Nonlinear effects and nonmonotonic relaxation of magnetization in ^{3}He-B

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Magnetic relaxation in spatially homogeneous ^{3}He-B is investigated within the framework of the Leggett-Takagi theory. The following results were obtained with the external magnetic field turned off: the averaged equation of motion of the magnetization, the spatial configuration of the magnetization and of the order parameter under conditions corresponding to the presence of an attractor in the solutions of the Leggett-Takagi equations, and the instability produced near the extinction of the nonlinear-ringing frequency by the interaction of the dissipative and nonlinear effects generated by the topological structure of the order-parameter space. A weak periodic external magnetic field induces a stochastic spin-dynamics regime.

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1. INTRODUCTION

A feature of the spin dynamics in superfluid ^{3}He-B is the presence of complicated nonlinear effects whose study is of great importance for the understanding of the nature of the superfluid state in P-pairing[1] (see Fomin's paper[2] concerning the asymptotic methods of analyzing the nonlinear problems of the spin dynamics of superfluid ^{3}He). One of the most outstanding examples in this respect is the wall-pinned (WP) mode,[3–5] theoretically explained in Ref. 3, and the presence of extinction in the nonlinear-ringing frequency of the magnetization,[6,7] which were investigated theoretically in Ref. 8. Both phenomena stem from the form of the dipole energy of ^{3}He-B, and can be theoretically explained even by the nondissipative spin-dynamics theory.[3,8] The more complete Leggett-Takagi (LT) theory,[4] now regarded as an adequate description of the spin-dynamics processes, makes it possible to take into account dissipative effects that are peculiar to superfluid ^{3}He.

We investigate in this paper two closely related phenomena whose existence is based on the interaction of the nonlinearities and of the internal dissipation mechanism[4] and which do not exist in the nondissipative regime.

We consider two spin-dynamics regimes: (1) with the external field turned off and (2) in the presence of a periodic external magnetic field. In a zero field it is assumed that the initial magnetic field was abruptly (i.e., within a time much shorter than the relaxation time) turned off and the system started to evolve on its own. We assume throughout validity of the hypothesis that the order parameter and the spin have spatially homogeneous distributions. The temporal evolution of the system is analyzed within the framework of the LT theory.[4]

We shall show that in a zero field the system tends to go near T_c into some attracting regime, corresponding to the attractor in the solutions of the LT equation. It was shown earlier[10] that an attractor exists at large values of the magnetization. We study here also the regions of medium and low magnetizations.

In the magnetization region corresponding to the extinction of the nonlinear-ringing frequency, we obtain an instability that causes the anomaly in the topology of the aforementioned attractor and the existence of random spin dynamics in the presence of an external field. It is shown namely that in a periodic external field with oscillation frequency and Larmor frequency of the order of the Leggett frequency the solutions of the LT equations are random (recalling the solutions for ergodic billiard balls[11,12]). This, in accord with modern ideas concerning turbulence,[13,14,15] allows us to advance the hypothesis that turbulent regimes of the spin dynamics of superfluid ^{3}He-B can exist.

2. THE LEGGETT-TAKAGI EQUATIONS

The system of LT equations in the nondissipative regime and in the absence of an external regime was integrated exactly in Ref. 16. In the general case, i.e., when dissipation is taken into account, the LT equations have no analytic solution in closed form and can be investigated either by numerical methods or approximately, within the framework of asymptotic methods. The latest developments for the case of strong external magnetic fields were expounded in a set of papers by Fomin.[2,17,18,19–21] Asymptotic solutions of the LT equations for large values of the magnetization and in the absence of an external field were investigated in Ref. 10. These asymptotic solutions led to the existence of the attractor regime investigated in the present paper (for a brief exposition of some of the results see Ref. 9).

The LT equations have been investigated in a number of recent studies[22–25] by numerical-analysis methods. It must be noted that these papers consider relaxation regimes in strong magnetic fields. It is shown in Ref. 25 that the results of the numerical analysis agree well with the asymptotic results obtained by Fomin.

In the present paper we study the LT system of equations (1) by asymptotic methods using the method of averaging the integrals of the basis system, and (2) by numerical-analysis methods. The combination of these two different methods yields, in closed form, the behavior of the relaxation in the entire range of magnetization. The numerical integration was carried out by a fourth-order Runge-Kutta

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algorithm, with the errors eliminating by using the doubled-accuracy regime and a variable-interval difference scheme.

The calculations were performed in the dimensionless variables

$$\tilde{S}_i=\omega_0^{-1}\gamma^2\chi^{-1}S_i, \quad \tilde{t}=\omega_0 t, \quad \omega_0=10^6 \text{ rad/sec},$$

where γ and χ are the gyromagnetic ratio and the susceptibility. The characteristic times are given in the test in dimensional units, msec. It should be noted that in dimensionless variables (the Leggett angular frequency Ω_L, the damping constant $\Gamma_\parallel$ corresponding to the longitudinal NMR line width, the dissipation and dipole constants μ and g_D, which are connected with $\Gamma_\parallel$ by the relation $\mu g_D = \Gamma_\parallel$, and the Larmor angular frequency ω_0 corresponding to the magnetization scale) enter into the LT equations only via the two ratios $\alpha = \Gamma_\parallel/\Omega_L$ and Ω_L/ω_0.

In the main calculations we have $\Omega_L = 10^5$ rad/sec and $\Gamma_\parallel = 10^4$ rad/sec, corresponding a ratio $\alpha = \Gamma_\parallel/\omega_L = 0.1$, shown in Ref. 4 to be the most realistic. From the approximate formula $\Omega_L = 10^7\times(1 - T/T_c)^{1/2}$ given in Ref. 4 it can be seen that the chosen value of Ω_L is close to T_c. Actually, since the LT equations depend only on the ratios $\Gamma_\parallel/\Omega_L$ and Ω_L/ω_0, the results can be extended to a wider range of temperatures than would follow directly from the formula given above for Ω_L.

It is assumed that the Larmor angular frequency is of order not larger than $\omega_0 = 10^6$ rad/sec, i.e., it can be assumed that $\omega_0\tau \ll 1$, where $\tau \lesssim 10^{-7}$ sec is the quasiparticle relaxation time. In this frequency range the hydrodynamic LT approximation[4] is applicable and the LT equations can be regarded as a Hamiltonian system with a dissipative function[26] specified by one of the following: (1) Poisson brackets between the coordinates of the spins S_1, S_2, S_3 and the order parameters A_{ij}, $j = 1,2,3$,

$$\{S_i, S_j\}=\varepsilon_{ijk}S_k, \quad \{S_i, A_{jm}\}=\varepsilon_{ijk}A_{km}, \quad \{A_{ij}, A_{km}\}=0;$$

(2) the Leggett Hamiltonian

$$\mathcal{H}={}^1/_2\gamma^2\chi^{-1}\mathbf{S}^2-\gamma\mathbf{HS}+U;$$

(3) the dissipative function $F = \frac{1}{2}\mu(U'_\theta)^{1/2}$ (Refs. 26 and 27). Here

$$U=g_D(\cos\theta+{}^1/_4)^2+\text{const}$$

is the dipole energy for ^{3}He-B. The order parameter for ^{3}He-B is of the form $A_{ij} = (\Delta/\sqrt{3})[\exp(i\varphi)]R_{ij}$, where R_{ij} is the three-dimensional rotation operator. We put hereafter $\varphi = 0$. It is convenient to parametrize the matrix R_{ij} with the aid of the angle θ and the rotation-angle unit vector c_i, $i = 1,2,3$:

$$R_{ij}=\delta_{ij}\cos\theta+c_ic_j(1-\cos\theta)-c_k\varepsilon_{ijk}\sin\theta.$$

It should be noted that the order parameter is invariant to the transformation

$$c_i\to-c_i, \quad \theta\to2\pi-\theta. \tag{1}$$

When account is taken of the dissipative function and of the Poisson brackets between S_i and A_{ij}, the LT equation can be written in the form[3]

$$\frac{d}{dt}\mathbf{S}=\gamma[\mathbf{S}\times\mathbf{H}]-\frac{dU}{d\theta}\mathbf{c},$$

$$\frac{d}{dt}\mathbf{c}=-{}^1/_2\operatorname{ctg}\frac{\theta}{2}((\gamma^2\chi^{-1}\mathbf{S}-\gamma\mathbf{H})\mathbf{c})\mathbf{c}+{}^1/_2[(\gamma^2\chi^{-1}\mathbf{S}-\gamma\mathbf{H})\mathbf{c}] \tag{2}$$

$$+\frac{1}{2}\operatorname{ctg}\frac{\theta}{2}(\gamma^2\chi^{-1}\mathbf{S}-\gamma\mathbf{H}), \quad \frac{d}{dt}\theta=(\gamma^2\chi^{-1}\mathbf{S}-\gamma\mathbf{H})\mathbf{c}-\mu\frac{dU}{d\theta}.$$

To describe the solutions of the LT equations in the absence of an external magnetic field, it is convenient to introduce the variables

$$S_\parallel=\mathbf{Sc}, \quad S_\perp=(\mathbf{S}^2-S_\parallel^2)^{1/2}, \quad \theta$$

It is shown in Ref. 10 that they satisfy a three-equation system that is a corollary of the system (2).

If we exclude the dissipation (by putting $\mu = 0$), the LT equations are transformed into a conservative Hamiltonian system that has in the absence of a magnetic field an additional scalar integral

$$B=S_\perp\sin(\theta/2) \tag{3}$$

(see Ref. 16) and three integrals that make up the vector

$$\mathbf{L}=\frac{\sin^2(\theta/2)}{B}\left(\operatorname{ctg}\frac{\theta}{2}[\mathbf{S}\times\mathbf{c}]-\mathbf{S}+S_\parallel\mathbf{c}\right) \tag{4}$$

(see the review by Brinkman and Smith in Ref. 6). The integrals (4) are interpreted geometrically; in the absence of an external magnetic field and dissipation, the vector **c** rotates in a fixed plane whose unit normal has as its coordinates the integrals (4). The end point of the vector **S** rotates in turn in a plane parallel to the plane of the vector **c**. Furthermore $\mathbf{L}\cdot\mathbf{S} = -B$, where B is the integral (3).

In the absence of an external field the system has an attractor that exists for all values of the magnetization (at $S_\perp \neq 0$), although its form changes significantly as a function of its magnitude (see Fig. 1). The attractor is a hypersurface in the space of the variables $S_1, S_2, S_3, c_1, c_2, c_3$, and θ, and the solutions of the system (2) tend to this surface at sufficiently long system-evolution times. In the region of large values of

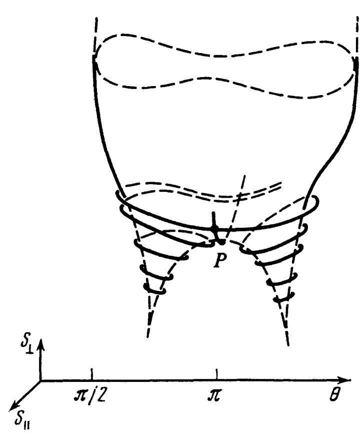

FIG. 1. Form of attractor in the space of the variables $S_\parallel$, $S_\perp$, and θ. For clarity, the scales of the axes are distorted and there is no factorization with respect to the transformation (1). It can be seen that the presence of the pinching point P leads to divergence of initially close trajectories.

the magnetization, corresponding to Larmor frequencies of the order of 10^6, the attractor is given approximately by the equation[10]

$$S_{\parallel}^2+S_{\perp}^2 \cos\theta=0.$$

A complete description of the attractor, including the region of weak fields, is obtained in this paper by a numerical analysis for which we have examined a rather large number of solutions of Eqs. (2) with various initial data. The attractor is a continuous hypersurface that becomes pinched at magnetization of the order of $6\gamma^2\chi^{-1}g_D$ (corresponding to extinction of the nonlinear-ringing frequency), see Fig. 1. The form of the attractor does not change when Ω_L is varied from 10^5 to 10^6 rad/sec and $\Gamma_{\parallel}$ is varied from 10^4 to 10^3 rad/sec.

One can distinguish the following characteristic stages of the relaxation from the large-magnetization region: (1) entry to the attractor, (2) the attractor regime above the pinching point, (3) region of extinction of the nonlinear-ringing frequency (pinching point), and (4) motion in the vicinity of the WP mode.

There can exist, independently, a WP mode corresponding to the prongs of the separatrix in Fig. 2 (Ref. 10) and a Leggett regime for which $S_{\perp}=0$.

The indicated regimes (1)–(4) differ greatly both in the configuration of the dynamic variables and in the characteristic times and frequencies. It is therefore best to consider them separately.

3. AVERAGING OF THE EQUATION OF MOTION OF THE MAGNETIZATION VECTOR

To understand better the magnetization relaxation it is useful to derive an averaged equation of motion of the vector **S**. It is assumed that the magnetization is large enough to

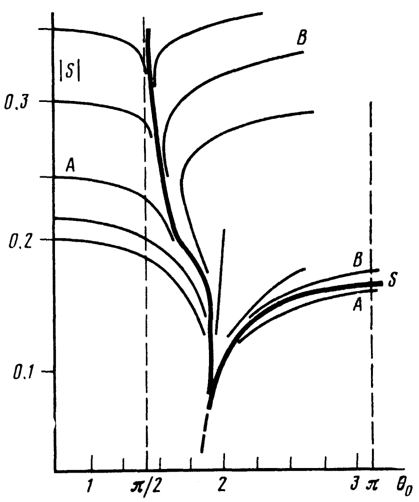

FIG. 2. Intersection of the plane $S_{\parallel}=0$ with the attractor, and projection of the separatrix on this plane. Exits from the attractor in the regions of small (curves of type A) and large (curves of type B) values of θ are shown. the WP-mode regime corresponds to the prongs of the separatrix S. The amplitude θ_0 corresponds to the abscissa of the attractor intersection point; it follows from the geometry of the attractor that $\theta_0>\pi/2$. It can also be seen that θ has a local minimum as it moves out of the region of large values.

neglect in the zeroth approximation the influence of the dissipation and of the dipole energy. For the basic solution we have then $\mathbf{S}=\text{const}$, i.e., the vector **S** is an integral of the motion. In addition, **L** is an integral of the motion.

It can be shown[10] that in the basic solution the time dependences of $S_{\parallel}$, $S_{\perp}$, and θ are given by

$$S_{\perp}=S_{\perp 0}\sin^{-1}(\theta/2),\quad S_{\parallel}=S_{\perp 0}[\sin^{-2}(\theta_0/2)-\sin^{-2}(\theta/2)],\quad (5)$$
$$\cos(\theta/2)=\cos(\theta_0/2)\sin\psi,\quad \psi=\psi_0+\omega_\theta t,$$
$$\theta_0=\text{const},\quad S_{\perp 0}=\text{const},\quad \psi_0=\text{const},\quad \omega_0=\text{const}.$$

Using (4) we can express the vector **c** as a function of the vectors **L** and **S** and of the scalar variables:

$$\mathbf{c}=(S^2-B^2)^{-1}(S_{\parallel}\mathbf{S}+BS_{\parallel}\mathbf{L}+B\operatorname{ctg}(\theta/2)[\mathbf{L}\times\mathbf{S}]).\qquad(6)$$

This equation is exact if dissipation is neglected. Substituting **c** from (6) in the first equations of the system (2) we obtain

$$\frac{d}{dt}S=-(S^2-B^2)^{-1}\frac{dU}{d\theta}\left(S_{\parallel}\mathbf{S}+BS_{\parallel}\mathbf{L}+B\operatorname{ctg}\frac{\theta}{2}[\mathbf{LS}]\right),\qquad(7)$$

where B is the Maki-Ebisawa integral (3). The dissipation enters in (7) implicitly via θ. Equation (7) is averaged over the period of the basic solution (5). The average of f is taken to be

$$\langle f\rangle=\frac{1}{2\pi}\int_0^{2\pi}f\,d\psi,$$

where ψ is the phase from (5). The averaged equation (7) is of the form (the averaging sign is omitted hereafter for simplicity).

$$\frac{d}{dt}\mathbf{S}=\frac{3}{2}g_D\cos^2\frac{\theta_0}{2}\left(2\cos^2\frac{\theta_0}{2}-1\right)B(S^2-B^2)^{-1}[\mathbf{L}\times\mathbf{S}].\qquad(8)$$

As shown in Ref. 10, at large magnetizations the attractor regime corresponds to $\theta_0\approx\pi/2$. In this case it follows from (8) that S is constant, in accord with the initial assumptions. Actually θ_0 turns out to be larger than $\pi/2$ (Ref. 9), therefore $d\mathbf{S}/dt\neq 0$. Moreover, on leaving the region with $\theta<\pi/2$ the sign of the coefficient in the right-hand side of (8) is reversed, and this leads to a change in the direction of rotation of the averaged vector **S** around the **L** axis when landing on the attractor. It must be remembered that averaged quantities are referred to throughout.

Similar reasoning leads to an averaged equation for the vector **L**:

$$\frac{d}{dt}\mathbf{L}=\frac{3}{2}\mu g_D\cos^2\frac{\theta_0}{2}\left(2\cos^2\frac{\theta_0}{2}-1\right)$$
$$\times\left[\left(1-\frac{1}{2}\frac{S^2}{S^2-B^2}\right)\mathbf{L}-\frac{1}{2}\frac{B}{S^2-B^2}\mathbf{S}\right].$$

It can be seen that after landing on the attractor ($\theta_0\to\pi/2$) the mean value of the vector **L** becomes stable: $d\mathbf{L}/dt\to 0$. This property is the principal asymptotic property of the attractor regime. It is very convenient from the veiwpoint of numerical analysis (see Sec. 5 below). From the observational viewpoint, a small change in the averaged vector **L** in the attractor regime makes it possible to describe the motion of the vector **S** after long time intervals of the order of dozens of milliseconds as rotation around a fixed axis, namely the average position of the vector **L**.

TABLE I.

H, G	δ (L)	$S_{\parallel}^2 + S_{\perp}^2 \cos\theta$	τ_{out}, msec	
			θ_{init} =1 rad	θ_{init} =2 rad
10	0.005	0.002	4.4	6.1
12.5	»	»	5.0	6.8
15	»	»	5.4	7.3
17.5	»	»	5.4	7.7
20	»	»	5.7	8.0
22.5	»	»	5.9	8.3

4. REGIME OF REACHING THE ATTRACTOR

The main test of the existence of an attractor regime prior to the region of the extinction of the nonlinear-ringing frequency is provided by the integrals **L**. In the presence of dissipation they are no longer conserved quantities, but their very change can describe the behavior of the system, as indicatd by the averaged equations for **S** and **L** and as is made especially clear in the numerical analysis of the solution of the system (2).

The numerical criterion of reaching the attractor regime at large magnetizations was that the deflection angle δ(**L**) of the vector **L** during one period of the angle [see the basic solution (5)] not exceed 0.005 rad. When reaching the attractor the planar character of the motion of the vector **c** is weakly pronounced and the position of the vector **L** is considerably altered upon averaging over one period. The times of landing on the attractor, calculated in accord with the criterion $\delta(\mathbf{L}) < 0.005$, are listed in Table I, where the first column contains the fields H corresponding to the initial magnetization, and $\Omega_L = 10^5$ rad/sec and $\Gamma_{\parallel} = 10^4$ rad/sec throughout.

As shown in Ref. 10, at large magnetizations the attractor regime can be characterized by the proximity of the amplitude of the angle θ to $\pi/2$ and by satisfaction of the condition $S_{\parallel}^2 + S_{\perp}^2\cos\theta \approx 0$. Estimates, in accord with these criteria, of the time to reach the attractor agree with those given above (see Table I). The features of the landing on the attractor at medium and low magnetizations are illustrated in Fig. 2.

5. THE ATTRACTOR REGIME UP TO THE PINCHING POINT

In this regime there are two scales of both the time and the spin. The first scale corresponds to rapid variables: it is determined by the frequency ω_c of the rotation of the vector **c** around the position of the vector **L**, a position that changes little in one revolution. The spin vector **S** traces in this case, in a plane parallel to that of the vector **c**, a curve with a characteristic dimension of the order of

$$0.3(\Omega_L/\omega_0)^2/(\gamma^2\chi^{-1}\omega_0^{-1}S).$$

The angle θ varies in the range $\theta_0 \leqslant \theta \leqslant \pi$, where θ_0 is close to $\pi/2$ (see Table I and Fig. 3). During one period of the vector **c**, the vector **L** is deflected as a result of dissipation by an angle not more than 0.005 rad (at $\omega_L = 10^5$ and $\Gamma_{\parallel} = 10^4$ rad/sec).

The second time scale corresponds to the spin lifetime on the attractor, of the order of the modulus of **S**, i.e., much larger than the size of the triangle in Fig. 3. The behavior of the spin vector in this scale is illustrated in Fig. 4. The lifetime on the attractor up to the pinching point is a linear function of the square of the initial magnetization, with a Larmor angular frequency of the order of 100 rad/sec and with $\omega_L = 10^5$ and $\Gamma_{\parallel} = 10^4$ rad/sec. The regime described continues up to magnetizations with characteristic Larmor angular frequencies on the order of 19×10^4 rad/sec, corresponding for a gyromagnetic ratio $\gamma = 2 \times 10^4$ rad/sec·G to fields of the order of 10 G.

Particular notice should be taken of the peculiarities of the landing of the system on the attractor. If this landing is from the region with initial data $\theta < \pi/2$, the direction of rotation of **S** is reversed on landing and the curvature of the trajectory is very large. The vector **S** is at standstill for a long time in the landing region, especially if the system had initially $\theta < \pi/2$. For example, on starting from a position with $\theta = 0.2$ and $\Omega_L = 10^5$ and $\Gamma_{\parallel} = 10^4$ rad/sec, one of the calculated trajectories stayed 80 msec in the landing region, and in this case θ and the modulus of **S** remained in the ranges from 1.571 to 1.573 rad and from 1.99 to 1.92, respectively, the direction of the vector **S** changed by 0.03 rad, and that of the vector **L** by 0.05 rad. These are typical data and **S** is in dimensionless units.

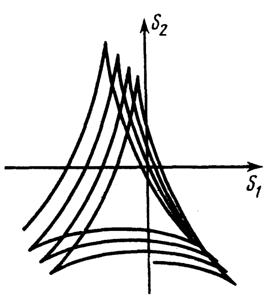

FIG. 3. Motion of spin vector in the coordinates S_1 and S_2 in the large-magnetization region.

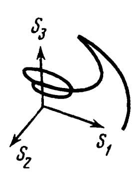

FIG. 4. Averaged motion of the spin vector in the space S_1, S_2, S_3, corresponding to an exit from the region $\theta < \pi/2$. The mean value of the vector **L** practically coincides with the OS_3 axis.

The properties indicated allow us to conclude that the relaxation has a quasistationary behavior in the region of landing on the attractor.

Motion in a large scale relative to the attractor involves the question, discussed in the literature,[22,23,28] of the nonmonotonic and nonexponential character of the magnetization relaxation. It can be seen from Fig. 4 that, at any rate in the absence of a magnetic field, there is no monotonic relaxation whatever. In particular, the reversal of the rotation of the vector **S** after the system reaches the attractor should lead to observable effects similar to the magnetization-relaxation reversal obtained[28] for ^{3}He-*A*.

6. VICINITY OF THE POINT OF ATTRACTOR PINCHING AND OF THE WP MODE

In what follows we shall find useful the following geometrical interpretation of the order parameter, which corresponds to homeomorphism of the group of rotations of three-dimensional space, $SO(3)$, on the three-dimensional projective space $RP(3)$. We introduce the vector $\theta\mathbf{c}$ which is collinear with $\mathbf{c}$, the angle θ varying in the range $0\leqslant\theta\leqslant\pi$. The end points of all such vectors fill a three-dimensional sphere of radius π. The diametrically opposite (or antipodal) points of the surface of this sphere are identified in accord with the transformation (1), as a result of which the values of the order parameter are in a one-to-one correspondence with the points of the thus obtained three-dimensional projective space $RP(3)$. The boundary surface with radius with the identified antipodal points is the projective plane imbedded as a submanifold in the manifold of the order parameter $RP(3)$.

The time variation of the order parameter corresponds now to a curve in $RP(3)$.

It is necessary in this connection to consider the relief of the surfaces of the dipole-energy levels U in the order-parameter space. It follows from the foregoing description that U has:

1) two maxima, namely the center and the surface of the sphere of radius π, with identifiable antipodal points, i.e., a projective plane $RP(2)$:

2) only one minimum—a sphere of radius $\arccos(-1/4)$.

It is useful to bear in mind also the following circumstance. In the absence of dissipation the integrals **L** are conserved, the vector **c** rotates in a fixed plane, and the vector $\theta\mathbf{c}$ moves on a circle with identifiable antipodal points of the boundary. The dipole energy has maxima at the center of the circle and on its periphery. It has a minimum on a circle of radius $\arccos(-1/4)$. The relief of its level surfaces resembles a crater or the surface of water on which a small drop has fallen.

The vicinity of the attractor pinching point corresponds to states with energy close to the maximum of the dipole energy at $\theta=\pi$. In terms of the vector $\theta\mathbf{c}$ these are trajectories that either cross the projective plane $RP(2)$ specified by the condition $\theta=\pi$, or come very close to it (see Fig. 5). The latter trajectories have an energy lower than $U(\theta=\pi)$, by virtue of which they are reflected so to speak from the projective plane $RP(2)$ and remain all the time inside the open three-dimensional sphere corresponding to the condition $\theta<\pi$. Trajectories with energy higher than $U(\theta=\pi)$ pierce through $RP(2)$ in accord with the foregoing interpretation of $RP(2)$ as a sphere of radius π with identifiable antipodal points. When such a trajectory enters through a point on the sphere, it emerges from the antipodal point (see Fig. 5).

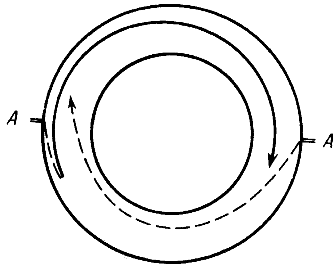

FIG. 5. Mapping of two close trajectories in the projective plane corresponding to the condition $\mathbf{L}=\text{const}$ (this condition is satisfied with good accuracy at low dissipation). The solid line is the trajectory reflected from $RP(2)$. The dashed trajectory pierces $RP(2)$ at the point A. The trajectories cease to be close starting with this instant.

The described behavior of the trajectories is based only on the topology of the order parameter and on the form of the dipole energy for ^{3}He-*B*. The manifold of the maxima $RP(2)$, in analogy with the semitransparent mirror of a Michelson interferometer (Ref. 29, p. 331 of translation), splits the beams of close trajectories of the system in the order-parameter space; these beams are partially reflected and partially transmitted through this space. As a result of dissipation each of the trajectories should be reflected from $RP(2)$ in the course of time and land in the interior of the sphere of radius π. The resultant confinement leads to scattering of trajectories that made up initially an almost homogeneous beam.

In the vicinity of the attractor pinching point, the described mechanism induces effectively an instability of the solutions of the system (2), as is clearly seen from the numerical data.

After passing through the attractor pinching point, the system, enters a regime wherein the trajectories are wound around the prongs of the separatrix (see Fig. 1). In the order-parameter space this corresponds to motion near the dipole-energy minimum—a sphere of radius $\arccos(-1/4)$. The linear time dependence of the square of the WP-mode period[4] is well confirmed numerically.

7. STOCHASTIC REGIME

The instability in the vicinity of the attractor pinching point is an indication that multiple passage of the trajectories through this region under the action of a periodic external field leads to actual randomization of the process. To obtain this effect, a periodic external field

$$H_1=0,\quad H_2=0,\quad H_3=a[1+\sin(bt)],$$

of constant direction and amplitude was used for activation, i.e., the external interference with the spin dynamics was minimal. However, as shown by calculations, the process

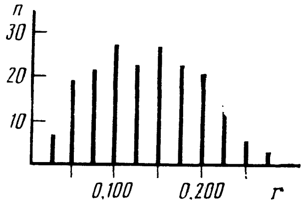

FIG. 6. Distribution of the distances between initially close lattice points after a time $\tau = 10$ msec. The abscissa is the distance r, and the ordinate the number of pairs of points diverging to the given distance. The histogram for distances between arbitrary pairs of points is similar.

becomes stochastic at amplitudes and frequencies in the ranges defined by the inequalities $0.05 \leqslant a \leqslant 0.13$ and $0.05 \leqslant b \leqslant 0.15$ (in dimensional units the field H varies between 2.5 and 6.5 G and the frequency between 7.9 and 23.8 kHz).

To cast light on the character of the process, we investigated first the instability in the indicated fields. The numerical criterion of instability was taken to be the degree of scatter of initially close points within a fixed time interval. A square of 121 points was considered in a plane described by the equations

$$S_1=-0.1;\quad \theta=3.8;\quad c_1=0;\quad c_2=1;\quad c_3=0,$$

with its center at the point with coordinates

$$S_1=-0.1;\quad S_2=-0.1;\quad S_3=0.15$$

and with a uniform lattice with mesh 5×10^{-5} (in dimensionless units). The average distance between initially neighboring points became equal to 0.125 within 10 msec. The external field chosen (in dimensionless units) was

$$H_1=H_2=0,\quad H_3=0.11[1+\sin(0.105t)].$$

The stochastic character of the process is well illustrated by the histogram in Fig. 6. For comparison, the scatter without the field was 0.0005.

8. CONCLUSIONS

It follows from our results that within the framework of the Leggett-Takagi theory it is possible to predict the existence of a new nonlinear magnetization-relaxation regime whose salient features (a tendency of the spin vector to assume a certain position in space, quite long characteristic times, and weak magnetic fields) make it experimentally observable. The investigated attractor regime of relaxation is in essence a dissipative process, i.e., it is not realized in a nondissipative approximation and is a direct consequence of the internal Leggett-Takagi relaxation mechanism. Its observation in experiment would therefore be one more confirmation of this theory. On the other hand, should its observation be difficult, this would mean that it is necessary to take into account in the Leggett-Takagi theory some additional factor, such as spatial inhomogeneity of the real system, as already pointed out earlier by Fomin.[17,18]

In this respect, the effective instability deduced by us for the vicinity of the extinction of the nonlinear-ringing frequency can initiate growth of the small inhomogeneities which are always present in a real system, and lead to the onset of quite large order-parameter gradients. These can ensure those texture effects to which the frequency extinction was previously attributed.

The ensuing situation is comparable with hydrodynamic instability and can be interpreted as the onset of turbulence. Similar phenomena are well known in the theory of liquid crystals.[30] A significant indication of the possibility of turbulent regimes in the spin dynamics of ^{3}He-B is the stochastic regime obtained in the present paper.

The quasistationary state corresponding to the start of relaxation in the attractor regime can be of special interest. It may have analogs in other magnetic system, as indicated by the analogies with ^{3}He-B in the theory of magnets, developed by Andreev and Marchenko.[31]

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