Inverted population of light-hole band pumped at cyclotron resonance

V. A. Kozlov, L. S. Mazov, and I. M. Nefedov

Institute of Applied Physics, USSR Academy of Sciences
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We investigate the behavior of a system of hot carriers inelastically scattered by optical phonons in a semiconductor in a strong microwave field and a magnetic field. A mechanism is proposed for the redistribution of the holes among the heavy and light subbands of the valence band; this mechanism is capable of controlling the relative hole densities. At cyclotron resonance of the heavy holes it is possible to produce inverted population of the light-hole subband with a relative density higher than in constant fields $\mathbf{E}\perp\mathbf{B}$. Cyclotron resonance of the light holes causes strong depletion of the light-carrier subband.

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1. Accumulation of hot carriers in singled-out momentum-space regions produces in semiconductors a strong disequilibrium of the distribution functions that can become even inverted in some cases.[1] Such an accumulation is possible in dynamic heating of the carriers, when a substantial role in the heating is played by the free-motion dynamics. Such a situation is realized, for example, in pure semiconductors at low lattice temperatures $T \ll \hbar\omega_0/k$, when the carrier scattering proceeds mainly on account of spontaneous emission of optical phonons, so that at an energy $\varepsilon < \hbar\omega_0$ the motion of the carriers can be regarded as almost free; here $\hbar\omega_0$ is the optical-phonon energy and k is Boltzmann's constant.

The most interesting carrier accumulation manifests itself in semiconductors with degenerate subbands of the heavy and light holes, for in this case scattering can be accompanied by transitions between the subbands. It is usually assumed that with increasing electric field the first to be heated are the light holes (since their effective mass is small), so that the relative density of the light and heavy holes n_l/n_h should be decreased by heating to a value $(m_l/m_h)^2$ less than the equilibrium value $(m_l/m_h)^{3/2}$ (Refs. 2–4). Here m_l, n_l, m_h, n_h are the effective masses and densities of the light holes (l) and the heavy ones (h), respectively. The situation changes substantially in fields $\mathbf{E}\perp\mathbf{B}$, where only light holes can accumulate, so that n_l/n_h can exceed $(m_l/m_h)^{3/2}$ (Ref. 5). The increase in the relative density of the light holes leads, in particular, to inversion of the population of the radiative transitions between the light and heavy hole subbands,[6] and also to corresponding singularities in the galvanomagnetic characteristics.[7] These effects were observed experimentally in Refs. 8–12.

We report here an investigation of carrier heating in an alternating field, including one at cyclotron resonance. We have determined for the values of the microwave and magnetic field strengths the conditions under which an accumulation region is produced, an effective redistribution of the carriers among the subbands sets in, and the population of the light-hole subband is inverted. Comparison with numerical calculations has shown that the concepts developed make possible a simple and sufficiently complete description of the structure of the distribution function in strong microwave fields.

2. To describe the distribution function when the principal scattering mechanism is spontaneous emission of optical phonons, the momentum space is usually divided into two regions, active with carrier energy $\varepsilon > \hbar\omega$ and passive with $\varepsilon < \hbar\omega$ (Ref. 13). In the passive region the collision frequency ν_i is determined by scattering from acoustic phonons and impurities, and by absorption of optical phonons; it is as a rule much lower than the collision frequency ν_0 in the active region, where spontaneous emission of optical phonons is turned on:

$$\nu_i \ll \nu_0. \tag{1}$$

If condition (1) is satisfied in sufficiently strong electric fields, the carrier motion in the passive region can be regarded as almost free, and the carriers that land in the active region emit rapidly (within a time ν_0^{-1}) optical phonons and return to the passive region.

The entire aggregate of trajectories of the free motion under conditions (1) can be divided into two classes: (a) internal trajectories, along which the moving carriers remain all the time in the passive region[1)]; (b) external trajectories, along which the moving carriers may turn out to be in the active regions. Particles that land on the internal trajectories accumulate on them because of the long lifetime τ on the internal trajectories ($\tau \sim \nu_i^{-1}$). We define the accumulation region $K(t)$ the set of momentum-space (or velocity-space) points on which the particles move at the instant of time t along internal trajectories.[2)] An important role in the process of carrier accumulation in the region K is played by its volume V_K. We note that although the shape of the region K can vary with time, the volume V_K is nevertheless independent of time. This can be proved with the aid of the Liouville theorem, since the division of the carrier trajectories into external and internal involves the presence of a *threshold* energy $\hbar\omega_0$ (and not of scattering), and is valid at all instants of time. In turn, the unit volume of momentum space does not change in size in the case of free motion.

The carrier populations of the different trajectories and the evolution of the distribution function are determined

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also by the width of the source of the carriers[3)] produced in the passive region after the scattering with emission of optical phonons.[15] A broad carrier source is produced in strong electric fields, when the carriers penetrate deeply enough into the active region. In this case the population of the region K is almost uniform and is large compared with the population of the external trajectories. If the carrier source is narrow, the population of K is accompanied by an increase of the population in the vicinity of the principal trajectories, whose topology assumes an important role in the formation of the distribution function.[4)] A narrow source requires moderate electric fields and semiconductors that are quite pure.

3. We consider the free motion of carriers in a constant magnetic field $\mathbf{B}(B_x = 0, B_y = 0, B_z = B)$ and an alternating electric field $\mathbf{E}_\omega$ of circular polarization ($E_x = E_\omega \cos \omega t$, $E_y = -E_\omega \sin \omega t, E_z = 0$) with amplitude E_ω. Depending on the ratio of the field frequency ω and the cyclotron frequency $\omega_B = qB/mc$, qualitatively different situations will be realized in the system of hot carriers[5)]: here q and m are the charge and effective mass of the carrier, and c is the speed of light.

We investigate first the case of cyclotron resonance, when $\omega = \omega_B$ and the direction of rotation of the electric field coincides with the direction of the free rotation of the carriers in the magnetic field. Solving the free motion equation $\dot{\mathbf{v}} = (q/m)\{\mathbf{E}_\omega + \mathbf{v}\times\mathbf{B}/c\}$ with initial conditions $v_x = v_{xi} = v_{\perp i} \cos\varphi_i; v_{yi} = v_{\perp i} \sin\varphi_i; v_z = v_{zi}$ at the instant of time $t = t_i$, we can easily find the square of the velocity of the singled-out carrier:

$$v^2(t) = v_{\perp i}^2 + v_{zi}^2 + [v_c\omega(t-t_i)]^2 + 2v_c v_{\perp i}\omega(t-t_i)\cos(\psi_i+\varphi_i), \tag{2}$$

where $v_{\perp i} = (v_{xi}^2 + v_{yi}^2)^{1/2}$ is the initial transverse velocity, φ_i is the initial angle, $\psi_i = \omega t_i$ the entry phase, i.e., the phase of the field at the instant of carrier entry; $v_c = cE_\omega/B$ is a characteristic velocity analogous to the carrier drift velocity in constant fields $\mathbf{E}$ and $\mathbf{B}$ (Ref. 19). In velocity space v_c corresponds to the position of the center of the carrier-revolution trajectories in constant fields $\mathbf{E}\perp\mathbf{B}$.[20] It can be seen from (2) that any free-motion trajectory starting out from the passive region leaves this region after some time, i.e., all the trajectories are external and consequently the region K does not exist for cyclotron resonance (Fig. 1a). The carriers are bunched in this case around the principal trajectories.[6)]

The time τ_E during which a carrier produced after scattering in the passive region, with velocity $v_i = 0$, reaches the optical-phonon energy is $\tau_E = mv_0/qE_\omega$, where $v_0 = (2\hbar\omega_0/m)^{1/2}$ is the speed corresponding to the boundary of the passive region. At $\tau_E < \nu_i^{-1}$ the carriers move in the passive region with practically no collisions, so that it is convenient to introduce here the transit frequency $\nu_E = \tau_E^{-1}$ (Refs. 22,23) at which a carrier moving under cyclotron resonance conditions reaches periodically an energy $\hbar\omega_0$, emits spontaneously an optical phonon, and returns to the passive region (we assume that $\nu_E \ll \nu_0$). Such a cyclic carrier motion at a frequency ν_E that depends on the amplitude E_ω and is generally speaking different from ω_B is analogous to carrier motion in a constant electric field E under streaming conditions.[24–27] By starting with this analogy we can find the depth of carrier penetration into the active region:

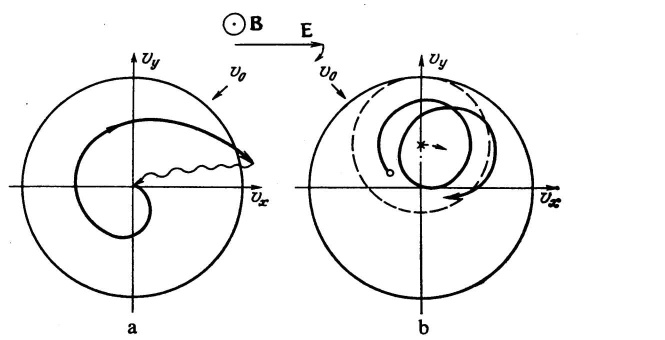

FIG. 1. Trajectory of carriers in velocity space: a) $\omega_B = \omega$, cyclotron resonance; b) $\omega_B > \omega$; the dashed circle shows the position of the accumulation region K at the instant of time t; the asterisk marks the center of the region K; the dashed arrow show the direction of rotation of K around the v_z axis.

$$\Delta v \approx v_0(3E_\omega/2^{5/2}E_0)^{2/3} = v_0(3\nu_E/2^{5/2}\nu_0)^{2/3}$$

and estimate the size of the source[15] of the carriers produced in the passive region after emission of an optical phonon:

$$\delta v \approx (2v_0\Delta v)^{1/2} = v_0(3E_\omega/2E_0)^{1/3} = v_0(3\nu_E/2\nu_0)^{1/3},$$

where $E_0 = mv_0\nu_0/q$ is the characteristic optical-scattering field. Such a weak (cube-root) dependence of δv on the ratio ν_E/ν_0 shows that under real conditions it is difficult to produce a narrow carrier source in the passive region, since $\nu_E > \nu_i$ in the case of streaming (cf. Ref. 28). For example, for pure p-Ge at $T = 4$ K the ratio $\nu_i/\nu_0 \sim 0.01$, therefore the width of the distribution function is of the order of the displacement $\sim 0.5v_0$ (Ref. 4) of the distribution center along the revolving principal trajectory even in a field $E_\omega \sim 0.01E_0$.

4. We proceed now to describe carrier motion in a circularly polarized field, when $\omega \neq \omega_B$. Here $v^2(t)$ is given by

$$v^2 = v_{\perp i}^2 + v_{zi}^2 + 2v_B v_{\perp i}\cos(\psi_i+\varphi_i)\sin[(\omega_B-\omega)(t-t_i)] + 2v_B[v_B + v_{\perp i}\sin(\psi_i+\varphi_i)]\{1-\cos[(\omega_B-\omega)(t-t_i)]\}, \tag{3}$$

from which it can be seen that $v^2(t)$ has an upper bound[7)]

$$v_{max}^2 = v_{zi}^2 + [v_B + \{v_B^2 + v_{\perp i}^2 + 2v_B v_{\perp i}\sin(\psi_i+\varphi_i)\}^{1/2}]^2, \tag{4}$$

where the characteristic speed is $v_B = v_c\omega_B/(\omega_B - \omega) = v_0\nu_E/(\omega_B - \omega)$.

With increasing magnetic field, owing to the decrease of v_B, the maximum velocity $v_{\max}$ becomes less than v_0, and this leads to the onset of an accumulation region K (Fig. 1b). The volume V_K of the region K can be calculated by integrating over the initial velocities v_{xi}, v_{yi}, v_{zi} (or $v_{\perp i}, \varphi_i, v_{zi}$) satisfying the condition $v_{\max}^2 \leqslant v_0^2$. We note that although the position of the region K depends on the entry phase ψ_i, the volume V_K is, naturally, independent of time, since the entry phase ψ_i enters in the expression for $v_{\max}^2$ only via $\sin(\psi_i + \varphi_i)$, and consequently variation of ψ_i produces simply rotation of K around the axis v_z. It follows from (4) that V_K can depend only on v_B and does not depend on ω and ω_B separately. This enables us to obtain an expression for V_K by making the substitution $v_c \to v_B$ in the analytic expression for V_K, which

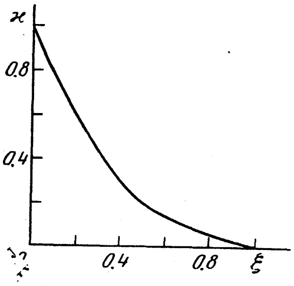

FIG. 2. Ratio of the volume of the accumulation region K and of the passive region $\varkappa = V_K/V_p$ vs the position of the center of the trajectories $\xi = v_B/v_0$; $v_B = v_c(1-\omega/\omega_B) = cE/B(1-\omega/\omega_B)$.

can be easily obtained for constant fields $\mathbf{E}\perp\mathbf{B}$.[13] Similar reasoning can be used by transforming into a rotating coordinate frame.[21]

The ratio of the volumes of the region K and the passive region is the same for momentum space and velocity space. It is therefore convenient to introduce the relative volume $\varkappa = V_K/V_p$ that depends only on the parameter

$$\xi=|v_B/v_0|=|cE_\omega/B(1-\omega/\omega_B)v_0|,$$

here $V_p = 4\pi v_0^3/3$ is the volume of the positive region. The dependence of $\varkappa$ on ξ is valid for arbitrary ω and ω_B (Fig. 2). For example: (a) if $\omega_B/\omega\to\infty$ at constant v_c, then $v_B\to v_c$, meaning that V_K in an alternating field coincides with the spindle-like volume calculated for a constant field $\mathbf{E}$ of amplitude E_ω; this is the consequence of the adiabatic motion at $\omega_B\gg\omega$; (b) if $\omega\to\omega_B$ at constant v_c, then $\xi\to\infty$ and consequently $V_K=0$, a reflection of the absence of a K region at cyclotron resonance. Putting $\xi=1$ and $v_c<v_0$ we can determine the range of frequencies (or of magnetic fields) where there is no region K in the vicinity of cyclotron resonance (cf. Refs. 21 and 29):

$$1-v_c/v_0\leqslant\omega/\omega_B\leqslant1+v_c/v_0;$$

(c) if $\omega_B=0$ (there is no magnetic field), the expression for $v^2(t)$ can be obtained from (3), and the corresponding value is

$$\xi=|v_B/v_0|=qE_\omega/\omega m v_0.$$

Thus, when the condition $qE_\omega/\omega m v_0<1$ is satisfied the region K exists in a circular-polarization field even at a zero magnetic field,[26,30–32] and the volume of this region can be obtained from Fig. 2.

The onset of the region K leads to an increase of the distribution function in this region because the carrier lifetime τ (the time between collisions) on the inner trajectories exceeds the lifetime on the outer ones. If the carrier source is broad enough ($\delta v/v_0\sim1$), knowing the value of V_K, we can estimate the relative density of the carriers located in the region K:

$$\frac{n_K}{n_s}=\frac{V_K}{V_p-V_K+\Delta V_a}\frac{\tau_K}{\tau_s}, \tag{5}$$

where n_K is the number of particles in the region K, n_s is the number of particles on the outer trajectories, and τ_K is the average lifetime in the region K ($\tau_K\simeq\nu_i^{-1}$); τ_s is the average lifetime on the outer trajectories and can be estimated to be equal to the transit time τ_E. The factor $V_K/(V_p-V_K+\Delta V_a)$ reflects the probability that the particles will land in the region K after scattering; ΔV_a is the additional volume due to the penetration of the particles into the active region, $\Delta V_a\simeq\pi(\delta v)^2\Delta v$. Assuming that the particles fill K approximately uniformly, we obtain directly an estimate of the ratio of the distribution functions in the region K and in the vicinity of this region:

$$f_K/f_s\approx\tau_K/\tau_s\approx\nu_E/\nu_i.$$

5. We now use the arguments advanced above to describe the behavior of carriers in degenerate heavy- and light-hole subbands.[8)] We are primarily interested in the redistribution of the carriers among the subbands in a strong high-frequency circularly polarized field. For a qualitative description we assume the frequencies of the collisions of the light and heavy holes in the passive regions to be the same and equal to ν_i; the frequencies ν_0 of collisions with spontaneous emission of optical phonons are accordingly also equal.[9)] We assume for the sake of argument that the hole source is broad[10)] (Ref. 15). Owing to the weak dependence of the source width on the electric field amplitude, $\delta v/v_0\sim(E_\omega/E_0)^{1/3}$, this assumption is valid in a large field interval $0.01E_0<E_\omega\lesssim E_0$. The hole source is produced in both subbands by carriers landing on the outer trajectories and penetrating into the active region. Scattering causes both intraband and interband transitions. It follows from the energy conservation law for scattering with optical-phonon emission that the relative source width is the same in the heavy- and light-hole bands and is determined by the subbands from which the scattering took place $\delta v_h/v_{0h}=\delta v_l/v_{0l}$, where $v_{0h,l}=(2\hbar\omega_0/m_{h,l})^{1/2}$ are the boundaries of the passive region in velocity space; $\delta v_{h,l}$ is the size of the source in the heavy-hole (h) and light-hole (l) band, respectively. We denote the accumulation regions for the heavy- and light-hole subbands by K_h and K_l, respectively. Since the conditions for the onset of K_h and K_l are different, we introduce for the heavy holes the parameter $\xi_h=|cE_\omega/B(1-\omega/\omega_{Bh})\omega_{0h}|$ and for the light ones $\xi_l=|cE_\omega/B(1-\omega/\omega_{Bl})v_{0l}|$; here $\omega_{Bh,l}=qB/m_{h,l}c$ are the cyclotron frequencies of the heavy and light holes, respectively.

The relative density n_l/n_h of the light and heavy holes in strong fields can be estimated from the simple equation[34]

$$\frac{n_l}{n_h}=\left(\frac{m_l}{m_h}\right)^{3/2}\frac{V_l/V_{pl}}{V_h/V_{ph}}\frac{\tau_l}{\tau_h}. \tag{6}$$

Here V_l is the volume occupied in velocity space by the principal part of the light holes; τ_l is the average lifetime—the free motion time; $V_{pl}=4\pi v_{0l}^3/3$ is the volume of the passive region for the light holes; V_h, V_{ph}, τ_h are the corresponding values for the principal part of the heavy holes; the factor $(m_l/m_h)^{3/2}$ is a reflection of the difference between the source amplitudes in the heavy and light hole bands as a result of the difference in the state densities. The factor V_l/V_{pl} is proportional to the probability of a light hole landing, after scattering, in the velocity region occupied by the principal part of the heavy holes, V_h/V_{ph} has the same meaning for the heavy holes, and finally the ratio τ_l/τ_h describes the difference between the lifetimes of the holes on the free-motion trajectories.

Let us consider the most typical cases:

(a) Cyclotron resonance of heavy holes ($\omega=\omega_{Bh},\omega_{Bl}\gg\omega$). For heavy holes $\xi_h\to\infty$, therefore there is no region K_h and the heavy holes are grouped in a rather broad vicinity of the principal trajectory ($V_h/V_{ph}\sim0.5$–1), where they move under streaming conditions with a characteristic frequency

$$\nu_{Eh}=qE_\omega/m_h v_{0h}(\tau_h\approx\nu_{Eh}^{-1}).$$

For the light holes the parameter is

$$\xi_l=|cE_\omega/B(1-m_l/m_h)v_{0l}|$$

and at field amplitudes E_ω satisfying the condition $\xi_l\leqslant1$ an accumulation region K_l is produced in the light-hole subband, with a relative volume $\varkappa(\xi_l)$ (Fig. 2). Since τ_l is large in the region K_l ($\tau_l=\nu_i^{-1}$), the bulk of the light holes is concentrated (see Fig. 5) at not too small $\varkappa(\xi_l)$ in K_l. Thus, n_l/n_h in the case of pumping at cyclotron resonance of heavy holes is

$$n_l/n_h=(m_l/m_h)^{3/2}(\nu_{Eh}/\nu_i)\varkappa(\xi_l).$$

With increasing E_ω, an increase of n_l/n_h is first observed, owing to the increase of ν_{Eh}, after which n_l/n_h decreases because of the decrease of $\varkappa(\xi_l)$. The maximum value of n_l/n_h can be larger here than at $\omega=0$, where the region K_h exists only at $v_c/v_{0l}>(m_l/m_h)^{1/2}$. For example, in p-Ge at $\omega=\omega_{Bh}=\nu_{Eh}=\nu_0(B=19\text{ kG})$ the parameter $\xi_l=0.346$, $\varkappa(\xi_l)=0.3$ and the ratio $n_l/n_h\approx1.2$, almost exactly as in the case $\omega=0$. At high pump frequencies $\omega=\omega_{Bh}=2\nu_0;\nu_{Eh}=\nu_0(B=38\text{ kG})$, however, the parameter $\xi_l=0.173$ $\varkappa(\xi_l)=0.59$ and $n_l/n_h=2.5$, i.e., much higher than in constant fields $\mathbf{E}\perp\mathbf{B}$ at the same ratio ν_i/ν_0 (Ref. 20); for pure p-Ge at $T=4$ K we have $\nu_i/\nu_0=0.01$.

(b) Cyclotron resonance of light holes ($\omega=\omega_{Bl}$, $\omega_{Bh}<\omega$). There is no region K_l ($V_l/V_{pl}\sim1$), and the light holes move under streaming conditions with a flight frequency

$$\nu_{El}=qE_\omega/m_l v_{0l}(\tau_l=\nu_{El}^{-1}).$$

For the heavy holes, the parameter

$$\xi_h=|cE_\omega/B(1-m_h/m_l)v_{0h}|,$$

and at $\xi_h\lesssim1$ there appears in the heavy-hole subband a region K_h with relative volume $\varkappa(\xi_h)$. The bulk of the heavy holes is concentrated in the region K_h, where their lifetime is $\tau_h=\nu_i^{-1}$. The ratio n_l/n_h when pumped at a frequency $\omega=\omega_{Bl}$ is equal to

$$n_l/n_h=(m_l/m_h)^{3/2}(\nu_i/\nu_{El})\varkappa^{-1}(\xi_h),$$

i.e., at $\nu_i\ll\nu_{El}$ the density of the light holes is anomalously low. In pure p-Ge, n_l/n_h decreases at $\nu_{El}\sim\nu_0$ by several dozen times compared with $(m_l/m_h)^{3/2}$. When K_h vanishes with increasing E_ω and both types of hole move under streaming conditions, we have $n_l/n_h\sim(m_l/m_h)^2$.

(c) Magnetic field $B=0$ $\xi_h=\nu_{Eh}/\omega$, $\xi_l=\nu_{El}/\omega$; $\xi_h/\xi_l=(m_l/m_h)^{1/2}$. With increasing E_ω, three different cases are possible:

1. If $\xi_h<\xi_l<1$, the regions K_h and K_l exist, and then $\tau_h=\nu_i^{-1}$; $\tau_l=\nu_i^{-1}$; $\varkappa(\xi_h)>\varkappa(\xi_l)$ and the density of the light holes is less than the equilibrium value

$$n_l/n_h=(m_l/m_h)^{3/2}(\varkappa(\xi_l)/\varkappa(\xi_h)).$$

2. If $\xi_h<1<\xi_l$, only K_h exists, therefore $\tau_h=\nu_i^{-1}$; $\tau_l=\nu_{El}^{-1}$; $V_h/V_{ph}=\varkappa(\xi_h)$, $V_l/V_{pl}\sim1$. At these amplitudes E_ω, a dip is observed in the relative density of the light holes:

$$n_l/n_h=(m_l/m_h)^{3/2}(\nu_i/\nu_{El})\varkappa^{-1}(\xi_h).$$

The value of n_l/n_h turns out to be here much less than $(m_l/m_h)^2$ reached under streaming conditions.[2–4]

3. If $1<\xi_h<\xi_l$, there is neither K_h nor K_l, therefore $\tau_h\approx\nu_{Eh}^{-1}$; $\tau_l\approx\nu_{El}^{-1}$; $V_h/V_{ph}\approx V_l/V_{pl}$. Since $\nu_{Eh}/\nu_{El}=(m_l/m_h)^{1/2}$, it follows that $n_l/n_h\approx(m_l/m_h)^2$, i.e., just as in streaming in a constant field $\mathbf{E}$ (Refs. 2–4).

Although the foregoing analysis is only approximate, the estimates obtained show that the effects considered are quite coarse and should manifest themselves in semiconductors with degenerate subbands and in multivalley semiconductors.

6. A detailed calculation of the populations of the light- and heavy-hole subbands in p-Ge was carried out by a Monte Carlo simulation of carrier motion under condition of heavy-hole cyclotron resonance. Scattering by optical and acoustic phonons, with intraband and interband transitions, was taken into account. Impurity scattering was neglected, so that the results pertain to p-Ge with low impurity density.

The following expressions[35–37] were used for the probabilities of scattering with spontaneous phonon emission:

$$w_{\alpha\alpha'}(\mathbf{p},\mathbf{p}')\,d^3\mathbf{p}'=\frac{(D_tK)^2}{2(2\pi\hbar)^2\rho\hbar\omega_0}\,\delta\left(\frac{\mathbf{p}'^2}{2m_{\alpha'}}+\hbar\omega_0-\frac{\mathbf{p}^2}{2m_\alpha}\right)d^3\mathbf{p}',\tag{7}$$

where $(D_tK)^2=9\cdot10^8$ eV/cm is the deformation-potential parameter; $\rho=5.32$ g/cm^3 is the density, $\Theta_D=430$ K is the Debye temperature, and $\hbar\omega_0=k\Theta_D$. The subscripts α and α' stand respectively for h and l for the heavy- and light-hole subbands, the quasimomenta $\mathbf{p}$ and $\mathbf{p}'$ and the subscript α and α' describe the states of the hole before and after scattering and

$$\nu_{0\alpha'}=(D_tK)^2m_{\alpha'}^{3/2}/2\hbar^2\pi\rho(2\hbar\omega_0)^{1/2}.$$

The corresponding field is $E_0=m_hv_{0h}\nu_{0h}/q=3.5$ kV/cm.

The acoustic scattering was described in the zero-oscillation approximation (cf. Refs. 4 and 38). This approximation is valid at lattice temperatures $T<20$ K, and in this case the scattering probabilities are[35–37]

$$w_{\alpha\alpha'}(\mathbf{p},\mathbf{p}')\,d^3\mathbf{p}'=\frac{E_1^2|\mathbf{p}-\mathbf{p}'|}{(2\pi\hbar)^2\rho s}G(\theta)\,\delta\left(\frac{\mathbf{p}'^2}{2m_{\alpha'}}-\frac{\mathbf{p}^2}{2m_\alpha}\right)d^3\mathbf{p}';\tag{8}$$

$E_1=4.6$ eV is the deformation-potential constant, $s=4\times10^5$ cm/sec is the speed of sound, $G(\theta)=(1+3\cos^2\theta)/4$ and $G(\theta)=3(1-\cos^2\theta)/4$ for intra-

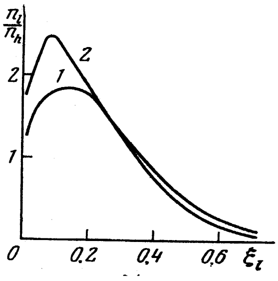

FIG. 3. Relative density of light and heavy holes vs $\xi_l = v_B/v_{0l}$: 1 — $\omega_{Bh} = \nu_{0h}$; 2 — $\omega_{Bh} = 2\nu_{0h}$; $\nu_{0h} = 9.24\cdot10^{11}$ sec^{-1}.

band and for interband transitions, and θ is the angle between **p** and **p**′.

Figure 3 shows the calculated n_l/n_h as a function of ξ_l, while Fig. 4 shows the corresponding dependence of the average light-hole collision frequency in the region K_l for $\omega = \nu_{oh}$ and $\omega = 2\nu_{oh}$; it can be seen that $n_l/n_h > 1$ in a wide range of electric fields, this being due to the absence of K_h. The maximum values of n_l/n_h are then ~ 40 times larger than the equilibrium value $(m_l/m_h)^{3/2}$ in the first case and ~ 60 times larger in the second. In weak fields, when $\nu_{Eh} \sim \nu_i$, the ratio n_l/n_h is close to the equilibrium value, but increases with increasing field because of the decrease of τ_h, which is equal to τ_{Eh} under streaming conditions. Finally, at large field amplitudes E the ratio n_l/n_h decreases mainly on account of the decrease of $\varkappa(\xi_l)$ (see Fig. 2), since the width of the source and the collision frequency increase negligibly in the K_l region in this case (see Fig. 4).

The pump frequency at $\omega \sim \nu_{0h}$ is in the millimeter band, for which sufficiently powerful radiation sources are available[39]; the required magnetic field ($B = 19$ kG) is also readily obtainable; the calculations presented are therefore of considerable experimental interest, since they make possible overpopulation of a radiative transition between the light- and heavy-hole subbands by a contactless method. Although these characters are by way of a model and allowance for impurity scattering lowers n_l/n_h, the estimates show that the overpopulation of the light-hole subband remains appreciable up to impurity densities $N_I \leqslant 10^{15}$ cm^{-3}. We note that the qualitative agreement between the overpopulations calculated by the Monte Carlo method and those obtained from the estimate (6) shows that the premises developed in Refs. 14 and 15 concerning the structure of the distribution function in inelastic scattering by optical phonons describe simply and accurately enough the electron distributions not only in constant but also in strong microwave fields.

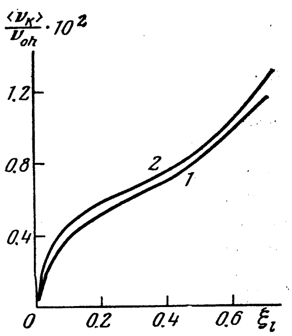

FIG. 4. Average frequency $\langle \nu_K \rangle$ of light-hole collisions in the region K as a function of $\xi_l = v_B/v_{0l}$: 1 — $\omega_{Bh} = \nu_{0h}$; 2 — $\omega_{Bh} = 2\nu_{0h}$; $\nu_{0h} = 9.24\cdot10^{11}$ sec^{-1}.

7. The redistribution of the carriers among the light- and heavy-hole subbands leads to a number of interesting effects. Owing to the population inversion of the radiative transition between the light- and heavy-hole subbands[6] it is possible to obtain amplification and generation in the far infrared. Distinctive features will also be observed in this band in the spontaneous emission.[10–12] The absorption coefficient is also changed at the frequencies of the transition between the detached subband and the light- and heavy-hole subbands.[9] The carrier redistribution among the light- and heavy-hole subbands should manifest itself in the amplitude and shape of the cyclotron-resonance line[40]: peculiarities appear also in the conductivity and in the static galvanomagnetic characteristics.[8,34,41,42] In contrast to the method of overpopulating the heavy- and light-band subbands in constant fields **E**⊥**B** by pumping at the cyclotron frequency of the heavy holes, it is possible to use the large volume of the region K to accumulate light holes and increase the overpopulation.

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[1)]A particular case of internal trajectories are closed trajectories produced in constant fields **E**⊥**B** (Ref. 13). The trajectory passing through the point **p** = 0 corresponds to the principal trajectory which is generally speaking different for different phases of an alternating field.

[2)]The region $K(t)$ is similar to the spindle-shaped region of closed trajectories 14 in constant fields **E**⊥**B** (cf. Ref. 13).

[3)]Another important feature of an alternating field, besides the source width, is the phase shift of the field relative to the field.

[4)]For example, if the principal trajectory becomes internal for certain phases of the field, bunching of the carriers will take place in these phases.

[5)]To simplify the description, we confine ourselves to a model with an isotropic and parabolic band, although at a threshold character of the scattering the qualitative picture of the effects considered will manifest itself in more complicated band structure, namely nonparabolic,[16] nonspherical,[17] or multivalley.[18]

[6)]In a coordinate frame (v_x', v_y', v_z) rotating at an angular velocity ω, the principal trajectories lie on a straight-line segment from $v_x' = 0$ to $v_x' = v_0$ (Ref. 21).

[7)]Expression (4) is valid at negative v_B, but in this case the sign of the radical must be reversed.

[8)]We are considering here a case when the energy Δ of the subband detached on account of spin-orbit interaction is much larger than $\hbar\omega_0$: the light- and heavy-hole subbands are assumed to be isotropic, parabolic and to have different effective masses (in p-Ge, e.g., $m_h = 0.35m_0$, $m_l = 0.043m_0$).

[9)]The assumption that the collision frequencies are equal (although it is strictly speaking incorrect) is frequency used in the literature (see e.g., Ref. 33). The justification is that in scattering from both the heavy- and light-hole subbands the transitions take place mainly into the heavy-hole subband, where the state density is high. In the next section, where the carrier motion is simulated by the Monte Carlo method, rigorous transition probabilites are used and this assumption is not made.

[10)]Effects similar to those considered occur also with a narrow source when $(\nu_i/\nu_0)^{1/3} \ll 1$ and $E_\omega \sim (\nu_i/\nu_0)E_0$, but the numerical values depend on the overlap integral of the source and the accumulation region. In the case of a broad source the overlap integral ~ 1.

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