Saturation in free-electron lasers at a large undulator length

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The gain for a free-electron laser is found in the case of a long undulator and comparatively large energy spread of the beam electrons. Two nonlinear regimes are shown to exist in this case, those of weak and of strong saturation. In the weak saturation regime the efficiency of the free-electron laser is shown to increase, although it differs from that found in the weak-field approximation. The free-electron laser efficiency in this case can considerably exceed that of a laser with a "short" undulator, the amplification level in a weak field being the same. In the weak saturation region the amplification decreases slowly (compared to the case of a "short" undulator) and experiences appreciable oscillations. The feasibility of lasing under hard self-excitation conditions is discussed.

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1. It is well known[1] that even in a weak field a free-electron laser (FEL), can operate in two different amplification regimes corresponding to the "short" and "long" undulator approximations. The quantitative parameter ζ which determines the effective length of the undulator and separates the two indicated regions is equal, apart from a coefficient of the order of unity, to the product of the number n of the undulator periods and the relative width of the electron energy distribution $\Delta\varepsilon/\epsilon$: $\zeta = 2\sqrt{\pi}n\Delta\varepsilon/\varepsilon$. The undulator is "long" or "short" depending on whether $\zeta > 1$ or $\zeta < 1$. Notice must be taken of a certain arbitrariness in the concepts of "long" and "short" undulator. A change in the parameter ζ and a transition from one regime to another can be ensured by a real change in the length L of the undulator, as well as by changing the electron energy spread $\Delta\varepsilon$ at a fixed length.

Saturation in FEL and the linear behavior of the gain and efficiency have been quite thoroughly investigated in the "short" undulator approximation[1] (see also the literature cited in the review[1]). In the present paper we shall investigate the features of saturation in an FEL that corresponds to the inverse approximation, i.e., to the approximation of the "long" undulator, when $\zeta > 1$. In this case, in contrast to the "short" undulator, regardless of the conditions of the applicability of the weak-signal approximation, two parameter ranges appear, corresponding to weak and strong saturations, in which the behavior of the gain and of the efficiency are significantly different.

It will be shown below that the transition from the linear approximation to different saturation regimes, just as the transition from the model of the "short" undulator to the model of the "long" one, lends itself to the following unified interpretation. Several mechanisms can broaden the gain contour in an FEL: the broadening can be due to the finite length of the undulator, to the electron-energy spread in the beam, or to the strong field of the electromagnetic wave. To each of these mechanisms there corresponds a separate width of the amplification band. The competition of the different broadening mechanisms is such that the mechanism that predominates is always the one corresponding to the largest width. With change of the system parameters, the ratio of the widths that are due to the different mechanisms changes. The conditions for realization of one approximation or another, or of one model or another, can be defined as the conditions under which the corresponding width prevails over all others. With changing width ratio a transition takes place from one broadening mechanism to another, i.e., from one model to another.

2. In the weak-signal approximation, the gain of an FEL with a "short" undulator ($\zeta < 1$) is known[1] to be

$$G_L = \frac{2\pi N_e e^4 B_0^2 L^3}{c^3 m^2 \varepsilon\omega} \frac{d}{du} \frac{\sin^2 u}{u^2}, \tag{1}$$

where N_e is the electron density in the beam, B_0 is the amplitude of the transverse magnetic field intensity of the helical undulator, ω is the frequency of the amplified signal, $u = -2\pi n\Delta/\varepsilon$, $\Delta = \varepsilon - \varepsilon_0$ is the detuning of the resonance relative to the deviation of the energy from the resonant value,

$$\varepsilon_0 = mc^2(\omega/2q_0 c)^{1/2}, \quad q_0 = 2\pi/\lambda_q,$$

and λ_q is the undulator period.

At large undulator length $\zeta > 1$, but also in the weak-signal approximation, the gain is well known[1] and is given by

$$G_{\Delta\varepsilon} = \frac{N_e e^4 B_0^2 L \lambda_q^2}{2c^3 m^2 \omega} \varepsilon \left.\frac{df}{d\varepsilon}\right|_{\varepsilon=\varepsilon_0}, \tag{2}$$

where $f(\varepsilon)$ is the electron-energy distribution function normalized by the condition $\int f(\varepsilon)d\varepsilon = 1$.

Formula (2) is obtained in elementary fashion from (1) at $\zeta > 1$ by averaging of G_L over the distribution function $f(\varepsilon)$.

Figure 1 shows qualitatively the resonant dependence of the gain on the detuning, $G(\Delta)$ at $\zeta < 1$ and $G(\overline{\Delta})$ at $\zeta > 1$, where $\overline{\Delta} = \overline{\varepsilon} - \varepsilon_0$ is the average detuning that determines the deviation of the average electron energy from the resonant value ε_0. The width of the amplification band Γ is different in these two cases: it is equal to $\Gamma_L = \varepsilon/2\pi n$ at $\zeta < 1$, and $\Gamma = \Gamma_{\Delta\varepsilon} \equiv \Delta\varepsilon$, i.e., the band width is equal to the width of the distribution function $f(\varepsilon)$, at $\zeta > 1$. The maximum (relative to changes of the detuning Δ) values of the gain in the cases of the "short" and "long" undulator are

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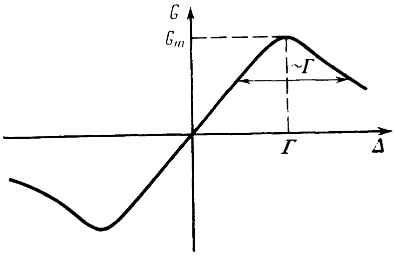

FIG. 1. Resonant dependence of the gain on the detuning.

$$G_{L,m}\approx\frac{\pi N_e e^4 B_0^2 L^3}{c^3 \varepsilon m^2 \omega}, \quad G_{\Delta\varepsilon,m}\approx\frac{N_e e^4 B_0^2 L \lambda_q^2 \varepsilon}{4c^3 m^2 \omega (\Delta\varepsilon)^2}, \tag{3}$$

where we assume

$$[d(\sin^2 u/u^2)/du]_{max}\approx 1/2, \quad [df/d\varepsilon]_{max}\approx 1/2(\Delta\varepsilon)^{-2}.$$

Equations (3) shows that the maximum values of the gain $G_{L,m}$ and $G_{\Delta\varepsilon,m}$ differ by the parameter ζ^2. This means that the smaller of the two gains $G_{L,m}$ and $G_{\Delta\varepsilon,m}$ is always realized (at the same value of B_0, N_e, and ε).

The parameter ζ, which separates the region of applicability of the "short" and "long" undulator approximations, is obviously equal to the ratio of the widths $\Gamma_{\Delta\varepsilon}$ and Γ_L, which are governed by the electron energy spread and by the finite undulator length: $\zeta = \Gamma_{\Delta\varepsilon}/\sqrt{\pi}\Gamma_L$. The width Γ of the $G(\Delta)$ curve is determined in the weak-signal approximation by the larger of the values $\Gamma_{\Delta\varepsilon}$ and Γ_L; this corresponds to the rule formulated in Sec. 1 for separating the broadening mechanism corresponding to the larger of the widths.

3. The region of applicability of the weak-signal approximation in the case of a "short" undulator ($\zeta < 1$) is limited by the condition that the saturation parameter be small.[1]

$$\mu = eL(2E_0B_0)^{1/2}/\varepsilon, \tag{4}$$

where E_0 is the amplitude of the electric field intensity of the amplified wave. At a large value of the saturation parameter μ, the gain decreases with increasing field E_0. In the approximation linear in the detuning Δ, the asymptotic expression for the gain is of the form[1]

$$G_E = \frac{4\pi N_e}{E_0^2}\Delta\left\{1-\frac{2}{(\pi\mu)^{1/2}}\sin\left[\frac{\pi}{4}+\mu\left(1-\frac{\Delta^2}{16\Delta_m^2}\right)\right]\right\}, \tag{5}$$

where

$$\Delta_m = e\lambda_q(E_0B_0/2)^{1/2}/2\pi. \tag{6}$$

Qualitatively, the $G(\Delta)$ dependence is characterized in this case again by the curve shown in Fig. 1 where, however, the band width Γ is determined by the value of the field E_0:

$$\Gamma = \Gamma_E \equiv \Delta_m.$$

Equation (5) is valid at $|\Delta| < \Gamma_E$, $\mu > 1$ (Ref. 2). The maximum value of the gain (relative to changes of the detuning Δ, but at a specified value of the field E_0) is reached at $\Delta\approx\Delta_{max}\simeq 1.4\Gamma_E$:

$$G_{E,max}\approx 0.8\frac{4\pi N_e}{E_0^2}\Gamma_E\left\{1-\frac{2}{(\pi\mu)^{1/2}}\sin\left(\frac{\pi}{4}+\mu\right)\right\}. \tag{7}$$

The numerical value μ_0 of the saturation parameter μ, at which the transition takes place from the weak-signal approximation [Eq. (1)] to saturation [Eqs. (5) and (7)], is determined most convincingly from the approximate-equality condition $G_{E,max}\simeq G_{L,m}$, which yields $\mu_0\approx 2.3$. Since it can be easily seen that $\mu = \Gamma_E/\pi\Gamma_L$, the condition $\mu \gtrsim 2.3$ means that at $\Gamma_E < \Gamma_L$, i.e., at $\zeta < 1$ we have under saturation conditions $\Gamma_{\Delta\varepsilon} < \Gamma_L < \Gamma_E$. Therefore the width of the gain band is in this case of the order of Γ_E, and the inequalities cited confirm again the rule for determining the larger of the widths (Sec. 1).

The numerical coefficients 1.4 and 0.8 in the definitions of the detuning Δ_{max} at which the gain is a maximum, $\Delta_{max}\approx 1.4\Gamma_E$ and of its maximum value G_{max} (7), can of course not be obtained from the conditions for the applicability of the analytic asymptotic formula (5). These values, however, can be obtained from analysis of the $G(\Delta)$ curves at $\mu > 1$, plotted from numerical calculations, such as in Ref. 3. Comparison of the maximum values of the gain, obtained in Ref. 3 at $\mu = 2$, 3, and 4 (in our notation) with the calculation by formulas (7) and (3) yields again the numerical factor ≈ 0.8 in expression (7). Next, according to Ref. 3, the gain $G(\Delta)$ at $\mu = 2$ and $\mu = 3$ is a maximum at the values of the detuning parameters $\mu\Delta/\Delta_m$ used in Ref. 3, namely $\mu\Delta/\Delta_m \approx 2.9$ and $\mu\Delta/\Delta_m\approx 3.95$, i.e., on the average at $\Delta\approx\Delta_{max}\approx 1.4\Delta_m$.

A more or less correct quantative estimate of the numerical coefficients Δ_{max} and G_{max} is important for a correct determination of the nonlinear gain in the FEL with "long" undulator and for the estimate of the region of applicability of the corresponding formulas (see Sec. 4 below).

4. We proceed now to describe the saturation in an FEL with a "long" undulator, i.e., under the condition $\zeta > 1$ (or $\Gamma_{\Delta\varepsilon} > \Gamma_L$). In this case it is clear that the qualitative criterion of the applicability of the weak-signal approximation is as before the condition that the saturation parameter μ (4) be small, or that the field width Γ_E be small compared with Γ_L.

At large values of the saturation parameter μ, the weak-signal approximation is inapplicable but, generally speaking, the asymptotic formulas (5)–(7) are also inapplicable. The reason why these aymptotic formulas are inapplicable is that even although the field width Γ_E at $\mu > 2.3$ indeed exceeds the value Γ_L governed by the finite length of the undulator, the condition $\zeta > 1$ it can make it smaller than the width $\Gamma_{\Delta\varepsilon}$ determined by the electron energy spread. Nonetheless, the asymptotic formulas (5)–(7) are valid in this case for a separate group of elecrons with a specified energy ε. This means that to obtain the gain of the entire electron beam as a whole it is necessary to average over the electron distribution function $f(\varepsilon)$, but what is averaged in this case is not Eq. (1), but the strong-field gain defined by formulas (5)–(7).

Bearing in mind the need for such an averaging and taking the condition $\Gamma_E > \Gamma_L$ into account, we shall assume the $G(\Delta)$ plot for an electron group with a given energy ε to be highly peaked, and approximated by a derivative of a δ function

$$G(\Delta)=-A\delta'(\Delta). \quad (8)$$

The constant A is expressed in terms of the width and height Γ_E and $G_{E,\max}$ of the $G(\Delta)$ curve, and is obviously equal to the area bounded by $\Delta G(\Delta)$ curve:

$$A=\int_{-\infty}^{+\infty}\Delta G(\Delta)\,d\Delta=\alpha\Gamma_E^2 G_{E,max}, \quad (9)$$

where α is a numerical parameter. An exact determination of α is possible obviously only on the basis of numerical calculations. A more or less satisfactory result, however, can apparently be obtained with the aid of the analytic formulas (5) and (7). We approximate the $G(\Delta)$ curve by the linear dependence (5) on the section $|\Delta|<0.5\Delta_m$, by a linear relation that satisfies the condition $G(\Delta=1.4\Delta_m)=G_{E,\max}$ (7) on the section $0.5\Delta_m<|\Delta|<1.4\Delta_m$, and by a linear decreasing dependence with a constant slope on the section $1.4\Delta_m<|\Delta|<2.8\Delta_m$, assuming that $G(\Delta=2.8\Delta_m)=0$. Calculation of the integral (9) with such an approximation yields $\alpha\approx 4.2$. Using this value of α in Eqs. (6)–(9), we average the gain $G(\Delta)$ over the distribution function $f(\varepsilon)$. As a result of the averaging we obtain the following expression for the average gain in the parameter region defined by the inequalities $\Gamma_L<\Gamma_E<\Gamma_{\Delta\varepsilon}$:

$$\bar{G}_E=3.4\frac{4\pi N_e}{E_0^2}\Gamma_E^3\left\{1-\frac{2}{(\pi\mu)^{1/2}}\sin\left(\frac{\pi}{4}+\mu\right)\right\}\frac{df}{d\varepsilon}\bigg|_{\varepsilon=\varepsilon_0}. \quad (10)$$

This is in fact the sought nonlinear gain of the FEL with a "long" undulator.

The width of the gain band after averaging becomes equal to $\Gamma_{\Delta\varepsilon}\equiv\Delta\varepsilon$ in accordance with the selection rule for the largest of the widths. The maximum value of the average gain (10) (relative to the changes of the average detuning $\bar{\Delta}=\bar{\varepsilon}-\varepsilon_0$, i.e., for given E_0, L, and μ) is of the order of

$$\bar{G}_{E,m}\approx 3.4\frac{N_e e^3 B_0^{1/2}\lambda_q^3}{8\pi^2\sqrt{2}E_0^{1/2}(\Delta\varepsilon)^2}\left\{1-\frac{2}{(\pi\mu)^{1/2}}\sin\left(\frac{\pi}{4}+\mu\right)\right\}$$
$$\sim\left(\frac{\Gamma_E}{\Gamma_{\Delta\varepsilon}}\right)^2 G_{E,max}\sim\frac{\Gamma_L}{\Gamma_E}G_{\Delta\varepsilon,m}. \quad (11)$$

The numerical value of the saturation parameter μ, starting with which Eq. (10) and (1) become valid, can again be obtained from the condition that $\bar{G}_{E,m}\approx G_{\Delta\varepsilon,m}$, which yields $\mu\approx 3$.

Equations (10) and (11) describe the nonlinear behavior of the gain in an FEL with a "long" undulator in the parameter range $\Gamma_L<\Gamma_E<\Gamma_{\Delta\varepsilon}$. With increasing field intensity E_0, the gain $\bar{G}_E$ decreases in this region in proportion to $E_0^{-1/2}$, i.e., much more slowly than in the range of values of the saturation parameter μ in the case of a "short" undulator, where in accord with (7) we have $G_{E,\max}\propto E_0^{-3/2}$. Since $\bar{G}_E$ decreases slowly, the parameter range $\Gamma_L<\Gamma_E<\Gamma_{\Delta\varepsilon}$ can be called the region of the weak saturation. The presence of this region is a distinguishing feature of the case of the "long" undulator, inasmuch as in an FEL with a "short" undulator the weak-saturation region, as is well known,[1] does not occur. In an FEL with a "long" undulator, the strong saturation arises in the case when the field E_0 is so strong that $\Gamma_E>\Gamma_{\Delta\varepsilon}>\Gamma_L$. The ratio of the widths Γ_E and $\Gamma_{\Delta\varepsilon}$ determines the new saturation parameter

$$\mu^*=\Gamma_E/\Gamma_L=\pi^{1/2}\mu/\zeta. \quad (12)$$

The meaning of this parameter is that it is precisely the condition $\mu^*>1$ (and not $\mu>1$) which is the condition for the transition into the strong-saturation region in an FEL with a "long" undulator. At $\mu^*>1$ the field width Γ_E becomes so large that it exceeds the width $\Delta\varepsilon$ of the electron distribution function. For this reason, at $\Gamma_E>\Gamma_{\Delta\varepsilon}>\Gamma_L$ there is no need for averaging over the energy distribution of the electrons, and the gain is determined again by the usual formulas (5)–(7) of the nonlinear theory, regardless of the value of the parameter ζ. The width of the gain band is euqal in this case, as before, to the largest of the widths Γ_L, $\Gamma_{\Delta\varepsilon}$, and Γ_E, i.e., in this case it is equal to the field width Γ_E.

5. Besides the gain of the FEL, great interest attaches also to its efficiency, defined as the ratio of the energy radiated by the electron in one pass to the initial electron energy:

$$\eta=\frac{E_0^2}{4\pi N_e\varepsilon}G.$$

We present now formlas for the efficiency of the FEL, η_m, maximized with respect to the detuning Δ (or $\bar{\Delta}$), in the weak field region ($\eta_{L,m}$ and $\eta_{\Delta\varepsilon,m}$) and under saturation conditions ($\eta_{E,m}$ and $\bar{\eta}_{E,m}$) in the cases of the "short" and "long" undulators respectively:

$$\eta_{L,m}=\frac{\mu^4}{64\pi n},\quad \eta_{\Delta\varepsilon,m}=\frac{\mu^4}{(16\pi)^2 n^3}\left(\frac{\varepsilon}{\Delta\varepsilon}\right)^2=\frac{1}{\zeta^2}\eta_{L,m}, \quad (13)$$

$$\eta_{E,m}=\frac{0.8\mu}{4\pi n}\left\{1-\frac{2}{(\pi\mu)^{1/2}}\sin\left(\frac{\pi}{4}+\mu\right)\right\}, \quad (14)$$

$$\bar{\eta}_{E,m}=\frac{3.4\mu^3(\varepsilon/\Delta\varepsilon)^2}{128\pi^3 n^3}\times\left\{1-\frac{2}{(\pi\mu)^{1/2}}\sin\left(\frac{\pi}{4}+\mu\right)\right\}=\frac{1.7\mu^2}{3.2\pi\zeta^2}\eta_{E,m}. \quad (15)$$

The $\eta(\mu)$ plots corresponding to these formulas are shown in Fig. 2. The solid and dashed lines in Fg. 2a shows the efficiency of FEL with "long" and "short" undulators. Actual-

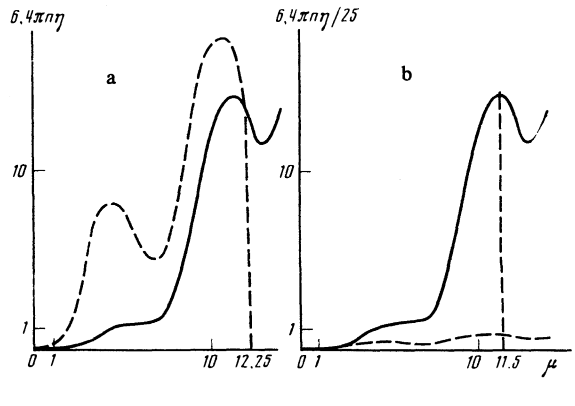

FIG. 2. Efficiency η as a function of the saturation parameter μ in FEL with "short" (dashed curves) and "long" (solid curves) undulators at different (a) and identical (b) gain levels in a weak field.

ly these curves correspond to systems in which all the parameters including the undulator length L, coincide, with the exception of the width $\Delta\varepsilon$ of the energy spread of the electrons in the beam. The difference between the values $\Delta\varepsilon$ ensures in fact the transition from the case $\zeta < 1$ to the case $\zeta = 5 > 1$ (the specific value $\zeta = 5$ was chosen for ease in illustration).

Formulas (13)–(15) and the curves in Fig. 2a point to a considerable qualitative difference between the behavior of the FEL efficiencies at $\zeta < 1$ and $\zeta > 1$. It is known[1] that the efficiency of an FEL with a "short" undulator increases with increasing field and oscillates (on the average in proportion to $E_0^{1/2}$), and the depth of the oscillations is far from small, as is illustrated by the dashed curve in Fig. 2a. The structure of the efficiency of the FEL with a "long" undulator in the weak saturation region (15) is at first glance quite similar to the structure of the efficiency of the FEL with a "short" undulator (14). However, the faster power-law growth (on the average proportional to $E_0^{3/2}$) compared with $\eta_m \propto E_0^{1/2}$ leads to a qualitatively new singularity. Up to a saturation-parameter value $\mu \approx 11.5$ the power-law factor μ^3 in (15) compensates for the decrease of the oscillating factor, as a result of which the FEL efficiency in the weak saturation region at $\mu < 11.5$ increases monotonically with increasing field intensity E_0. The fact that $\bar{\eta}_m(\mu)$ does not decrease at $\mu \lesssim 11.5$ distinguishes qualitatively the FEL with the "long" undulator ($\zeta > 1$) from systems whose parameters correspond to the "short" undulator model. The oscillatory dependence $\bar{\eta}_m(\mu)$ sets in the case of $\zeta > 1$ only at $\mu > 11.5$ or else on going to a stronger saturation $\mu > 2.45\,\zeta$. In the region of strong saturation, the gain decreases rapidly and becomes less than the loss level in the cavity (Fig. 3). Therefore in practically the entire range of parameters of real interest, the efficiency of the FEL with a "long" undulator increases monotonically, although a $\mu > 3$ the $\bar{\eta}(\mu)$ dependence does differ from the $\bar{\mu} \propto \mu^4$ dependence whcih is valid in the weak-signal approximation.

The quantitative difference between the efficiencies of FEL with "short" and "long" undulators can also become quite noticeable at large values of the saturation parameter μ, as illustrated in Fig. 2b. The dashed curve in Fig. 2b are, in a different scale, the same plot as in Fig. 2a, i.e., the efficiency of an FEL with a "short" undulator. The solid curve of Fig. 2b is the efficiency of an FEL with a "long" undulator at $\zeta = 5$. It is assumed that on going from the "short" to the "long" undulator the electron energy spread $\Delta\varepsilon$ increases. It is also assumed that the electron current in the beam increases in this case by a factor $\zeta^2 = 25$, the other parameters remaining unchanged. The increase of the current ensures an identical gain in the FEL with the "short" and "long" undulators in a weak field (as $\mu \to 0$). As can be easily seen from Fig. 2b, at identical gain in a weak field, the efficiency of an FEL with a "long" undulator in the weak-saturation region exceeds considerably the efficiency of an FEL with a "short" undulator. In the general case, the maximum increase of the efficiency is determined by the factor $\zeta^2 \gg 1$ and is reached at the boundary of the regions of the weak and strong saturation, i.e., at $\mu = 2.45\,\zeta = 12.25$.

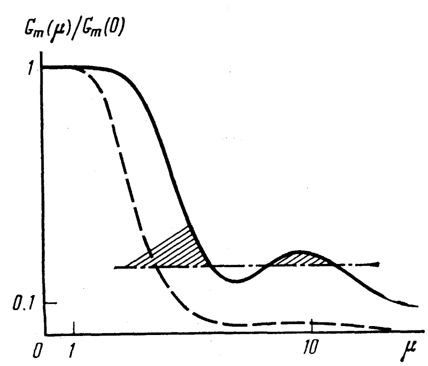

FIG. 3. Gain, normalized to unity as $\mu \to 0$, in an FEL with a "short" (dashed curves) and a "long" (solid) undulators.

Figure 3 shows the dependence, on the saturation parameter μ, of the FEL gain referred to its value in a weak field ($G_{L,m}$ or $G_{\Delta\varepsilon,m}$), in the "short" (dashed curve) and "long" undulator (solid curve) regimes. The parameters of the systems in these two cases are again chosen such that the difference in the value of the parameter ζ is determined exclusively by the difference between the values of the width $\Delta\varepsilon$ of the distribution function $f(\varepsilon)$, with all other conditions equal. The qualitative distinguishing features of the saturation in the "long" undulator regime are the following: 1) on the average, the gain decreases with increasing field E_0 much more slowly than in the "short" undulator regime; 2) as a result, the oscillations of the gain as functions of E_0 become much more noticeable, as can be easily seen in the figure. The last circumstance makes more realistic the possiblity of observing two lasing bands in the case of a "long" undulator. Lasing in an FEL is possible if the gain exceeds the loss level in the cavity. If the loss level is determined by the horizontal dash-dot line of Fig. 3, lasing is obviously possible both at field intensities starting from zero, and in another band of stronger fields (hard self-excitaiton), but is impossible in the intermediate region. In Fig. 3, the regions corresponding to the possiblity of lasing are shaded.

In principle, a similar possibility exists also in the "short" undulator region.[1] In this case, however, the gain $G(0)$ must greatly exceed the loss level, since the second maximum of the gain is quite small ($\sim 0.02 G_{L,m}$, Ref. 1). In contrast, in the case of a "long" undulator the height of the second maximum of the gain is only slightly smaller than its value in a weak field. Therefore in this case lasing in this hard self-excitation regime is possible if, for example, the loss level in the cavity, referred to the weak-field gain $G_{\Delta\varepsilon,m}$, is in the interval from 0.18 to 0.3. Observation of the hard regime of self-excitation in the FEL is possible, naturally, on passage of a priming electromagnetic wave whose intensity is so large that the system is immediately in the lasing region adjacent to the second maximum of the gain.

From the practical point of view it is clear that, depending on the beam parameters obtained with different accelerators and on the undulator parameters, any of the amplification regimes, with "short" and "long" undulators, can be realized. For example, in the case of the Stanford experiment,[4] the undulator length was $L = 5$ m at a periodicity pitch $\lambda_q = 3.2$ cm and a relative width of the electron distribution in energy $\Delta\varepsilon/\varepsilon \sim 10^{-3}$, corresponding to a parameter value $\zeta \approx 0.554$, i.e., to the "short" undulator regime. It is clear that the "long" undulator regime can be realized, for example, at the same values of L and λ_q, but the larger value of $\Delta\varepsilon$, i.e., in beams that are less monoenergetic, which are in fact those realized in many accelerators. For example, at $\Delta\varepsilon/\varepsilon = 0.93 \times 10^{-5}$ we hve $\zeta = 5$. In order for lasing to be possible in such a system, it is necessary to ensure preservation of the previous value of the gain ($\sim 10\%$ in Ref. 4). This condition can be satisfied not withstanding the decrease of the gain by a factor ζ^2 on going over to the "long" undulator, if the electron current is increased by the same number of times. In the example considered, this means that the current must be 25 times larger than under the conditions of the experiment of Ref. 4, i.e., it should amount to $I = 65$ A. This value is perfectly realistic at an electron energy spread $\Delta\varepsilon/\varepsilon \sim 10^{-2}$. In such a system, all the above singularities of the weak saturation regime should take place, namely a monotonic increase of the efficiency, as well as a slow decrease and noticeable oscillations of the gain.

We note finally that an analysis of the properties of saturation in an FEL with a "long" undulator can be important for the development of a nonlinear theory of amplification in a Compton laser for noncollinear propagation of electrons and of the amplified wave, inasmuch as optimization of the gain in such a scheme is determined in the linear regime precisely by the relation between the long region of the interaction and the electron-energy scatter in the beam.[5]

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