Diagram technique in the theory of relaxation processes

Yu. A. Kukharenko and S. G. Tikhodeev

P. N. Lebedev Physics Institute, USSR Academy of Sciences
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A diagram technique is constructed for the calculation of the Green's functions of a macroscopic nonequilibrium system with allowance for correlations. The theory is free of secular divergences and permits a complete description of the relaxation to the equilibrium state. A system of transport equations for the long-lived correlators is derived and describes the influence of the fluctuations on the system relaxation. An equation for the pair correlation function of a rarefied gas is obtained and contains in addition to the known terms also others that can play a substantial role in the hydrodynamic stage.

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§1. INTRODUCTION

Problems in physical kinetics can be naturally divided into two classes. The first is connected with finding the constrained solutions describing the response to an external field that can be directly included in the Hamiltonian of the system. Abrikosov, Gor'kov, and Dzyaloshinskiĭ,[1] Fradkin,[2] Konstantinov and Perel',[3] and Dzyaloshinskiĭ[4] have developed in their papers a diagram technique that makes it possible to calculate in the linear approximation the response in the external field. Keldysh[5], Gor'kov and Éliashberg,[6] and Kadanoff and Baym[7] have constructed a diagram technique for the description of essentially nonequilibrium states that evolve from the equilibrium state as a result of adiabatic application of a strong external field.

The second class of problems is connected with the determination of the relaxation of an arbitrarily specified (at the initial instant of time) nonequilibrium density matrix of the system to the equilibrium state in the absence of external field. In this case, the initial state of the system is described by a set of correlation functions (correlators) of all orders, as a result of which Wick's theorem cannot be used and there is no conventional diagram technique. Fujita[8] and Prigogine[9] have constructed a diagram technique that makes it possible to develop a perturbation theory directly for the multiparticle density matrix of a system satisfying the Liouville equation. Unfortunately, this technique differs substantially from the Feynman technique and is very complicated, since it does not make use of the second-quantization formalism to take into account the symmetry of the density matrix with respect to permutation of the particle coordinates.

A diagram technique with allowance for the initial correlations, which generalizes the Keldysh technique, was formulated in the papers by Hall.[10,11] This diagram technique, however, is not suitable for long times of the order of the relaxation time, and consequently does not describe the relaxation of the system to the equilibrium state.

In the present paper, on the basis of the approach proposed by Hall, we develop a diagram technique free of secular divergences, which makes it possible to describe completely the relaxation of a macroscopic system to the equilibrium state. A system of kinetic equations is obtained for the correlation functions. An equation is derived for the pair correlation function. An equation for the pair correlation function in a rarefied gas was first obtained by Kadomtsev.[12] An analogous approach to the theory of fluctuation was developed in the work by Gor'kov, and Dzyaloshinskiĭ and Pitaevskii.[13] Kogan and Shul'man,[14] Gantsevich, Gurevich, and Katilyus,[15] and Klimontovich[16] have shown that in the nonequilibrium cases it is necessary to take into account an additional fluctuation source. The equation obtained by us for the two-particle correlator contains the additional terms that influence the temporal evolution of the fluctuations.

In §2 we introduce a diagram technique for a nonequilibrium system with account taken of the initial correlations. It is shown in §3 that this diagram technique contains secular divergences at long times; these can be reduced to singular diagrams that renormalize the single-particle density and the long-lived correlators. In §4 is developed a method for renormalizing the diagram technique and a system of transport equations is derived for long-lived correlators. An equation for the pair correlation function of a rarefied gas is obtained in §5.

§2. DIAGRAM TECHNIQUE WITH ALLOWANCE FOR THE INITIAL CORRELATIONS

We consider the evolution of a macroscopic system of Bose or Fermi particles, whose initial state is determined by an arbitrary nonequilibrium density matrix $\rho(t_0)$. Complete information on the system is given by the single-particle and multiparticle Green's functions G and $K^{(n)}$, $n = 2, 3, \ldots$ (Refs. 5 and 17), equal in the momentum representation to

$$G(1, 1') = \frac{1}{i\eta}\langle T_c a_H^+(1') a_H(1)\rangle = \frac{1}{i\eta}\,\mathrm{Sp}\rho(t_0) T_c a_H^+(1') a_H(1), \tag{2.1}$$

$$K^{(n)}(1, \ldots, n; n', \ldots, 1') = \frac{1}{(i\eta)^n}\langle T_c a_H^+(1') \ldots a_H^+(n') a_H(n) \ldots a_H(1)\rangle. \tag{2.2}$$

Here $1 = (p_1, t_1, \alpha_1)$; $p_1 = (p_1, \sigma_1)$ is the aggregate of the momentum and spin of the particle, whose Heisenberg annihila-

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tion operator is

$$a_H(1)=e^{iH(t_1-t_0)}a_{p_1}e^{-iH(t_1-t_0)};$$

H is the Hamiltonian of the system; $\eta=1$ for bosons and -1 for fermions. The boson (fermion) creation and annihilation operators a_p and a_p^+ satisfy the usual (anti) commutation relations. α_1 is a temporal index indicating whether the point belongs to the upper[1)] ($\alpha_1=-$) or lower ($\alpha_1=+$) part of the close contour c that goes from t_0 to

$$t^m=\max(t_1,t_1',\ldots,t_n,t_n')$$

and back. In the contour c we carry out T_c ordering that coincides with the chronological ordering T on the lower part of the contour and with the antichronological $\tilde{T}$ on the upper. Any time t_+ located on the lower part precedes by definition t'_-.

From the Green's functions (2.1) and (2.2) we calculate all the mean values, including the distribution functions at the current instant of time

$$f^{(n)}(p_1,\ldots,p_n;p_n',\ldots,p_1';t)=\langle a_H^+(p_1',t)\ldots a_H^+(p_n',t)a_H(p_n,t)\ldots a_H(p_1,t)\rangle \quad (2.3)$$

and the correlators $g^{(n)}(p_1,p_1';\ldots;p_n,p_n';t)$ connected with them by the following relations:

$$f^{(1)}(p_1,p_1',t)=g^{(1)}(p_1,p_1'\,t),$$
$$f^{(2)}(p_1,p_2;p_2',p_1',t)=g^{(1)}(p_1,p_1',t)\times g^{(1)}(p_2,p_2',t)+\eta g^{(1)}(p_1,p_2',t)g^{(1)}(p_2,p_1',t)+g^{(2)}(p_1,p_1';p_2,p_2',t)+\eta g^{(2)}(p_1,p_2';p_2,p_1',t), \quad (2.4)$$
$$f^{(n)}(p_1,\ldots,p_n;p_n',\ldots,p_1';t)=\sum\eta^{[P_n']}P_n'P_{n,q,r,s}\times\underbrace{g^{(q_1)}\ldots g^{(q_1)}}_{r_1}\ldots\underbrace{g^{(q_s)}\ldots g^{(q_s)}}_{r_s},$$

where the summation is over all the breakdowns of n into $1\leqslant r_1+\ldots+r_s\leqslant n$ terms of the form $n=r_1q_1+\ldots+r_sq_s$, $q_1,\ldots q_s\geqslant 1$, over all $n!$ permutations of P_n' coordinates with primes, and over all $n!/[(q_1!)^{r_1}\cdot r_1!\ldots(q_s!)^{r_s}r_s!]$ nonidentical permutations $P_{n,q,r,s}$ pairs of coordinates between different correlators, with account taken of the parity of the permutation $[P_n']$ in the case of fermions. The correlators (2.4) have the following symmetry property with respect to permutation of the arguments: $g^{(n)}=\eta^{[P_n']}P_n'g^{(n)}$, from which it follows that all the terms in (2.4), that differ only in the order of the primed arguments inside the correlators, coincide. This definition of the correlators turns out to be convenient for the construction of the diagram technique.

We introduce the connective Green's functions $G^{(n)}(\mathbf{1},\mathbf{1}';\ldots;\mathbf{n},\mathbf{n}')$, $n=2,3,\ldots$, expressed in terms of $G=G^{(1)}$ and $K^{(n)}$ with the aid of relations similar to (2.4), with the replacements $f^{(1)}\to G^{(1)}$, $f^{(n)}\to K^{(n)}$, $g^{(n)}\to G^{(n)}$, $p_1\to 1$. We note that the values of the functions $G^{(n)}$, $n>1$, at coinciding instants of time $t_1=\ldots=t_n'=t$ do not depend, in contrast to G, on the temporal indices and on the order of the tendency of the times to t (Ref. 18):

$$\mathbf{G}^{(n)}(1,1';\ldots;n,n')|_{t_1=\ldots=t_n'=t}=\frac{1}{(i\eta)^n}g^{(n)}(p_1,p_1';\ldots;p_n,p_n';t)\begin{pmatrix}1,1\\1,1\end{pmatrix}\otimes\ldots\otimes\begin{pmatrix}1,1\\1,1\end{pmatrix} \quad (2.5)$$

where $\otimes$ is the symbol for the direct product of matrices whose number is equal to n, and $\mathbf{1}=(1,\alpha_1)$.

Assume for the sake of argument that the system is described by the Hamiltonian

$$H=H_0+V,\quad H_0=\sum_p\varepsilon_pa_p^+a_p, \quad (2.6)$$

$$V=\frac{1}{2\mathscr{V}}\sum_{p_1,p_1';p_2,p_2'}V(p_1,p_1';p_2,p_2')a_{p_1'}^+a_{p_2'}^+a_{p_2}a_{p_1}, \quad (2.7)$$

where $\varepsilon_p=\mathbf{p}^2/2m$ is the kinetic energy of the particles,

$$V=V(|\mathbf{p}_1-\mathbf{p}_1'|)\delta_{\mathbf{p}_1+\mathbf{p}_2,\,\mathbf{p}_1'+\mathbf{p}_2'}\delta_{\sigma_1\sigma_1'}\delta_{\sigma_2\sigma_2'}$$

is the pair-interaction potential, and $\mathscr{V}$ is the volume of the system.

A diagram technique for nonequilibrium systems, based on the known Keldysh technique[5,17] and making it possible to calculate the Green's functions with allowance for the initial correlations, was proposed by Hall in Refs. 10 and 11. Hall has shown that among the diagrams for the functions G and $G^{(n)}$ are included all the connective Keldysh diagrams for these quantities, which depend via of the free Green's function on the initial density $f(p,t_0)$, and in which the integration with respect to time in the internal vertices is carried out from t_0 to $t^m=\max(t_1,\ldots,t_n')$. All the remaining diagrams contain the correlation functions

$$G_0^{(k)}(1,1';\ldots;k,k')=\frac{1}{(i\eta)^k}g^{(k)}(p_1,p_1';\ldots;p_k,p_k';t_0)\times\exp\left\{\sum_{j=1}^{k}i\varepsilon_{p_j'}(t_j'-t_0)-i\varepsilon_{p_j}(t_j-t_0)\right\}\begin{pmatrix}1,1\\1,1\end{pmatrix}\otimes\ldots\otimes\begin{pmatrix}1,1\\1,1\end{pmatrix}, \quad (2.8)$$

to which corresponds the diagram shown in Fig. 1. These diagrams are obtained from the Keldysh diagram in which all the possible correlation lines are drawn. Figure 2 shows the Keldysh diagram of second order in the interaction for the Green's function $\mathbf{G}$ and examples of the corresponding correlation diagrams, which contain $\mathbf{G}_0^{(2)}$, $\mathbf{G}_0^{(2)}\cdot\mathbf{G}_0^{(2)}$ and $\mathbf{G}_0^{(3)}$. It is necessary to discard here the correlationally-related diagrams[19] which are equal to zero and containing some block connected with the remaining part of the diagram only via the correlation functions. Examples of such diagrams, corresponding to the Keldysh diagram shown in Fig. 2(a), are given in Fig. 3.

We shall find it convenient to transform to the retarded and advanced Green's functions G^r and G^a (Refs. 5 and 17). In this representation the matrices $\begin{pmatrix}1,1\\1,1\end{pmatrix}$ are replaced in (2.8)

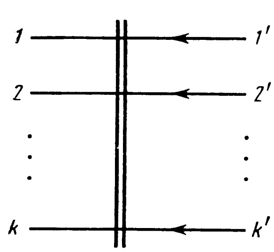

FIG. 1. Diagram corresponding to the correlation function $G_0^{(k)}$ $(1,1';\ldots;k,k')$ (2.8).

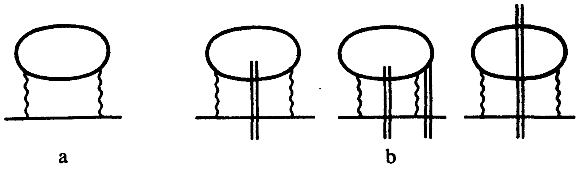

FIG. 2. Keldysh diagram (a) and the corresponding correlation diagrams (b) containing $\mathbf{G}_0^{(2)}$, $\mathbf{G}_0^{(2)}\mathbf{G}_0^{(2)}$, and $\mathbf{G}_0^{(3)}$.

by the matrices $\begin{pmatrix}0,0\\0,2\end{pmatrix}$, so that the correlation function $\mathbf{G}_0^{(n)}$ has only one nonzero component, with indices "2". The free Green's function is

$$\mathbf{G}_0(1,1')=\begin{pmatrix}0, & G_0{}^a\\ G_0{}^r, & F_0\end{pmatrix}=-i\exp[i\varepsilon_{p_1}(t_1'-t_1)]$$

$$\times\delta_{p_1p_1'}\begin{pmatrix}0, & -\Theta(t_1'-t_1)\\ \Theta(t_1-t_1'), & 1+2\eta f(p_1,t_0)\end{pmatrix}. \tag{2.9}$$

The relations for the Green's functions with coinciding times take the form

$$\mathbf{G}_0(1,1')|_{\substack{t_1'=t+\delta\\ t_1=t}}=-i\begin{pmatrix}0, & -\Theta(\delta)\\ \Theta(-\delta), & 1+2\eta f(p_1,t)\end{pmatrix}\delta_{p_1p_1'}, \tag{2.10}$$

$$\mathbf{G}^{(n)}(1,1';\dots;n,n')|_{t_1=\dots=t_n'=t}$$

$$=\left(\frac{2}{i\eta}\right)^n g^{(n)}(p_1,p_1';\dots;t)\begin{pmatrix}0,0\\0,1\end{pmatrix}\otimes\dots\otimes\begin{pmatrix}0,0\\0,1\end{pmatrix}. \tag{2.11}$$

§3. SECULAR DIVERGENCES

The diagram technique introduced in the preceding section, which takes into account the initial correlations, contains secular divergences at large values of $|t_0|$, and consequently does not describe the irreversible transition of the system to the equilibrium state. To demonstrate this, we consider the n-particle inset $\mathbf{I}^{(n)}(1,1';...;n,n')$ (Fig. 4)—a singly-connected part of the diagram, some of the n incoming and n outgoing lines of which can enter in the correlation functions. The insert whose external momenta satisfy the condition

$$\omega_n=\sum_{j=1}^{n}\varepsilon_{p_j}-\varepsilon_{p'_j}=0, \tag{3.1}$$

will be called isoenergetic and will be designated $\mathbf{I}_\varepsilon^{(n)}$. We note that any single-particle insert is isoenergetic, in view of the spatial homogeneity.

We consider the component with indices "2" of the diagram shown in Fig. 4 (we write out only the time-dependent arguments):

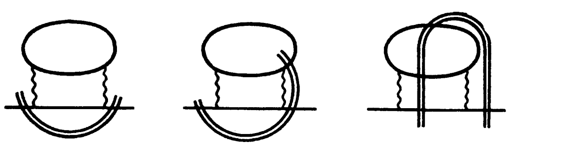

FIG. 3. Examples of zero correlation diagrams corresponding to the diagrams shown in Fig. 2a. (The second diagram of Fig. 2b is also correlationally connective and is equal to zero).

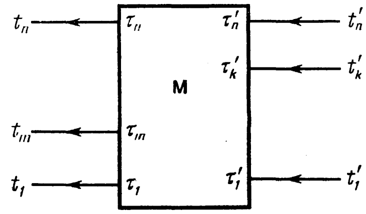

FIG. 4. Inset $\mathbf{I}^{(n)}$

$$\mathbf{I}^{(n)}(t_1,t_1';\dots;t_n,t_n')_{2\dots2}$$

$$=\exp\Sigma(i\varepsilon_{p'_j}t_j'-i\varepsilon_{p_j}t_j)\int_{t_0}^{t^m}\{d\tau\}\Theta(t_k'-\tau_k')$$

$$\times\Theta(t_m-\tau_m)\dots\mathbf{M}(\tau_1,\tau_1';\dots;\tau_n,\tau_n')_{\dots1\dots}\exp\Sigma(i\varepsilon_{p_j}\tau_j-i\varepsilon_{p'_j}\tau_j'), \tag{3.2}$$

where $t^m=\max(t_1,...,t_n')$, and the Θ-functions arise when any one of the external lines is $G_0^a(\tau_k',t_k')$ or $G_0^r(t_m,\tau_m)$. At least one such line is always present, for otherwise (3.2) would contain the block $\mathbf{M}_{2...2}=0$ (the last equality is proved in Ref. 19). It is important that the Θ functions do not limit from below the regions of integration with respect to τ. We note that integration with respect to the internal times τ is carried out up to the largest of the external times of the given inset, and not with respect to the diagram whose internal part it is, since the integration with respect to times longer than t^m yields a zero contribution to (3.2).

If the insert $\mathbf{I}^{(n)}$ is simple (i.e., contains no singularities) and isoenergetic, then in the limit $t_0\to-\infty$ the integrand in (3.2) depends on $2n-1$ difference times; whereas the integration is carried out over $2n$ time between infinite limits. Therefore, as $t_0\to\infty$ the integral in (3.2) diverges in proportion to $|t_0|$ on account of the "extra" integration over the total time. If $\mathbf{I}_\varepsilon^{(n)}$ is not simple and contains diverging parts, then the singularity as $t_0\to-\infty$ is of higher order: expression (3.2) is proportional to $|t_0|^k, k>1$. On the other hand, if the simple inset is not isoenergetic, the integrand in (3.2) depends explicitly on the total time and the integral converges.

We consider now that component of the simple inset $\mathbf{I}^{(n)}$ which has at least one external index "1". Such a component does not diverge at $t_0\to-\infty$, since at least one function $\Theta(\tau_k'-t_k')$ or $\Theta(\tau_m-t_m)$ appears in the corresponding integral of type (3.2) and restricts from below the limits of integration with respect to τ_k' or τ_m. (As $t_0\to-\infty$ all the external times are regarded as fixed.)

We shall show that any simple inset $\mathbf{I}_\varepsilon^{(n)}$, which diverges as $t_0\to-\infty$, can be broken up into a sum of regular and singular parts in such a way that the singular part is the corresponding diagram for the density ($n=1$) or for the correlator ($n=2, 3, ...$) at the running instant of time. Writing $\Theta(\tau)=1-\Theta(-\tau)$, we separate from (3.2) the part that is regular as $t_0\to-\infty$, in which integration with respect to at least one of the internal times is cut off from below. The remaining singular part is a diagram for the equal-time n-particle inset $\mathbf{I}^{(n)}(t^m, t^m;...; t^m, t^m)_{2...2}$. The latter, being a certain diagram for the n-particle Green's function $\mathbf{G}^{(n)}$, $n=1,2,...$, with coinciding times, is as a result of relations

(2.10) and (2.11) the corresponding diagram for the density or the correlator at the instant of time t^m. As a result we obtain

$$\mathbf{I}_\varepsilon^{(n)}(1,1';\ldots;n,n')=\mathbf{I}_\varepsilon^{(n)}(1,1';\ldots;n,n')_R+(2/i\eta)^n\exp\{i\Sigma[\varepsilon_{p_j'}(t_j'-t^m)-\varepsilon_{p_j}(t_j-t^m)]\}\Delta g_\varepsilon^{(n)}(p_1,p_1';\ldots;t^m)\begin{pmatrix}0,0\\0,1\end{pmatrix}\otimes\ldots\otimes\begin{pmatrix}0,0\\0,1\end{pmatrix}, \tag{3.3}$$

where the first term in the right-hand side is the regularized diagram, in which all the components with at least one external index "1" are equal to the components of the initial diagram, while in the components with the indices "2" the external functions $G_0^a(\tau_k', t_k)$ are replaced by $G_0^r(\tau_k', t_k')$ and $G_0^r(t_m, \tau_m)$ are replaced by $G_0^a(t_m, \tau_m)$.

We note that by resolving the Θ functions in (3.2) into the corresponding terms we can separate a singular part similar to (3.3), but with a time $t_{\min} = \min(t_1, \ldots, t_n')$, and generally, with an arbitrary "running" time t^p, which remains finite as $t_0 \to -\infty$.

For the process considered by us, namely the relaxation of the system to the state of equilibrium, the condition $t_0 \to -\infty$ means physically that $|t_0 - t^m| \sim \tau_r$, where τ_r is the relaxation time. From the statements proved in this section it follows that the initial correlators $g^{(m)}(t_0)$, whose momenta satisfy the condition $|\omega_m| > 1/\tau_r$ over times of the order of τ_r, make no contribution. The "long-lived" correlators $g^{(m)}$, the characteristic frequency of which is $|\omega_m| \lesssim 1/\tau_r$, together with the single-particle distribution function, determine the evolution of the system during the kinetic stage. However, the number of long-lived correlators is relatively small and they are determined by the volume of the momentum space that satisfies the condition $|\omega_m| \lesssim 1/\tau_r$. It is easy to verify that as a result each long-lived correlator $g^{(m)}$ introduces into the diagram a contribution proportional to the small factor $(1/nl^3)^{m-1}$, where $l \sim v\tau_r$ is the characteristic relaxation length, v is the average particle velocity and n is the density. It follows therefore that the correlation diagrams in the kinetic stage of the relaxation process make a small contribution in comparison with the corresponding Keldysh diagram. We note that this statement is applicable only to a stable system. In the case of an unstable system, in which the fluctuations can grow to a level greatly exceeding the equilibrium level, allowance for diagrams containing long-lived correlators is essential.

§4. RENORMALIZATION OF THE DIAGRAM TECHNIQUE, AND SYSTEM OF TRANSPORT EQUATIONS FOR LONG-LIVED CORRELATORS

We consider first the case when the contribution of the diagrams containing long-lived correlations can be neglected. In this case it is necessary to take into account the single-particle insets that diverge secularly at long times (of the order of the relaxation time), and are the diagrams for the Green's function G. In final analysis these divergences are due to the fact that in the nonequilibrium case the Green's functions contain the small parameter ε not only in explicit form, but also in the form of the product εt:

$$G(t,t',\varepsilon)=G(t-t',\varepsilon(t+t')/2,\varepsilon).$$

Bogolyubov[20] has shown that these divergences can be eliminated if $f(p, t_0)$ is eliminated and all the quantities are expressed in terms of the slowly varying distribution function $f(p, t)$ at the running instant of time. Berezinskiĭ[21] has shown that this procedure can be carried out with the aid of a renormalization of the diagrams. However, the renormalization method proposed by Berezinskiĭ does not permit partial summation and makes it impossible to obtain the renormalized Keldysh equations for the Green's functions. To eliminate the secular divergences we shall use another method, which leads to renormalized Keldysh matrix equations.

We represent the free Green's function in the form

$$\mathbf{G}_0(t,t')=\begin{cases}\mathbf{R}_0'+\mathbf{S}_0', & t>t'\\ \mathbf{R}_0''+\mathbf{S}_0'', & t<t'\end{cases}, \tag{4.1}$$

$$\mathbf{S}_0'=-ie^{i\varepsilon_p(t'-t)}\begin{pmatrix}0, & 0\\ 1, & 1+2\eta f(p,t_0)\end{pmatrix},\qquad \mathbf{S}_0''=-ie^{i\varepsilon_p(t'-t)}\begin{pmatrix}0, & -1\\ 0, & 1+2\eta f(p,t_0)\end{pmatrix}, \tag{4.2}$$

$$\mathbf{R}_0'=G_0^a(t,t')\begin{pmatrix}0, & 1\\ 1, & 0\end{pmatrix},\quad \mathbf{R}_0''=G_0^r(t,t')\begin{pmatrix}0, & 1\\ 1, & 0\end{pmatrix}. \tag{4.3}$$

We consider a simple single-particle insert (in operator form)

$$\mathbf{I}^{(1)}(t,t')=\mathbf{G}_0\boldsymbol{\Sigma}\mathbf{G}_0, \tag{4.4}$$

where

$$\boldsymbol{\Sigma}=\begin{pmatrix}\Omega, & \Sigma^r\\ \Sigma^a, & 0\end{pmatrix}$$

is the self-energy matrix.[5,17] Then, using the results obtained in §3, we can easily show that the regular part of (4.4) is equal to

$$\mathbf{I}_R^{(1)}(t,t')=\begin{cases}\mathbf{S}_0'\boldsymbol{\Sigma}\mathbf{R}_0'', & t>t',\\ \mathbf{R}_0'\boldsymbol{\Sigma}\mathbf{S}_0'', & t<t',\end{cases} \tag{4.5}$$

and the singular part takes the form of a correction to the density at the instant of time $t^m = \max(t, t')$:

$$\mathbf{I}_S^{(1)}(t,t')=\mathbf{S}_0'\boldsymbol{\Sigma}\mathbf{S}_0''=\frac{2}{i\eta}e^{i\varepsilon_p(t'-t)}\Delta f(p,t^m)\begin{pmatrix}0. & 0\\ 0. & 1\end{pmatrix}. \tag{4.6}$$

Diagrams of the type $\mathbf{R}_0'\boldsymbol{\Sigma}\mathbf{R}_0''$ equal to zero, since the integration in (4.4) is with respect to t^m. It is now easy to regularize the sum of the single-particle reducible diagrams for $\mathbf{G}$ in the form

$$\mathbf{G}_k=\mathbf{G}_0+\mathbf{G}_0\boldsymbol{\Sigma}\mathbf{G}_0+\ldots\mathbf{G}_0(\boldsymbol{\Sigma}\mathbf{G}_0)^k,\quad k=2,3,\ldots. \tag{4.7}$$

We assume first that $t > t'$. Then $\mathbf{G}_0(t, t') = \mathbf{S}_0'$ in (4.7) can be regrouped in the following manner:

$$\mathbf{G}_k=\mathbf{S}_k'+\mathbf{S}_{k-1}'\boldsymbol{\Sigma}\mathbf{R}_0''\ldots+\mathbf{S}_0'(\boldsymbol{\Sigma}\mathbf{R}_0'')^k. \tag{4.8}$$

The function $\mathbf{S}_m'$ is connected here with the renormalized (accurate to the m-th power of Σ) momentum density $f_m(p,t)$ by the relation

$$\mathbf{S}_m'=\mathbf{S}_0'+\mathbf{S}_0'\boldsymbol{\Sigma}\mathbf{S}_0''+\ldots+\mathbf{S}_0'\boldsymbol{\Sigma}(\mathbf{G}_0\boldsymbol{\Sigma})^{m-1}\mathbf{S}_0''=-ie^{i\varepsilon_p(t'-t)}\begin{pmatrix}0, & 0\\ 1, & 1+2\eta f_m(p,t)\end{pmatrix} \tag{4.9}$$

In the limit as $k\to\infty$, $\mathbf{G}_k\to\mathbf{G}, f_k\to f$ and (4.8) takes the form of the integral equation for the exact Green's function

$$\mathbf{G}(t>t')=\mathbf{S}'+\mathbf{G}\boldsymbol{\Sigma}\mathbf{R}_0'', \qquad (4.10)$$

where $\mathbf{S}'$ is obtained from (4.9) by the substitution $f_m\to f(p,t)$, and the lower limit of integration with respect to time can be set equal to $-\infty$.

It is easy to obtain also the corresponding equation for the case $t<t'$:

$$\mathbf{G}(t<t')=\mathbf{S}''+\mathbf{R}_0'\boldsymbol{\Sigma}\mathbf{G}, \qquad (4.11)$$

$$\mathbf{S}''(t,t')=-ie^{i\varepsilon_p(t'-t)}\begin{pmatrix}0, & -1\\ 0, & 1+2\eta f(p,t')\end{pmatrix}. \qquad (4.12)$$

The components of Eqs. (4.10) and (4.11) for the advanced and retarded Green's functions coincide with the corresponding Keldysh equations.[5,17] The equation for the two-time correlation function is of the form

$$F(t,t')=-i(1+2\eta f(p,t))e^{i\varepsilon_p(t'-t)}+(G^r\Omega+F\Sigma^a)G_0{}^r, \quad t>t',$$
$$(4.13)$$
$$F(t,t')=-i(1+2\eta f(p,t'))e^{i\varepsilon_p(t'-t)}+G_0{}^a(\Omega G^a+\Sigma^r F), \quad t<t',$$

In which, compared with the Keldysh equation for this function, first, the term that depends on the initial density has been replaced by an expression of the same form as F_0, but dependent on the running density ($t^p=t^m$) and, second, the functions G_0^a and G_0^r are replaced by G_0^r and G_0^a, respectively.

Equations (4.10) and (4.11) must be supplemented by an equation for the density $f(p,t)$ at the running instant of time. This equation is obtained as the consistency equation for the renormalized integral equations (4.10) and (4.11). If the first approximation is chosen to be

$$\mathbf{G}(t,t')=\Theta(t-t')\mathbf{S}'(t,t')+\Theta(t'-t)\mathbf{S}''(t,t') \qquad (4.14)$$

and account is taken of the diagram shown in Fig. 2(a), we obtain the Boltzmann transport equation for a gas of weakly interacting particles.

The regularization method used by us leads in the general case to a transport equation of non-Markov type. The retardation is the result of separation of the secular inset in the form of a correction to the density at the instant of time t^m that is maximal for this inset. However, as indicated in §3, the choice of this instant is arbitrary. In particular, it would be possible to separate the singular part in the form of a correction to the density at some fixed (different for each diagram) instant of time t. The equation for $f(p,t)$ would then contain the values of the density at the same instant of time, and the Boltzmann equation would have a Markov character. It is precisely this result which was obtained by Berezinskiĭ.[21] However, as already noted, such an approach does not permit partial summation of the diagrams, and consequently does not yield the renormalized Keldysh equations for the Green's functions. From the results obtained above it follows also that the Markov or non-Markov character of the transport equation is determined simply by where we include the time dependence—in the distribution function $f(p,t)$ or in the kernel of the operator contained in the collision integral.

We now take into account the long-lived correlations. Generalizing the described renormalization method, we can sum all the diverging diagrams and construct a diagram technique that takes into account the long-lived correlations and is free of secular divergences.

We turn to Eq. (3.3). We see that the matrix structure of the singular inserts coincides with the structure of the free Green's functions G_0 and $G_0^{(n)}$. Therefore summation of all the singular inserts leads to a renormalization of the density and of the long-lived correlators. In this case the first derivatives of the simple insets with respect to time are finite and their summation yields in fact the transport equations for the density and for the long-lived correlators. The following statements hold:

1) The exact Green's functions are determined by the sum of all the simple diagrams in which the initial density and the long-lived correlators are replaced by the density correlators at the running instant of time.

2) The density $f(p,t)$ and the long-lived correlators $g^{(n)}(p_1,p_1';\ldots;t)$ at the running instant of time are determined by the sum of all the simple singular insets and satisfy the system of equations

$$\left(i\frac{\partial}{\partial t}-\omega_n\right)g^{(n)}(p_1,p_1';\ldots;t)=\left(\frac{i\eta}{2}\right)^n\{[G_0{}^{-1}(1)+\ldots$$
$$\ldots+G_0{}^{-1}(n)-G_0{}^{-1}(1')^*-\ldots-$$
$$-G_0{}^{-1}(n')^*]\mathbf{G}_R^{(n)}(1,1';\ldots;n,n')_{2\ldots2}\}_{t_1=\ldots=t_n'=t}; \qquad (4.15)$$

$$G_0{}^{-1}(1)=i\frac{\partial}{\partial t_1}-\varepsilon_{p_1}, \quad \omega_n=\sum_{j=1}^{n}\varepsilon_{p_j}-\varepsilon_{p_j'}, \quad n=1,2,\ldots,$$

$\mathbf{G}_R^{(n)}$ is the sum of all the diagrams calculated in accordance with the statement (1).

The kinetic equation (4.15) for the density ($n=1$) and the long-lived correlators ($n=2,3,\ldots$) are the consistency conditions for the renormalized diagram technique. The diagrams contained in $\mathbf{G}_R^{(n)}$ can be divided into three classes: 1) Keldysh diagrams, which depend functionally on the momentum density $f(p,t)$ at the running instant of time (Fig. 5a), 2) diagrams containing renormalized long-lived correlators

$$g^{(n)}(\mathbf{p}_1,\mathbf{p}_1+\mathbf{k}_1;\ldots;\mathbf{p}_n,\mathbf{p}_n+\mathbf{k}_n;t), \quad \mathbf{k}_1+\ldots+\mathbf{k}_n=0,$$

in which there is no summation over any of the momenta k [Fig. 5(b), and 3] diagrams containing long-lived correlators

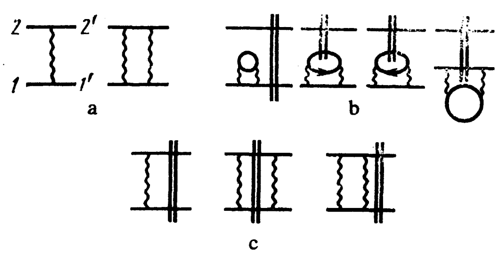

FIG. 5. Examples of simple diagrams $\mathbf{G}_R^{(2)}$ which enter in the kinetic equation for $g^{(2)}$: a) source $S^{(2)}\{f\}$, b) linearized collision integral $I_1 g^{(2)}$, c) cross term $R^{(2)}\{f,g^{(2)}\}$.

of any order, summed over at least one of the momenta k (Fig. 5c). The first class of diagrams, which does not contain correlators, gives the collision integral $S^{(1)}\{f\}$ in the equation for the single-particle density and the source $S^{(n)}\{f\}$ in the equation for the correlator $g^{(n)}$, $n>1$. The diagrams of the second class give the linearized collision integral in the equation of the type $(I_1+\ldots+I_n)g^{(n)}$ (Ref. 17). The third class of diagrams depends functionally on the density and on all the long-lived correlator. The term $R^{(n)}\{f,g\}$ corresponding to it in Eq. (4.15) describes the influence of the correlators on one another and on the single-particle density. For stable systems it is proportional, generally speaking, to the small parameter $1/nl$.[3] As a result, the system (4.15) takes the form

$$\partial f/\partial t=S^{(1)}\{f\}+R^{(1)}\{f,g\}, \tag{4.16}$$

$$(\partial/\partial t+i\omega_n-I_1-\ldots-I_n)g^{(n)}=S^{(n)}\{f\}+R^{(n)}\{f,g\}. \tag{4.17}$$

Equations (4.16) and (4.17) make it possible to describe completely the influence of the fluctuations on the kinetics of the system. To prevent misunderstandings, we note that they differ from the known Bogolyubov chain of equations[20] in the following aspects: 1) Eqs. (4.16) and (4.17) hold only for long-lived correlations; 2) these equations are suitable only for long times, comparable with the relaxation time of the system to the equilibrium states; 3) these equations constitute a closed system, and the integral operators I, S, and R it contains can be calculated with any degree of accuracy with the aid of the diagram technique.

§5. EQUATION FOR TWO-PARTICLE CORRELATOR

We present the explicit form of the equation for the two-particle correlator $g^{(2)}(p,p-k;p',p'+k,t)$ in a gas of weakly interacting particles in the limit of small occupation numbers $f_p\ll 1$. We confine ourselves to diagrams of first and second order in the interaction, which are shown in Fig. 5. We turn to the $G^{\pm}$ representation, which is preferable for actual calculations, since the vertex matrix in it differs from zero and is equal to unity only when the time indices of all the lines that enter the vertex coincide. The renormalized Green's functions (4.14) corresponding to the solid lines in Fig. 5 are equal, when account is taken of $f_p\ll 1$, to

$$\begin{aligned} G_p^+(t,t')&=-i\eta f(p,t^m)e^{i\varepsilon_p(t'-t)},\\ G_p^-(t,t')&=-ie^{i\varepsilon_p(t'-t)}, \end{aligned} \tag{5.1}$$

and the renormalized correlation function corresponding to the diagram on Fig. 1 with $k=2$ is given by

$$\mathbf{G}^{(2)}(1,1';2,2')=-g^{(2)}(p,p-k;p',p'+k;t^m)\exp\{i\varepsilon_{p-k}(t_1'-t^m)+i\varepsilon_{p'+k}(t_2'-t^m)-i\varepsilon_p(t_1-t^m)-i\varepsilon_{p'}(t_2-t^m)\}\begin{pmatrix}1, & 1\\ 1, & 1\end{pmatrix}\otimes\begin{pmatrix}1, & 1\\ 1, & 1\end{pmatrix}. \tag{5.2}$$

We calculate now the contribution made to the equation for $g^{(2)}$ by the components of the diagrams shown in Fig. 5, with indices $\alpha_1=\alpha_2=+$, $\alpha_1'=\alpha_2'=-$, putting (after applying the operators G_0^{-1}) $t_1=t_1'=t_2-0=t_2'-0=t$. As a result of relations (2.5), the integral expression does not depend on this choice. The result of the action of the operator $G_0^{-1}(1)+G_0^{-1}(2)$ on the diagrams shown in Figs. 5a and 5c at $f_p\ll 1$ is proportional to $G_{p'}^c$, $(t+0,t)+G_p^c(t,t+0)=-i$. Therefore the contribution of these diagrams to the equation for $g^{(2)}$ is determined by the diagrams shown in Fig. 6 and 7a. The diagrams shown in Figs. 7b and 7c are proportional respectively to $f^2g^{(2)}$ and $fg^{(2)}$, and are small at $f_p\ll 1$. As a result, we obtain the following equations for the two-particle correlator [where $\omega(\mathbf{p},\mathbf{p}',\mathbf{k})=\varepsilon_\mathbf{p}+\varepsilon_{\mathbf{p}'}-\varepsilon_{\mathbf{p}'+\mathbf{k}}-\varepsilon_{\mathbf{p}-\mathbf{k}}$]:

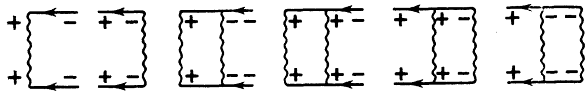

FIG. 6. Diagrams for the source $S^{(2)}\{f\}$.

$$\left(\frac{\partial}{\partial t}+i\omega(\mathbf{p},\mathbf{p}',\mathbf{k})-I_1-I_2\right)g^{(2)}(\mathbf{p},\mathbf{p}-\mathbf{k};\mathbf{p}',\mathbf{p}'+\mathbf{k};t)=S^{(2)}\{f\}+R^{(2)}\{f,g^{(2)}\}; \tag{5.3}$$

$$S^{(2)}=S_1^{(2)}+S_2^{(2)},\quad R^{(2)}=R_1^{(2)}+R_2^{(2)},\quad S_1^{(2)}\{f\}=\frac{i}{\mathscr{V}}V(k)\left(f_\mathbf{p}f_{\mathbf{p}'}-f_{\mathbf{p}-\mathbf{k}}f_{\mathbf{p}'+\mathbf{k}}\right). \tag{5.4}$$

$$S_2^{(2)}\{f\}=\frac{i}{\mathscr{V}}\left(\frac{1}{\mathscr{V}}\sum_\mathbf{q}\right)V(q)V(|\mathbf{k}+\mathbf{q}|)\times\left[\frac{f_{\mathbf{p}'+\mathbf{q}+\mathbf{k}}f_{\mathbf{p}-\mathbf{q}-\mathbf{k}}-f_{\mathbf{p}'+\mathbf{k}}f_{\mathbf{p}-\mathbf{k}}}{\omega(\mathbf{p}'+\mathbf{k},\mathbf{p}-\mathbf{k},\mathbf{q})+i\delta}+\frac{f_{\mathbf{p}'+\mathbf{q}+\mathbf{k}}f_{\mathbf{p}-\mathbf{k}-\mathbf{q}}-f_\mathbf{p}f_{\mathbf{p}'}}{-\omega(\mathbf{p},\mathbf{p}',\mathbf{k}+\mathbf{q})+i\delta}\right], \tag{5.5}$$

$$R_1^{(2)}\{f,g^{(2)}\}=\frac{i}{\mathscr{V}}\left(\frac{1}{\mathscr{V}}\sum_\mathbf{q}\right)V(q)\times[\mathscr{V}g^{(2)}(\mathbf{p},\mathbf{p}-\mathbf{q}-\mathbf{k};\mathbf{p}',\mathbf{p}'+\mathbf{q}+\mathbf{k})-\mathscr{V}g^{(2)}(\mathbf{p}+\mathbf{q},\mathbf{p}-\mathbf{k};\mathbf{p}'-\mathbf{q},\mathbf{p}'+\mathbf{k})], \tag{5.6}$$

$$\begin{aligned} R_2^{(2)}\{f,g^{(2)}\}=\frac{i}{\mathscr{V}}\left(\frac{1}{\mathscr{V}^2}\sum_{\mathbf{q},\mathbf{Q}}\right)V(q)V(Q)\times\Bigg[&\mathscr{V}g^{(2)}(\mathbf{p}-\mathbf{Q},\mathbf{p}-\mathbf{k}-\mathbf{q};\mathbf{p}'+\mathbf{Q},\mathbf{p}'+\mathbf{k}+\mathbf{q})\\ &\times\left(\frac{1}{\omega(\mathbf{p}-\mathbf{k},\mathbf{p}'+\mathbf{k},\mathbf{q})+i\delta}+\frac{1}{-\omega(\mathbf{p},\mathbf{p}',\mathbf{Q})+i\delta}\right)\\ &-\frac{\mathscr{V}g^{(2)}(\mathbf{p}-\mathbf{q}-\mathbf{Q},\mathbf{p}-\mathbf{k};\mathbf{p}'+\mathbf{Q}+\mathbf{q},\mathbf{p}'+\mathbf{k})}{-\omega(\mathbf{p}-\mathbf{q},\mathbf{p}'+\mathbf{q},\mathbf{Q})+i\delta}\\ &-\frac{\mathscr{V}g^{(2)}(\mathbf{p},\mathbf{p}-\mathbf{k}-\mathbf{Q}-\mathbf{q};\mathbf{p}',\mathbf{p}'+\mathbf{k}+\mathbf{Q}+\mathbf{q})}{\omega(\mathbf{p}-\mathbf{k}-\mathbf{Q},\mathbf{p}'+\mathbf{k}+\mathbf{Q},\mathbf{q})+i\delta}\Bigg], \end{aligned} \tag{5.7}$$

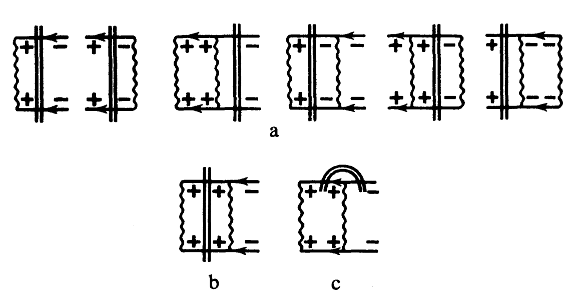

FIG. 7. Diagrams for $R^{(2)}\{f,g^{(2)}\}$.

$$I_1 g^{(2)}(\mathbf{p},\mathbf{p}-\mathbf{k};\dots)$$

$$=\frac{i}{\mathcal{V}^2}\sum_{\mathbf{q},\mathbf{p}''}\Big\{V^2(q)\Big[f_{\mathbf{p}''}g^{(2)}(\mathbf{p}-\mathbf{q},\mathbf{p}-\mathbf{k}-\mathbf{q};\dots)$$

$$\times\left(\frac{1}{\omega(\mathbf{p}-\mathbf{k},\mathbf{p}''-\mathbf{q},\mathbf{q})+i\delta}+\frac{1}{-\omega(\mathbf{p},\mathbf{p}''-\mathbf{q},\mathbf{q})+i\delta}\right)$$

$$-f_{\mathbf{p}''-\mathbf{q}}g^{(2)}(\mathbf{p},\mathbf{p}-\mathbf{k};\dots)$$

$$\times\left(\frac{1}{-\omega(\mathbf{p}-\mathbf{k},\mathbf{p}''-\mathbf{q},\mathbf{q})+i\delta}+\frac{1}{\omega(\mathbf{p},\mathbf{p}''-\mathbf{q},\mathbf{q})+i\delta}\right)\Big]$$

$$+V(q)V(|\mathbf{k}+\mathbf{q}|)\Big[f_{\mathbf{p}-\mathbf{k}-\mathbf{q}}g^{(2)}(\mathbf{p}''+\mathbf{k}+\mathbf{q},\mathbf{p}''+\mathbf{q};\dots)$$

$$\times\left(\frac{1}{\omega(\mathbf{p}-\mathbf{k},\mathbf{p}'',\mathbf{q})+i\delta}+\frac{1}{-\omega(\mathbf{p},\mathbf{p}'',\mathbf{k}+\mathbf{q})+i\delta}\right)$$

$$-g^{(2)}(\mathbf{p}''-\mathbf{q},\mathbf{p}''-\mathbf{q}-\mathbf{k};\dots)\left(\frac{f_{\mathbf{p}}}{-\omega(\mathbf{p},\mathbf{p}''-\mathbf{k}-\mathbf{q},\mathbf{k}+\mathbf{q})+i\delta}\right.$$

$$\left.+\frac{f_{\mathbf{p}-\mathbf{k}}}{\omega(\mathbf{p}-\mathbf{k},\mathbf{p}''-\mathbf{q},\mathbf{q})+i\delta}\right)\Big]\Big\}. \qquad (5.8)$$

Equation (5.3) becomes much simpler if the transfer momentum $\mathbf{k}$ is small compared with the average particle momentum. Introducing the function

$$g(\mathbf{p},\mathbf{p}',\mathbf{k})=\mathcal{V}g^{(2)}(\mathbf{p}+\mathbf{k}/2,\mathbf{p}-\mathbf{k}/2;\mathbf{p}'-\mathbf{k}/2,\mathbf{p}'+\mathbf{k}/2),$$

which is the Fourier transform with respect to the difference of the coordinates $\mathbf{R}-\mathbf{R}'$ of the pair correlation function g $(\mathbf{p},\mathbf{R};\mathbf{p}',\mathbf{R}')$ in the Wigner representation, we obtain in this case

$$\left[\frac{\partial}{\partial t}+i\mathbf{k}(\mathbf{v}-\mathbf{v}')-I_1-I_2\right]g(\mathbf{p},\mathbf{p}',\mathbf{k})$$

$$=S_1^{(2)}\{f\}+S_2^{(2)}\{f\}+R_1^{(2)}\{f,g\}+R_2^{(2)}\{f,g\}, \qquad (5.9)$$

$$S_2^{(2)}\{f\}=\frac{1}{\mathcal{V}}\sum_{\mathbf{q}}V^2(q)\delta(\mathbf{q}\mathbf{v}-\mathbf{q}\mathbf{v}')[f_{\mathbf{p}-\mathbf{q}}f_{\mathbf{p}'+\mathbf{q}}-f_{\mathbf{p}}f_{\mathbf{p}'}], \qquad (5.10)$$

$$R_2^{(2)}\{f,g\}=\frac{1}{\mathcal{V}^2}\sum_{\mathbf{q},\mathbf{Q}}V^2(Q)\delta(\mathbf{Q}\mathbf{v}-\mathbf{Q}\mathbf{v}')$$

$$\times[g(\mathbf{p}-\mathbf{Q},\mathbf{p}'+\mathbf{Q},\mathbf{q})-g(\mathbf{p},\mathbf{p}',\mathbf{q})], \qquad (5.11)$$

$$I_1 g=\frac{1}{\mathcal{V}^2}\sum_{\mathbf{p}'',\mathbf{q}}V^2(q)\delta(\mathbf{q}\mathbf{v}-\mathbf{q}\mathbf{v}'')[f_{\mathbf{p}''}g(\mathbf{p}-\mathbf{q},\mathbf{p}',\mathbf{k})$$

$$+f_{\mathbf{p}-\mathbf{q}}g(\mathbf{p}'',\mathbf{p}',\mathbf{k})-f_{\mathbf{p}''-\mathbf{q}}g(\mathbf{p},\mathbf{p}',\mathbf{k})-f_{\mathbf{p}}g(\mathbf{p}''-\mathbf{q},\mathbf{p}',\mathbf{k})], \qquad (5.12)$$

$\mathbf{v}=\mathbf{p}/m$ is the particle velocity and $\delta\ (a-b)=\mathcal{V}^{1/3}\delta_{a,b}$.

If, however, we do not neglect f_p compared with unity, we obtain (in the case of fermions with $\eta=-1$) an equation of the same form as (5.9), but with the quantity in square brackets replaced in (5.10) by

$$(1-f_{\mathbf{p}}-f_{\mathbf{p}'})[f_{\mathbf{p}-\mathbf{q}}f_{\mathbf{p}'+\mathbf{q}}(1-f_{\mathbf{p}})(1-f_{\mathbf{p}'})-f_{\mathbf{p}}f_{\mathbf{p}'}(1-f_{\mathbf{p}-\mathbf{q}})(1-f_{\mathbf{p}'+\mathbf{q}})],$$

and in formula (5.11) by

$$(1-f_{\mathbf{p}}-f_{\mathbf{p}'})[(1-f_{\mathbf{p}})(1-f_{\mathbf{p}'})g(\mathbf{p}-\mathbf{Q},\mathbf{p}'+\mathbf{Q},\mathbf{q})$$
$$-g(\mathbf{p},\mathbf{p}',\mathbf{q})(1-f_{\mathbf{p}-\mathbf{Q}})(1-f_{\mathbf{p}'+\mathbf{Q}})].$$

The theory of fluctuations in a nonequilibrium rarefied gas, developed in Refs. 12–16 is based on equations, containing a derivative with respect to the time difference, for the two-time correlation function of the fluctuations of the occupation numbers. These equations are derived directly with the aid of the diagram technique developed by us, and, since they coincide fully with those obtained in Refs. 14 and 16, they will not be cited here. The pair correlator $g(\mathbf{p},\mathbf{p}',\mathbf{k},t)$ which depends on the running time plays the role of the initial condition for the equation for the two-time correlation function. Equations (5.3) and (5.9), which determine its variation in the course of relaxation of the system to the equilibrium state, differ from the corresponding equation obtained in Refs. 14–16 by containing additional terms $S_1^{(2)}$, $R_1^{(2)}$, and $R_2^{(2)}$. The term $S_1^{(2)}$ differs from zero only at $\mathbf{k}\neq 0$ and is a source of both equilibrium and nonequilibrium long-wave fluctuations due to the particle interaction. The terms $R_1^{(2)}$ and $R_2^{(2)}$ describe the change of the pair correlation function as a result of the dynamic interaction of the particles. It is easy to verify that the operator $R_2^{(2)}$ has five additive invariants $(1,\mathbf{p}+\mathbf{p}',\epsilon_p+\epsilon_{p'})$. Therefore $R_2^{(2)}$ must be taken into account, particularly for an unambiguous determination of the equilibrium value of the pair correlation functions of an ideal gas contained in a finite volume:

$$g(p,p',k)=-(\delta_{k,0}/n)f_p f_{p'}.$$

In addition, it is important to take into account the operator $R_2^{(2)}$ during the hydrodynamic stage, since it determines the relaxation of the projections of the pair correlator on the subspace of the zeros of the linearized Boltzmann collision integral I_1+I_2. The term $R_2^{(2)}$ can also contribute during the kinetic stage of relaxation of the system, in the limit of low density, inasmuch as in contrast to the operator I_1+I_2 it does not contain the momentum distribution function of the particles. In this case the relaxation time of the correlation function may turn out to be small compared with the relaxation time of the distribution function f, and Eq. (5.9) can be solved for a fixed nonequilibrium distribution function of the particles. This steady-state solution for g is determined completely by the source $S^{(2)}$ and "follows quasistatically" the variation of the function f.

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[1a]We use the notation of Ref. 5. The functions in the matrix representation with the respect to the time indices are designated by bold-face symbols.

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