Dynamics of an atom in a field of quantized radiation and an elliptically polarized wave

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In the framework of Weisskopf-Wigner microscopic theory, a Fokker-Planck equation describing the dynamics of an atom in a field of quantized radiation and a strong, elliptically polarized monochromatic wave that couples the levels $p_{3/2}$ with $s_{1/2}$ or $p_{1/2}$ with $s_{1/2}$ is obtained. In the limiting case of momentum-independent relaxation constants, the force of the light pressure and the rate of growth of the temperature of the atom are determined as functions of the polarization of the strong field.

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Much attention is currently devoted to the dynamics of atoms in strong resonance fields[1] and, in particular, to the study of the forces that arise as a result of spontaneous transitions.[2] In the theoretical treatment of this phenomenon, an atom is, as a rule, simulated by a system consisting of two levels nondegenerate with respect to the energies.[3–5] In the present paper, the behavior of an atom is investigated with allowance for the actual degeneracy of the levels with respect to the directions of the total angular momentum, which makes it possible to determine the dependence of the theoretical quantities on the polarization of the strong radiation. The microscopic theory of Weisskopf and Wigner[6] is used, which makes it possible to avoid the phenomenological approach generally employed and consisting of the introduction of a relaxation operator in the equation for the density matrix.

We shall study the case of atoms of alkali metals, which is of practical interest, restricting ourselves to $p_{3/2}$–$s_{1/2}$ and $p_{1/2}$–$s_{1/2}$ transitions, since for such transitions the problem of the behavior of an atom in a monochromatic field of arbitrary intensity and polarization has an exact solution.

Writing the expression for the electric field of a monochromatic wave as an expansion with respect to the circularly polarized components,

$$\mathbf{E}={}^1\!/_2 E(\alpha\mathbf{e}_+ + \beta\mathbf{e}_-)e^{i(\mathbf{kR}-\omega t)} + \text{c.c.}, \tag{1}$$

where $\alpha^2+\beta^2=1$, $\mathbf{e}_\pm = \mathbf{e}_x \pm i\mathbf{e}_y$, and E is the field amplitude in the case of circular polarization, we represent the Hamiltonian of the complete system in the form

$$H=H_A+H_Q+V_C+V_Q, \tag{2}$$

where

$$H_A=H_C+\hat{p}^2/2M, \quad H_Q=\sum_{\mathbf{q}\sigma}\hbar\omega_{\mathbf{q}}b_{\mathbf{q}\sigma}{}^+b_{\mathbf{q}\sigma}$$

are the energy operator of the atom with allowance for the translational degree of freedom and the Hamiltonian of the quantized radiation, and V_C and V_Q are, respectively, the operators of the interaction of the atom with the strong wave and the quantized radiation:

$$V_C=-d\hat{E}(E^-e^{i(\mathbf{kR}-\omega t)}+\text{h.c.}), \tag{3}$$

$$V_Q=id\sum_{\mathbf{q}\sigma}(2\pi\hbar\omega_{\mathbf{q}})^{1/2}E_{\mathbf{q}\sigma}{}^+b_{\mathbf{q}\sigma}e^{i\mathbf{qR}}+\text{h.c.}$$

We shall distinguish transitions by specifying the change Δj in the total angular momentum and, choosing the quantization axis along the direction of propagation of the wave, we write down the matrix elements of E^+ and $E_{\mathbf{q}\sigma}^+$ that do not vanish in the resonance approximation (first assumption of Weisskopf-Wigner theory):

$$\begin{aligned}\langle pm'|E^+|sm\rangle &= \alpha\delta_{m',m+1}(\delta_{m,1/2}+3^{-1/2}\delta_{m,-1/2}) \\ &\quad -\beta\delta_{m',m-1}(\delta_{m,-1/2}+3^{-1/2}\delta_{m,1/2}),\end{aligned} \tag{4}$$

$$\begin{aligned}\langle pm'|E_{\mathbf{q}\sigma}{}^+|sm\rangle &= e_{\mathbf{q}\sigma}{}^-\delta_{m',m+1}(\delta_{m,1/2}+3^{-1/2}\delta_{m,-1/2}) \\ &\quad -e_{\mathbf{q}\sigma}{}^+\delta_{m',m-1}(\delta_{m,-1/2}+3^{-1/2}\delta_{m,1/2})-2\cdot 3^{-1/2}e_{\mathbf{q}\sigma}{}^z\delta_{m',m}\end{aligned}$$

for $\Delta j=1$ transitions, and

$$\begin{aligned}\langle pm'|E^+|sm\rangle &= -(2/3)^{1/2}(\alpha\delta_{m',1/2}\delta_{m,-1/2}+\beta\delta_{m',-1/2}\delta_{m,1/2}), \\ \langle pm'|E_{\mathbf{q}\sigma}{}^+|sm\rangle &= -(2/3)^{1/2}[e_{\mathbf{q}\sigma}{}^-\delta_{m',1/2}\delta_{m,-1/2} \\ &\quad +e_{\mathbf{q}\sigma}{}^+\delta_{m',-1/2}\delta_{m,1/2}+e_{\mathbf{q}\sigma}{}^z(\delta_{m',1/2}\delta_{m,1/2}-\delta_{m',-1/2}\delta_{m,-1/2})]\end{aligned} \tag{5}$$

for $\Delta j=0$ transitions.

In Eqs. (3), $d=-e\langle p|r|s\rangle/\sqrt{6}$, where $\langle p|r|s\rangle$ is the radial part of the matrix element of the dipole moment operator. In Eqs. (4) and (5), $e_{\mathbf{q}\sigma}{}^\pm = e_{\mathbf{q}\sigma}{}^x \pm ie_{\mathbf{q}\sigma}{}^y$, where $e_{\mathbf{q}\sigma}$ are the polarization vectors of the quantized radiation. The normalization volume is taken equal to unity.

In what follows, to simplify the notation, we label the internal states of the atom by a single index ν, so that $\nu=1$ will correspond to the excited state $\langle pj|$ of the atom with maximal projection of the angular momentum, $\nu=2$ corresponds to the state $\langle pj-1|$, and so forth.

Having determined the Hamiltonian of the complete system, we turn to the solution of the problem, for which we represent the statistical operator of the system in the form[7]

$$\begin{aligned}\mathbf{P}(\mathbf{R},\mathbf{R}';t) &= \mathbf{P}_0(\mathbf{R},\mathbf{R}';t) \\ &\quad -\frac{i}{\hbar}\int_0^t U(\mathbf{R},t;\mathbf{R}'t')[V_Q(\mathbf{R}')\mathbf{P}_0(\mathbf{R}';\mathbf{R}'';t) \\ &\quad -\mathbf{P}_0(\mathbf{R}',\mathbf{R}'';t')V_Q(\mathbf{R}'')]U(\mathbf{R}'',t';\mathbf{R}'t)\,d,d\mathbf{R}'\,d\mathbf{R}'',\end{aligned} \tag{6}$$

where U is the evolution operator of the system, and P_0 is chosen in the form

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$$P_0(\mathbf{R}, \mathbf{R}'; t) = \rho_0(\mathbf{R}, \mathbf{R}'; t)\rho_Q,$$

where ρ_Q is the canonical distribution of the quantized radiation, and ρ_0 is an arbitrary normalized solution of the equation

$$i\hbar\partial\rho_0/\partial t = [H_A + V_C, \rho_0].$$

The choice of such a ρ_0 is dictated by our desire to restrict the calculation to the stochastic part of the density matrix, which determines the steady state of the system. Calculations show that a different choice of ρ_0 leads to the appearance of oscillating terms in the density matrix of the atom, these being damped over times long compared with the spontaneous decay time.

All the following calculations will be made by the methods of perturbation theory in the interaction with the quantized radiation; i.e., under the assumption that the interaction parameter, which is of the order of the reciprocal time of spontaneous decay, is small compared with the remaining frequencies of the system (second assumption of Weisskopf-Wigner theory). For this reason, it is convenient from the very beginning to go over to a representation in which the operator $H_A + V_C$ is diagonal; i.e., to represent

$$P(\mathbf{R}, \mathbf{R}'; t) = \sum_{pp'} L_p(\mathbf{R}, t) P_{pp'}(t) L_{p'}^{+}(\mathbf{R}', t),$$
$$U(\mathbf{R}, t; \mathbf{R}', t') = \sum_{pp'} L_p(\mathbf{R}, t) U_{pp'}(t, t') L_{p'}^{+}(\mathbf{R}', t'), \tag{7}$$

etc. The unitary operator by means of which this transition is realized is determined by the equation

$$(-i\hbar\partial/\partial t + H_A + H_Q + V_C) L_p(\mathbf{R}, t) = L_p(\mathbf{R}, t)\hat{E}_p,$$

where $\hat{E}_p$ is diagonal. It is easy to see that $L_p(\mathbf{R}, t)$ can be represented in the form

$$L_p(\mathbf{R}, t) = \exp\left\{ i\left[\mathbf{pR} + \frac{H_0}{\omega_0}(\mathbf{kR} - \omega t) - H_Q t \right] \Big/ \hbar \right\} T_p, \tag{8}$$

where $\hbar\omega_0$ is the difference between the levels of the atom, and the eigenvalues $\hat{E}_p$ and the operator T_p for the transition $\Delta j = 1$ are given by

$$E_{pv} = E_p + \hbar\Omega_{pv}/2, \quad E_p = [\mathbf{p}^2 + (\hbar\mathbf{k}/2)^2]/2M,$$
$$\Omega_{p1} = -\Omega_{p5}, \quad \Omega_{p2} = -\Omega_{p6}, \quad \Omega_{p3} = \Omega_{p4} = \omega_0 - \omega,$$
$$\Omega_{p1} = [(\omega_0 - \omega + \mathbf{pk}/M)^2 + 4(dE/\hbar)^2(\alpha^2 + \beta^2/3)]^{1/2},$$
$$\Omega_{p2} = [(\omega_0 - \omega + \mathbf{pk}/M)^2 + 4(dE/\hbar)^2(\beta^2 + \alpha^2/3)]^{1/2},$$

$$T_p = \begin{pmatrix} -c_\beta u_{p1} & 0 & s_\beta & 0 & c_\beta v_{p1} & 0 \\ 0 & -s_\alpha u_{p2} & 0 & c_\alpha & 0 & s_\alpha u_{p2} \\ s_\beta u_{p1} & 0 & c_\beta & 0 & -s_\beta v_{p1} & 0 \\ 0 & c_\alpha u_{p2} & 0 & s_\alpha & 0 & -c_\alpha v_{p2} \\ v_{p1} & 0 & 0 & 0 & u_{p1} & 0 \\ 0 & v_{p2} & 0 & 0 & 0 & u_{p2} \end{pmatrix}$$

with amplitudes u and v determined by the relations

$$u_i^2 + v_i^2 = 1, \quad (u_i^4 + v_i^4)/2(u_i v_i)^2 = \Delta_i, \quad i = \{p1, p2\},$$

where

$$\Delta_{p1} = 1 + \hbar^2(\omega_0 - \omega + \mathbf{pk}/M)^2/2(dE)^2(\alpha^2 + \beta^2/3),$$
$$\Delta_{p2} = 1 + \hbar^2(\omega_0 - \omega + \mathbf{pk}/M)^2/2(dE)^2(\beta^2 + \alpha^2/3), \tag{9}$$
$$c_\alpha = \beta(\beta^2 + \alpha^2/3)^{-1/2}, \quad c_\beta = \alpha(\alpha^2 + \beta^2/3)^{-1/2}, \quad c_{\alpha(\beta)}^2 + s_{\alpha(\beta)}^2 = 1.$$

The corresponding quantities for the transition $\Delta j = 0$ are

$$\Omega_{p1} = \Omega_{p4}, \quad \Omega_{p2} = \Omega_{p3},$$
$$\Omega_{p1} = [(\omega_0 - \omega + \mathbf{pk}/M)^2 + {}^8/_3(\alpha dE/\hbar)^2]^{1/2},$$
$$\Omega_{p2} = [(\omega_0 - \omega + \mathbf{pk}/M)^2 + {}^8/_3(\beta dE/\hbar)^2]^{1/2},$$

$$T_p = \begin{pmatrix} u_{p1} & 0 & 0 & -v_{p1} \\ 0 & u_{p2} & -v_{p2} & 0 \\ 0 & v_{p2} & u_{p2} & 0 \\ v_{p1} & 0 & 0 & u_{p1} \end{pmatrix},$$

$$\Delta_{p1} = 1 + 3\hbar^2(\omega_0 - \omega + \mathbf{pk}/M)^2/4(\alpha dE)^2, \tag{10}$$
$$\Delta_{p2} = 1 + 3\hbar^2(\omega_0 - \omega + \mathbf{pk}/M)^2/4(\beta dE)^2.$$

Finally, determining the advanced and retarded Green's functions of the atom,

$$G_{pp'}^{A}(t, t') = \begin{cases} 0 & t > t' \\ iU_{pp'}(t, t') & t < t' \end{cases}$$
$$G_{pp'}^{R}(t, t') = \begin{cases} -iU_{pp'}(t, t') & \text{for } t > t' \\ 0 & \text{for } t < t', \end{cases} \tag{11}$$

which satisfy the equations

$$(id/dt - E_p/\hbar) G_{pp'}(t, t') = \delta_{pp'}\delta(t - t') + \sum_{p''} V_{pp''}(t) G_{p''p'}(t, t'),$$
$$V_{pp'}(t) = \frac{1}{\hbar}\sum_{q\sigma} E_{pp'}^{-}(\mathbf{q}, \sigma)\delta_{p-p', \hbar(q-k)} e^{-i(\omega_q - \omega)t} + \text{h.c.}, \tag{12}$$
$$E_{pp'}^{+}(q\sigma) = T_p^{+} E_{q\sigma}^{+} T_{p'},$$

and taking the trace with respect to the variables of the quantized radiation, we obtain for the density matrix

$$\tilde{\rho}_{pv} = \mathrm{Tr}_\delta P_{pv,\,pv}$$

of the atom the representation

$$\tilde{\rho}_{pv}(t) = \sum_{p'v'} \tilde{\rho}_{p'v'}(0)\tilde{P}_{p'v',pv}(t), \tag{13}$$

where the matrix $\tilde{P}$ of the conditional probabilities is given by

$$\tilde{P}_{p'v',pv}(t) = \delta_{p'p}\delta_{v'v}$$
$$-i\sum_{p''v''}\int \langle G_{p'v',pv}^{A}(t', t) G_{pv,p''v''}^{R}(t, t') V_{p''v'',p'v'}(t') - V_{p'v',p''v''}(t') G_{p''v'',pv}^{A}(t', t) G_{pv,p'v'}^{R}(t, t')\rangle dt', \tag{14}$$

in which $\langle ... \rangle = \mathrm{Tr}_b\, \rho_Q \{...\}$.

Expanding the Green's functions in (14) with respect to the interaction with the quantized radiation and using Wick's theorem for the mean over the canonical distribution of the product of an arbitrary number of photon operators, we arrive at a diagram form of the perturbation theory. Some of the diagrams which then arise are shown in Fig. 1 (cases *a* and *b*), in which the continuous lines correspond to the Green's functions of the atom averaged over the initial distribution of the quantum radiation,

$$\langle G_{pv,p'v'}^{A(R)}(t, t')\rangle = \frac{1}{2\pi}\delta_{pp'}\delta_{vv'}\int g_{pv}^{A(R)}(\varepsilon) e^{-i\varepsilon(t-t')} d\varepsilon, \tag{15}$$

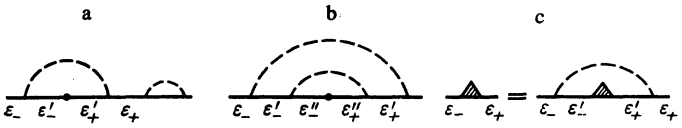

FIG. 1.

and the broken lines correspond to the radiation function

$$\langle V_{\mathbf{pv},\mathbf{p'v'},\mathbf{pv}}(t)\,V_{\mathbf{p'v'},\mathbf{pv}}(t')\rangle=\frac{1}{2\pi}\int D_{\mathbf{pv},\mathbf{p'v'}}(\varepsilon)\,e^{-i\varepsilon(t-t')}d\varepsilon. \tag{16}$$

Calculating the self-energy part in the first order of perturbation theory under the assumption that the spectrum of the quantized radiation has an almost thermal nature (the third and last assumptions of Weisskopf-Wigner theory), and retaining in it only the imaginary part, which we shall need in what follows, we obtain for the Green's functions

$$g_{\mathbf{pv}}^{-1}(\varepsilon)=\varepsilon-E_{\mathbf{pv}}/\hbar-\Sigma_{\mathbf{pv}},$$

$$\Sigma_{\mathbf{pv}}^{A(R)}=\overset{+}{(-)}\,i\gamma_{\mathbf{pv}}/2,\quad \gamma_{\mathbf{pv}}=\sum_{\mathbf{p'v'}}\gamma_{\mathbf{pv},\mathbf{p'v'}},$$

$$\gamma_{\mathbf{pv},\mathbf{p'v'}}$$

$$=\frac{2\pi}{\hbar}\sum_{\mathbf{q}\sigma}\{|E^{+}_{\mathbf{pv},\mathbf{p'v'}}(\mathbf{q}\sigma)|^2(N_{\mathbf{q}}+1)\delta(E_{\mathbf{pv}}-E_{\mathbf{p'v'}}-\hbar(\omega_{\mathbf{q}}-\omega))$$

$$\times\delta_{\mathbf{p}-\mathbf{p'},\hbar(\mathbf{q}-\mathbf{k})}$$

$$+|E^{+}_{\mathbf{p'v'},\mathbf{pv}}(\mathbf{q}\sigma)|^2N_{\mathbf{q}}\delta(E_{\mathbf{p'v'}}-E_{\mathbf{pv}}-\hbar(\omega_{\mathbf{q}}-\omega))\delta_{\mathbf{p'}-\mathbf{p},\hbar(\mathbf{q}-\mathbf{k})}\},$$

$$N_{\mathbf{q}}=[\exp(\hbar\omega_{\mathbf{q}}/k_BT)-1]^{-1}. \tag{17}$$

Turning now directly to the calculation of $\tilde{\rho}_{\mathbf{pv}}$, we note that the perturbation series for $\tilde{P}$ contains not only diagrams of order $\gamma/\Omega_{\mathbf{p}1(2)}$, where γ is of the order of the reciprocal time of spontaneous decay, but also diagrams that make a contribution of order $\gamma t/\max\{1,\gamma t\}$, which arises on integration of the product of the advanced and retarded Green's functions for values of their arguments that are nearly equal; when $\gamma t\gtrsim 1$, such diagrams must be summed.

The "singular" diagrams are conveniently summed by introducing the vertex function Λ, which is related to the Fourier transform of the mean value of the product of the advanced and retarded Green's functions,

$$\langle G^{A}_{\mathbf{p'v'},\mathbf{pv}}(t_1,t)\,G^{R}_{\mathbf{pv},\mathbf{p'v'}}(t,t_2)\rangle=\frac{1}{(2\pi)^2}\int\Pi_{\mathbf{p'v'},\mathbf{pv}}(\varepsilon_-,\varepsilon_+)$$

$$\times e^{-i\varepsilon_-(t_1-t)-i\varepsilon_+(t-t_2)}d\varepsilon_-d\varepsilon_+,\quad \varepsilon_\pm=\varepsilon\pm\chi/2, \tag{18}$$

by the relation

$$\Pi_{\mathbf{p'v'},\mathbf{pv}}(\varepsilon_-,\varepsilon_+)=g^{A}_{\mathbf{p'v'}}(\varepsilon_-)\,[\delta_{\mathbf{p'p}}\delta_{\mathbf{v'v}}+\Lambda_{\mathbf{p'v'},\mathbf{pv}}(\varepsilon_-,\varepsilon_+)]\,g^{R}_{\mathbf{p'v'}}(\varepsilon_+). \tag{19}$$

The vertex function satisfies the equation shown graphically in Fig. 1 (case *c*), the analytic form of which is

$$\Lambda_{\mathbf{p'v'},\mathbf{pv}}(\varepsilon_-,\varepsilon_+)=\frac{1}{2\pi}\sum_{\mathbf{p''v''}}\int D_{\mathbf{p'v'},\mathbf{p''v''}}(\varepsilon-\varepsilon')\,g^{A}_{\mathbf{p''v''}}(\varepsilon_-')$$

$$\times[\delta_{\mathbf{p''p}}\delta_{\mathbf{v''v}}+\Lambda_{\mathbf{p''v''},\mathbf{pv}}(\varepsilon_-',\varepsilon_+')]\,g^{R}_{\mathbf{p''v''}}(\varepsilon_+')\,d\varepsilon',$$

$$\varepsilon_\pm'=\varepsilon'\pm\chi/2. \tag{20}$$

Because of the weak dependence of the vertex function on its arguments in the region of integration, this last reduces to the equation

$$\Lambda_{\mathbf{p'v'},\mathbf{pv}}=\sum_{\mathbf{p''v''}}\gamma_{\mathbf{p'v'},\mathbf{p''v''}}(\delta_{\mathbf{p''p}}\delta_{\mathbf{v''v}}+\Lambda_{\mathbf{p''v''},\mathbf{pv}})\,(\gamma_{\mathbf{p''v''}}-i\chi)^{-1},$$

$$\Lambda_{\mathbf{p'v'},\mathbf{pv}}\equiv\Lambda_{\mathbf{p'v'},\mathbf{pv}}(\varepsilon_-,\varepsilon_+)\,|\,\varepsilon=E_{\mathbf{p'v'}}/\hbar. \tag{21}$$

Using the quantities determined above, we can write the matrix of conditional probabilities in the form

$$\tilde{P}_{\mathbf{p'v'},\mathbf{pv}}(t)=\delta_{\mathbf{pp'}}\delta_{\mathbf{vv'}}-i\int dt'e^{i\chi(t'-t)}\Big[\Pi_{\mathbf{p'v'},\mathbf{pv}}(\varepsilon_-,\varepsilon_+)\Sigma^{R}_{\mathbf{pv}}(\varepsilon_+)$$

$$-\Sigma^{A}_{\mathbf{p'v'}}(\varepsilon_-)\,\Pi_{\mathbf{p'v'},\mathbf{pv}}(\varepsilon_-,\varepsilon_+)+g^{A}_{\mathbf{p'v'}}(\varepsilon_-)\,\Lambda_{\mathbf{p'v'},\mathbf{pv}}(\varepsilon_-,\varepsilon_+)$$

$$-\Lambda_{\mathbf{p'v'},\mathbf{pv}}(\varepsilon_-,\varepsilon_+)\,g^{R}_{\mathbf{pv}}(\varepsilon_+)\Big]d\varepsilon\,d\chi/(2\pi)^2,$$

whence, after integration over t' and ε, we obtain

$$\tilde{P}_{\mathbf{p'v'},\mathbf{pv}}(t)=\frac{1}{2\pi}\int d\chi\,e^{-i\chi t}(\Lambda_{\mathbf{p'v'},\mathbf{pv}}+\delta_{\mathbf{pp'}}\delta_{\mathbf{vv'}})\,(\gamma_{\mathbf{p'v'}}-i\chi)^{-1}. \tag{22}$$

Comparing (22) with (21), we find that the density matrix of the atom describes a stochastic process with matrix of conditional probabilities satisfying the Kolmogorov equation

$$\frac{d}{dt}\tilde{P}_{\mathbf{p'v'},\mathbf{pv}}=\sum_{\mathbf{p''v''}}(\tilde{P}_{\mathbf{p'v'},\mathbf{p''v''}}\gamma_{\mathbf{p''v''},\mathbf{pv}}-\tilde{P}_{\mathbf{p'v'},\mathbf{pv}}\gamma_{\mathbf{pv},\mathbf{p''v''}}) \tag{23}$$

with initial condition $\tilde{P}_{\mathbf{p'v'},\mathbf{pv}}(0)=\delta_{\mathbf{p'p}}\delta_{\mathbf{v'v}}$.

By means of $\tilde{\rho}$, we can find the dynamical quantities for the atom. For example, the momentum is given by

$$\bar{\mathbf{p}}=\sum_{\mathbf{pv}}(\mathbf{p}+\hbar\mathbf{k}_{\mathbf{v}})\,\tilde{\rho}_{\mathbf{pv}}, \tag{24}$$

where $\mathbf{k}_\nu$ for the transition $\Delta j=1$ is

$$\mathbf{k}_1=-\mathbf{k}_5=\mathbf{k}(u_{\mathbf{p}_1}^2-v_{\mathbf{p}_1}^2)/2,\ \mathbf{k}_2=-\mathbf{k}_6=\mathbf{k}(u_{\mathbf{p}_2}^2-v_{\mathbf{p}_2}^2)/2,$$

$$\mathbf{k}_3=\mathbf{k}_4=\mathbf{k}/2, \tag{25}$$

and for the transition $\Delta j=0$

$$\mathbf{k}_1=-\mathbf{k}_4=\mathbf{k}(u_{\mathbf{p}_1}^2-v_{\mathbf{p}_1}^2)/2,\quad \mathbf{k}_2=-\mathbf{k}_3=\mathbf{k}(u_{\mathbf{p}_2}^2-v_{\mathbf{p}_2}^2)/2. \tag{26}$$

It can be seen from (24) that it is convenient to make the transformation

$$\tilde{\rho}_{\mathbf{pv}}(t)=\rho_{\mathbf{v}}(\mathbf{p}+\hbar\mathbf{k}_{\mathbf{v}},t),\quad \tilde{P}_{\mathbf{p'v'},\mathbf{pv}}(t)=P_{\mathbf{v'v}}(\mathbf{p'}+\hbar\mathbf{k}_{\mathbf{v'}},\ \mathbf{p}+\hbar\mathbf{k}_{\mathbf{v}};\ t),$$

etc., after which (24) becomes $\bar{\mathbf{p}}=\Sigma_{\mathbf{pv}}\mathbf{p}\rho_{\mathbf{v}}(\mathbf{p})$.

Finally, using the relative smallness of the momentum transferred to the atom, we can go over from the Kolmogorov equations to a Fokker-Planck equation. Following the well-known procedure,[8] we obtain the representation

$$\rho_{\mathbf{v}}(\mathbf{p},t)=\sum_{\mathbf{p'v'}}\rho_{\mathbf{v'}}(\mathbf{p'},0)P_{\mathbf{v'v}}(\mathbf{p'},\mathbf{p};t),$$

where P satisfies the equation

$$\frac{\partial}{\partial t}P(\mathbf{p}',\mathbf{p};t)=P(\mathbf{p}',\mathbf{p};t)\,\Gamma^{(0)}_{(\mathbf{p})}$$
$$+\frac{\partial}{\partial \mathbf{p}_l}[P(\mathbf{p}',\mathbf{p};t)\Gamma^{(1)}_l(\mathbf{p})]+\frac{1}{2}\frac{\partial^2}{\partial \mathbf{p}_l\,\partial \mathbf{p}_m}[P(\mathbf{p}',\mathbf{p};t)\Gamma^{(2)}_{lm}(\mathbf{p})] \tag{27}$$

with matrices of the relaxation parameters given by

$$\Gamma^{(0)}_{\nu\nu'}(\mathbf{p})=\sum_{\mathbf{q}\sigma}\Big[\Gamma_{\nu\nu'}(\mathbf{p},\mathbf{q}\sigma)-\delta_{\nu\nu'}\sum_{\nu''}\Gamma_{\nu\nu''}(\mathbf{p},\mathbf{q}\sigma)\Big],$$
$$\Gamma^{(1)}_{\nu\nu',l}(\mathbf{p})=\hbar\sum_{\mathbf{q}\sigma}(\mathbf{q}-\mathbf{k}+\mathbf{k}_\nu-\mathbf{k}_{\nu'})_l\,\Gamma_{\nu\nu'}(\mathbf{p},\mathbf{q}\sigma), \tag{28}$$
$$\Gamma^{(2)}_{\nu\nu',lm}(\mathbf{p})=\hbar^2\sum_{\mathbf{q}\sigma}(\mathbf{q}-\mathbf{k}+\mathbf{k}_\nu-\mathbf{k}_{\nu'})_l(\mathbf{q}-\mathbf{k}+\mathbf{k}_\nu-\mathbf{k}_{\nu'})_m\,\Gamma_{\nu\nu'}(\mathbf{p},\mathbf{q}\sigma).$$

In the case of greatest interest, namely, optical frequencies $\omega, \omega_0 \gg k_B T/\hbar$, we have

$$\Gamma_{\nu\nu'}(\mathbf{p},\mathbf{q}\sigma)=\frac{2\pi}{\hbar}|E^{+}_{\mathbf{p}\nu,\mathbf{p}\nu'}(\mathbf{q}\sigma)|^2\,\delta(E_{\mathbf{p}\nu}-E_{\mathbf{p}\nu'}-\hbar(\omega_{\mathbf{q}}-\omega)). \tag{29}$$

An analytic solution of Eq. (27) for the case of strong "effective" fields, i.e., for $\Omega_{\mathbf{p}1(2)} \gg \mathbf{kp}/M$, when the dependence of the relaxation parameters on the momentum can be ignored, can be readily obtained by going over to Fourier transforms:

$$P(\mathbf{p},\mathbf{p}';t)=\frac{1}{2\pi}\int P(\chi,\mathbf{r})\,e^{-i[\chi t+(\mathbf{p}-\mathbf{p}')\mathbf{r}]}\,d\mathbf{r}\,d\chi.$$

Analysis of the exact solution shows that the establishment of the distribution with respect to the internal and translational degrees of freedom proceeds simultaneously over times of the order of the spontaneous decay time.

We give the expression for $P(\chi,\mathbf{r})$ when $\chi/\gamma \sim \mathbf{kr} \ll 1$; i.e., for times long compared with the spontaneous decay time:

$$P_{\nu'\nu}(\chi,\mathbf{r})=\rho_\nu^\infty[-i\chi+i\mathbf{rF}+(x^2D_x+y^2D_y+z^2D_z)/2]^{-1}, \tag{30}$$

from which it follows that when $t \gg 1/\gamma$ an initial Maxwellian distribution of the atom in the momentum space evolves into the distribution

$$\rho(t)=\rho^\infty\prod_i[2\pi(k_B MT_0+D_i t)]^{-1/2}$$
$$\times\exp\Big[-\sum_i(\mathbf{p}-\mathbf{F}t)_i^2/2(k_B MT_0+D_i t)\Big], \tag{31}$$

i.e., besides the establishment of a steady distribution with respect to the internal degrees of freedom a distribution is established in the momentum space with center that is displaced with acceleration; i.e., the atom behaves as if it were subject to the force $\mathbf{F}$. The diffusion constants D_i determine the rate of growth of the temperature of the atom, which is determined in terms of the variance of the momentum.

In the resonance case $\omega_0-\omega \ll \omega$, the quantities ρ^∞, $\mathbf{F}$, and D_i in (31) can be found for the transition $\Delta j=1$ from the relations

$$\rho_{\mathbf{p}}{}^{\infty}=\Big\{\lambda\Big(\frac{v_{\mathbf{p}1}}{u_{\mathbf{p}1}}\Big)^2,\quad \mu\Big(\frac{v_{\mathbf{p}2}}{u_{\mathbf{p}2}}\Big)^2,\ 0,\ 0,\ \lambda\Big(\frac{u_{\mathbf{p}1}}{v_{\mathbf{p}1}}\Big)^2,\ \mu\Big(\frac{u_{\mathbf{p}2}}{v_{\mathbf{p}2}}\Big)^2\Big\}$$
$$\times\Big[\lambda\Big(\frac{v_{\mathbf{p}1}}{u_{\mathbf{p}1}}\Big)^2+\mu\Big(\frac{v_{\mathbf{p}2}}{u_{\mathbf{p}2}}\Big)^2+\lambda\Big(\frac{u_{\mathbf{p}1}}{v_{\mathbf{p}1}}\Big)^2+\mu\Big(\frac{u_{\mathbf{p}2}}{v_{\mathbf{p}2}}\Big)^2\Big]^{-1},$$
$$\mathbf{F}(\mathbf{p})=\hbar\mathbf{k}\gamma(\lambda+\mu)/2(\lambda\Delta_{\mathbf{p}1}+\mu\Delta_{\mathbf{p}2}),$$
$$D_x(\mathbf{p})=D_y(\mathbf{p})=\frac{1}{5}\hbar\mathbf{k}\mathbf{F}(\mathbf{p})\Big(\frac{3}{2}+\frac{2\lambda\mu}{3(\lambda+\mu)}\Big),$$
$$D_z(\mathbf{p})=\hbar\mathbf{k}\mathbf{F}(\mathbf{p})\Big\{\frac{1}{5}\Big[7-\frac{4\lambda\mu}{3(\lambda+\mu)}\Big]+\frac{\lambda+\mu}{(\lambda\Delta_{\mathbf{p}1}+\mu\Delta_{\mathbf{p}2})^2}$$
$$\times\Big[\frac{3\lambda\mu(\Delta_{\mathbf{p}1}-\Delta_{\mathbf{p}2})^2}{4(\lambda+\mu)^2}+\lambda(\Delta_{\mathbf{p}1}-1)+\mu(\Delta_{\mathbf{p}2}-1)\Big]\Big\}, \tag{32}$$

where $\lambda=\alpha^2/(\alpha^2+3\beta^2)$, $\mu=\beta^2/(\beta^2+3\alpha^2)$, and $\gamma=8\omega_0{}^2d^2/3\hbar c^3$ is the reciprocal decay time of the excited state, and $\Delta_{\mathbf{p}1}$ and $\Delta_{\mathbf{p}2}$ are defined in (9).

For the transition $\Delta j=0$, the corresponding quantities are

$$\rho_{\mathbf{p}}{}^{\infty}=\Big\{\Big(\frac{v_{\mathbf{p}1}}{u_{\mathbf{p}1}}\Big)^2,\ \Big(\frac{v_{\mathbf{p}2}}{u_{\mathbf{p}2}}\Big)^2,\ \Big(\frac{u_{\mathbf{p}2}}{v_{\mathbf{p}2}}\Big)^2,\ \Big(\frac{u_{\mathbf{p}1}}{v_{\mathbf{p}1}}\Big)^2\Big\}$$
$$\times\Big[\Big(\frac{v_{\mathbf{p}1}}{u_{\mathbf{p}1}}\Big)^2+\Big(\frac{v_{\mathbf{p}2}}{u_{\mathbf{p}2}}\Big)^2+\Big(\frac{u_{\mathbf{p}2}}{v_{\mathbf{p}2}}\Big)^2+\Big(\frac{u_{\mathbf{p}1}}{v_{\mathbf{p}1}}\Big)^2\Big]^{-1},$$
$$\mathbf{F}(\mathbf{p})=\hbar\mathbf{k}\gamma/(\Delta_{\mathbf{p}1}+\Delta_{\mathbf{p}2}),\quad D_x(\mathbf{p})=D_y(\mathbf{p})={}^1/_3\hbar\mathbf{k}\mathbf{F}(\mathbf{p}), \tag{33}$$
$$D_z(\mathbf{p})=\hbar\mathbf{k}\mathbf{F}(\mathbf{p})$$
$$\{{}^1/_3+[3(\Delta_{\mathbf{p}2}-\Delta_{\mathbf{p}1})^2+2(\Delta_{\mathbf{p}1}+\Delta_{\mathbf{p}2}-2)]\,(\Delta_{\mathbf{p}1}+\Delta_{\mathbf{p}2})^{-2}\}$$

with $\Delta_{\mathbf{p}1}$ and $\Delta_{\mathbf{p}2}$ from (10).

We recall that the solution (31) is valid in the limit of strong fields, when the dependence on the momentum can be ignored. In the general case, the dynamics of the atom after establishment of the distribution with respect to the internal degrees of freedom is described by the scalar Fokker-Planck equation for $\rho(\mathbf{p})=\Sigma_\nu\rho_\nu(\mathbf{p})$, namely,

$$\frac{\partial}{\partial t}\rho(\mathbf{p})=-\Big(\frac{\partial}{\partial p_l}\Big)[\rho(\mathbf{p})F_l(\mathbf{p})]$$
$$+\frac{1}{2}\Big(\frac{\partial^2}{\partial p_l\,\partial p_m}\Big)[\rho(\mathbf{p})D_{lm}(\mathbf{p})],\quad D_{lm}=\delta_{lm}D_l \tag{34}$$

with $\mathbf{F}(\mathbf{p})$ and $D_l(\mathbf{p})$ determined by Eqs. (32) and (33).

It can be seen from (32) that for the transition $\Delta j=1$ in the case of circular polarization (when either $\lambda=1, \mu=0$ or $\lambda=0, \mu=1$) we obtain, as was to be expected, the well-known result,[1,3] since in this case the transition when $t>\gamma^{-1}$ takes place only between two states. The dependence of the force on the polarization of the strong field for the transition $\Delta j=1$ is weak and vanishes in the case of saturation, i.e., when $\Delta_{\mathbf{p}1(2)}\to 1$. For the transition $\Delta j=0$, the force takes maximal value in the case of linear polarization and vanishes for circular polarization. This last is explained by the depopulation of the excited state when $t>1/\gamma$.

We formulate the main result of the paper. On the basis of the microscopic theory, we have found a Fokker-Planck equation that describes the dynamics of the atom in the strong field with allowance for the degeneracy of the levels with respect to the directions of the total angular momentum. In accordance with the assumption of Weisskopf-

Wigner theory, the relaxation parameters are found in the first nonvanishing order of perturbation theory, in which the expansion parameter is $\gamma/\Omega_{\mathbf{p}1(2)}$.

In Ref. 9, the dynamics of an atom in a strong field was considered in a general formulation, i.e., for arbitrary orbital angular momenta of the levels, arbitrary polarizations and numbers of modes of the strong field, etc. However, it is readily seen that the result obtained in Ref. 9 has no bearing on the force of the light pressure due to the spontaneous transitions[2]; this follows from the vanishing of the force obtained in Ref. 9 in the case of a single mode of the strong field with frequency equal to the transition frequency of the atom.

Finally, we note that the restriction to strong fields justifies the neglect of the hyperfine structure, since, as is well known, the line width increases with increasing intensity of the field and for sufficiently strong fields it exceeds the hyperfine splitting.

The present paper is part of a program of study of the gas dynamics of resonance atoms in strong fields in the framework of microscopic theory. In what follows, it is intended to take into account collisional processes. It is hoped that much attention will be devoted to the investigation of the viscosity, thermal conductivity, and other kinetic characteristics of such systems.

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