# Ground state of superdense quark-lepton matter

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The ground-state energy of cold quark-lepton matter is calculated in the SU(2)×U(1) weak-interaction model in the density interval $n \sim m_W{}^3 - m_X{}^3$ ($m_{W,X}$ are the characteristic electroweak and grand-unification energies). It is shown that at a baryon/lepton charge ratio $B/L \neq 4/3$ the $W$- and $\varphi$-boson condensates produced in the system lead to the appearance of a term proportional to $\alpha^{1/3}$ in the energy density ($\alpha = e^2/4\pi$). The behavior of quark-lepton matter is qualitatively analyzed within the framework of the SU(5) model at extremely high densities.

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## 1. INTRODUCTION

Modern gauge theories of electroweak and strong interactions allow us to calculate the ground-state energy of quark-lepton matter up to extremely high densities. The main circumstance that guarantees the success of such calculations in smallness of the coupling constants of electroweak interactions and the asymptotic-freedom property of strong interactions, so that perturbation theory can be used in the calculation.

The first to calculate the thermodynamic potential of a cold electron plasma in the two-loop approximation within the framework of quantum electrodynamics (QED) were Akhiezer and Peletminskiĭ.[1] The development of the field theory of strong interactions—quantum chromodynamics (QCD)—made it possible to solve the analogous problem for a quark plasma (see, e.g., the review by Shuryak.[2] In full analogy with QED, at high densities, when nonlinear effects in QCD can be neglected, the quark-quark correlations make a small contribution [on the order of $\alpha_s = g^2(\mu_F)/4\pi$, $g$ is the strong-interaction constant and $\mu_F$ is the chemical potential of the quarks] to the energy of the quark matter.

When densities $F$ corresponding to the electroweak-unification energy $F \sim m_W{}^3$ are reached, allowance for the weak interaction becomes essential. At present the universally accepted theory of electroweak interactions is taken to be the SU*(2)×U(1) Weinberg-Salam model, in which spontaneous breaking of the symmetry is reached on account of the appearance of the vacuum mean value of the scalar field $\varphi$ (see, e.g., Ref. 3).

The question of the behavior of superdense matter within the framework of gauge models with spontaneous symmetry breaking was first considered by Harrington and Yildiz,[4] who deduced the existence of a critical fermion density at which the mean scalar field vanishes. Further investigations[5,6] have shown that this statement is not always valid for gauge theories. In particular, the converse can occur, where the mean scalar field increases with increasing density.

For the Weinberg-Salam model, the question of the phase transition in a cold quark-lepton plasma depends on the ratio of the lepton ($L$) and baryon ($B$) charges (Ref. 7).[1)] If[2)] $(B/L)_c = \frac{4}{3}$, we have at a density $L_c$ (Ref. 8)

$$L_c \approx (m_H/m_f)^3 f^3$$

[$m_H$ is the Higgs-boson mass, $m_f$ is the heavy lepton (quark) mass, $f \approx 250$ GeV] the SU(2) symmetry in electrically neutral quark-lepton matter is restored ($\varphi = 0$) and the effective masses of the vector bosons and fermions vanish. The partial densities of quarks (leptons) of different sorts and helicities are then equal.[8] The first correction to the kinetic energy of a fermion gas, up to grand-unification energies, is determined by the strong quark-quark correlations ($\sim \alpha_s$). At $B/L \neq 4/3$ there is no phase transition in cold matter and the mean scalar field increases monotonically with increasing density.[7,8]

The weakness of the interaction constants of gauge and Higgs fields causes the main contribution to the quark-lepton matter energy to be made by the fermion kinetic energies

$$\varepsilon \sim \sum_{i=q,l} n_{(i)}^{4/3} .$$

The contribution of these terms, in turn, is a minimum when, given $L$ and $B$, the partial densities of different sorts of quarks and leptons are equal (with distinction between right and left-hand particles). In electrically neutral matter at $B/L \neq 4/3$ this can be made possible only because of the formation of a condensate of charged $W$ bosons[9,8] ($W$ condensate).

The ground-state energy of a superdense ($B/L \gtrsim m_W{}^3$) quark-lepton plasma at $B/L \neq 4/3$ can thus depend significantly on the energies of the boson ($W$, $\varphi$) condensates. It will be shown below that their contribution is proportional to $\alpha^{1/3}$ ($\alpha = e^2/4\pi$) and exceeds, in the indicated density region, the contribution due to the quark-quark correlations, even though in this energy region the strong-interaction constant $\alpha_s$ is still considerably larger than $\alpha$.

What happens to contracting quark-lepton matter when the grand-unification density $m_X{}^3$ is reached? At these extremal densities, owing to formation of a leptoquark condensate, transitions of quarks (leptons) into antiquarks and antileptons become energetically favored. In the SU(5) model[10] (see also the reviews[11]) only the difference $B - L \equiv A$ is rigorously conserved. In accordance with this conservation law there are produced in a cold quark-lepton plasma, at

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densities higher than $m_X{}^3$, either antileptons ($A > 0$) or antiquarks ($A < 0$) with density $\bar{L}_f = A/2 + L_i$ ($\bar{B}_f = |A|/2 + B_i$), i.e., particle-antiparticle pairs $n_{l\bar{l}} = L_i$ ($n_{q\bar{q}} = B_i$) are produced in the system ($L_i$ and $B_i$ are the initial densities of the lepton and baryon charges). This means that once thermodynamic equilibrium sets in the system is characterized by an additional thermodynamic parameter, the temperature $T \sim L_i^{1/3}$ ($T \sim B_i^{1/3}$, $A < 0$). If the inequality $|L_i - B_i| \ll B_i$ is satisfied for the bare densities of the leptons and quarks we inevitably arrive at a hot quark-lepton plasma with restored SU(5) symmetry.

In the energy region considered, the effects of the chemical potential of the conserved charges may turn out to be important for the choice of the initial state of a nonadiabatically expanding universe and for the study of the relativistic collapse of cold matter.

In Secs. 2–4 of this paper we calculate, for the standard $SU_c(3) \times SU(2) \times U(1)$ model of strong and electroweak interactions, the ground-state energy of quark-lepton matter in the density interval $F \sim m_W{}^3 - m_X{}^3$ ($m_W \sim 10^2$ GeV and $m_X \sim 10^{15}$ GeV are the characteristic energies of the electroweak and grand unifications). The behavior of an extremely dense ($F \gtrsim m_X{}^3$) quark-lepton plasma is analyzed in Sec. 5 within the framework of the SU(5) grand-unification model.

## 2. BOSON CONDENSATES IN A SUPERDENSE QUARK-LEPTON PLASMA

The Lagrangian of the $SU(2) \times U(1)$ model of electroweak interactions of leptons and quarks is of the following standard form[3]:

$$\mathscr{L} = -\frac{1}{4}(G_{\mu\nu}{}^a)^2 - \frac{1}{4}F_{\mu\nu}{}^2$$
$$+ \sum_{j=l,q} i\bar{L}_{(j)}\left(\hat{\partial} - i\frac{g}{2}\tau\hat{\mathbf{A}} - i\frac{g'}{2}Y_{(j)}\hat{B}\right)L_{(j)}$$
$$+ \sum_{j=l,q} \bar{R}_{(j)}(\hat{\partial} - ig'Q_{(j)}\hat{B})R_{(j)} + \left|\left(\partial_\mu - i\frac{g}{2}\tau\mathbf{A}_\mu - i\frac{g'}{2}B_\mu\right)\Phi\right|^2$$
$$-2^{1/2}\sum_{j=l,q} h_{(j)}(\bar{L}_{(j)}R_{(j)}\Phi + \bar{R}_{(j)}L_{(j)}\Phi^+) - \lambda\left(\Phi\Phi^+ - \frac{1}{2}f^2\right)^2. \tag{1}$$

Here $G_{\mu\nu}{}^a = \partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a + g\varepsilon^{abc}A_\mu{}^b A_\nu{}^c$, $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. $L_{(j)}$ is an isodoublet of left-hand quarks $q$ (of leptons $l$), $R_{(j)}$ is an isosinglet of right-hand $q(l)$, $Q_{(j)}$ and $Y_{(j)}$ are the electric charge and weak hypercharge

$$Y_{(j)} = \begin{cases} -1; & j=l \\ 1/3; & j=q \end{cases}, \quad Q_{(j)} = \begin{cases} 0, \quad -1; & j=l \\ 2/3; \quad -1/3; & j=q \end{cases}, \tag{2}$$

and $f$ is the vacuum expectation value of the scalar field $\Phi$. In a unitary gauge

$$\langle\Phi\rangle = 2^{-1/2}\begin{pmatrix}0\\ \varphi\end{pmatrix}, \quad \varphi = f.$$

At nonzero temperature and nonzero density, the mean scalar field $\varphi$ differs from $f$ and becomes a function of the temperature and of the densities of the conserved (electric, lepton, baryon) charges.[12]

It is clear from physical considerations that at weak-interaction constants $g, g', \lambda, h_{(j)} \ll 1$ (at $B \gtrsim m_W{}^3$ the quark strong-interaction constant is also small, $\alpha_s \ll 1$), the main contribution to the energy of a cold quark-lepton plasma is made by the fermion energies

$$\varepsilon^{(0)} = {}^3/_4(6\pi^2)^{1/3}\sum_{\substack{j=L,R\\ i=l,q}}(n_i{}^j)^{4/3}. \tag{3}$$

In Eq. (3), $n_i{}^j$ are the fermion densities, the superscript labels the fermion helicity ($L, R$), and the subscript its type. Given the total density of the lepton and baryon charges, the sum (3) is a minimum when the partial densities of the leptons (quarks) are equal. In electrically neutral matter, at an arbitrary ratio of the baryon and lepton charges, this is possible only on account of production of a $W^\pm$ condensate[9,8] that ensures transition between the ano- and cathofermions of each doublet of the family ($\nu_L \rightleftarrows e_L^- + W^+$, etc). Equalization of the densities of the right- and left-hand fermions is due to the existence of a $\varphi$ condensate ($e_L \rightleftarrows e_R + \varphi$, $u_L \rightleftarrows u_R + \varphi$, etc.).

In what form should the $W$-condensate field be chosen? Finding the equilibrium form of the condensate is always a variational problem. In our case physical consideration allows us to choose four varied variables pertaining to boson condensates. All the condensates are spatially homogeneous,[8] and this simplifies greatly the solution of the problem.

A condensate of charged $W$ bosons cannot be static[3)] and is described by two parameters, the amplitude $\omega$ and the chemical potential $\mu \geqslant m_W$:

$$\langle W_j{}^\pm\rangle \equiv \frac{1}{\sqrt{2}}\{\langle A_j^{(1)}\rangle \pm i\langle A_j^{(2)}\rangle\} = \omega_j e^{\pm i\mu t}, \quad \langle A_j^{(3)}\rangle = 0. \tag{4}$$

The condensate of Higgs mesons is characterized by one parameter $\varphi$. Finally, in the general case we must assume a nonzero mean value of the zeroth component of the field $Z_\mu$ ($\langle Z_0\rangle \equiv Z$, $\langle Z_j\rangle = 0$):

$$Z_\mu = -B_\mu \sin\theta_W + A_\mu^{(3)}\cos\theta_W \tag{5}$$

($\theta_W$ is the Weinberg mixing angle), generated by a neutral weak charge $J_0{}^Z$ (Refs. 5 and 6):

$$J_0{}^Z = \frac{e}{\sin 2\theta_W}\left\{n_\nu + 2\sin^2\theta_W n_e{}^R - \cos 2\theta_W n_e{}^L\right.$$
$$+ \left(1 - \frac{4}{3}\sin^2\theta_W\right)n_u{}^L - \frac{4}{3}\sin^2\theta_W n_u{}^R$$
$$\left. - \left(1 - \frac{2}{3}\sin^2\theta_W\right)n_d{}^L + \frac{2}{3}\sin^2\theta_W n_d{}^R\right\}. \tag{6}$$

Using (1), (4), and (6) we easily find an expression for the energy densities of the boson ($W$, $\varphi$) condensates:

$$\varepsilon_B = \frac{1}{2}(G_{0j}{}^a)^2 - \frac{1}{2}m_Z{}^2 Z_\mu{}^2 - m_W{}^2|W_\mu|^2 + \frac{\lambda}{4}(\varphi^2 - f^2)^2$$
$$= \omega^2(\mu + eZ\operatorname{ctg}\theta_W)^2 + m_W{}^2\omega^2 - \frac{1}{2}m_Z{}^2 Z^2 + \frac{\lambda}{4}(\varphi^2 - f^2)^2. \tag{7}$$

Here $m_W$ and $m_Z$ are the effective masses of the $W$ and $Z$ bosons:

$$m_W{}^2=e^2\varphi^2/4\sin^2\theta_W,\qquad m_Z{}^2=e^2\varphi^2/\sin^2 2\theta_W. \tag{8}$$

The field $Z$ is not an independent dynamic variable, and in accord with the coupling condition that follows from (1) it satisfies the equation

$$Z(m_Z{}^2+2e^2\omega^2\operatorname{ctg}\theta_W)+en\operatorname{ctg}\theta_W+J_0{}^z=0. \tag{9}$$

The electric-charge density $n$ of the $W$ condensate equals

$$n=i\left\{W_j{}^*\frac{\partial W_j}{\partial t}-W_j\frac{\partial W_j{}^*}{\partial t}\right\}=2\mu\omega^2. \tag{10}$$

## 3. FERMION SPECTRUM IN THE FIELD OF BOSON CONDENSATES

The next task is to determine the spectrum of the leptons and quarks in the field of $W$, $Z$, and $\varphi$ condensates. The condensate of the Higgs mesons causes the fermions to acquire mass. Assuming the Fermi momenta of the particles to be much larger than their effective masses $m_{(j)}=h_{(j)}\varphi$ (it will be seen subsequently that this is indeed the case at $h\ll 1$), we regard the quarks and leptons as ultrarelativistic.

The spectrum is easiest to obtain for right-hand fermions. The corresponding Dirac equation takes according to (1) the form

$$\{i\gamma_\mu(\partial_\mu-ig'Q_{(j)}B_\mu)\}R_{(j)}=0, \tag{11}$$

where $\langle B_j\rangle=0$, $\langle B_0\rangle=-\sin\theta_W Z$. Since $Z$ is independent of the coordinates, Eq. (11) is solved in elementary fashion:

$$R_{(j)}=\exp\{-i[E_R^{(j)}t-\mathbf{px}]\}r_j,\quad E_R^{(j)}=eQ_{(j)}\operatorname{tg}\theta_W Z+p \tag{12}$$

($r_j$ is a constant spinor). When the $\varphi$ condensate is taken into account

$$p\to(\mathbf{p}^2+h_{(j)}^2\varphi^2)^{1/2}. \tag{13}$$

To determine the energy of the left-hand fermions we must solve the Dirac equation in the field of the $W$ and $Z$ condensates. We consider a more general problem, assuming a nonzero fermion mass $m$,

$$\{i\gamma_\mu(\partial_\mu-{}^1/_2ig\boldsymbol{\tau}\mathbf{A}_\mu-{}^1/_2ig'Y_{(j)}B_\mu)-m\}\Psi_{(j)}=0. \tag{14}$$

The spectrum of the phonons in the field of a $W$ condensate of the form (4) can be determined exactly. Indeed, choosing $\Psi_{(j)}$ in the form

$$\Psi_{(j)}=e^{i\mathbf{px}}\exp\{-iE'_{(j)}t\}u_{(j)},\qquad E'_{(j)}=E+\frac{\mu}{2}\tau_3 \tag{15}$$

($\tau_3$ is a Pauli matrix), we obtain for $u_j$ a system of homogeneous algebraic equations

$$Du_{(j)}=0,\quad D\equiv\hat{p}+{}^1/_2\tau_3\hat{a}+\tau_1\hat{q}-m, \tag{16}$$

where $\hat{x}\equiv x_\mu\gamma_\mu$,

$$p_\mu=\left(E-\frac{e}{2}Y_{(j)}\operatorname{tg}\theta_W Z,\mathbf{p}\right),$$
$$a_\mu=(\mu+eZ\operatorname{ctg}\theta_W,0),\quad q_\mu=\left(0,\frac{e}{2^{1/2}\sin\theta_W}\boldsymbol{\omega}\right). \tag{17}$$

The spectrum of the energies $E$ is obtained from the condition $\det D=0$. In our case it is more convenient to consider the following equivalent equation (see also Ref. 13, where a similar method was used to determine the nucleon spectrum on a $\pi$-condensate field):

$$\det\{\gamma_5 D\gamma_5 D\}=0. \tag{18}$$

After a number of simple transformations we obtain

$$\{(p_\mu+{}^1/_2a_\mu)^2+q_\mu{}^2-m^2\}\{(p_\mu-{}^1/_2a_\mu)^2+q_\mu{}^2-m^2\}$$
$$=4(p_\mu q_\mu)^2-{}^1/_4[\hat{a},\hat{q}]^2. \tag{19}$$

Using for the vectors the explicit expressions (17), we obtain ultimately for the energies of the left-hand ano- and cathofermions

$$E_L{}^a=E+{}^1/_2\mu,\qquad E_L{}^c=-{}^1/_2\mu+E, \tag{20}$$

where $E$ are positive-frequency solutions (at $Z=0$, $\mu=0$, $\omega=0$) of the equation

$$\left(E-\frac{e}{2}Y_{(j)}\operatorname{tg}\theta_W Z\right)^2=\frac{1}{4}(\mu+eZ\operatorname{ctg}\theta_W)^2+\frac{e^2\omega^2}{2\sin^2\theta_W}$$
$$+\mathbf{p}^2+m^2\pm\left\{(\mu+eZ\operatorname{ctg}\theta_W)^2(\mathbf{p}^2+m^2)+\frac{2e^2}{\sin^2\theta_W}(\mathbf{p}\boldsymbol{\omega})^2\right\}^{1/2}. \tag{21}$$

The exact form of the fermion spectrum (20), (21) is too unwieldy, but for our purpose an approximate spectrum is sufficient. Physical considerations allow us to assume

$$p_F\gg\mu,\ eZ,\ e\omega,\ m. \tag{22}$$

In this case we have for the energies of the left-hand ano- and cathofermions

$$E_L^{a(c)}=p+\frac{e}{2}Y_{(j)}\operatorname{tg}\theta_W Z\pm\frac{\mu}{2}$$
$$\mp\frac{1}{2}\left\{(\mu+eZ\operatorname{ctg}\theta_W)^2+\frac{2e^2\omega^2}{\sin^2\theta_W}\cos^2\gamma\right\}^{1/2}, \tag{23}$$

where $\gamma$ is the angle between the vectors $p$ and $\omega$.

## 4. GROUND-STATE ENERGY OF SUPERDENSE QUARK-LEPTON MATTER

Knowing the fermion spectrum, it is easy to calculate in the single-loop approximation the energy density of the gas of quarks and leptons

$$\varepsilon_F=\sum_i\int\frac{d^3p}{(2\pi)^3}E^{(i)}. \tag{24}$$

The summation here is over all types of fermions (but with a distinction made between particles of opposite helicity). To simplify the calculations, we shall consider hereafter only one generation of quarks ($u$, $d$) and leptons ($\nu$, $e$).

For the spectrum $E^{(i)}$ [(11), (23)] it is easy to integrate in (24) with respect to the momenta. The system energy density $\varepsilon=\varepsilon_B+\varepsilon_F$ expressed in terms of the parameters of the boson condensates and the lepton and quark densities, is of the form

$$\varepsilon=\frac{3}{4}(6\pi^2)^{1/3}\sum_{\substack{j=L,R\\ i=l,q}}(n_i^j)^{4/3}+\omega^2(\mu+eZ\operatorname{ctg}\theta_W)^2$$
$$+m_W^2\omega^2-\frac{1}{2}m_Z^2Z^2+\frac{\lambda}{4}(\varphi^2-f^2)^2+\frac{eZ\operatorname{tg}\theta_W}{2}$$
$$\times\left(n_\nu+n_e^L+2n_e^R-\frac{1}{3}n_u^L-\frac{1}{3}n_d^L-\frac{4}{3}n_u^R+\frac{2}{3}n_d^R\right)$$
$$+(n_\nu-n_e^L+n_u^L-n_d^L)\left\{\frac{\mu}{2}-\frac{1}{4}\left[\left((\mu+eZ\operatorname{ctg}\theta_W)^2\right.\right.\right.$$
$$\left.+\frac{2e^2\omega^2}{\sin^2\theta_W}\right)^{1/2}+\frac{(\mu+eZ\operatorname{ctg}\theta_W)^2\sin\theta_W}{2^{1/2}e\omega}$$
$$\left.\left.\times\ln\frac{((\mu+eZ\operatorname{ctg}\theta_W)^2+2e^2\omega^2/\sin^2\theta_W)^{1/2}}{\mu+eZ\operatorname{ctg}\theta_W}\right]\right\}. \qquad (25)$$

To solve our problem we must find the minimum of (25) subject to the additional conditions: conservation of the lepton and baryon charges, and electroneutrality of the system

$$L=\sum_{\substack{j=L,R\\ i=\nu,e}}n_i^j,\qquad B=\frac{1}{3}\sum_{\substack{j=L,R\\ i=u,d}}n_i^j, \qquad (26)$$

$$n-(n_e^L+n_e^R)+\frac{2}{3}(n_u^L+n_u^R)-\frac{1}{3}(n_d^L+n_d^R)=0. \qquad (27)$$

We choose, using (26) and (27), the independent variables of the variational problem to be, besides the three parameters ($\varphi$, $\mu$, $\omega$) that characterize the boson condensates, the fermion densities $n_e^L$, $n_e^R$, $n_u^L$, $n_d^R$. We recall that the field $Z$ is expressed in terms of the independent variables in accordance with Eq. (9).

Before we write down the system of equations that determine the equilibrium parameters of the boson condensates, we analyze expression (25). The smallness of the interaction constants in the entire considered energy range allows us to assume the main contribution to come from the first term of (25) (the sum of the kinetic energies of the right- and left-hand leptons and quarks). We thus have in the lowest-order approximation[8,9]

$$n_\nu=n_e^L=n_e^R={}^1/_3L,\qquad n_u^L=n_u^R=n_d^L=n_d^R={}^3/_4B, \qquad (28)$$

$$n=F_0\equiv{}^2/_3(L-{}^3/_4B),\quad \mu\sim eZ\sim e\omega\sim(n_i-n_j)_{i\neq j}\sim o(1)F_0^{1/3}. \qquad (29)$$

Taking (28) and (29) into account, we easily see that to obtain the equations in the next order of perturbation theory we can neglect the term in the curly brackets of (25). It is multiplied by $\Sigma_{i\neq j}(n_i^L-n_j^L)$ and has therefore a higher order of smallness.

We note also that, at the accuracy indicated (we omit terms proportional to $eZ(n_i-n_j)$

$$\begin{aligned}{}^1/_2e\operatorname{tg}\theta_WZ(n_\nu+n_e^L+2n_e^R-{}^1/_3n_u^L-{}^1/_3n_d^L-{}^4/_3n_u^R)\\ \approx-eZJ_0^z/\sin 2\theta_W\end{aligned} \qquad (30)$$

and (25) takes on an easy-to-interpret form: It is the sum of the kinetic energies of the fermions and of the energies of the boson condensates, plus the term that accounts for the energy of the interaction of the field with the neutral weak charge. The system energy in this form is exact in the investigated approximation at $\omega=0$ (there is no $W$ condensate). In the latter case Eq. (25) can be easily seen, when account is taken of (9), to go over into the standard expression for the energy density of ultrarelativistic fermions interacting via a massive vector field.[5,7,14]

Varying (25) with respect to all the independent variables, we easily obtain a system of equations for the equilibrium amplitudes of the boson condensates and for the fermion densities. We write out the the equations that enable us to find the lowest approximation the parameters of the boson condensates $Z$, $\omega$, $\mu$, and $\varphi$:

$$Z(m_Z^2+2e^2\operatorname{ctg}^2\theta_W\omega^2)=-\frac{2}{\sin 2\theta_W}eF_0, \qquad (31)$$

$$\frac{e^2\omega^2}{4\sin^2\theta_W}-\frac{1}{2}\frac{e^2Z^2}{\sin^2 2\theta_W}+\frac{\lambda}{2}(\varphi^2-f^2)-\frac{(m_Z^2Z+eF_0\operatorname{tg}\theta_W)eZ^2}{F_0\sin 2\theta_W}$$
$$=-\frac{3}{4}(6\pi^2)^{-1/3}\left[h_l^2\sum_{j=L,R}(n_e^j)^{2/3}+h_q^2\sum_{\substack{j=L,R\\ i=u,d}}(n_i^j)^{2/3}\right], \qquad (32)$$

$$\mu^2-m_Z^2\cos^2\theta_W-e^2\operatorname{ctg}^2\theta_WZ^2+\frac{4e\cos^3\theta_WZ^2(m_Z^2Z+eF_0\operatorname{tg}\theta_W)}{F_0\sin\theta_W}$$
$$=0, \qquad (33)$$

$$2\mu\omega^2=F_0. \qquad (34)$$

In the derivation of Eq. (32) $\partial\varepsilon/\partial\varphi^2=0$ account was taken of the fermion masses $m_j=h_j\varphi$. The lepton and quark densities $n_{l,q}^{L,R}$ in (32) satisfy Eq. (29).

It follows from (31) that the field $Z$ is equal to zero only at $B/L=4/3$. In this case there is no $W$ condensate, $\omega=0$ (34), and according to Eq. (32) there exists a critical density of the lepton (baryon) charge at which restoration of the SU(2)×U(1) symmetry is restored, $\varphi=0$ (Ref. 8).

We shall be interested in the general case $B\neq 4L/3$, when there is no phase transition into a state with restored symmetry, and a $W$ condensate is produced in the system at ultrahigh densities $B(L)\gtrsim m_W^3$ (Refs. 8 and 9). We assume also that the Higgs-boson mass is much less than the characteristic masses of the electroweak unification and there are no superheavy fermions in nature:

$$\lambda\ll e^2,\qquad h_{(j)}\ll e^{2/3}. \qquad (35)$$

The solution of the system (31)–(34) takes then the form

$$Z=x^{1/3}e^{-1/3}F_0^{1/3},\qquad \omega=ye^{-1/3}|F_0|^{1/3}, \qquad (36)$$

$$\mu=pe^{2/3}F_0^{1/3},\qquad m_Z=ke^{2/3}|F_0|^{1/3}, \qquad (37)$$

where $x$ satisfies the equation

$$\frac{\cos^2\theta_W}{(1-4\cos^2\theta_W)^2}(1+4\operatorname{ctg}\theta_Wx)^2=x^2\left(\frac{1}{\sin^2\theta_W}-4\right)$$
$$-\frac{2x(1-4\operatorname{ctg}\theta_Wx)\left[1+\operatorname{ctg}\theta_W(1+4\sin^2\theta_W)x\right]}{\sin 2\theta_W(1+4\operatorname{ctg}\theta_Wx)} \qquad (38)$$

and the condition $1 + 4 \cot\theta_W x < 0$. Taking into account the smallness of the Weinberg mixing angle, $\sin^2\theta_W \approx 0.23$, we have

$$x \approx -\frac{\operatorname{tg}\theta_W}{4}(2\sin^2 2\theta_W - 1)^{-1}[16\cos^4\theta_W - (1-4\sin^2\theta_W)(1+4\cos^2\theta_W)^2] \approx -3.0, \tag{39}$$

$$y \approx \frac{|x|^{-1/6}}{2^{4/3}}\left(\frac{4\cos^2\theta_W - 1}{\operatorname{ctg}\theta_W\cos^2\theta_W}\right)^{1/2} \approx 0.36,$$

$$p \approx -\frac{4x^{1/3}\operatorname{ctg}\theta_W\cos^2\theta_W}{4\cos^2\theta_W - 1} \approx 3.9, \tag{40}$$

$$k = |x|^{-1/6}\left(\frac{1+4\sin^2\theta_W}{2\sin 2\theta_W}\right)^{1/2} \approx 0.9. \tag{41}$$

Using (36), (37), and the equations that follow from (25) we can determine in first order in the developed perturbation theory the densities of the fermions and quarks. The corresponding equations are

$$n_l = \frac{L}{3}\left\{1 + e^{2/3} f_l\left(\frac{B}{L}\right)\right\}, \quad n_q = \frac{3B}{4}\left\{1 + e^{2/3} f_q\left(\frac{B}{L}\right)\right\}, \tag{42}$$

where $f_{l,q}(B/L)$ are unwieldy functions of the ratio $B/L$.

The expressions obtained for the parameters of the boson condensate and the equilibrium densities of the leptons and quarks enable us to determine the energy density of quark-lepton matter

$$\varepsilon = \varepsilon^{(0)} + 0.4\alpha^{1/3} F_0^{4/3}. \tag{43}$$

Here $\varepsilon^{(0)}$ is the kinetic energy of the gas of leptons and quarks (3) with partial densities satisfying (28). We note that Eq. (43) corresponds to the single-loop approximation in the presence of classical $W$- and $\varphi$-condensate fields. Under the condition $\Sigma_j h_{(j)} \ll e^{2/3}$ ($j$ is the generation index) Eq. (43) is valid also when the next generations of the leptons and quarks are taken into account.

Thus, in the absence of a phase transition ($T = 0$, $B/L \neq 4/3$), starting with densities corresponding to the electroweak unification, terms proportional to $\alpha^{1/3}$, due to the presence of boson condensates, appear in the energy of cold quark-lepton matter. At densities higher than $m_W{}^3$ these increments exceed the terms due to the quark-quark correlations.[2]

## 5. QUARK-LEPTON PLASMA IN GRAND-UNIFICATION SU(5) MODEL

When densities corresponding to the characteristic grand-unification energies $L, B \sim m_X{}^3$, reactions with nonconservation of the baryon and lepton charges set in, in addition to the processes described by the standard $SU_c(3) \times SU(2) \times U(1)$ model. In particular, transitions of quarks (leptons) into antiquarks and antileptons become possible. This situation takes place in a cold quark-lepton plasma when the Fermi energy of the quarks (leptons) becomes comparable with the mass of the lepto-quarks ($m_X \sim m_Y \sim 10^{15}$ GeV).

In the SU(5) model, only the difference $B - L \equiv A$ is rigorously globally conserved. The energy density of three generations of colored quarks and leptons, in terms of the densities $B$ and $L^{(j)}$ ($j = e, \mu, \tau$), is

$$\varepsilon^{(0)} = \frac{3}{4}(6\pi^2)^{1/3}\left\{3\sum_j\left(\frac{L^{(j)}}{3}\right)^{4/3} + 12\left(\frac{B}{12}\right)^{4/3}\right\}. \tag{44}$$

The minimum of (44) at a given $A$ leads, as can be easily seen, to the following equilibrium fermion (antifermion) densities:

$$A>0:\ B_f = \bar{L}_f = A/2; \quad A<0:\ L_f = \bar{B}_f = |A|/2. \tag{45}$$

The system contains in this case, in addition to lepton-antilepton ($A > 0$) or quark-antiquark ($A < 0$) pairs with respective densities $n_{l\bar{l}} = L_i$, $n_{q\bar{q}} = B_i$ ($L_i$ and $B_i$ are the initial charge densities. This means that a temperature $T \sim L_i^{1/3}$ ($T \sim B_i^{1/3}$) sets in after establishment of thermal equilibrium, and if the inequalities $|A| \ll L_i, B_i$ are satisfied we arrive ultimately at a hot quark-lepton plasma with SU(5) symmetry restored via a temperature phase transition.[12,15]

Expression (45) establishes a natural relation (corresponding to an energy minimum) between the densities of the baryons and leptons of the cold quark-lepton plasma within the framework of the SU(5) grand-unification model. It is easily seen that in this case the neutral weak charge $J_0{}^Z$ [Eq. (6)] differs from zero and the scalar field responsible for the spontaneous breaking of the $SU(2) \times U(1)$ symmetry does not vanish at any density whatever. Owing to the Higgs-field-potential terms that mix the 5- and 24-plet of the Higgs bosons, density symmetry in cold matter is likewise not restored in the sector of heavy Higgs fields.

Which are the physical processes to which the foregoing results can be useful? Such high densities are reached in relativistic collapse of matter and could occur during the early stages of the expansion of the universe. The model universally accepted at present is that of a hot (big-bang) homogeneous and isotropic universe. Although the observational data do not contradict a relatively large lepton chemical potential contained in the neutrino ($\mu_\nu \lesssim 0.1\ T$, $T$ is the temperature),[16] from the point of view of the grand unification theory (GUT) models, the most natural is the relation $\mu_L/T \sim \mu_B/T$ (Ref. 17) ($\mu_{L,B}$ are the chemical potentials of the leptons and baryons). The currently observed value is $\mu_B/T \sim 10^{-8}$, so that for an adiabatically expanding universe the effects of the chemical potential are negligibly small. At certain periods of its evolution, however, the early universe could be far from adiabatic expansion, if the symmetry breaking processes [$SU(2) \times U(1)$, SU(5)] in it were of the first-order phase-transition type.[15,18] When the ground state is chosen in this case, account must be taken of the effect of the chemical potential of the conserved charges. In particular, the scenario observed does not admit of an initial state in which $\mu_{B-L} \gtrsim T$. Indeed, in this case the effects of the chemical potential are decisive[5,7] and, according to the foregoing analysis, the minimum of the thermodynamic potential always corresponds to a state with broken SU(5) symmetry.

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1)A numerical error was made in the calculation of $(B/L)_c$ in Ref. 7.

2)$(B/L)_c = 1$ formally for the model with a massive Dirac neutrino. However, in view of the extreme suppression of lepton transition into right-hand neutrinos in the considered energy region ($E \sim m_W$) the number of the latter can be regarded as constant over times determined by the rate of the gravitational collapse of matter. Therefore in realistically conceivable processes the presence of a small Dirac or Majorana neutrino mass does not change the value of $(B/L)_c$.

3)For gauges with a charged component $\chi$ of the Higgs field $\Phi$ the entire time dependence can be transferred to the $\chi$ condensate.[9]


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