# Limits on electrodynamics: paraphotons?

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The accuracy to which the electromagnetic interaction at large distances has been investigated is discussed. For a quantitative parametrization of possible deviations from electrodynamics a model with two paraphotons is used, the mass of one of them not being negligible.

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**1.** The possible existence of new long-range interactions has attracted increasing attention during recent times. To a certain degree, this is related to various symmetry and supersymmetry schemes in elementary particle theory, for which the predictions are, unfortunately, at present far from unique (see, e.g., the discussion of a superlight vector boson which simulates antigravity,[1] of a light axial-vector boson,[2] superlight scalar and pseudoscalar bosons,[3] macroscopic confinement[4]). To a certain degree the interest in new long-range interactions is due to natural scientific curiosity, the desire to find out[5] to what level the standard conceptions about long-range forces (gravitational and electromagnetic) are verified experimentally.

Searches for very light and very weakly interacting new particles became promising after it was understood that the mass scale in elementary particle physics is defined probably by the Planck mass $m_{\rm Pl} \approx 10^{19}$ GeV (see, e.g., the review[6]). At this scale the masses of the electron and of the proton are very small, and the next ranks of the mass hierarchy may indeed correspond to particles which are by 20 to 40 orders of magnitude lighter than the electron.

From a phenomenological point of view there exist rigid bounds on the coupling constants of new superlight bosons (i.e., long-range fields) to stable matter. These bounds are derived essentially from the very precise Eötvös experiments and their latter-day analogs (Ref. 7). Thus, for the hypothetical baryonic and leptonic photons the analogs corresponding to the electromagnetic fine structure constant $\alpha = 1/137$ are respectively $\lesssim 10^{-45}$ and $\lesssim 10^{-47}$ (see, Ref. 8). The bounds on the interactions of hypothetical long-range scalar fields with electrons and nucleons are at the same level.

From this point of view an exceptional position would be assumed by a superlight hypothetical vector particle which would interact, just as an ordinary photon, only with the electromagnetic current, for owing to the electric neutrality of ordinary matter experiments of the Eötvös type do not yield any constraints on the coupling constants of such particles.

**2.** We shall consider a modified electrodynamics, containing two photons $A_1$ and $A_2$ (we shall call these paraphotons) with masses $m_1 < m_2$ and coupling constants $e_1$ and $e_2$, described by the Lagrangian

$$L = -{}^1/_4 F^2_{1\mu\nu} - {}^1/_4 F^2_{2\mu\nu} + {}^1/_2 m_1^2 A_{1\mu}^2 + {}^1/_2 m_2^2 A_{2\mu}^2 + j_\mu (e_1 A_{1\mu} + e_2 A_{2\mu}), \tag{1}$$

where $j_\mu$ is the ordinary electromagnetic current, and

$$F_{i\mu\nu} = \partial_\mu A_{i\nu} - \partial_\nu A_{i\mu}, \quad i = 1,\ 2. \tag{2}$$

The data on the magnetic field of Jupiter indicate[9] (see, also the reviews[10]) that $1/m_1 \gtrsim 10^6$ km (we use units with $\hbar = c = 1$), whereas data on galactic magnetic fields indicate[11] that $1/m_1 \gtrsim 10^{17}$ km. It is easy to see that these data also indicate that $e_1$ cannot be substantially smaller than $e$.

Let us find out what kind of constraints exist on $e_2$ at various values of $m_2$.

**3.** For small distances $r \ll 1/m_2$ the Coulomb interaction between charges is due to exchange of the fields $A_1$ and $A_2$, so that

$$\alpha = \alpha_1 + \alpha_2; \tag{3}$$

here $\alpha = e^2/4\pi = 1/137$, $\alpha_1 = e_1^2/4\pi$, and $\alpha_2 = e_2^2/4\pi$. For $1/m_1 \gg r \gtrsim 1/m_2$ the Coulomb potential $U(r) = \alpha/r$ is modified:

$$U(r) = \alpha_1/r + \alpha_2 e^{-m_2 r}/r. \tag{4}$$

In particular, this modification must lead to nonvanishing of the field inside of a uniformly charged sphere. As was shown by Maxwell,[12] for a potential $U(r)$ of arbitrary form produced by a point charge, the potential $V(r)$ of a uniformly charged sphere of radius $R$, at a distance $r$ from the center of the sphere is of the form

$$V(r) = \frac{1}{2Rr}[f(R+r) - f(|R-r|)], \quad f(r) = \int_0^r sU(s)\,ds. \tag{5}$$

For the potential (4) we have

$$f(r) = \alpha_1 r + \frac{\alpha_2}{m_2}(1 - e^{-m_2 r}).$$

As a result, the potential difference between a charged sphere of radius $R_1$ and a concentric uncharged sphere of smaller radius $R_2$ must be equal to

$$V(R_1) - V(R_2) = \frac{\alpha_2}{\alpha_1} e^{-m_2 R_1}\left(\frac{\operatorname{sh} m_2 R_2}{m_2 R_2} - \frac{\operatorname{sh} m_2 R_1}{m_2 R_1}\right) \times \left[1 + \frac{\alpha_2}{\alpha_1} e^{-m_2 R_1}\left(\frac{\operatorname{sh} m_2 R_1}{m_2 R_1}\right)\right]^{-1} V(R_1). \tag{6}$$

This yields, in the limit of small or large masses

$$\frac{V(R_1) - V(R_2)}{V(R_1)} = \begin{cases} \dfrac{\alpha_2}{\alpha}\dfrac{m_2^2}{6}(R_2^2 - R_1^2), & \text{if} \quad m_2 R_1 \ll 1 \quad \text{(7a)} \\ \dfrac{\alpha_2}{\alpha_1}\dfrac{1}{2 m_2 R_1}, & \text{if} \quad m_2 R_1 \gg 1. \quad \text{(7b)} \end{cases}$$

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Searches for such a potential difference were carried out by Plimpton and Lawton,[13] and later by Bartlett, Goldhagen, and Phillips.[14] In the latter experiment $R_1 = 46$ cm, $R_2 = 38$ cm, $[V(R_1) - V(R_2)]/V(R_1) < 10^{-14}$. This yields the best upper bound on $\alpha_2$ ($\alpha_2 \lesssim 10^{-16}$) obtained for $m_2 \approx 10^{-2}$ cm$^{-1}$. For $m_2 \lesssim 10^{-6}$ cm$^{-1}$ and $m_2 \gtrsim 10^{12}$ cm$^{-1}$ this experiment yields no bound on $\alpha_2$.

**4**. On the side of small values $m_2 \lesssim 10^{-6}$ cm$^{-1}$ bounds on the value of $\alpha_2$ can be obtained starting from the constancy of the observed spectra of stars. The reason for this is that a nonzero mass difference between the two photons must lead to specific oscillations in the observed intensity of light rays, the period of these oscillations increasing linearly with the frequency of the light.[1)]

As we have noted, the linear superposition

$$B_1 = (e_1 A_1 + e_2 A_2)/e \tag{8}$$

interacts with the electromagnetic current. The superposition orthogonal to it

$$B_2 = (-e_2 A_1 + e_1 A_2)/e \tag{9}$$

is sterile and does not interact with matter.

Consider a light wave with definite frequency (energy) $\omega$. On account of the mass difference the photons $A_1$ and $A_2$ will have different wave vectors (momenta):

$$k_1 = (\omega^2 - m_1^2)^{1/2} \approx \omega - m_1^2/2\omega, \tag{10a}$$

$$k_2 = (\omega^2 - m_2^2)^{1/2} \approx \omega - m_2^2/2\omega, \tag{10b}$$

$$q = k_1 - k_2 \approx (m_2^2 - m_1^2)/2\omega \approx m_2^2/2\omega. \tag{10c}$$

Therefore at some distance $r$ from the point of emission the usual (active) photon $B_1$ will be in a state containing the sterile component $B_2$.

$$\begin{aligned} B_1 &\to e^{-1}(e_1 A_1 e^{-i(\omega t - k_1 r)} + e_2 A_2 e^{-i(\omega t - k_2 r)}) \\ &= e^{-i(\omega t - k_1 r)}(e_1 A_1 + e_2 A_2 e^{-iqr}) e^{-1} \\ &= e^{-i(\omega t - k_1 r)}[e_1(e_1 B_1 - e_2 B_2) + e_2(e_1 B_2 + e_2 B_1) e^{-iqr}] e^{-2} \\ &= e^{-i(\omega t - k_1 r)}[(e_1^2 + e_2^2 e^{-iqr}) B_1 + e_1 e_2(-1 + e^{-iqr}) B_2] e^{-2}. \end{aligned} \tag{11}$$

It follows from this that $\rho_s$ (the relative intensity of the sterile component $B_2$ at the distance $r$) will be equal to:

$$\rho_s = 4\alpha_1 \alpha_2 \alpha^{-2} \sin^2(qr/2), \tag{12a}$$

and the relative intensity of the active component is:

$$\rho_a = 1 - 4\alpha_1 \alpha_2 \alpha^{-2} \sin^2(qr/2). \tag{12b}$$

If, for instance, $\alpha_1 = \alpha_2 = \alpha/2$, then at $r = r_0 \equiv \pi/q$ the intensity of the active component would vanish, and a monochromatic wave would become invisible. (For $1/m_2 = 10^8$ cm and $1/\omega = 10^{-5}$ cm $r_0$ will be approximately $10^{22}$ cm $\approx 10^4$ light years.) As a result of this the observable spectra of remote ($r \gtrsim r_0$) stars and galaxies would be significantly distorted. Assuming that such distortions of the spectra of stars do not exceed one percent, we must conclude that $\alpha_2/\alpha \lesssim 10^{-2}$ for $m_2 \approx 10^{-8}$ cm$^{-1}$.

It would be interesting to search for shallow ($\sim 10^{-3}$) periodic pulsations of the spectra of radio sources. In the radio region the values of $r_0$ are by 5 to 8 orders of magnitude smaller than in the optical region, and for $1/m_2 \lesssim 10$ to 100 km $r_0$ becomes smaller than the radius of the orbit of the Earth.

**5**. Oscillating transitions between the active and sterile photons in vacuum, produced by the nonzero mass difference between the paraphotons, could lead, in principle, to a peculiar light transmission effect through completely opaque screens, e.g., the light from a star eclipsed by the Moon could be transmitted through the Moon, or the light from a laser could pass through a mountain range. This refers, of course, not only to light, but also to radio waves, e.g., to the radio emissions from the Crab Nebula, when occluded by the Moon. This effect is based on the fact that on the path $L_1$ from the source to the screen a sterile component appears in the beam with relative intensity

$$\rho_s = 4\alpha_1 \alpha_2 \alpha^{-2} \sin^2(qL_1/2).$$

This component passes through the opaque screen practically unabsorbed, and then on the path $L_2$ from the screen to the detector is again partially converted into active photons, so that the ratio between the number of active photons reaching the detector to the number of active photons emitted by the source will be:

$$\rho = 16\alpha_2^2 \alpha_1^2 \alpha^{-4} \sin^2(qL_1/2) \sin^2(qL_2/2). \tag{13}$$

It is obvious that the effect disappears when either $L_1$ or $L_2$ are equal to zero. For $qL_1 \ll 1$ and $qL_2 \ll 1$ the effect is maximal when the screen is equidistant to source and detector.

Unfortunately, a real observation of such effects seems unreliable owing to the insufficient intensity of the sources and the insufficient sensitivity of the receivers. Translunar observations are also hampered by the emission of the Moon itself (see, e.g., Ref. 17).

If one assumes for an emitter in the wavelength band between 30 and 40 cm a power of one GW (gigawatt), and for the receiver a sensitivity of 10 fW (femtowatt), then the power ratio is $10^{-23}$. On the other hand, for $m_2 = 10^{-4}$ cm$^{-1}$, $\lambda = 30$ cm, $\omega = 2\pi/\lambda = 0.2$ cm$^{-1}$, $L_1 = L_2 = 80$ km the expected $\rho$ is of the order of $10^{-3}(\alpha_2/\alpha)^2 \lesssim 10^{-23}$ since experiment [14] yields for $m_2 = 10^{-4}$ cm$^{-1}$ the ratio $\alpha_2/\alpha \lesssim 10^{-10}$. However, this estimate seems to be too high by several orders of magnitude, since it does not take account of the angular divergence of the beam.

In the case of laser beams the situation is even worse. Consider an emitter with a wavelength $\lambda = 10^{-3}$ cm, with a power of the order of one kilowatt, and a receiver capable of recording one photon per second. Here again the ratio of sensitivity to power is of the order of $10^{-23}$. However, since the effect we are interested in falls off in proportion to $\lambda^4$, its expected magnitude, everything else remaining equal, will be by 16 orders of magnitude lower than in the decimeter radio band.

**6**. It seems that in the optical range the expected effect would be considerably larger if we would take larger values of $m_2$ (the effect is proportional to $m_2^8$). However, for values of $m_2$ larger than 10 cm$^{-1}$ there exists a very low bound for $\alpha_2$ because intensive emission of sterile photons would cause an inadmissibly rapid evolution of the Sun.

It is easy to obtain an appropriate estimate starting from the usual mechanism of photon diffusion from the center of the Sun to its periphery (see, e.g., Ref. 18). As the roughest approximation we neglect the change of plasma density and temperature with the distance from the solar center. We regard the Sun as a homogeneous ball of radius $R_\odot = 7\times 10^{10}$ cm with a density $\rho = 1.4$ g/cm$^3$ containing $n \approx 7\times 10^{23}$ electrons per cubic centimeter. For the photon-electron scattering cross section we assume the Thomson cross section

$$\sigma = 8\pi\alpha^2/3m_e^2 \approx 0.7\times 10^{-24}\ \text{cm}^2. \tag{14}$$

Then we obtain for the mean free path of the electron $l_0 = 1/n\sigma \approx 2$ cm. This implies that the diffusion path of the photon contains $N = (R_\odot/l_0)^2 \approx 10^{21}$ links. In order for the effect of emission of sterile photons to be acceptably small it is necessary that the probability $P$ for their production on each link be smaller than $1/N$. It is obvious that

$$P = \rho_s = \frac{4\alpha_1\alpha_2}{\alpha^2}\sin^2\left(\frac{ql_0}{2}\right) \approx \left(\frac{\alpha_2}{\alpha}\right)(ql_0)^2 = \frac{1}{4}\frac{\alpha_2}{\alpha}\frac{m_2^4 l_0^2}{\omega^2}. \tag{15}$$

Choosing the mean frequency of the photon, $\omega$, of the order of the mean temperature of the Sun $T = 2\times 10^5$ K $= 20$ eV $= 10^6$ cm$^{-1}$, for $m_2^2 \ll 2\pi\omega/l_0 \approx 3\times 10^6$ cm$^2$ one arrives at the bounds

$$\frac{\alpha_2}{\alpha}(m_2^4\cdot 10^{-12}\ \text{cm}^4) \lesssim 10^{-21} \quad \text{or} \quad \frac{\alpha_2}{\alpha}m_2^4 < 10^{-9}\text{cm}^{-4}. \tag{16}$$

For $m_2 \gtrsim 10^3$ cm$^{-1}$ we obtain $\alpha_2/\alpha \lesssim 10^2$. Let us compare this with the restrictions following from the experiment[14]:

$$\frac{\alpha_2}{\alpha}m_2^2 < 10^{-16}\ \text{cm}^{-2}, \quad \text{if} \quad m_2 < 10^{-2}\ \text{cm}^{-1}, \tag{17a}$$

$$\frac{\alpha_2}{\alpha}\frac{1}{m_2} < 10^{-12}\ \text{cm}, \quad \text{if} \quad m_2 > 10^{-2}\ \text{cm}^{-1}. \tag{17b}$$

For $m_2 \gtrsim 10$ cm$^{-1}$ the solar bound on $\alpha_2$ becomes stricter than the electrostatic one. Taking into account the data on the nonhomogeneity of the Sun (see Ref. 19), which can be done making use of the results of Ref. 18, will modify these bounds somewhat.

In the estimates given above we assume that the "sterile luminosity" of the Sun may be comparable to its ordinary luminosity. The solar limit on $\alpha_2$ could be considerably improved if one takes into account that such a large flux of sterile photons incident on Earth would be easily detectable. In this connection it is interesting to search for solar photons in dark underground laboratories.

7. The scope of this article does not include a discussion of the region of values $m_2 \gtrsim 10^2$ eV. We just make a few remarks.

We note, in particular, that the bounds which follow from the data on the electron magnetic moment $(g-2)_e$ are considerably weaker than those yielded by the absence of strong emission of sterile photons by the Sun. This is due to the fact that the paraphoton correction to $(g-2)_e$ is of the order $\alpha_2(m_2/m_e)^2\ln(m_e/m_2)$, and it is not observable for $m_2/m_e \lesssim 10^{-4}$, even if $\alpha_2/\alpha$ is not very small.

It is interesting to compare the astrophysical restrictions on $\alpha_2$ with those derived from atomic spectroscopy by looking at supernarrow resonances low-energy $e^+e^-$ annhilation, as well as from searches for anomalously penetrating photons in x-ray experiments, in synchrotron radiation, experiments with nuclear gamma rays, photon beams from neutral pion decays, etc.

8. Having discussed the possible phenomenological manifestations of paraphotons, it is appropriate to turn to more general questions.

We have considered above a model with two paraphotons. It is quite obvious that similar pheonomena would occur in models with a larger number of paraphotons. Only their description will be more cumbersome and will involve a larger number of free parameters. I have been unable to think of any other type of modification of electrodynamics without entering into contradiction with the fundamental principles of contemporary quantum field theory. Thus, an attempt to introduce nonconservation of electric charge (no matter how small) into the theory cannot be achieved without a violation of causality (see Ref. 20).

Let us now discuss to what extent it is realistic to expect any deviations at all from standard electrodynamics. After all, electrodynamics (both classical and particularly quantum electrodynamics) is unique in its theoretical beauty. And any conceivable deviations from it can hardly be labeled as beautiful. This is, of course, a strong argument. But we know that this argument "does not work" at short distances. Inspired by the beauty of the classical theories of electromagnetism and gravitation, Einstein has tried to join them into a unified theory describing the whole world. Nevertheless, the beauty of electrodynamics has not prevented the existence of quite different physics at short distances, that of the weak and strong interactions. Moreover, as we understand things today, a higher beauty resides in the unification of all four interactions.

The most developed theoretical models of the so-called grand unification predict a large range of distances between $10^{-16}$ cm and $10^{-28}$ cm which should not contain any new fundamental physics. This region has been christened "the gauge desert." The majority of physicists treat the idea of such a desert with suspicion. At the same time, as far as the larger distances are concerned, the common viewpoint is that nothing new is to be expected either in electrodynamics or in gravitation theory up to distances of $10^{28}$ cm. To this substantially larger desert one has become used: one does not notice its aridity.

Thus, the crux of the matter is not beauty, but the fact that we have the impression of having studied everything concerning the large distances. We have indeed not discovered so far any phenomena contradicting standard electrodynamics. But, as we have seen above, this can be explained to a large degree by the fact that the accuracy of the corresponding measurements is insufficient (if, for example, $\alpha_2$ and (or) $m_2$ are small).

Here we again return to the question of the mass hierarchy, mentioned at the beginning of the paper. As is well known, during recent years, the main progress in the study of fundemental interactions is related to gauge symmetries. In the framework of gauge theories the problem of the particle masses cannot be separated from the problem of the mechanism of symmetry breaking, and will probably not be solved as long as the scalar bosons (see Ref. 21) will not be discovered experimentally. Can different paraphotons have masses which differ from one another and from the masses of the other elementary particles by many orders of magnitude? So far, the theory cannot give a definite answer to this question. Taking into account the already known hierarchy (from the Planck mass down to the masses of the electron and the neutrino), we have no basis for considering such a possibility as unlikely.

As regards the relation between the fine-structure constant $\alpha$ and the coupling constant $\alpha_2$ (as well as the similar gauge charges in the case of a larger number of paraphotons) one cannot exclude here either the existence of a hierarchical ladder, the rungs of which are separated from each other by many orders of magnitude.

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[1)] The photon oscillations under discussion are similar to the long familiar oscillations of neutral kaons.[15] They are even more reminiscent of neutrino oscillations,[16] for which searches are now under way in several laboratories.

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