ s^a\end{pmatrix}, \tag{8}$$
$$\bar{\psi}=(C\psi)^T,\quad a=1,2.$$

In the expressions (8) the superscript a indicates affiliation with the retarded or advanced Green function; χ and χ^* are the Grassman anticommuting variables (the rules of computation with these variables can be found in Ref. 10); and s and s^* are complex numbers. The letter T denotes the operation of transposition. The elements of the matrix C have the form

$$C^{ab}=\Lambda^{ab}\begin{pmatrix}c_1&0\\ 0&c_2\end{pmatrix}, \tag{9}$$

where Λ is a diagonal matrix with elements $\Lambda^{11}=-\Lambda^{22}=1$. The matrices c_1 and c_2 are equal to

$$c_1=\begin{pmatrix}0&-1\\ 1&0\end{pmatrix},\quad c_2=\begin{pmatrix}0&1\\ 1&0\end{pmatrix}.$$

In the absence of external fields H_0 in (7) has the form

$$H_0=-\varepsilon-\frac{1}{2m}\frac{\partial^2}{\partial r^2}.$$

If only elastic scattering by the ordinary impurities is possible in the metal, then $H_1=U(r)$, where $U(r)$ is the impurity potential. Let us assume that the potential $U(r)$ is a random quantity distributed according to the Gauss δ-correlated law:

$$\langle U(r)\rangle=0,\quad \langle U(r)\,U(r')\rangle=\frac{1}{\pi\nu\tau}\delta(r-r'), \tag{10}$$

where τ is the mean free time.

We average in (6) with the aid of (10). The expression (6) has the same form after the averaging if by the Lagrangian L we mean the following quantity:

$$L=\int\left[-i\bar{\psi}H_0\psi+\frac{1}{2\pi\nu\tau}(\bar{\psi}\psi)^2+\frac{i(\omega-i\delta)}{2}\bar{\psi}\Lambda\psi\right]dX. \tag{11}$$

The Lagrangian (11) is similar to the Lagrangian studied in field theory. It is shown in the author's previous paper[6] that the supersymmetry that is possessed by the Lagrangian (11) at zero frequency is spontaneously broken, as a result of which the averages $Q_{\alpha\beta}=\langle\psi_\alpha\bar{\psi}_\beta\rangle$ arise. The quantity Q is a supermatrix, containing both Bose and Fermi elements. This matrix satisfies the charge-self-adjointness and hermiticity conditions:

$$Q=\bar{Q}=KQ^{S+}K,\quad K=\begin{pmatrix}1&0\\ 0&k\end{pmatrix}, \tag{12}$$

where k is a superelement of the form

$$k=\begin{pmatrix}1&0\\ 0&-1\end{pmatrix}.$$

As in Ref. 6, by the operation of charge conjugation and the operation of Hermitian conjugation we mean the following transformations:

$$\bar{Q}=CQ^{ST}C^T,\quad Q^{S+}=(Q^{ST})^*.$$

The indices ST and $*$ denote supertransposition and complex conjugation. Upon the separation of the averages in the term $(\bar{\psi}\psi)^2$ in (11), the Lagrangian L goes over into

$$\tilde{L}=\int\left[\bar{\psi}\left(-iH_0+\frac{i(\omega-i\delta)}{2}+\frac{Q}{\tau}\right)\psi+\frac{\pi\nu}{4}\,\mathrm{S\,Sp}\,Q^2\right]dr, \tag{13}$$

where $\mathrm{SSp}\,M\equiv\mathrm{Sp}\,kM$ for any matrix M. The supertrace SSp plays in the theory of supermatrices the same role played by trace in the theory of ordinary matrices.

At low impurity concentrations, i.e., at concentrations such that the condition $\tau\varepsilon_0\gg1$ is fulfilled, the eigenvalues of the supermatrix Q fluctuate slowly, and are determined by the saddle-point value of the free-energy functional corresponding to the Lagrangian $\tilde{L}$, (13). At the same time, the ground state is highly degenerate at $\omega=0$. The general form of the supermatrix Q corresponding to the ground state can be written as follows:

$$Q=W+\Lambda(1-W^2)^{1/2}, \tag{14}$$

where

$$W=\begin{pmatrix}0&Q^{12}\\ -\overline{Q^{12}}&0\end{pmatrix},\quad Q^{12}=i\begin{pmatrix}A&i\Sigma\\ \rho^+&iB\end{pmatrix}. \tag{15}$$

The matrices A, B, Σ, and ρ are 2×2 matrices whose elements are ordinary numbers in the cases of A and B and Grassman variables in the cases of Σ and ρ. The matrices A, B, Σ, and ρ that guarantee the fulfillment of the condition (12) have the form

$$A=\begin{pmatrix}A_1&A_2\\ -A_2^*&A_1^*\end{pmatrix},\quad B=\begin{pmatrix}B_1&B_2\\ B_2^*&B_1^*\end{pmatrix},$$
$$\Sigma=\begin{pmatrix}\sigma_1&\sigma_2\\ -\sigma_2^*&-\sigma_1^*\end{pmatrix},\quad \rho=\begin{pmatrix}\rho_1&\rho_2\\ -\rho_2^*&-\rho_1^*\end{pmatrix}. \tag{16}$$

The strong degeneracy of the ground state at zero frequency leads to the existence of Goldstone modes connected with the Q fluctuations. These modes and their interaction are described by a nonlinear generalized σ model in whose free energy the frequency plays the role of an external field[6]:

$$F=\frac{\pi\nu}{8}\int\mathrm{SSp}[D(\nabla Q)^2-2i(\omega-i\delta)\Lambda Q]\,dr, \tag{17}$$

where $D=\overline{v_x^2}\tau/2$ is the diffusion coefficient.

At sufficiently low temperatures, at which the diffusion length determined by the inelastic processes is greater than the dimensions of the system, only the states with coordinate-independent Q are important. Allowance for the higher

harmonics leads to a great increase in the energy, and can therefore be discarded. The evaluation of the integral over ψ with the Lagrangian $\tilde{L}$, (13), in the formula (6) is performed similarly to the computation of the density correlator in Ref. 6, and leads to the expression

$$\sum_{m_1,m_2} \langle G_{m_1}{}^R(\varepsilon+\omega) G_{m_2}{}^A(\varepsilon)\rangle$$

$$= \frac{1}{64}\int \mathrm{SSp}\, k(1+\Lambda) Q\, \mathrm{SSp}\, k(1-\Lambda) Q e^{-F_0[Q]}\, dQ,$$

$$F_0[Q] = -(\pi/4\Delta)(\omega - i\delta)\,\mathrm{SSp}\,\Lambda Q. \qquad (18)$$

The expression (18) has been derived in the limit of sufficiently small mean level spacing, i.e., under the assumption that

$$\Delta \ll \tau^{-1}. \qquad (19)$$

The inequality (19) allows us to go over in the sums of the type

$$\sum_m (\varepsilon_m - \varepsilon_0 + iQ/\tau)^{-1}$$

arising in the computation of (18) to integration over $\xi = \varepsilon_m - \varepsilon_0$. In (18) we evaluate a simple, and not a functional, integral over Q, which corresponds to the retention of only the zeroth harmonic of Q. A simple analysis shows that such an approximation is admissible if the frequency is not very high:

$$\omega \ll \min\{Da^{-2}, \tau^{-1}\}. \qquad (20)$$

The ratio $\omega/\Delta = \omega\nu V$ can be either much greater or much smaller than unity, since the condition (19) and the inequality

$$\Delta \ll D/a^2 \qquad (21)$$

are satisfied.

Formally, in deriving (18), we used the assumption that the disorder in the system is created by the impurities that are located in the interior, and are distributed according to the δ-correlated law (10). The particle boundary was considered in this case to be regular. At the same time, we need not, in deriving (18), impose an additional limitation on the quantity $v\tau/a$. The only exception is the case of a pellet of perfectly spherical shape, in which the levels are highly degenerate and widely spaced. For a particle of any other shape only the conditions (19) and (21) need be fulfilled. Therefore, it may be inferred that an irregular shape of the surface will also lead to the expression (18). Notice that the quantity τ is not contained in this expression at all. All the formulas obtained above are for the case in which both magnetic and spin-orbit interactions do not occur in the system. The presence of an external magnetic field leads to the replacement in the Lagrangian (17) of ∇Q by $(\nabla Q + c^{-1}e\mathbf{A}[Q,\tau_3])$ (Ref. 6), where τ_3 is a matrix of the form $\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$ in the space of the matrices A, B, Σ, and ρ. As a result, the symmetry of the ground state is lowered, and some of the modes acquire a gap even at zero frequency. Such modes can be neglected in the limit $\omega \ll DeH/c$. In this case the expressions (14), (15), and (18) preserve their form if by A, B, Σ, and ρ we mean the following matrices:

$$A = \begin{pmatrix} A_1 & 0 \\ 0 & A_1^* \end{pmatrix}, \quad B = \begin{pmatrix} B_1 & 0 \\ 0 & B_1^* \end{pmatrix},$$

$$\Sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1^* \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho_1 & 0 \\ 0 & -\rho_1^* \end{pmatrix}. \qquad (22)$$

To allow for the magnetic impurities and the spin-orbit interactions, we must double the dimensions of ψ and Q because of the allowance for the spin structure. These interactions also manifest themselves in the form of effective fields that lower the symmetry of the Lagrangian (17), and freeze out some of the diffusional modes. When the condition $\omega\tau_s \ll 1$ or $\omega\tau_{s0} \ll 1$ (where τ_s and τ_{s0} are the mean free times corresponding respectively to the magnetic and spin-orbit impurities) is fulfilled, we neglect these modes. The expressions (14), (15), and (18) remain valid in this case. When $\omega\tau_s \ll 1$, the matrices A, B, Σ, and ρ are given by the formulas (22), in which the quantities A_1, B_1, σ_1, and ρ_1 should be taken to be 2×2 unit matrices. When $\tau_{s0}/\tau_s \ll \omega\tau_{s0} \ll 1$, the freezing of the modes is milder. Accordingly, for the matrices A, B, Σ, and ρ we have

$$A = \begin{pmatrix} A_1 & A_2 \\ A_2^* & A_1^* \end{pmatrix}, \quad B = \begin{pmatrix} B_1 & -B_2 \\ B_2^* & B_1^* \end{pmatrix}, \qquad (23)$$

$$\Sigma = \begin{pmatrix} \sigma_1 & \sigma_2 \\ -\sigma_2^* & \sigma_1^* \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho_1 & \rho_2 \\ -\rho_2^* & \rho_1^* \end{pmatrix}. \qquad (24)$$

In (23) and (24), the A_i, B_i, Σ_i, and ρ_i are 2×2 unit matrices.

Thus, the problem of the computation of the level correlation function for a particle with small dimensions reduces to the evaluation of the definite integral (18) over Q. The symmetry of the supermatrix Q depends on the presence of magnetic and spin-orbit interactions.

Although the problem under consideration has been reduced to the definite integral (18), the evaluation of this integral is not simple because of the large number of variables (8 for the case with magnetic interactions, 16 for potential scattering and spin-orbit interactions). In the following section we shall show how this calculation can be simplified by going over to "polar" matrix coordinates.

3. EVALUATION OF INTEGRAL OVER THE SUPERMATRIX BY GOING OVER TO INTEGRAL OVER THE EIGENVALUES

The independent variable in the integration over the supermatrix Q is the superelement Q^{12}. The superelement Q^{12} with symmetry given by the relations (16), (22), (23), and (24) can be reduced to the quasidiagonal form

$$Q^{12} = iu \sin\hat{\theta}\bar{v}, \quad \hat{\theta} = \begin{pmatrix} \theta_{11} & 0 \\ 0 & \theta_{22} \end{pmatrix}. \qquad (25)$$

The elements θ_{11} and θ_{22} are equal to

$$\theta_{11}=\begin{pmatrix}\theta & 0\\ 0 & \theta\end{pmatrix}, \quad \theta_{22}=i\begin{pmatrix}\theta_1 & \theta_2\\ \theta_2 & \theta_1\end{pmatrix},$$

$$0<\theta<\pi, \quad \theta_1>0, \quad \theta_2>0$$

in the case of potential scattering; to

$$\theta_{11}=\begin{pmatrix}\theta & 0\\ 0 & \theta\end{pmatrix}, \quad \theta_{22}=i\begin{pmatrix}\theta_1 & 0\\ 0 & \theta_1\end{pmatrix}, \quad 0<\theta<\pi, \quad \theta_1>0$$

in the case of magnetic interactions; and to

$$\theta_{11}=\begin{pmatrix}\theta_1 & \theta_2\\ \theta_2 & \theta_1\end{pmatrix}, \quad \theta_{22}=i\begin{pmatrix}\theta & 0\\ 0 & \theta\end{pmatrix},$$

$$\theta>0, \quad 0<\theta_1<\pi, \quad 0<\theta_2<\pi$$

in the case of spin-orbit interactions.

The matrices u and v in (25) satisfy the conditions

$$\bar{u}u=1, \quad \bar{v}v=1, \quad \bar{v}=kv^{s+}k, \quad \bar{u}=u^{s+}. \tag{26}$$

Let us recall that the operation of conjugation in the spin-orbit case differs somewhat from the corresponding operation in the case of potential scattering. Using the results of the author's previous paper,[6] we can easily make the appropriate changes. To do this, it is sufficient to interchange in all the formulas containing the operation of conjugation and pertaining to the spin-orbit case the matrices c_1 and c_2 in (9).

Using (25), we can write the supermatrix Q in the form

$$Q=U\begin{pmatrix}\cos\hat{\theta} & i\sin\hat{\theta}\\ -i\sin\hat{\theta} & -\cos\hat{\theta}\end{pmatrix}\bar{U}, \quad U=\begin{pmatrix}u & 0\\ 0 & v\end{pmatrix}. \tag{27}$$

It is not difficult to verify that the conditions (26) ensure the fulfillment of the equality (12). The expression (27) describes the transition to the "polar" coordinates. To effect such a transition, we must write down the corresponding Jacobian. (It is customary to call the Jacobian arising in the simultaneous transformations of the boson and fermion variables the Berezinian. Below we shall use this terminology.) We can write down the Berezinian of the transformation if we know the quadratic form $(ds)^2$ formed by all the integration variables:

$$(ds)^2=\sum_{n=1}^{N} d\Phi_n^{*}\, d\Phi_n=\sum_{m,n=1}^{N} g_{mn}\, d\tilde{\Phi}_m^{*}\, d\tilde{\Phi}_n. \tag{28}$$

In (28) $d\Phi_n$ and $d\Phi_n^{*}$ are the old variables, $d\tilde{\Phi}_n$ and $d\tilde{\Phi}_n^{*}$ are the new variables, and g_{mn} is a supermatrix. Let us assume that the components $d\Phi_n$ are ordinary numbers for $1<n<p$ and Grassman differentials for $p<n<N$. If the supermatrix g_{mn} is diagonal, then the Berezinian reduces to the product

$$\prod_{n=1}^{p} g_{nn}\prod_{n=p}^{N} g_{nn}^{-1}.$$

This formula follows directly from the rules for the change of variables in the integrals over the Grassman and ordinary variables. If g_{mn} is a nondiagonal supermatrix, then the above-obtained product goes over into S det g. The superdeterminant S det g is connected with the supertrace by the following relation:

$$\text{S det } g=\exp(\text{SSp ln } g).$$

The properties of superdeterminants and other useful relations can be found in Berezin's review article.[11]

Let us, in accordance with the foregoing, construct the quadratic form from the elements of the matrix dQ, and go over to the variables $\hat{\theta}$ and u through the use of the formula (27):

$$\begin{aligned}\text{SSp } (dQ)^2=&2\text{ SSp } (d\hat{\theta})^2\\ &+2\text{ SSp } (2\delta u\sin\hat{\theta}\delta v\sin\hat{\theta}+(\delta u\cos\hat{\theta})^2\\ &+(\delta v\cos\hat{\theta})^2)-2\text{ SSp } (\delta u)^2-2\text{ SSp } (\delta v)^2,\end{aligned} \tag{29}$$

where $\delta u=\bar{u}du=-\delta\bar{u}$, $\delta v=\bar{v}dv=-\delta\bar{v}$.

In deriving (29) we used the invariance property of the supertrace under cyclic transpositions of the supermatrices.

Let us rewrite the expression (29), explicitly separating the Grassman δu_{12}, δu_{21}, δv_{12}, δv_{21} and ordinary δu_{11}, δu_{22}, δv_{11}, δv_{22} elements of the supermatrices δu and δv:

$$\begin{aligned}\text{SSp } (dQ)^2=&2\text{ SSp } (d\hat{\theta})^2\\ &+2\text{ Sp } (-(\delta u_{11})^2+(\delta u_{11}\cos\hat{\theta}_{11})^2-(\delta v_{11})^2\\ &+(\delta v_{11}\cos\hat{\theta}_{11})^2+(\delta u_{22})^2-(\delta u_{22}\cos\hat{\theta}_{22})^2\\ &+(\delta v_{22})^2-(\delta v_{22}\cos\hat{\theta}_{22})^2\\ &-2\delta u_{22}\sin\hat{\theta}_{22}\delta v_{22}\sin\theta_{22}+4\delta u_{12}\sin\hat{\theta}_{22}\delta v_{21}\sin\hat{\theta}_{11}\\ &+2\delta u_{12}\cos\hat{\theta}_{22}\delta u_{21}\cos\hat{\theta}_{11}+2\delta u_{11}\sin\hat{\theta}_{11}\delta v_{11}\sin\hat{\theta}_{11}\\ &+2\delta v_{12}\cos\hat{\theta}_{22}\delta v_{21}\cos\hat{\theta}_{11}-2\delta u_{12}\delta u_{21}-2\delta v_{12}\delta v_{21}).\end{aligned} \tag{30}$$

The quadratic form (30) consists of two parts, one of which contains only the boson elements of the matrices δu, δv, and $d\hat{\theta}$; the other, only the fermion elements. Therefore, the general Berezinian is equal to the ratio of the two Jacobians corresponding to these parts. For the specific calculations we should write the matrices u and v in their explicit forms. Let us represent these matrices as follows:

$$u=u_1u_2, \quad v=v_1v_2, \quad \bar{u}_1u_1=\bar{u}_2u_2=\bar{v}_1v_1=\bar{v}_2v_2=1,$$

$$u_1=\begin{pmatrix}1-2\eta\bar{\eta}+6(\eta\bar{\eta})^2 & 2\eta(1-2\bar{\eta}\eta)\\ -2(1-2\eta\bar{\eta})\bar{\eta} & 1-2\bar{\eta}\eta+6(\bar{\eta}\eta)^2\end{pmatrix}, \quad u_2=\begin{pmatrix}F_1 & 0\\ 0 & F_2\end{pmatrix},$$

$$v_1=\begin{pmatrix}1+2\varkappa\bar{\varkappa}+6(\varkappa\bar{\varkappa})^2 & 2i\varkappa(1+2\bar{\varkappa}\varkappa)\\ -2i(1+2\bar{\varkappa}\varkappa)\bar{\varkappa} & 1+2\bar{\varkappa}\varkappa+6(\bar{\varkappa}\varkappa)^2\end{pmatrix}, \quad v_2=\begin{pmatrix}\Phi_1 & 0\\ 0 & \Phi_2\end{pmatrix}. \tag{31}$$

In (31) the symmetry of the matrices η, $\varkappa$, F_1, F_2, Φ_1, and Φ_2 depends on which of the three models in question we are carrying out the calculations for. Below, for brevity, we shall call the model with potential scattering the model I; the model with magnetic interactions, the model II; and the model with spin-orbit interactions, the model III. The η matrices for these three models are equal to

$$\eta=\begin{pmatrix}\eta_1 & \eta_2\\ -\eta_2^* & -\eta_1^*\end{pmatrix}\quad \text{I},$$
$$\eta=\begin{pmatrix}\eta_1 & 0\\ 0 & -\eta_1^*\end{pmatrix}\quad \text{II}, \qquad (32)$$
$$\eta=\begin{pmatrix}\eta_1 & \eta_2\\ -\eta_2^* & \eta_1^*\end{pmatrix}\quad \text{III}.$$

The $\bar{\eta}$ matrices coincide with the Hermitian conjugates η^+. (It must be borne in mind here that $(\eta_i^*)^* = -\eta_i$.) The $\varkappa$ matrices have the same structure as the η matrices.

The diagonality of the η and $\varkappa$ matrices in the model with magnetic interactions leads to a situation in which only the terms linear and quadratic in η and $\varkappa$ remain in u_1 and v_1 (31). It is not difficult to verify that in all the cases $\bar{u}_1 u_1 = \bar{v}_1 v_1 = 1$. It is no coincidence that we have written the matrices u_1 and v_1 in the form (31). The coefficients have been chosen so as to ensure the satisfaction of the equality

$$\int \prod_{i,k} (\delta v_{12})_{ik} = \int \prod_{i,k} (\delta u_{12})_{ik} = 0.$$

Figuring in this equality are the products of the independent elements of the matrices δu and δv. The vanishing of these integrals allows us to regard the elements $(\delta v_{12})_{ik}$ and $(\delta u_{12})_{ik}$ as Grassman differentials. The matrices u_2 and v_2 can be parametrized as follows:

$$F_1=\frac{1-iM}{1+iM}, \quad F_2=e^{\varphi\tau_3}, \quad \Phi_1=1, \quad \Phi_2=e^{\chi\tau_3} \quad \text{I},$$
$$F_1=e^{\varphi\tau_3}, \quad F_2=e^{\chi\tau_3}, \quad \Phi_1=\Phi_2=1 \quad \text{II}, \qquad (33)$$
$$F_1=e^{\varphi\tau_3}, \quad F_2=\frac{1-iM}{1+iM}, \quad \Phi_1=e^{\chi\tau_3}, \quad \Phi_2=1 \quad \text{III}.$$

Here

$$\tau_3=\begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix}, \quad M=\begin{pmatrix}m & m_1^*\\ m_1 & -m\end{pmatrix},$$

and m, φ, and χ are real numbers.

The substitution of (31)–(33) into (30) enables us to write the quadratic form in terms of the independent variables η, $\varkappa$, M, φ, and χ. Because of the fact that the Grassman variables η and $\varkappa$ separate from the ordinary variables, the volume element dQ can be written in the form of the product

$$dQ = J_1 \cdot J_2 dR_1 dR_2 d\theta, \qquad (34)$$

where $J_1 dR_1$ is the volume element in fermion space; $J_2 dR_2$, the volume element in boson space.

Constructing the Jacobians with the aid of the fermion and boson quadratic forms, we obtain

$$J_1^{(\mathrm{I})}=\frac{1}{2^{20}}(\mathrm{ch}\,(\theta_1+\theta_2)-\cos\theta)^{-2}(\mathrm{ch}\,(\theta_1-\theta_2)-\cos\theta)^{-2},$$
$$dR^{(\mathrm{I})}=d\eta_1 d\eta_1^* d\eta_2 d\eta_2^* d\varkappa_1 d\varkappa_1^* d\varkappa_2 d\varkappa_2^*,$$
$$J_2^{(\mathrm{I})}=\frac{2^9}{\pi^4}|\mathrm{sh}\,\theta|^3|\sin\theta_1||\mathrm{sh}\,\theta_2|,$$
$$dR_2^{(\mathrm{I})}=dm\,dm_1\,dm_1^*\,d\varphi\,d\chi, \quad d\theta^{(\mathrm{I})}=d\theta\,d\theta_1\,d\theta_2,$$
$$J_1^{(\mathrm{II})}=\frac{1}{2^{10}}(\mathrm{ch}\,\theta_1-\cos\theta)^{-2}, \quad dR_1^{(\mathrm{II})}=d\eta_1 d\eta_1^* d\varkappa_1 d\varkappa_1^*, \qquad (35)$$
$$J_2^{(\mathrm{II})}=\frac{2^2}{\pi^2}|\sin\theta||\mathrm{sh}\,\theta_1|, \quad dR_2^{(\mathrm{II})}=d\varphi\,d\chi, \quad d\theta^{(\mathrm{II})}=d\theta\,d\theta_1,$$
$$J_1^{(\mathrm{III})}=\frac{1}{2^{20}}(\mathrm{ch}\,\theta-\cos(\theta_1+\theta_2))^{-2}(\mathrm{ch}\,\theta-\cos(\theta_1-\theta_2))^{-2},$$
$$J_2^{(\mathrm{III})}=\frac{2^9}{\pi^4}|\mathrm{sh}\,\theta|^3|\sin\theta_1||\sin\theta_2|,$$
$$dR_1^{(\mathrm{III})}=d\eta_1 d\eta_1^* d\eta_2 d\eta_2^* d\varkappa_1 d\varkappa_1^* d\varkappa_2 d\varkappa_2^*,$$
$$d\theta^{(\mathrm{III})}=d\theta\,d\theta_1\,d\theta_2, \quad dR_2^{(\mathrm{III})}=dm\,dm_1\,dm_1^*\,d\varphi\,d\chi.$$

Substituting the expressions (27), (31)–(35) into (18), we obtain an expression in which the integration over R_1 and R_2 can easily be performed. Carrying out the appropriate calculations, and substituting (18) and (5) into (3), we reduce the expressions for the correlation functions to the form

$$R^{(\mathrm{I})}(x)=1+\mathrm{Re}\int_{-1}^{1}\int_{1}^{\infty}\int_{1}^{\infty}\frac{(\lambda_1\lambda_2-\lambda)^2(1-\lambda^2)\exp(i(x-i\delta)(\lambda-\lambda_1\lambda_2))\,d\lambda\,d\lambda_1\,d\lambda_2}{(\lambda_1^2+\lambda_2^2+\lambda^2-2\lambda\lambda_1\lambda_2-1)^2},$$

$$R^{(\mathrm{II})}(x)=1+\frac{1}{2}\mathrm{Re}\int_{-1}^{1}\int_{1}^{1} e^{i(x-i\delta)(\lambda-\lambda_1)}\,d\lambda\,d\lambda_1, \qquad (36)$$

$$R^{(\mathrm{III})}(x)=1+\mathrm{Re}\int_{1}^{\infty}\int_{-1}^{1}\int_{0}^{1}\frac{(\lambda-\lambda_1\lambda_2)^2(\lambda^2-1)\exp(i(x-i\delta)(\lambda_1\lambda_2-\lambda))\,d\lambda\,d\lambda_1\,d\lambda_2}{(\lambda^2+\lambda_1^2+\lambda_2^2-2\lambda_1\lambda_2\lambda-1)^2},$$

where $x = \pi\omega/\Delta$.

In reducing the integrals to the form (36), we made the following change of variables: $\lambda = \cos\theta$, $\lambda_1 = \cosh\theta_1$, and $\lambda_2 = \cosh\theta_2$ for the case I; $\lambda = \cos\theta$ and $\lambda_1 = \cosh\theta_1$ for the case II; and $\lambda = \cosh\theta$, $\lambda_1 = \cos\theta_1$, and $\lambda_2 = \cos\theta_2$ for the case III. Notice that, to find the $x\to 0$ limit, we must evaluate the integral for nonzero x values before passing to the limit. Passage to the $x\to 0$ limit directly in the integrand results in divergent integrals.

The investigation performed in this section shows that integration over the group of a supermatrix Q having quite large dimensions reduces to double or triple definite integrals. In the following section we shall evaluate these integrals and consider physical applications.

4. THE TWO-LEVEL CORRELATION FUNCTION AND THE RESPONSE TO AN ELECTROMAGNETIC FIELD

The integrals (36) completely determine the solution to the problem of the computation of the two-level correlation function (2) in all the three possible cases. It is remarkable that in all the three cases the integrals (36) can be computed analytically. This assertion is obvious for the case II, but is not trivial for the cases I and III. Therefore, let us show how the integral I in (36) can be computed. The integral III can be computed, using a similar scheme.

Let

$$I(x)=\int_{-1}^{1}\int_{1}^{\infty}\int_{1}^{\infty}\frac{(\lambda_1\lambda_2-\lambda)^2(1-\lambda^2)\exp(i(x-i\delta)(\lambda-\lambda_1\lambda_2))\,d\lambda\,d\lambda_1\,d\lambda_2}{(\lambda_1^2+\lambda_2^2+\lambda^2-2\lambda\lambda_1\lambda_2-1)^2}. \qquad (37)$$

Let us go over to the Fourier representation with the aid of the formula

$$\tilde{I}(t)=\int_{-\infty}^{\infty} I(x)\,e^{ixt}dx.$$

For $I(t)$ we immediately obtain

$$\tilde{I}(t)=2\pi\int_{-1}^{1}\int_{1}^{\infty}\int_{1}^{\infty}\frac{(\lambda_1\lambda_2-\lambda)^2(1-\lambda^2)\,\delta(\lambda-\lambda_1\lambda_2+t)\,d\lambda d\lambda_1 d\lambda_2}{(\lambda^2+\lambda_1^2+\lambda_2^2-2\lambda\lambda_1\lambda_2-1)^2}. \tag{38}$$

Going over to the variables $u=\lambda_1\lambda_2$ and $z=\lambda_1^2$, we reduce the integral $\tilde{I}(t)$, (38), to the form

$$\tilde{I}(t)=\pi t^2\int_{1}^{\infty}\int_{1}^{u^2}\frac{(1-(u-t)^2)\,\theta(u-t+1)\,\theta(1-u+t)\,z\,dz\,du}{(z^2+z(t^2-u^2-1)+u^2)^2}, \tag{39}$$

where

$$\theta(x)=\begin{cases}1, & x>0\\ 0, & x<0\end{cases}.$$

The integrand turns out to be a rational function of z, which allows us to perform the integration over z. As a result, we obtain

$$\tilde{I}(t)=\tilde{I}_1(t)+\tilde{I}_2(t),$$

$$\tilde{I}_1(t)=2\pi\int_{1}^{\infty}\frac{\theta(u-t+1)\,\theta(1-u+t)\,(u^2-1)}{(u+t+1)\,(u+t-1)}\,du, \tag{40}$$

$$\tilde{I}_2(t)=2\pi t^2\int_{1}^{\infty}\frac{\theta(u-t+1)\,\theta(1-u+t)\,(t^2-u^2-1)}{(1+u-t)^{1/2}(1+u+t)^{3/2}(1-u+t)^{1/2}(u+t-1)^{3/2}}$$
$$\times\left(\operatorname{arctg}\frac{t^2-u^2-1}{(4u^2-(t^2-u^2-1)^2)^{1/2}}-\operatorname{arctg}\frac{u^2+t^2-1}{(4u^2-(t^2-u^2-1)^2)^{1/2}}\right)du.$$

The subsequent integration in $\tilde{I}_1(t)$ is fairly simple. The integration in $\tilde{I}_2(t)$ can, in spite of the rather unwieldy integrand, also be performed analytically. To do this, we use the equality

$$\frac{d}{du}\left[\frac{(1+u-t)^{1/2}(1-u+t)^{1/2}}{(1+u+t)^{1/2}(t+u-1)^{1/2}}\right]$$
$$=\frac{2t(t^2-u^2-1)}{(1+u-t)^{1/2}(1+u+t)^{3/2}(1-u+t)^{1/2}(u+t-1)^{3/2}}.$$

Integration by parts with the use of this equality reduces $\tilde{I}_2(t)$, (40), to the form

$$\tilde{I}_2(t)=\pi t\int_{1}^{\infty}\frac{\theta(u-t+1)\,\theta(1-u+t)\,(3u^2-t^2+1)\,du}{(1+u+t)\,(u+t-1)\,u}. \tag{41}$$

Evaluating the integrals of the rational functions in $\tilde{I}_1(t)$, (40), and $\tilde{I}_2(t)$, (41), we obtain

$$\tilde{I}(t)=\pi t\begin{cases}\dfrac{4}{t}-\ln\dfrac{t+1}{t-1}, & t>2\\ 2-\ln(t+1), & 0<t<2.\\ 0, & t<0\end{cases}$$

Going over to the x representation, and separating out the real part, we reduce the correlation function $R^{(I)}(x)$, (36), to the form

$$R^{(I)}(x)=1-\frac{\sin^2 x}{x^2}-\frac{d}{dx}\left(\frac{\sin x}{x}\right)\int_{1}^{\infty}\frac{\sin xt}{t}\,dt. \tag{42}$$

Performing the corresponding calculations for the remaining two models, i.e., for $R^{(II)}$ and $R^{(III)}$ in (36), we obtain

$$R^{(II)}(x)=1-\frac{\sin^2 x}{x^2}, \tag{43}$$

$$R^{(III)}(x)=1-\frac{\sin^2 x}{x^2}+\frac{d}{dx}\left(\frac{\sin x}{x}\right)\int_{0}^{1}\frac{\sin xt}{t}\,dt. \tag{44}$$

The correlation functions $R^{(I)}$, $R^{(II)}$, and $R^{(III)}$, (42)–(44), coincide exactly with the corresponding correlation functions obtained for the orthogonal, unitary, and symplectic Dyson ensembles.[2] Let us note that this coincidence appears only in the final result (42)–(44). No similarity occurs at any intermediate stage. In the $x\to 0$ and $x\to\infty$ limits the functions $R(x)$, (42)–(44), have entirely different asymptotic forms:

$$R^{(I)}(x)\approx\frac{\pi}{6}x,\quad x\to 0;\quad 1-\frac{1}{x^2}+\frac{1}{x^4}(1+\cos^2 x),\quad x\to\infty,$$
$$R^{(II)}(x)\approx\frac{x^2}{3},\quad x\to 0;\quad 1-\frac{\sin^2 x}{x^2},\quad x\to\infty, \tag{45}$$
$$R^{III}(x)\approx\frac{x^4}{135},\quad x\to 0;\quad 1+\frac{\pi}{2}\frac{\cos x}{x},\quad x\to\infty.$$

Long-range correlations manifest themselves most strongly in the case III; least strongly in the case I. The formulas in (45) indicate the existence of a level-repulsion effect [$R(x)\to 0$ as $x\to 0$]. This effect also manifests itself most strongly in the case III.

As has already been noted above, the two-level correlation function determines such quantities as the response to an electric field, the magnetic susceptibility, etc. Using Dyson's results for the orthogonal, unitary, and symplectic ensembles, Gor'kov and Éliashberg[4] computed these quantities. The coincidence of the formulas (42)–(44) with the corresponding Dyson expressions allows us to use the results obtained in Ref. 4. Let us briefly give the main points of the computations. The investigation is limited to the case of a single pellet with dimension a, located in a weak electric field, such that the inequality $eEa\ll\Delta$ is satisfied. For the computations we used the Kubo formula

$$\langle d_\alpha\rangle=i\int\ll[d_\alpha(t),d_\beta(t')]\gg E_\beta(t')\,dt', \tag{46}$$

where the d_α are the components of the dipole-moment operator and [,] is a commutator. The angle brackets denote averaging over the states of the system and over the impurities. Performing the averaging over the states of the system in (46), we reduce the expression to the form

$$d_{\alpha\omega}=\frac{e^2E_{\beta\omega}}{2}\sum_{k,l}\left\langle\frac{\operatorname{th}(\varepsilon_k-\varepsilon_0)/2T-\operatorname{th}(\varepsilon_l-\varepsilon_0)/2T}{\varepsilon_k-\varepsilon_l-(\omega-i\delta)}r_{kl}^{\alpha}r_{lk}^{\beta}\right\rangle, \tag{47}$$

where $d_{\alpha\omega}$ is the Fourier transform of the α-th component of the dipole moment and r_{kl}^{α} is the coordinate matrix element connecting the states k and l. The summation is performed

over all the states of the system. The angle brackets in (47) now denote only averaging over the impurities. The expression (47) allows us to obtain the polarizability α_ω:

$$\alpha_\omega=\alpha_0+\frac{e^2\omega^2}{6}\sum_{k,l}\frac{\mathrm{th}(\varepsilon_k-\varepsilon_0)/2T-\mathrm{th}(\varepsilon_l-\varepsilon_0)/2T}{\varepsilon_k-\varepsilon_l}\times\frac{r_{kl}r_{lk}}{(\varepsilon_k-\varepsilon_l)^2-(\omega-i\delta)^2}, \tag{48}$$

where α_0 is the static polarizability, equal to

$$\alpha_0=\frac{e^2}{6}\sum_{k,l}\frac{r_{kl}r_{lk}(\mathrm{th}(\varepsilon_k-\varepsilon_0)/2T-\mathrm{th}(\varepsilon_l-\varepsilon_0)/2T)}{\varepsilon_k-\varepsilon_l}=\frac{4e^2a^5mp_0}{15\pi}.$$

In the limit $\omega\ll v/a$ the averaging of the matrix elements and everything else can be performed independently.

Recalling the definition of the correlation function R, (2), and computing the matrix elements $r_{kl}r_{lk}$ in the same way as is done in Ref. 7, we obtain

$$\alpha_\omega=\alpha_0+{}^1/_3e^2\beta\nu A(\omega), \tag{49}$$

$$A(\omega)=\omega^2\nu\int_0^\infty\frac{R(\varepsilon)}{\varepsilon^2-(\omega-i\delta)^2}.$$

The coefficient β in (49) depends on the relation between the mean free path l and the sample dimension a. In the limiting cases this coefficient is equal to[7]

$$\beta=\begin{cases}\dfrac{139}{150}\dfrac{1}{2\pi\nu}\dfrac{a^3}{v} & \text{for } l\gg a,\\ \dfrac{48}{175}\dfrac{1}{2\pi\nu}\dfrac{a^4}{D} & \text{for } l\ll a.\end{cases}$$

The expression (49) establishes the connection between the two-level correlation function R and the polarizability in the complex frequency region. In Ref. 7 the integral (49) is evaluated, and the frequency dependences of the real and imaginary parts of the polarizability are obtained. A characteristic of the expressions obtained is the fact that they preserve the oscillations when the frequency is varied.

5. CONCLUSION

The investigation carried out in the preceding sections shows that the random potential preserves the correlation between the levels in a metallic particle. The coincidence of the two-level correlation functions (42)–(44) with the results obtained for the orthogonal, unitary, and symplectic Dyson ensembles seems remarkable. At the same time we cannot avow that the formulas (42)–(44) have a universal character, and are applicable to any system in which the various types of interactions are realized. The assertion that they do not have a universal character is made, for example, in Ref. 12, where the applicability of the Dyson formulas is on the whole called into question. At the same time there exist a class of systems to which the formulas (42)–(44) are applicable. In the case of a single particle located in a random potential, it seems probable that a necessary condition for the applicability of the Dyson hypothesis is the possibility of a prolonged travel over the whole sample. This corresponds to the above-considered limit of long mean free times $\tau\varepsilon_0\gg 1$. In the limit of sufficiently high impurity concentration, the particle is localized, and does not feel the boundaries. This leads to the disappearance of the correlations in the level disposition. For this same reason, apparently, the Dyson formulas are also inapplicable even in the limit of long mean free times to a one-dimensional chain of finite length because of the localization. Let us recall that, in the limit of short mean free paths, the above-developed theory is again inapplicable to one-dimensional chains. Above we assumed that the disorder is created by the impurities located inside the sample. It may be inferred that the obtained results remain valid for pure samples with surfaces of irregular form.

In the preceding sections the main attention was given to the computation of the two-level correlation function. The density of states is also an important characteristic of the system. But in the model in question this quantity is identically equal to ν. The random potential in the limit under investigation completely smooths out all the oscillations in the density of states. Let us note that this quantity can be obtained even in the standard first-order perturbation theory.[5] For the two-level correlation function calculations even allowance for the diagrams the summation of which yields the diffusion modes is not sufficient.[7] This is due to the fact that such diagrams give rise to an expansion in powers of x^{-1}, where $x=\pi\omega/\Delta$. The terms of the type $x^{-n}\cos 2x$ that are oscillating functions of x cannot be obtained within the framework of such a perturbation theory, since they are nonanalytic in x^{-1}. The consideration of the problem with the aid of the σ models obtained by the replica method[13,14] also leads to unsurmountable mathematical difficulties due to the complexity of the computation of the integral over Q under the constraint $Q^2=1$. Therefore, at present the computational procedure proposed above and based on the method of supersymmetry[6] seems to be the only possible method of solving the formulated problem.

Experimentally, the level correlation could be observed by measuring the polarizability of metallic particles in a weak alternating electric field. Quite a large number of different granular materials exist. Unfortunately, quite a large spread in the particle dimensions is usually found. This leads to large fluctuations in the mean level spacing. Numerical estimates of the washing out of the oscillations for actually existing granular materials are given in Ref. 15. It is concluded in that paper that the observation of the oscillations is at present practically impossible. The production of granular materials with a smaller spread in the granule dimensions will be of great interest from the point of view of the observation of the correlations between the levels.

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