# Nonequilibrium plasmons in a two-dimensional electron gas

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An explanation is proposed of the experiments of Tsui *et al.* [Sol. St. Commun. **35**, 875 (1980)] and Höpfel *et al.* [Surf. Sci. **113**, 118 (1982)], in which electromagnetic radiation was observed corresponding to radiative decay of two-dimensional plasmon in metal-insulator-semiconductor (MIS) structures based on silicon or in GaAs–GaAlAs heterojunctions. Bremsstrahlung of plasma waves by electrons scattered by charged and neutral impurities, as well as by deformation and piezoacoustic phonons, is suggested as the physical mechanism of the effect.

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## 1. INTRODUCTION

Electromagnetic radiation corresponding to radiative decay of two-dimensional plasmon in metal-insulator-semiconductor (MIS) structures based on silicon and in GaAs–GaAlAs heterojunction was observed in recent experiments.[1,2] The radiation appears when a constant electric field is applied and causes the electrons to drift at a velocity $v_0$ of the order of $(1–4)\times 10^5$ cm/sec. This radiation is characterized by a strong anisotropy: the maximum of the radiation corresponding to drift directed along the axis of the periodic electrode system that sets the momentum **k** of the two-dimensional plasmon and ensures a coupling between the plasma and electromagnetic oscillations. The radiation frequency does not depend on the applied field and is determined, in accordance with the plasmon dispersion law, by the momentum $k$ and by the surface charge density $N_s$. The effect can therefore not be attributed to the onset of transition radiation when the charge moves near the lattice of metallic electrodes. On the other hand, the attained drift velocities are much lower than the phase velocity of the generated plasmons ($\omega\sim 10^{13}$ sec$^{-1}$, $k\sim 10^4$ cm$^{-1}$), so that Čerenkov emission of plasma waves is likewise impossible. The question of the physical nature of the effect and of its quantitative description thus remains open.

In this article we propose for the observed effect a theory that corresponds, in our opinion, to the experiment at least when it comes to the dependences of the radiation intensity on the direction and magnitude of the drawing field.

We assume that the physical mechanism of the effect is bremsstrahlung of plasma waves by electrons. Plasmon emission and absorption are allowed, irrespective of the electron velocity, if they are accompanied by electron scattering from impurities, phonons, etc. These bremsstrahlung effects produce a plasmon gas described in the thermodynamic equilibrium state by a Planck distribution function $n_0(\omega)$. The radiative decay of the equilibrium plasmons contributes to the background ("black") radiation. The electron drift upsets the equilibrium of the plasmon subsystem. If the nonequilibrium distribution function of the plasmons is such that $n_k > n_0$ in the mode "followed" by the experiment,[1)] an excess of electromagnetic radiation is observed at the frequency $\omega(k)$. Its intensity is obviously proportional to $\delta n_k = n_k - n_0$. The problem is thus reduced to calculation of the nonequilibrium distribution function of the plasmons.

## 2. BASIC EQUATIONS

The plasma oscillations of electrons located in the $z=0$ plane are described by the equations

$$\Delta\varphi=-4\pi e\tilde{N}_s(\boldsymbol{\rho})\delta(z),\quad \tilde{N}_s=-N_s\operatorname{div}\mathbf{R}(\boldsymbol{\rho}),\tag{1}$$

where $\mathbf{R}(\boldsymbol{\rho})$ is the particle-displacement vector and $\varphi$ is the electrostatic potential. We expand all quantities in Fourier series and introduce in standard fashion the normal coordinates $Q_k$:

$$\mathbf{R}=\sum_{\mathbf{k}}\mathbf{Q}_{\mathbf{k}}e^{i\mathbf{k}\boldsymbol{\rho}}+\text{c.c.}$$

The Fourier component of the potential in the $z=0$ plane is then

$$\varphi_k=-2\pi iN_se(\mathbf{k}\mathbf{Q}_{\mathbf{k}})/k,$$

and the Hamiltonian of the free plasmon field is

$$H=\frac{1}{2}\sum_{\mathbf{k}}\left\{\frac{P_{\mathbf{k}}P_{-\mathbf{k}}}{mN_s}+m\omega^2(k)N_sQ_{\mathbf{k}}Q_{-\mathbf{k}}\right\},\tag{2}$$

where $\omega^2=2\pi e^2N_sk/m$, $m$ is the electron mass, and $P_k$ is the momentum operator. The interaction of the electrons with the plasmons is described by the operator

$$\hat{V}_{int}=2\pi ie^2N_s\sum_{\mathbf{k}}Q_{\mathbf{k}}e^{i\mathbf{k}\boldsymbol{\rho}}+\text{c.c.}\tag{3}$$

As already mentioned, first-order processes are forbidden by the energy-momentum conservation laws, so that it is necessary to take into account in the transport equation for the function $n_k$ plasmon creation and annihilation processes described by $\hat{V}_{\text{int}}$, with simultaneous scattering of the electrons by impurities or by phonons. We denote by $W(\mathbf{p},\mathbf{p}',\mathbf{k})$ the probability of a process in which the transition of an electron from a state **p** into **p**′ is accompanied by creation of a plasmon having a momentum **k**.

We write down the transport equation for $n_k$:

$$\frac{\partial n_{\mathbf{k}}}{\partial t}=\sum_{\mathbf{p},\mathbf{p}'}W(\mathbf{p},\mathbf{p}',\mathbf{k})\left[n_{\mathbf{k}}(f_{\mathbf{p}}-f_{\mathbf{p}'})+f_{\mathbf{p}}(1-f_{\mathbf{p}'})\right],\tag{4}$$

$$p^2-p'^2=2m\omega,\quad \hbar=1.$$

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Here $f_p$ is the electron-momentum distribution function.

In the experiments of Refs. 1 and 2 the crystal lattice was at helium temperature, and the electron gas heated up substantially. It is known from Ref. 3 that at the drawing fields used in Refs. 1 and 2 (10–30 V/cm) the temperature of the two-dimensional electrons in the Si–$SiO_2$ system is of the order of 10–20 K at a Fermi energy of the order of $E_F \sim 100$ K. It is perfectly evident, however, that heating effects alone cannot explain the observed radiation, for in this case its intensity would not depend on the angle between the electron drift velocity and the plasmon momentum. Therefore, without determining here the true form of the electron distribution function, we substitute in (4) a function $f_p$ that described a drift with specified velocity $\mathbf{v}_0$. We then obtain $n_k$ from the stationarity conditions. The simplest procedure is to put $f_p = f_0(\mathbf{p}-\mathbf{p}_0)$, where $f_0$ is the equilibrium distribution function with a temperature $T$ that is generally speaking different from the lattice temperature.

We note that in principle the plasma oscillations can be excited by another mechanism connected with instability development. The instability sets in if the coefficient of $n_k$ in the right-hand side of Eq. (4) becomes positive. We shall show below that this possibility is realized at drift velocities much higher than in the experiments of Refs. 1 and 2, and can therefore have no bearing on the effect observed.

### 3. IMPURITY SCATTERING

We begin with calculation of $W(\mathbf{p}, \mathbf{p}', \mathbf{k})$ in the lowest Born approximation in the electron—impurity interaction. Within the framework of standard perturbation theory, account must be taken of two processes, in which the plasmon emission either precedes or follows the scattering by the impurity. We obtain for the probability $W$ averaged over the impurity locations

$$W(\mathbf{p}, \mathbf{p}', \mathbf{k}) = \frac{8\pi N_i m^4 \omega^3}{k^2} \left| \frac{U(\mathbf{p}-\mathbf{p}'-\mathbf{k})}{p^2-(\mathbf{p}'+\mathbf{k})^2} + \frac{U(\mathbf{p}'-\mathbf{p}-\mathbf{k})}{p'^2-(\mathbf{p}-\mathbf{k})^2} \right|^2 \times \delta(p^2-p'^2-2m\omega), \qquad (5)$$

where $U(\mathbf{p})$ is the Fourier component of the impurity potential and $N_i$ is the impurity density. We simplify (5) by neglecting $\mathbf{k}$ in the argument of $U(\mathbf{p}-\mathbf{p}')$, as well as the terms $kp/m$ compared with $\omega(k)$. The validity of these simplifications follows from the condition for the existence of weakly damped plasma waves. We have

$$W = \frac{2\pi N_i}{\omega(k)} |U(\mathbf{p}-\mathbf{p}')|^2 \left[ \frac{\mathbf{k}}{k} (\mathbf{p}-\mathbf{p}') \right]^2 \delta(p^2-p'^2-2m\omega). \qquad (6)$$

It follows from the energy conservation law that at $T=0$ the plasmon frequency cannot exceed $2v_0k_F$, where $k_F$ is the Fermi wave number. Estimates show that $\omega/2k_Fv_0 \sim 3-4$, in the experiments of Refs. 1 and 2, i.e., the radiation is due only to thermal smearing of the Fermi distribution. We consider first the region $k_Fv_0 \ll \omega$, $T$. The distribution function $f_0(\mathbf{p}-\mathbf{p}_0)$ should be expanded in this case in powers of $\mathbf{p}_0$. Obviously, when Eq. (6) is used for the transition probability, all the odd terms of this expansion vanish. A contribution linear in $\mathbf{p}_0$ would appear if corrections of order $kp/m\omega$ were retained in Eq. (5). The corresponding increment to $n_k$ is obviously proportional to $(\mathbf{k}\cdot\mathbf{p}_0)$ and cannot contribute to the observed radiation, since the total number of plasmons with specified frequency $\omega(k)$ does not change.

We obtain the stationary solution of Eq. (4) by substituting $W(\mathbf{p}, \mathbf{p}', \mathbf{k})$ from (5), and $f_p$ and $f_{p'}$ in the form of expansions up to terms of order $p_0^2$ inclusive. It is easy to verify that for any isotropic impurity potential (i.e., $U(\mathbf{p}-\mathbf{p}')$ depends only on the angle between $\mathbf{p}$ and $\mathbf{p}'$) the plasmon stationary distribution function is of the form

$$n_k = n_0(\omega) + \text{const}\cdot p_0^2(\cos^2\alpha + 1/2), \qquad (7)$$

where $\alpha$ is the angle between the vector $\mathbf{k}$ and the drift velocity. The actual value of the constant in (7) depends on the form of the potential and on the relations between $T$, $\omega$, and $E_F$. In the Boltzmann case, when it is natural to assume also that $\omega \ll T$, scattering by short-range impurities and two-dimensional Rutherford scattering lead to the same result

$$\delta n = \frac{p_0^2}{2m\omega}\left(\cos^2\alpha + \frac{1}{2}\right), \qquad p_0^2 \ll mT. \qquad (8)$$

It can be shown that for scattering by short-range impurities Eq. (8) is valid in a larger region, namely $P_0^2 \ll mT^2/\omega$. For degenerate electrons scattered by charged impurities we have

$$\delta n = \frac{p_0^2 p_F^2}{2(mT)^2} e^{-\omega/T}\left(\cos^2\alpha + \frac{1}{2}\right), \qquad E_F \gg T, \qquad \omega \gg T, \qquad (9a)$$

$$\delta n = \frac{1}{12}\frac{p_0^2 p_F^2}{m^2\omega T}\left(\cos^2\alpha + \frac{1}{2}\right), \qquad E_F \gg T, \qquad \omega \ll T. \qquad (9b)$$

For short-range impurity centers the results differ from (9a) and (9b) by a coefficient 3/2.

We consider now a region in which $k_Fv_0/T$ is arbitrary, but in accord with the experimental conditions we have $k_Fv_0 \ll \omega$, $T \ll \omega \ll E_F$. to calculate the integrals in (4) it is convenient to transform to the variables $\mathbf{q} = \mathbf{p} - \mathbf{p}_0$, $\mathbf{q}' = \mathbf{p}' - \mathbf{p}_0$. In the $(q^2. q'^2)$ plane the effective integration region is then a strip of width $4p_0p_F$ and length $2^{3/2}m\omega$ (see Fig. 1). The following approximations are valid in this region:

$$f_0(\mathbf{q}) - f_0(\mathbf{q}') = -1,$$

$$f(\mathbf{q})(1-f(\mathbf{q}')) \approx \exp\left\{\frac{q'^2-q^2}{2mT}\right\}.$$

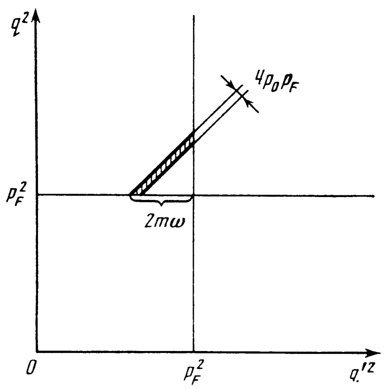


FIG. 1.

The calculations lead to the following results:

for short-range impurities

$$n_k = e^{-\omega/T}\{I_0^2(\gamma)+I_1^2(\gamma)+\cos 2\alpha[I_0(\gamma)I_2(\gamma)+I_1^2(\gamma)]\}, \quad (10a)$$

for Coulomb centers

$$n_k = e^{-\omega/T}\{I_0^2(\gamma)+\cos 2\alpha I_1^2(\gamma)\}, \quad (10b)$$

where $\gamma = v_0 p_F/T$ and $I_{0,1,2}$ are Bessel functions of imaginary argument.

At $\gamma \ll 1$ Eqs. 10(a) and 10(b) go over into the earlier results for low drift velocities. If, however, $\gamma \gg 1$, then

$$n_k = \frac{A}{\pi\gamma}\cos^2\alpha \exp\left\{-\frac{(\omega-2k_F v_0)}{T}\right\}; \quad (11)$$

$A = 2$ in the case [10(a)] and $A = 1$ for [10(b)]. Thus, with increasing drift velocity the anisotropy of the effect become stronger (the factor $\cos^2\alpha$ in place of $\cos^2\alpha + 1/2$). We note in this connection that in the experiment of Ref. 1 the effect has a large anisotropy and the parameter $\gamma$, insofar as can be estimated from the author's data, is of the order of unity. We present the value of the anisotropy parameter $\eta$, defined as the ratio of the radiation intensity at $\mathbf{k}\|\mathbf{v}_0$ to the analogous quantity at $\mathbf{k}\perp\mathbf{v}_0$, for scattering by Coulomb centers

| $\gamma$ | 0,5 | 0,8 | 1 | 1.2 | 1.5 | 2 |
|---|---|---|---|---|---|---|
| $\eta$ | 3.06 | 3.17 | 3.25 | 3.36 | 3.58 | 4.03 |

The temperature dependence of $n_k$, as expected, is activation-governed in the region $\omega \gg T$, with an activation energy equal to the excess of the energy of the emitted plasmons above the threshold value $2k_F v_0$.

If both types of scattering are simultaneously present, it is obvious as a result of the random distribution of the impurities that the total probability $W$ is made up additively of the quantities $W_c$ and $W_n$, which describe respectively the contributions of the Coulomb and neutral centers. In the absence of degeneracy and at drift velocities much lower than thermal, we again obtain Eq. (8) for $\delta n$, this being obvious beforehand, since both types of impurity give the same result in this case. The situation is more complicated for degenerate electrons. We denote by $R$ the ratio of the integrated probabilities of plasmon emission in scattering by two types of impurity:

$$R = \sum_{\mathbf{p},\mathbf{p}'} W_n(\mathbf{p},\mathbf{p}',\mathbf{k}) \Big/ \sum_{\mathbf{p},\mathbf{p}'} W_c(\mathbf{p},\mathbf{p}',\mathbf{k}).$$

At low drift velocity (in the sense of $\gamma \ll 1$) we have then

$$n_k - n_0 = \frac{1+{}^3/_2 R}{1+R}\delta n,$$

where $\delta n$ is taken from (9a) or (9b), depending on the ratio of $\omega$ and $T$. If, however, $\gamma \gg 1$, then

$$n_k = \frac{\cos^2\alpha}{\pi\gamma}\frac{1+2R}{1+R}\exp\{(2k_F v_0-\omega)/T\}$$

[see (11)]. Thus, in both limiting cases the dependences of $n_k$ on $\alpha$ and $v_0$ remain the same as before, and all that depends on $R$ is the total radiation intensity. At arbitrary $\gamma$ the formula obtained is more cumbersome, but the minimum of the effect corresponds as before to $\alpha = \pi/2$, while the maximum is reached at $\alpha = 0$ and $\alpha = \pi$.

## 4. SCATTERING BY PHONONS

Under the experimental conditions (lattice temperatures 4 K, electron temperature not higher than 10–20 K) we can neglect electron scattering by optical phonons. It suffices to consider interaction with acoustic phonons via the deformation potential, and in the case of GaAs also via the piezoeffect. It can be easily seen that owing to the low sound speed $s \ll v_F \ll \omega/k$, the electrons lose energy mainly on plasmons and momentum on phonons, i.e., $|\mathbf{p}-\mathbf{p}'| \sim \omega/v_F \sim q$, where $q$ is the phonon momentum, $sq \sim s\omega/v_F \ll \omega$. We can therefore neglect the plasmon momentum $k$ and the phonon energy $sq$ in the $\delta$-functions that express the energy and momentum conservation laws. In the transport equation for $n_k$ we must now take into account contributions of four processes, creation and annihilation of a plasmon accompanied by emission or absorption of a phonon. Taking the foregoing into account, we obtain for the total probability of the process $\mathbf{p}\to\mathbf{p}', \mathbf{k}$:

$$W = \sum_{q,\sigma} g^2(q)(2N_q+1)\frac{[\mathbf{k}\times(\mathbf{p}-\mathbf{p}')]^2}{(m\omega_k^2)^2}\delta(\mathbf{p}-\mathbf{p}'+\sigma\mathbf{q}) \times\delta(E_p-E_{p'}-\omega), \quad \sigma=\pm1. \quad (12)$$

Here $g(q)$ is the electron-phonon interaction constant and $N_q$ is the distribution function of the phononts and can be regarded as close to equilibrium, at least if the electron drift is slower than the longitudinal sound ($8\times10^5$ cm/sec in Si and $5\times10^5$ cm/sec in GaAs). In the essential region of $q$, the phonon energy can be easily estimated to be much lower than the lattice temperature $T_L$, i.e., we can put $N_q \approx T_L/sq \gg 1$. Substituting in (12) the corresponding expressions for $g(q)$ we find that $W \sim [\mathbf{k}\times(\mathbf{p}-\mathbf{p}'')]^2$ for interaction via the deformation potential and $W \sim [\mathbf{k}\times(\mathbf{p}-\mathbf{p}')]^2(\mathbf{p}-\mathbf{p}')^{-2}$ for the piezoelectric coupling. The former case reduces to scattering by short-range impurities, and the latter to two-dimensional Rutherford scattering. Therefore the interaction with the phonons does not call for a separate treatment.

## 5. EXACT ALLOWANCE FOR COULOMB SCATTERING

The wave function that describes scattering by a Coulomb center in the takes in the two-dimensional case the form (see Ref. 4).

$$\psi_p = \frac{1}{\sqrt{\pi}}e^{\pi e^2 m/2p}\Gamma\left(\frac{1}{2}-i\frac{me^2}{p}\right)e^{i\mathbf{p}\boldsymbol{\rho}}\Phi\left[i\frac{me^2}{p},\frac{1}{2}i(p\rho-\mathbf{p}\boldsymbol{\rho})\right], \quad (13)$$

where $\Phi$ is a confluent hypergeometric function. The Born approximation considered above corresponds to the limit $me^2/p \ll 1$, $\Phi \approx 1$. The matrix element $M(\mathbf{p},\mathbf{p}',\mathbf{k})$ that describes plasmon emission can be calculated in the dipole approximation ($\mathbf{k}\cdot\boldsymbol{\rho} \ll 1$), as in nonrelativistic bremsstrahlung theory, since the electron velocity is much lower than the plasmon phase velocity. We shall not present here the rather long calculations, which are perfectly analogous to the aforementioned problem of bremsstrahlung of an electron in a nucleus (see Ref. 5, § 90). The result is

$$|M(\mathbf{p},\mathbf{p}',\mathbf{k})|^2=|\langle\mathbf{p}',\mathbf{k}|\mathbf{k}\rho|\mathbf{p}\rangle|^2$$
$$=\left(\frac{4\pi}{m\omega}\right)^2\frac{\exp\{-2\pi me^2/p\}}{(\mathbf{p}-\mathbf{p}')^2(p^2-p'^2)^2\mathrm{ch}(\pi me^2/p)\,\mathrm{ch}(\pi me^2/p')}$$
$$\times\left|me^2(\mathbf{kp}-\mathbf{kp}')F(Z)+i\frac{(\mathbf{p}-\mathbf{p}')^2}{(p-p')^2}\mathbf{k}(\mathbf{p}p'-\mathbf{p}'p)\frac{dF}{dZ}\right|^2. \quad (14)$$

Here $F(Z)\equiv F(ime^2/p, ime^2/p'; 1/2; Z)$ is a complete hypergeometric function and $Z=2(\mathbf{pp}'-pp')(p-p')^{-2}$.

We consider only the case $\omega \ll p_F^2/m$, for which a simple result can be obtained. We note that $m\omega/p_F^2 \sim 0.1-0.2$. in the experiments of Refs. 1 and 2. Putting $p\approx p' \approx p_F$, $p-p'\approx m\omega/p_F$ in (14) and using the asymptotic expansion of $F(Z)\gg 1$, we obtain for the probability $W$

$$W(\mathbf{p},\mathbf{p}',\mathbf{k})=W_0C\left(\frac{me^2}{p_F}\right)\times\left[1+\left(\frac{2e^2m}{p_F}\right)^2\ln^2\frac{2p_F^4(1-\cos\sphericalangle\mathbf{pp}')}{m^2\omega^2}\right], \quad (15)$$

where $W_0$ is the Born probability of the process and $C(me^2/p_F)$ stands for all the factors in (14) that do not depend on the directions of the vectors $\mathbf{p}$, $\mathbf{p}'$, and $\mathbf{k}$. Thus, in the approximation considered the exact probability differs only by a logarithmic factor from the perturbation-theory result. At a low drift velocity ($\gamma \ll 1$), as already stated, any probability of the form $W_0 f(\cos\sphericalangle \mathbf{pp}')$ leads to a relation $\delta n_{\mathbf{k}} \sim p_0^2(\cos^2\alpha+1/2)$. At $\gamma \gg 1$ (and as before $\omega \gg T$), the ratio $n_k/n_0$ is exponentially large. The main contribution to $n_k$ is made by the region of direction over the directions of $\mathbf{p}$ and $\mathbf{p}'$, a region in which $\cos\sphericalangle\mathbf{kp}\approx 1$, $\cos\sphericalangle\mathbf{kp}'\approx -1$. We can therefore put in (15) $\cos\sphericalangle\mathbf{pp}'=-1$, after which we obtain again Eq. 10(b).

## 6. POSSIBILITY OF INSTABILITY DEVELOPMENT

We have so far taken into account in the transport equation (4) only the electronic plasmon-relaxation mechanism. In a real situation there are also other mechanisms, one of which is the observable radiative decay. If all the "non-electronic" relaxation processes are described by a phenomenological time $\tau$, it is necessary to add to the right-hand side of (4) a term $(n_0-n)/\tau$. We then easily obtain for $\delta n_k$

$$\delta n_{\mathbf{k}}=\delta n_{\mathbf{k}}^{(0)}[1+1/\Omega\tau]^{-1},\quad \Omega\equiv\sum_{\mathbf{p},\mathbf{p}'}W(\mathbf{p},\mathbf{p}',\mathbf{k})(f_{\mathbf{p}'}-f_{\mathbf{p}}). \quad (16)$$

$\Omega$ is positive in all the cases considered above. If the electron gas is degenerate, $\Omega$ does not depend on either the magnitude or the velocity of the drift in the region $k_F v_0 \ll \omega$, that is most important for the experiment. All the derived relations remain therefore in force, and only the absolute value of $\delta n_k$ decreases.

Instability sets in at $\Omega < -1/\tau$. It turns out that for the model considered by us, in which we assume $f_p = f_0(\mathbf{p}-\mathbf{p}_0)$, the onset of instability depends significantly on the electron-scattering mechanism. This is easiest to verify with a nondegenerate electron gas as the example. We calculate the first term in the right-hand side of (4) without assuming $p_0$ to be small. It turns out that for short-range impurities we have $\Omega>0$ at all drift velocities, i.e., no instability sets in.

In scattering by charged impurities we have

$$\Omega\sim\int_0^{\omega/T}e^{-x}I_0(2\beta x^{1/2})\,dx-\frac{\omega}{T}\cos 2\alpha\int_{\omega/T}^{\infty}e^{-x}I_2(2\beta x^{1/2})\frac{dx}{x};$$
$$\beta=\left(\frac{p_0^2}{2mT}\right)^{1/2}. \quad (17)$$

It is easy to verify that in the region $\omega/T\ll 1$, $\beta(\omega/T)^{1/2}\ll 1$ reversal of the sign of $\Omega$ takes place when $\beta$ is the root of the equation

$$\cos 2\alpha(e^{\beta^2}-\beta^2-1)=\beta^2. \quad (18)$$

Thus, instability is possible in the sectors $0\leqslant\alpha\leqslant\pi/4$ and $3\pi/4<\alpha\leqslant\pi$, if $\beta>\beta_{\min}$. At $\alpha=0$ we have $\beta_{\min}\approx 1.26$, which does not contradict the condition $\beta(\omega/T)^{1/2}\ll 1$.

In the case of degenerate electrons, the calculations become much more cumbersome, but it can be stated that there is no instability up to drift velocities of the order of the electron Fermi velocity. For short-range centers and at $T=0$ it is possible to calculate $\Omega$ for any ratio of $p_0 p_F$ and $m\omega$, by assuming only that the conditions $p_0\ll p_F$ and $m\omega\ll p_F^2$ are satisfied. It turns out that $\Omega>0$ in the entire indicated region, i.e., no instability can develop.

[1)] The periodic system of the electrodes serves as a resonator that separates momenta $k=\pm 2/l$, where $l$ is the period of the structure.


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