Theory of adiabatic surface scattering of electrons in metals

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A new electron-imperfect surface collision integral is obtained which takes account of the conservation of the adiabatic invariant in a system in which there is periodic motion along one of the coordinates. The surface relaxation rate and the impedance of a metal in a parallel magnetic field are computed with the aid of this integral.

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1. INTRODUCTION

There arises in the investigation of many physical phenomena occurring in metals the problem of the solution of the Boltzmann kinetic equation for the electron distribution function. Its general solution contains an arbitrary function, for the determination of which we must formulate boundary conditions at the sample surface. From the physical point of view, such conditions describe the interaction of the conduction electrons with the surface.

The simplest of the known boundary conditions is the relation (proposed by Fuchs[1])

$$f_+ = \rho f_- \tag{1.1}$$

between the nonequilibrium parts of the distribution functions of the electrons impinging on (f_-), and flying away from (f_+), the boundary. In Eq. (1.1) the properties of the boundary are characterized by the parameter ρ, which is the probability for specular reflection of an electron from the surface of the metal.

The phenomenological condition (1.1) correctly describes the limiting cases of diffuse ($\rho = 0$) and specular ($\rho = 1$) reflections. In the remaining cases it is necessary to take into account both the dependence of the coefficient ρ on the magnitude of the electron-momentum component normal to the surface and the possibility of the scattering of the electron in the "nonspecular" directions. The latter circumstance leads to a situation in which the boundary condition is transformed into an integral—with respect to the momenta—relation between the functions f_- and f_+. In the case of elastic scattering of the electrons at the surface of the metal, this relation can be written in the form

$$f_+(x=0, \mathbf{p}) = f_-(x=0, \mathbf{p}) - \int \frac{d\mathbf{p}'}{(2\pi\hbar)^2} V(\mathbf{p}, \mathbf{p}') [f_-(x=0, \mathbf{p}) - f_-(x=0, \mathbf{p}')]. \tag{1.2}$$

Here the x axis is the inner normal to the $x = 0$ boundary and $\mathbf{p}$ is the two-dimensional electron momentum in the plane of the sample (the yz plane). The quantity $V(\mathbf{p},\mathbf{p}') = V(\mathbf{p}',\mathbf{p})$ is the probability density for the occurrence of the electron transition from the state $\mathbf{p}$ into the state $\mathbf{p}'$ during a single scattering. The current state of the theory of diffraction allows us to calculate the kernel $V(\mathbf{p},\mathbf{p}')$ in its explicit form only in the case when the electron reflection is nearly specular, i.e., when the integral term in (1.2) is small compared to $f_-(x = 0,\mathbf{p})$. Thus far, the function $V(\mathbf{p},\mathbf{p}')$ has been determined for different types of surface defects (see Andreev's[2] and Okulov and Ustinov's[3] review articles).

In the model of a statistically homogeneous rough boundary with a root-mean-square roughness height σ and for an isotropic quadratic conduction-electron dispersion law the probability $V(\mathbf{p},\mathbf{p}')$ is described by the formula[4]

$$V(\mathbf{p}, \mathbf{p}') = \frac{4\sigma^2}{\hbar^2} p_x p_x' W(\mathbf{p}-\mathbf{p}'), \quad p_x = (p_F^2 - p^2)^{1/2}, \quad p_x' = (p_F^2 - p'^2)^{1/2}. \tag{1.3}$$

Here $W(\mathbf{p})$ is the spatial Fourier transform of the binary correlation function for the roughnesses. The characteristic variation scale for $W(\mathbf{p})$ is $2\pi\hbar/L$ where L is the mean length of the inhomogeneities along the boundary (the correlation length). The domain of the integration in (1.2) is bounded by the Fermi momentum $p_F(|\mathbf{p}'| \leqslant p_F)$.

A general feature of all the known collision integrals[2,3] is the fact that they are valid in the approximation of single electron scattering on a random surface. In the presence of periodic motion in the direction perpendicular to the surface, the electrons can repeatedly return to the sample boundary, i.e., multiple scattering occurs. In this situation there arises the question of the correlation of the successive reflections. The application of the boundary condition (1.2) in systems with periodic motion reduces to a simple summation of individual scattering events, and yields the correct results if there is no correlation. The absence of correlation implies that the distance Λ between two successive collisions of an electron with the boundary is much greater than the mean length L of the roughnesses. In the opposite case (i.e., for $\Lambda \lesssim L$) the successive reflections of the electron are not independent, the boundary condition (1.2) is inapplicable, and, thus, there arises the need for a new boundary condition.

In the present paper we derive for the surface collision integral a quasiclassical expression that takes account of the correlation in the case when multiple electron scattering events occur at the sample boundary. The periodic motion is secured by a constant and uniform magnetic field $\mathbf{H}$ oriented along the metal-vacuum boundary (parallel to the z axis). We show that, in the case of strong correlations (i.e., for $L \gg \Lambda$),

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the electron scattering on the random surface potential is adiabatic, and conserves the adiabatic invariant in the magnetic field. We compute in this limiting case the transport surface relaxation rate ν^s for the electron and the high-frequency impedance of the metal. The obtained functional dependences of the relaxation rate ν^s and the surface impedance on the magnetic field differ significantly from the corresponding dependences obtained in the case of weak correlations.[5-7]

2. THE AVERAGED ELECTRICAL CONDUCTIVITY OF A METAL IN A MAGNETIC FIELD

The electron-rough boundary collision integral arises naturally in the computation of the current excited by a plane electromagnetic wave incident from vacuum on the surface of a metal. According to Konstantinov and Perel',[8] the general quantum-mechanical expression for the current $\mathbf{j}=\mathbf{j}(x,\mathbf{r})e^{-i\omega t}$ in the approximation linear in the field $\mathbf{E}=\mathbf{E}(x,\mathbf{r})e^{-i\omega t}$ has the form

$$j_\alpha(x,\mathbf{r})=\int dx'\,d\mathbf{r}'\chi_{\alpha\beta}(x,\mathbf{r};x',\mathbf{r}')E_\beta(x',\mathbf{r}'); \qquad \alpha,\beta=y,z,\quad \mathbf{r}=\{y,z\}. \tag{2.1}$$

The integration is performed over the volume of the sample. The response function $\chi_{\alpha\beta}(x,\mathbf{r};x',\mathbf{r}')$ is given by the expression

$$\chi_{\alpha\beta}(x,\mathbf{r};x',\mathbf{r}') =2\hbar i\sum_{a,a'}\frac{F(\varepsilon_a)-F(\varepsilon_{a'})}{\varepsilon_a-\varepsilon_{a'}}\frac{(a|J_\alpha(x,\mathbf{r})|a')\,(a'|J_\beta(x',\mathbf{r}')|a)}{\varepsilon_{a'}-\varepsilon_a-\hbar\omega-i0}. \tag{2.2}$$

Here the factor 2 takes account of the spin degeneracy; each of the symbols a and a' denotes the complete set of quantum numbers in the steady state; ε_a and $\varepsilon_{a'}$ are the eigenvalues of the electron Hamiltonian in the magnetic field, which depend on the specific realization of the shape of the surface; $F(\varepsilon_a)$ is the equilibrium Fermi distribution function; and $(a|J_\alpha(x,\mathbf{r})|a')$ is the matrix element of the α component of the current-density operator in the magnetic field $\mathbf{H}$.

As a result of the multiple collisions with the surface, there occurs self-averaging of the electron state over the ensemble of random-boundary realizations. Let us write the equation of the boundary in the form $x=\xi(\mathbf{r})$, where $\xi(\mathbf{r})$ is a random function. Thus, the observable quantities should be averaged over the set of functions $\xi(\mathbf{r})$.

As a result of the statistical homogeneity of the surface, the field $\mathbf{E}(x,\mathbf{r})$ in the metal depends only on the coordinate x, while the average value $\langle\chi_{\alpha\beta}(x,\mathbf{r};x',\mathbf{r}')\rangle$ of the response function depends on x,x', and the difference $\mathbf{r}-\mathbf{r}'$. Taking this fact into account, we find from (2.1) that

$$\mathbf{j}_\alpha(x)=\int_0^\infty dx'Q_{\alpha\beta}(x,x')\mathbf{E}_\beta(x'), \tag{2.3}$$

$$Q_{\alpha\beta}(x,x')=\int d\mathbf{r}'\langle\chi_{\alpha\beta}(x,\mathbf{r};x',\mathbf{r}')\rangle. \tag{2.4}$$

To average $\chi_{\alpha\beta}$, we express its dissipation parts in terms of the product of the two single-particle Green functions of the Schrödinger equation in much the same way as is done by Edwards[9] for the static conductivity. Below we shall determine the nondissipative part of the conductivity with the aid of the Kramers-Krönig dispersion relation. Let us, omitting the standard transformations, give the expression for the dissipative part $Q_{\alpha\beta'}(x,x')$ of the electrical conductivity tensor:

$$Q_{\alpha\beta'}(x,x')=-4\pi e^2\hbar^3\int_0^\infty d\varepsilon\,\frac{F(\varepsilon+\hbar\omega)-F(\varepsilon)}{\hbar\omega} \times\int_{-\infty}^\infty dp\int_{-\infty}^\infty dq v_\alpha(\mathbf{q},x)\int_{-\infty}^\infty dp' v_\beta(\mathbf{p}',x') \times\mathrm{Re}\langle G_+(\mathbf{p}',\mathbf{p};x',x)\bar{G}_-(\mathbf{q},\mathbf{p}';x,x')\rangle. \tag{2.5}$$

Here $v_\alpha(\mathbf{p},x)=(p_\alpha+eA_\alpha/c)/m$ is the α component of the electron velocity in the magnetic field; e and m are the absolute value of the charge and the mass of the electron; $\mathbf{A}=\{0,Hx,0\}$ is the vector potential of the magnetic field $\mathbf{H}$; and c is the velocity of light. The Green functions satisfy the Schrödinger equation:

$$\left[-\mu^2\frac{d^2}{dx^2}+\left(\frac{x-X}{2\mu}\right)^2-(\eta\mp i0)\right]G_\pm(p,p';x,x') =-\frac{\delta(\mathbf{p}-\mathbf{p}')\delta(x-x')}{\hbar\Omega(2\pi\hbar)^2} \tag{2.6}$$

Here $\mu=(\hbar/2m\Omega)^{1/2}$ is the magnetic length; $X=-cp_y/eH$ is the coordinate of the center of the electron orbit in the magnetic field; and $\eta=(\varepsilon-p_z^2/2m)/\hbar\Omega$; $\Omega=eH/mc$ is the Larmor frequency. We do not write the energy variable ε of the Green function, and the tilde mark denotes an $\hbar\omega$ energy shift.

We must formulate boundary conditions for the differential equation (2.6). One of them is the requirement that the G function vanish at $x\to\infty$:

$$G_\pm(\mathbf{p},\mathbf{p}';\ x\to\infty,\ x')=0. \tag{2.7}$$

The condition on the surface of the metal is formulated in the coordinate representation:

$$\mathcal{J}_\pm(\mathbf{r},\mathbf{r}';\ x=\xi(\mathbf{r}),\ x')=0. \tag{2.8}$$

The transition to the momentum representation is effected in accordance with the formula

$$G_\pm(\mathbf{p},\mathbf{p}';x,x')=(2\pi\hbar)^{-4} \times\int_{-\infty}^\infty d\mathbf{r}\,d\mathbf{r}'\exp\left[\frac{i}{\hbar}(\mathbf{p}'\mathbf{r}'-\mathbf{p}\mathbf{r})\right]\mathcal{J}_\pm(\mathbf{r},\mathbf{r}';x,x'). \tag{2.9}$$

In order to rewrite the condition (2.8) in the momentum representation, we must bear in mind that the electron scattering on the rough surface is assumed to be weak, i.e., that the metal boundary $x=\xi(\mathbf{r})$ is assumed to differ little from the $x=0$ plane. This circumstance allows us to expand (2.8) in powers of the small magnitude of the deviation $\xi(\mathbf{r})$ of the real boundary from the perfect plane:

$$\mathcal{J}_\pm(\mathbf{r},\mathbf{r}';\ x=0,\ x')+\xi(\mathbf{r})\mathcal{J}_\pm'(\mathbf{r},\mathbf{r}';\ x=0,\ x')=0. \tag{2.10}$$

The primes denote the x derivatives of the quantities in question.

The approximate boundary condition (2.10) approximates the exact condition well if the first term in it is much greater than the second term. For this to be the case, the x "component" $\hbar/p_x$ of the de Broglie wavelength must be much greater than the root-mean-square height σ of the roughnesses:

$$(p_x\sigma/\hbar)^2 \ll 1. \tag{2.11}$$

In typical metals this inequality is fulfilled for those electrons which "graze" almost parallel to the surface. It is precisely these electrons that determine the conductivity in the case, characteristic of metals, of strong spatial dispersion, and therefore the approximation (2.10) is valid.

The condition (2.10) in the momentum representation has the form

$$G_\pm(\mathbf{p},\mathbf{p}';x=0,x')+\int_{-\infty}^{\infty}\frac{d\mathbf{q}}{(2\pi\hbar)^2}\xi(\mathbf{p}-\mathbf{q})G_\pm'(\mathbf{q},\mathbf{p}';x=0,x')=0, \tag{2.12}$$

$$\xi(\mathbf{p})=\int_{-\infty}^{\infty}d\mathbf{r}\exp(-i\mathbf{p}\mathbf{r}/\hbar)\xi(\mathbf{r}).$$

The formal solution to Eq. (2.6) with the boundary conditions (2.7) and (2.12) can be represented in the form of an iterative series containing moments (of different orders) of the function $\xi(\mathbf{p})$. The combination $G_+(\mathbf{p}',\mathbf{p};x',x)\tilde{G}_-(\mathbf{q},\mathbf{p}';x,x')$ is obtained as a result of the multiplication of two such series. To average such a product, we used a diagrammatic technique similar to the one that has been developed in the theory of wave scattering on statistically rough surfaces.[10] The use of the diagrammatic technique enables us to sum the iterative series, which turns out to be equivalent to an integral equation. Let us, omitting the intermediate calculations, give the final expression for the averaged two-particle Green function:

$$\langle G_+(\mathbf{p}',\mathbf{p};x',x)\tilde{G}_-(\mathbf{q},\mathbf{p}';x,x')\rangle=(2\pi\hbar)^{-4}(\hbar\Omega)^{-2}\delta(\mathbf{p}-\mathbf{q})\times\frac{D_{\eta(\varepsilon,p_z)-1/2}((x-X)/\mu)\,D_{\eta(\varepsilon+\hbar\omega,p_z)-1/2}((x-X)/\mu)}{D'_{\eta(\varepsilon,p_z)-1/2}(-X/\mu)\,D'_{\eta(\varepsilon+\hbar\omega,p_z)-1/2}(-X/\mu)}\times\frac{D_{\eta(\varepsilon,p_z')-1/2}((x'-X')/\mu)\,D_{\eta(\varepsilon+\hbar\omega,p_z')-1/2}((x'-X')/\mu)}{D'_{\eta(\varepsilon,p_z')-1/2}(-X'/\mu)\,D'_{\eta(\varepsilon+\hbar\omega,p_z')-1/2}(-X'/\mu)}g(\mathbf{p},\mathbf{p}'). \tag{2.13}$$

Here $D_{\eta-1/2}[(x-X)/\mu]$ is a parabolic cylinder function, being a solution to Eq. (2.6) with the right-hand side replaced by zero; $X'=-cp_y'/eH$; and a prime on a parabolic cylinder function denotes differentiation with respect to the argument. The function $g(\mathbf{p},\mathbf{p}')$ is determined from an equation of the Bethe-Salpeter type, that in the ladder approximation has the form

$$g(\mathbf{p},\mathbf{p}')=R_+(\mathbf{p})R_-(\mathbf{p})\times\left\{\delta(\mathbf{p}-\mathbf{q})+\sigma^2\int_{-\infty}^{\infty}\frac{d\mathbf{q}}{(2\pi\hbar)^2}W(\mathbf{p}-\mathbf{q})g(\mathbf{q},\mathbf{p}')\right\}. \tag{2.14}$$

In Eq. (2.14) the quantity $R_\pm(\mathbf{p})$ is the pole part of the averaged single-particle Green function:

$$R_\pm(\mathbf{p})=\{[\mu D_{\eta(\varepsilon,p_z)-1/2}(-X/\mu)/D'_{\eta(\varepsilon,p_z)-1/2}(-X/\mu)]\pm iM(\mathbf{p})\}^{-1}. \tag{2.15}$$

The mass operator $M(\mathbf{p})$ determines the single-particle damping of the electron states, that results from the scattering of the electrons by the rough surface. The mass operator is computed in the Born approximation in Ref. 11:

$$M(\mathbf{p})=\frac{\sigma^2}{4\mu^2}\int_{-q_0}^{q_0}\frac{dq_z}{2\pi\hbar}\sum_{n=1}^{N(q_z)}W(p_y-p_y^n(q_z);p_z-q_z),$$

$$q_0=[2m(\varepsilon-{}^1/_2\hbar\Omega)]^{1/2}. \tag{2.16}$$

The summation over the magnetic quantum number n in (2.16) terminates at the integral part, denoted by $N(q_z)$, of the quantity $[(\varepsilon-q_z^2/2m)/\hbar\Omega]+1/4$.

As usual, the poles of the single-particle Green function determine the spectrum of the quantum states. In metals with perfect boundaries (i.e., for which $\sigma=0$) the mass operator is equal to zero, and from (2.15) we obtain the quantization condition:

$$D_{\eta(\varepsilon,p_z)-1/2}(cp_y/\mu eH)=0. \tag{2.17}$$

If the transverse energy η of the electron is fixed, then Eq. (2.17) leads to the quantization of the y component $p_y=p_y^n(p_z)$ $[n=1,2,\ldots,N(p_z)]$ of the electron momentum. The quantum states determined by the dispersion equation (2.17) are strictly steady states. The presence of a random surface potential leads to a nonzero probability for transition of an electron from one state into another, i.e., the "lifetime" in a definite state becomes finite. If the electron reflection is nearly specular, then we can consider the states to be quasistationary, and retain the previous classification for them. For this reason, the external parameters (ε,p_y,p_z) entering into the right member of (2.16) correspond to the unperturbed spectrum (2.17) of the electrons.

The dissipative conductivity (2.5) of the metal can be represented in the form of a double sum over the discrete quantum numbers n and n'. To do this, we must resolve the single-particle Green functions $R_+(\mathbf{p})$ and $\tilde{R}_-(\mathbf{p})$ into the simplest fractions, and perform the integration over p_y, using the theory of residues. A similar procedure should be followed in Eq. (2.14). For the computation of the nondissipative part of the conductivity we shall use the Kramers-Krönig relation. As a result, the expression for $Q_{\alpha\beta}(x,x')$ will have a structure typical of quantum conductivity.

In the presence of the effects of spatial dispersion, it is convenient to analyze the conductivity of the metal in the Fourier representation, in which the relation (2.3) has the form

$$j_\alpha(k)\equiv 2\int_0^\infty dx\cos(kx)j_\alpha(x)=\frac{1}{\pi}\int_0^\infty dk'Q_{\alpha\beta}(k,k')\mathscr{E}_\beta(k'). \tag{2.18}$$

In the majority of cases the conduction electrons in the metal can be considered in the quasiclassical approximation. Therefore, let us at once give the quasiclassical asymptotic form of the tensor[1)] $Q_{\alpha\alpha}(k,k')$:

$$Q_{\alpha\alpha}(k,k')=\frac{1}{\pi^2\hbar^2\Omega}\int_{-p_F}^{p_F}dp_z\sum_{n=1}^{N(p_z)}\sum_{s=-\infty}^{\infty}\frac{\varphi_n}{R_\perp\sin\varphi_n}\times(n|I_\alpha(k,p_z)|n+s)A_s(k',\varphi_n,p_z). \tag{2.19}$$

The summation over the index s takes account of the transitions between the $(n+s)$-th and n-th quantum states of the surface electrons. The transition frequency $\omega_{ns} = \pi s\Omega/\varphi_n$. The angle φ_n of impact of the electron with the boundary is connected with the momentum components by the relations $p_x^n = p_\perp \sin\varphi_n$ and $p_y^n = p_\perp \cos\varphi_n$, where $p_\perp = (p_F^2 - p_z^2)^{1/2}$. From (2.17) we obtain the following quasiclassical quantization rule for the angle φ (see, for example, Ref. 12):

$$\varphi - \sin\varphi\cos\varphi = \pi(n - {}^1/_4)\hbar\Omega/(\varepsilon_F - p_z^2/2m). \tag{2.20}$$

The matrix element of the current-density operator can be represented in the form

$$(n|I_\alpha(k,p_z)|n+s) = -\frac{e}{m\varphi_n}\int_0^{\varphi_n} d\lambda\, p_\alpha(p_z,\lambda) \times\cos(\pi s\lambda/\varphi_n)\cos[kR_\perp(\cos\varphi_n - \cos\lambda)].$$

Here $R_\perp = cp_\perp/eH$ is the radius of gyration of an electron in the magnetic field; $p_y(p_z,\lambda) = p_\perp\cos\lambda$ and $p_z(p_z,\lambda) = p_z$ are the components of the electron momentum; and λ is the azimuthal angle.

The function $A_s(k,\varphi_n,p_z)$ should be found from the equation

$$[-i(\omega - \omega_{ns}) + \nu]A_s(k,\varphi_n,p_z) + \frac{\sigma^2\Omega}{2\mu^2\hbar\varphi_n} p_x^n(p_z) \times\sum_{\bar{s}=-\infty}^{\infty}\int_{-p_F}^{p_F}\frac{dp_z'}{2\pi\hbar}\sum_{\bar{n}=1}^{N(p_z')} W(p_y^n(p_z) - p_y^{\bar{n}}(p_z'); p_z - p_z') \times[A_s(k,\varphi_n,p_z) - (-1)^{s+\bar{s}}A_{\bar{s}}(k,\varphi_{\bar{n}},p_z')] = (n|I_\alpha(k,p_z)|n+s). \tag{2.21}$$

This equation has a structure that is characteristic of the linearized kinetic equation. The first term in the left member is similar to the field derivative of the nonequilibrium distribution function [with allowance made for the volume collisions in the τ approximation ($\nu = \tau^{-1}$)]; the remaining terms in the left member have the form "departure minus arrival," and is the quantum electron-surface collision integral; finally, figuring on the right is the usual inhomogeneity arising from $e\mathscr{E}_\alpha v_\alpha$. Allowance for the arrival term leads, as always, to the replacement of the total scattering cross section by the transport cross section.

Let us emphasize that the formula (2.19) gives the conductivity of the electrons that collide with the surface of the metal. The conductivity of the volume electrons naturally does not depend on the properties of the surface, and, when the reflection is nearly specular, its consideration leads only to small corrections.

The quasiclassical asymptotic form of $Q_{\alpha\alpha}(k,k')$ is valid if

$$\hbar/p_x \ll R_\perp\varphi^2/2 \text{ and } |s| \ll n, \tag{2.22}$$

i.e., if the dimension of the trajectory of a surface electron along the x axis is much greater than the de Broglie wavelength $\hbar/p_x$ ($n \gg 1$), and we can then consider only the transitions to the neighboring levels.

In deriving the results of this section, we made essential use of the fact that the electron reflection from the surface is nearly specular. This requirement imposes definite limitations on the geometrical dimensions σ and L of the roughnesses. One of them is expressed by the inequality (2.11). It can be shown that, besides the inequality (2.11), the following conditions should be fulfilled:

$$(\sigma/L)^2 \ll 1, \quad p_F\sigma^2/\hbar L \ll 1. \tag{2.23}$$

The first of them implies that the mean slopes of the inhomogeneities should be small, while the second allows us to neglect the diffraction occurring in a single electron scattering. The simultaneous satisfaction of the inequalities (2.11) and (2.23) ensures the applicability of the Born approximation to the surface collision integral in (2.21). If we replace the collision integral by the effective surface-relaxation rate ν^s, then the requirement that the reflection be nearly specular implies that ν^s is low compared to the frequency of the periodic electron motion along the x axis:

$$\nu^s \ll \Omega/2\varphi. \tag{2.24}$$

The classical result for the conductivity of the surface electrons is obtained from the formula (2.19) by going over in it from the summation over n to integration over the angle φ with the aid of the dispersion equation (2.20):

$$\sum_{n=1}^{N(p_z)} \ldots = \frac{p_\perp^2}{\pi\hbar m\Omega}\int_0^\pi d\varphi\sin^2\varphi\ldots. \tag{2.25}$$

In the kinetic equation (2.21) the sum over $\bar{n}$ can be replaced by an integral over p_y', i.e., we can set

$$\int_{-p_F}^{p_F}\frac{dp_z'}{2\pi\hbar}\sum_{\bar{n}=1}^{N(p_z')}\ldots = \frac{4\mu^2}{\hbar}\int\frac{d^2p'}{(2\pi\hbar)^2}p_x'\ldots, \tag{2.26}$$

if the correlator $W[p_y^n(p_z) - p_y^{\bar{n}}(p_z'); p_z - p_z']$ is a smooth function of the number $\bar{n}$. When the number $\bar{n}$ is changed by one, the argument of the correlation function changes by $|\partial p_y^{\bar{n}}/\partial\bar{n}|$. Taking into account the fact that W changes significantly over momenta of the order of $2\pi\hbar/L$, we find that the replacement (2.26) of the sum by an integral is possible if

$$|\partial p_y^n/\partial n| \ll 2\pi\hbar/L. \tag{2.27}$$

From (2.20) we easily find that $|\partial p_y^n/\partial n| = 2\pi\hbar/\Lambda_\perp$, where $\Lambda_\perp = 2R_\perp\sin\varphi_n$ is the distance traversed by an electron along the y axis between two successive collisions with the surface. Thus, the condition (2.27) implies that $L \ll \Lambda_\perp$, i.e., that the successive electron-scattering events occurring at the interface are independent events. In this case the collision integral in (2.21) is equivalent to the expressions (1.2) and (1.3) given in the Introduction. The weak-correlation case (2.27) is investigated in detail in Refs. 5–7, and therefore we shall not discuss it here.

3. THE SURFACE IMPEDANCE OF A METAL IN THE PRESENCE OF STRONG CORRELATION OF THE SUCCESSIVE REFLECTIONS

Let us use the results obtained in the preceding section to compute the surface impedance of a metal located in a

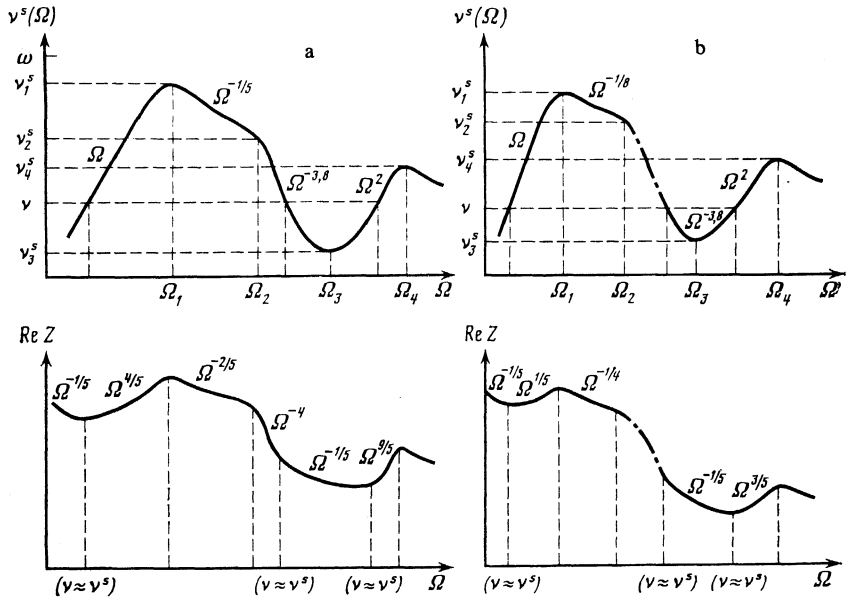

FIG. 1. Schematic dependences of the surface relaxation rate ν^s and the real part Re Z of the impedance on the cyclotron frequency Ω (the magnetic field). Indicated beside the curves are the laws of variation with the frequency Ω; the other symbols are defined in the text.

magnetic field. We shall consider the case of the anomalous skin effect,

$$R/\delta \gg 1+|\gamma|^2, \quad \gamma=(\nu-i\omega)/\Omega, \tag{3.1}$$

in which the electrodynamic properties of the metal are determined by the group of grazing electrons[13] with characteristic angles of hop $\varphi \sim (\delta/R)^{1/2}$ (δ is the skin depth and $R = cp_F/eH$ is the maximum cyclotron radius). The grazing electrons move inside the skin layer along small arcs of the Larmor circle, and repeatedly collide with the surface. The condition $\omega \ll \pi\Omega/\varphi$, which follows from (3.1), implies that noncollisional resonance absorption of the wave energy by the grazing electrons does not occur. Consequently, in this situation the electromagnetic absorption occurs in collisions with both the volume and surface scatterers. The inequality (3.1) also contains the condition $\nu \ll \Omega/2\varphi$, which, together with (2.24), ensures the periodicity of the motion of the grazing electrons.

The surface impedance of the metal

$$Z_\alpha = (4\pi i\omega/c^2) E_\alpha(0)/E_\alpha'(0) \tag{3.2}$$

is determined by solving the Maxwell equation for the Fourier transform of the electric field:

$$k^2 \mathscr{E}_\alpha(k) + 2E_\alpha'(0) = 4\pi i\omega c^{-2} j_\alpha(k). \tag{3.3}$$

Here the prime denotes the x derivative of the quantity in question.

An analysis of this equation can be performed only after extreme simplification of the kernel of the integral conductivity tensor $Q_{\alpha\alpha}(k,k')$. The condition (3.1) for anomalous skin effect allows us to limit ourselves in the sums over s and $\bar{s}$ [see the formulas (2.19) and (2.21)] to the consideration of the terms with $s = \bar{s} = 0$. Furthermore, because of the rapid oscillations of the matrix elements, the dominant contribution to the φ integral [after the substitution (2.25)] will be made by the region of small angles $\varphi \sim (kR)^{-1/2} \ll 1$. As a result, the kernel $Q_{\alpha\alpha}(k,k')$ assumes the form

$$Q_{\alpha\alpha}(k,k') = -\frac{eR}{(\pi\hbar)^3} \int_{-p_F}^{p_F} p_\perp n_\alpha(p_z)\, dp_z \int_0^\infty \varphi\, d\varphi \times \int_0^\varphi d\lambda \cos\left[\frac{kR_\perp}{2}(\varphi^2-\lambda^2)\right] A_0(k', \varphi, p_z);$$

$$n_y(p_z) = (1-n_z^2)^{1/2}, \quad n_z(p_z) = p_z/p_F. \tag{3.4}$$

In the case of extremely strong correlation of the successive reflections the inequality

$$\left|\frac{\partial p_y^n}{\partial n}\right| \gg \frac{2\pi\hbar}{L}; \quad \left|\frac{\partial p_y^n}{\partial p_z}\right| \frac{2\pi\hbar}{L} \tag{3.5}$$

is satisfied. After computing the derivatives, we can write it in the equivalent form

$$L \gg \Lambda, \quad \Lambda^2 = \Lambda_\perp^2 + \Lambda_H^2, \tag{3.6}$$

where $\Lambda_H = 2\varphi c|p_z|/eH$ is the distance traversed by an electron along the magnetic field H between two successive collisions. It follows from (3.5) that the correlation function $W(p_y^n(p_z) - p_y^{\bar{n}}(p_z'); p_z - p_z')$ is a "sharp" function of the difference $\bar{n} - n$, which allows us to limit ourselves in (2.21) to the consideration of only the term with $\bar{n} = n$. Thus, in the case of strong correlations the discrete quantum number n is conserved in the surface electron scattering.

The quantum-mechanical explanation of this effect is that the width of the scattering indicatrix is not "enough" to change the number n of the state in one scattering event. The strong-correlation condition in the form (3.6) can be interpreted as being a requirement that the state (classical or quantum) of the electron be changed adiabatically slowly by the succession of correlated reflections from the boundary.

According to the general theory of adiabatic perturbations,[14] the invariability of the discrete quantum number implies that the continuous quantum numbers should change in the course of the scattering. In our case such a quantum number is the momentum p_z. In order for the correlation to be observed in the classical magnetic-field region, the length of the roughnesses should be sufficiently large, i.e., we must have

$$L \gg \hbar/p_F. \tag{3.7}$$

Under this condition, the dominant contribution to the p_z' integral in (2.21) is made by a small neighborhood of the point $p_z' = p_z$. Consequently, the collision integral in (2.21) can be evaluated in the Fokker-Planck approximation. The expansion in powers of the small momentum transfer $p_z' - p_z$ up to the quadratic terms yields the following equation for the function A_0 in the strong-correlation limit:

$$8\pi^2 \frac{\sigma^2}{R_\perp L}\Omega\left(\frac{p_\perp}{p_F}\right)^3 \int_0^\infty \tau^2 w(\tau)\,d\tau \times\left\{\frac{\varphi}{3}\left(1+\frac{8p_z^2}{3p_\perp^2}\right)\frac{\partial A_0}{\partial\varphi}+\frac{2p_z^2\varphi^2}{9p_\perp^2}\frac{\partial^2 A_0}{\partial\varphi^2}\right.$$

$$\left.+\frac{2p_z\varphi}{3}\frac{\partial^2 A_0}{\partial\varphi\partial p_z}+\frac{p_\perp^2}{2}\frac{\partial^2 A_0}{\partial p_z^2}\right\}-(\nu-i\omega)A_0$$

$$=\frac{ep_F n_\alpha(p_z)}{m\varphi}\int_0^\varphi d\lambda\cos\left[\frac{kR_\perp}{2}(\varphi^2-\lambda^2)\right]. \tag{3.8}$$

Here $w(\tau) = L^{-2}W(2\pi\hbar\tau/L)$ is a dimensionless correlation coefficient.

It is not possible to obtain the general solution to Eq. (3.8). But this equation lends itself easily to a qualitative analysis. The first term in the left member is a surface collision integral, which, in the Fokker-Planck approximation, is a differential operator. If we take into account the fact that the function A_0 has a characteristic scale of variation in the angle $\varphi \sim (kR)^{-1/2}$, while the momentum p_z is always of the order of p_F, then, up to a numerical factor a_α, the action of this operator can be replaced by multiplication by the surface relaxation rate

$$\nu_\alpha^s = a_\alpha \frac{\sigma^2}{RL}\Omega\int_0^\infty \tau^2 w(\tau)\,d\tau. \tag{3.9}$$

After this replacement, the quantity $-(\nu+\nu_\alpha^s - i\omega)A_0$ will stand on the left-hand side of Eq. (3.8). The subsequent procedure for computing the asymptotic form (3.4), solving the Maxwell equation, and determining the impedance essentially coincides with the procedure followed in Refs. 13 in the $\nu^s = 0$ case. Let us, omitting the intermediate computations, give the final expression for the surface impedance of the metal:

$$Z_\alpha = -\frac{7i\omega}{c^2 k_\alpha}, \qquad k_\alpha = \left[\frac{3\sqrt{2}\,\pi C_\alpha}{5\Gamma^2(1/4)}\frac{\omega\omega_0^2}{c^2R^{1/2}(\omega+i\nu+i\nu_\alpha^1)}\right]^{2/5}. \tag{3.10}$$

Here k_α is a complex wave number, which determines the depth of penetration of the electromagnetic field into the metal ($\delta = |k_\alpha|^{-1}$), ω_0 is electron plasma frequency, $C_y = 3/2$, and $C_z = 1$. It follows from (3.10) that, depending on the relationship between the relaxation rates ν and ν^s, the electrodynamic properties of the metal are governed by either the volume ($\nu > \nu_\alpha^s$), or the surface ($\nu_\alpha^s > \nu$), relaxation of the gliding electrons.

If the frequency ω of the external wave is much higher than the sum $\nu + \nu_\alpha^s$, then the impedance (3.10) is essentially purely imaginary. The small real part, which determines the electromagnetic absorption, is proportional to $(\nu + \nu_\alpha^s)/\omega$. Let us note that, for such a relationship between the relaxation rates, the coefficient a_α in the formula (3.9) can be determined exactly, since Eq. (3.8) and the Maxwell equation (3.3) can be solved by the method of successive approximations (see Ref. 6). For the coefficient a_α we obtain the following result: $a_y = 163\pi^2/81$, $a_z = 118\pi^2/27$. In the opposite case of low frequencies (i.e., for $\omega \ll \nu + \nu_\alpha^s$), ω can be neglected in comparison with $\nu + \nu_\alpha^s$ in the expression for k_α, and the real and imaginary parts of Z_α are found to be of the same order of magnitude.

In the case of weak correlations the Fokker-Planck approximation is applicable in the region of sharp decrease (i.e., in the region $\varphi^2 \gg 2\pi\hbar/p_F L$), where the surface relaxation rate is given by the formula[5-7]

$$\nu_\alpha^s \sim \frac{\sigma^2}{L^2}(k_\alpha R)^{1/2}\Omega. \tag{3.11}$$

It can be seen from a comparison of (3.11) and (3.9) that these expressions cannot be matched with each other when $L \sim \Lambda$. The reason is that the formula (3.11) takes account of the contribution of a large number of terms in the $\bar{n}$ sum figuring in Eq. (2.21), and, what is more, the components $p_z' - p_z$ and $p_y' - p_y$ of the momentum transfer are in no way related with each other. On the other hand, the expression (3.9) takes account of the conservation of the quantum number n in the scattering [i.e., of the fact that $n(p_y, p_z) = n(p_y', p_z')$], and does not contain terms with $\bar{n} \neq n$. The transition from one formula to the other is effected precisely with the aid of such terms in (2.21). The decrease of the correlation function with increasing $|\bar{n} - n|$ and the simultaneous increase of the difference $A_0(k,\varphi_n, p_z) - A_0(k,\varphi_{\bar{n}}, p_z')$ give rise to competition in the off-diagonal—in n and $\bar{n}$—terms. If we take the off-diagonal terms into account, then in the strong-correlation limit (3.6) there appears in ν_α^s, besides (3.9), a term $\Delta\nu_\alpha^s$ that is sensitive to the form of the correlation function. For example, for the power correlation function

$$W(\mathbf{p}) = \pi^{-1}L^2(\beta-1)(1+p^2L^2/4\pi^2\hbar^2)^{-\beta}, \quad \beta > 2$$

the relaxation rate $\Delta\nu_\alpha^s$ is given by the formula

$$\Delta\nu_\alpha^s = b_\alpha\zeta(2\beta-3)\frac{\sigma^2}{L^2}(k_\alpha R)^{1/2}\left(\frac{\Lambda_\alpha}{L}\right)^{2\beta-4}\Omega, \tag{3.12}$$

where $\zeta(x)$ is the Riemann function, $\Lambda_\alpha = (k_\alpha R)^{-1/2}R$, and b_α is a numerical factor. It can be seen from the formula (3.12) that, in the transition region, the surface relaxation rate decreases rapidly as H (i.e., the parameter L/Λ_α) increases. The cause of this decrease of $\Delta\nu_\alpha^s$ is the adiabaticity of the surface-scattering potential in the case of strong correlations. The indicated decrease occurs until the terms (3.9) and (3.12) are of the same order of magnitude. As the mag-

netic-field intensity is increased further, the term (3.9), which increases with H, begins to play the dominant role. For a Gaussian correlation function, the relaxation rate $\Delta\nu_\alpha^s$ coincides up to a numerical factor with (3.12) with $\beta = 5$. This result is valid in the high-frequency region $\nu + \nu_\alpha^s \ll \omega$, where the method of successive approximations can be used. In the opposite ($\omega \ll \nu + \nu_\alpha^s$) case it can be shown that the relaxation rate $\Delta\nu_\alpha^s$ decreases rapidly with increasing magnetic-field intensity H, but that it is not possible to determine the functional dependence $\Delta\nu_\alpha^s(H)$.

Reference 7 contains plots of the relaxation rate ν^s and the real part Re Z of the impedance as functions of the magnetic field in the case of weak correlation of the successive reflections (i.e., for $L \ll \Lambda$). In Fig. 1 these curves are continued into the region of stronger magnetic fields (i.e., into the $\Omega > \Omega_2$ region, where Ω_2 is the point at which $L = \Lambda$), where neighboring reflections are no longer independent. In the region $\Omega_2 < \Omega < \Omega_3$, the surface relaxation rate is given by the formula (3.12) (in Fig. 1 we give the result for the Gaussian correlation function, i.e., for $\beta = 5$). The minimum of the surface scattering is attained at the point $\Omega = \Omega_3$, where the relaxation rates (3.12) and (3.9) are equal. To the right of the point $\Omega = \Omega_3$ the scattering conserves the quantum number n ($\bar{n} = n$), and the rate ν^s is given by the formula (3.9). The curves a) and b) correspond respectively to the high- and low-frequency cases. The dot-dash lines are in the regions where the functional dependences could not be determined.

A nonmonotonic dependence Re $Z(H)$ is realized in those magnetic-field regions where $\nu^s(H) > \nu$. If the volume scattering predominates [i.e., if $\nu^s(H) \ll \nu$], then the impedance is proportional to $H^{-1/5}$, which corresponds to the specular-reflection (i.e., $\nu^s = 0$) case.

Order-of-magnitude estimates of the characteristic field-intensity values at which the transition from one functional dependence to the other occurs, as well as of the corresponding surface relaxation rates, can be carried out. Such estimates are given for Ω_1 and ν_1^s in Ref. 7. Using the formulas (3.9)–(3.12), we easily find that

$$\Omega_2 \approx \left(\frac{v_F}{\omega_l L}\right)^{5/3} \omega_l, \quad \nu_2^s \approx \frac{\sigma^2}{L^2}\left(\frac{\omega_l L}{v_F}\right)^{1/3} \omega_l,$$

$$\omega_l = \omega_0 \frac{v_F}{l},$$

$$\Omega_3 \approx \left(\frac{v_F}{\omega_l L}\right)^{35/29} \omega_l, \quad \nu_3^s \approx \frac{\sigma^2}{L^2}\left(\frac{v_F}{\omega_l L}\right)^{41/29} \omega_l.$$

These results are valid when $\omega \gg \nu + \nu_\alpha^s$. The estimates for Ω_3 and ν_3^s are also valid in the opposite ($\omega \ll \nu + \nu_\alpha^s$) case, but no estimates could be obtained for Ω_2 and ν_2^s in this case.

4. A NEW BOUNDARY CONDITION

In this section we give a new boundary condition for the nonequilibrium distribution function for the conduction electrons. We shall proceed from the quasiclassical collision integral contained in the kinetic equation (2.21). Let us formally introduce integration over the momentum p_y' into this collision integral, and write down the appropriate δ function that removes this integration. The argument of the δ function should contain a quantity that gives rise to the same relation between the momentum components p_y' and p_z' that follows from the quasiclassical quantization rule (2.20). Such a classical quantity is evidently the adiabatic invariant

$$F(\mathbf{p}) = \frac{1}{2\pi}\oint p_x dx. \tag{4.1}$$

After this transformation, the collision integral will contain the classical momenta $\mathbf{p}$ and $\mathbf{p}'$, and the boundary condition that takes account of the periodic motion of the electrons along the x axis can be represented in the form

$$f_+(0,\mathbf{p}) = f_-(0,\mathbf{p}) - \int \frac{d\mathbf{p}'}{(2\pi\hbar)^2} V(\mathbf{p},\mathbf{p}') \times \sum_{l=-\infty}^{\infty} \delta\left[\frac{F(\mathbf{p}) - F(\mathbf{p}')}{\hbar} + l\right] [f_-(0,\mathbf{p}) - f_-(0,\mathbf{p}')]. \tag{4.2}$$

The boundary condition (4.2) has been written in the general form, and is valid for any quasiparticles and an arbitrary (static) surface scattering potential. In a magnetic field, the adiabatic invariant $F(\mathbf{p})$ is given by the well-known formula

$$F(\mathbf{p}) = (c/2\pi eH) S(\mathbf{p}),$$

where $S(\mathbf{p}) = p_1^2(\varphi - \sin\varphi\cos\varphi)$ is the area enclosed by the surface-electron orbit in momentum space.

In the case of weak correlations a large number of δ-function peaks with different l fit into the characteristic interval of variation of the transition probability $V(\mathbf{p},\mathbf{p}')$ (i.e., into the scattering indicatrix). Therefore, the sum over l can be replaced by an integral over l. This integral is evidently equal to unity, and we thereby arrive at the usual form (1.2) of the collision integral.

In the opposite limiting case of strong correlations only one δ function with $l = 0$ is "positioned" inside the scattering indicatrix. In other words, the scattering conserves the adiabatic invariant. We obtain from the conservation of the adiabatic invariant, i.e., from the relation $F(\mathbf{p}) = F(\mathbf{p}')$, a single-valued relation that connects the components of the momentum $\mathbf{p} - \mathbf{p}'$ transferred during the scattering, and decreases the dimensionality of the phase space by one, transforming this space from a two- into a one-dimensional space. It is clear that the adiabatic-invariant conservation law is the classical analog of the quantum condition $\bar{n} = n$. The conservation law $F(\mathbf{p}) = F(\mathbf{p}')$ becomes obvious when we take into account the fact that the condition $L \gg \Lambda$ for strong correlations allows us to treat the scattering potential as an adiabatic perturbation. Let us emphasize that, in the case of strong correlations, the boundary condition (4.2) with $l = 0$ is actually not an integral, but a differential, condition, since the Fokker-Planck approximation, as illustrated by the formula (3.8), is valid.

The boundary condition (4.2) again goes over into (1.2) in the limit as $H \to \infty$. Indeed, as the magnetic-field intensity increases, the total scattering cross section increases, and the width of the electronic levels eventually reaches a value of the order of, or greater than, the level spacing. This leads to the "smearing out" of the δ functions entering into (4.2), i.e.,

the dominant contribution to the sum over l is then made by a large number of terms, and the sum can again be replaced by an integral. It is easy to estimate the characteristic magnetic-field value at which the indicated transition will occur. The surface-relaxation rate computed without allowance for the arrival term in (4.2) [i.e., the rate proportional to the total scattering cross section, which, under strong-correlation conditions, is $(p_F L/2\pi\hbar)^2$ times greater than the transport cross section] should have a value of the order of $\Omega/2\varphi$. In the case of high frequencies we find from this condition that

$$\Omega_4 \approx \left(\frac{v_F}{\omega_l L}\frac{\hbar^2}{p_F^2\sigma^2}\right)^{5/7}\omega_l, \quad \nu_4^s \approx \frac{\sigma^2}{v_F L}\Omega_4^2.$$

We can use the boundary condition (1.2) when $\Omega > \Omega_4$. In this magnetic-field region the surface-relaxation rate and the impedance will begin to decrease as H is increased, just as they do in the case of weak correlations in the range from Ω_1 to Ω_2.

In conclusion, let us note that similar adiabatic effects should exist for the kinetic coefficients of thin conducting plates even in the absence of a magnetic field, as well as for the volume scattering in the presence of a long-range perturbation potential.

[1)]The off-diagonal element Q_{yz} is equal to zero as a result of the central symmetry of the Fermi surface.

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