Contribution to the theory of flicker noise

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An analysis is made of thermodynamic (equilibrium) fluctuations of the resistance due to fluctuations of the average (over the volume of a cylindrical sample) temperature because of heat exchange with an external circuit. The simplest models are used to investigate the problem of whether the spectral intensity of such fluctuations can be proportional to $1/\omega$, i.e., whether these fluctuations can be the cause of the flicker noise (known also as the excess or residual noise) and also of the $1/\omega$ or $1/f$ noise.

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INTRODUCTION

In the last 10–15 years the flicker noise in conductors and many other materials has been subjected to intensive experimental investigations, the results of which are summarized in many reviews (see, for example, Refs. 1–3). However, the state of the theory of this noise leaves much to be desired; in the first investigations a spectral intensity of the flicker noise proportional to $1/\omega$ has been obtained by superimposition of relaxation spectra proportional to $\nu/(\omega^2 + \nu^2)$ subject to fairly artificial assumptions about the distribution of the damping coefficient ν. In the recent work of Klimontovich[4] the approach was essentially the same and no physical justification was given for the assumed distribution.

Since many authors attribute the flicker noise in conductors to equilibrium fluctuations of the temperature or the number of carriers, we shall use very simple models to consider the possibility of whether such fluctuations give rise to a spectrum of the $1/\omega$ type.

Fluctuations of the temperature ΔT obey the well-known relationship[5]

$$\overline{(\Delta T)^2} = kT^2/C, \tag{1}$$

where k is the Boltzmann constant, T is the equilibrium temperature, and C is the specific heat of a sample. Bearing in mind the temperature dependence of the resistance, we can rewrite Eq. (1) in the form

$$\overline{(\Delta R)^2}/R^2 = k\gamma^2/C, \qquad \gamma = d\ln R/d\ln T, \tag{2}$$

where γ/T is the temperature coefficient of the resistance.

Introducing the spectral intensity $S_R(\omega)$ related to $\overline{(\Delta R)^2}$ by

$$\overline{(\Delta R)^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_R(\omega)\,d\omega = \frac{1}{\pi}\int_0^{\infty} S_R(\omega)\,d\omega, \tag{3}$$

we find that as a result of slow heat exchange between a sample and the external circuit the spectral intensity $S_R(\omega)$ rises in the limit $\omega \to 0$ in accordance with a definite law. Our aim will be to derive this law.

In some systems (liquid electrolytes, cell membranes, etc.) the fluctuations of the resistance may be due to fluctuations of the number of particles (carriers), i.e., they may be due to diffusion rather than heat conduction; moreover, combined fluctuations are also possible. We shall consider the specific case of just the temperature fluctuations but in view of the analogy between diffusion and heat conduction, all the relationships obtained are easily applied to fluctuations of the particle (carrier) density.

1. MAIN EQUATIONS

The fluctuation equations for heat conduction are[6]

$$\mathbf{j} = -\sigma \operatorname{grad} u + \mathbf{f}, \qquad c\frac{\partial u}{\partial t} + \operatorname{div}\mathbf{j} = 0, \tag{4}$$

where $\mathbf{j}$ is the heat flux density, σ is the thermal conductivity, u are fluctuations of the temperature [in this case the temperature is $T + u(\mathbf{r},t)$], c is the specific heat per unit volume, $\mathbf{f}$ is the density of a random heat flux which creates temperature fluctuations and which is uncorrelated along directions, in space, or in time; $\mathbf{f}$ obeys the relationship

$$\overline{f_\alpha(\mathbf{r},t) f_\beta(\mathbf{r}',t')} = \Theta(\mathbf{r})\,\delta_{\alpha\beta}\,\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t'), \tag{5}$$

where the bar denotes, as in Eqs. (1)–(3), the procedure of statistical averaging, and the indices α and β correspond to the Cartesian components along the x, y, and z axes.

The random field $\mathbf{f}$ is introduced like the random force in the Langevin equation for the Brownian motion. The function $\Theta(\mathbf{r})$ is introduced from a consideration of an unbounded homogeneous medium in which the function u satisfies the equation

$$c\frac{\partial u}{\partial t} - \sigma\Delta u = -\operatorname{div}\mathbf{f}; \tag{6}$$

its spectral intensity, governed by the four-dimensional Fourier integral

$$S_u(\mathbf{k},\omega) = \iint \overline{u(\mathbf{r},t)\,u(\mathbf{r}-\mathbf{s},t-\tau)}\, e^{-i(\mathbf{ks}-\omega\tau)}\,(d\mathbf{s})\,d\tau, \tag{7}$$

is obtained in the form

$$S_u(\mathbf{k},\omega) = \frac{\Theta\mathbf{k}^2}{c^2\omega^2 + \sigma^2\mathbf{k}^4}, \tag{8}$$

from where we find that

$$\overline{u(\mathbf{r},t)\,u(\mathbf{r}-\mathbf{s},t)} = \frac{\Theta}{(2\pi)^4}\iint \frac{\mathbf{k}^2}{c^2\omega^2+\sigma^2\mathbf{k}^4}\, e^{i\mathbf{ks}}\,(d\mathbf{k})\,d\omega = \frac{\Theta}{16\pi^3 c\sigma}\int e^{i\mathbf{ks}}\,(d\mathbf{k}) = \frac{\Theta}{2c\sigma}\,\delta(\mathbf{s}). \tag{9}$$

Since the quantity ΔT occurring in Eq. (1) is

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$$\Delta T=\frac{1}{V}\int u(\mathbf{r},t)\,(d\mathbf{r}), \tag{10}$$

where the integration is carried out over a volume V identified in the medium, we obtain

$$\overline{(\Delta T)^2}=\frac{\Theta}{2c\sigma V^2}\iint\delta(\mathbf{r}-\mathbf{r}')\,(d\mathbf{r})\,(d\mathbf{r}')=\frac{\Theta}{2C\sigma}, \tag{11}$$

and arrive at Eq. (1) if we assume that

$$\Theta=2kT^2\sigma, \tag{12}$$

where in the case of an inhomogeneous medium we should take local values of $T(\mathbf{r})$ and $\sigma(\mathbf{r})$.

The formula (8) represents the special case of the following relationship known from the spectral theory of random processes. Let us assume that a random function $v(t)$ is defined by the equation

$$v(t)=\sum_\lambda L_\lambda[f_\lambda(t)], \tag{13}$$

where $f_\lambda(t)$ are uncorrelated random functions satisfying the condition

$$\overline{f_\lambda(t)f_\mu(t')}=\Theta_\lambda\delta_{\lambda\mu}\delta(t-t'), \tag{14}$$

and L_λ are linear stationary (homogeneous in time) operators. The spectral intensity

$$S_v(\omega)=\int_{-\infty}^{\infty}\overline{v(t)v(t-\tau)}\,e^{i\omega\tau}\,d\tau \tag{15}$$

is calculated from the formulas

$$S_v(\omega)=\sum_\lambda|M_\lambda(\omega)|^2\Theta_\lambda,\qquad M_\lambda(\omega)=L_\lambda[e^{-i\omega t}]e^{i\omega t}, \tag{16}$$

whereas the functions $M_\lambda(\omega)$ representing the frequency characteristics corresponding to the operators L_λ are found by considering *harmonic oscillations* when $f_\lambda(t)$ and $v(t)$ are taken in the form

$$f_\lambda(t)=\tilde{f}_\lambda e^{-i\omega t},\qquad v(t)=\tilde{v}e^{-i\omega t} \tag{17}$$

and Eq. (13) becomes

$$\tilde{v}=\sum_\lambda M_\lambda(\omega)\tilde{f}_\lambda. \tag{18}$$

Therefore, instead of the system (4), we have to solve the equations

$$\tilde{\mathbf{j}}=-\sigma\,\mathrm{grad}\,\tilde{u}+\tilde{\mathbf{f}},\qquad ic\omega\tilde{u}=\mathrm{div}\,\tilde{\mathbf{j}}, \tag{19}$$

where instead of time we have the frequency ω and which contain the spectral intensity of interest to us. If necessary, inversion of the integral (15) can be used to calculate also the correlation function.

2. ONE-DIMENSIONAL PROBLEM

We shall consider an electric and a thermal circuit shown in Fig. 1 and consisting of a noisy conductor of length l and a homogeneous conductor (external circuit) of length l_e (see Sec. 4). However, we shall initially investigate a simpler one-dimensional system in which the transverse cross sections of the conductor and external circuit are the same. We shall be interested in fluctuations of the resistance $R=\rho l/S$,

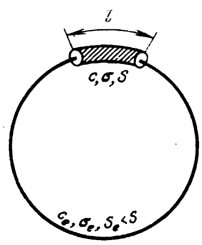

FIG. 1. Closed electric and thermal circuits.

where ρ is the resistivity and S (without the argument ω) is the cross-sectional area. We shall introduce

$$v(t)=\frac{\Delta R}{R}=\frac{\gamma}{TV}\int u\,dV \tag{20}$$

and we shall assume that the side surface of our system is impermeable to heat, i.e., that the condition $j_n=0$ is satisfied on this surface. Then, "rectifying" the system as shown in Fig. 2a, introducing a longitudinal coordinate z, and assuming initially that $l_e=\infty$, we find the spectral intensity $S_v(\omega)$ by a method represented by Eqs. (13)–(19). Averaging over the transverse cross section $z=$ const, we obtain functions

$$j=\frac{1}{S}\int\tilde{j}_z\,dS,\qquad u=\frac{1}{S}\int\tilde{u}\,dS,\qquad f=\frac{1}{S}\int\tilde{f}\,dS, \tag{21}$$

which depend only on z and ω (we shall not stress explicitly the dependence on ω). The functions of Eq. (21) satisfy the equations

$$j=-\sigma\frac{du}{dz}+f,\qquad ic\omega u=\frac{dj}{dz}, \tag{22}$$

and it follows from Eqs. (20) and (22) that

$$\tilde{v}=\frac{\gamma}{ic\omega T}\cdot\frac{j(l/2)-j(-l/2)}{l}=\frac{\gamma}{\sigma T}\cdot\frac{j(l/2)-j(-l/2)}{K^2l}. \tag{23}$$

In the case of the function $j(z)$ with $|z|<l/2$ we obtain

$$d^2j/dz^2+K^2j=K^2f,\qquad K=(ic\omega/\sigma)^{1/2}=(c\omega/2\sigma)^{1/2}(1+i), \tag{24}$$

where K is a complex wave number governing the propagation of temperature waves of frequency ω. If $|z|>l/2$, Eq. (24) should be modified by replacing K with $K_e=(ic_e\omega_e/\sigma_e)^{1/2}$. If $z=\pm l/2$, then at the contact between conductors with different values of c and ω the quantities u (i.e., $c^{-1}dj/dz$) and j should be continuous.

Elementary operations give

$$\tilde{v}=\frac{\gamma}{Tl}\left[\frac{1}{\sigma}\int_{|z|<l/2}\Gamma(z)f(z)\,dz+\frac{1}{\sigma_e}\int_{|z|>l/2}\Gamma(z)f(z)\,dz\right], \tag{25}$$

where

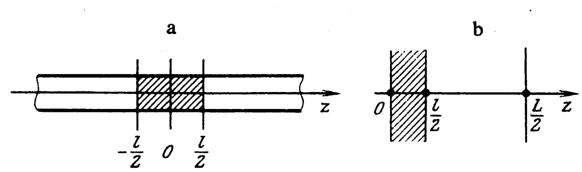

FIG. 2. Plane-parallel systems: a) cylindrical sample in an infinite circuit; b) film on a substrate.

$$\Gamma(z)=-\frac{\sin Kz}{K\cos(Kl/2)+H\sin(Kl/2)}\approx -z \quad \text{for} \quad |z|<\frac{l}{2},$$

$$\Gamma(z)=-\Gamma(-z)=-\frac{\sin(Kl/2)\exp\{iK_e(z-l/2)\}}{K\cos(Kl/2)+H\sin(Kl/2)}$$
$$\approx -\frac{l}{2}\exp\left\{iK_e\left(z-\frac{l}{2}\right)\right\} \quad \text{for} \quad z>\frac{l}{2} \tag{26}$$

and

$$H=-i\frac{c}{c_e}K_e=\frac{c}{c_e}\left(-i\frac{c_e\omega}{\sigma_e}\right)^{1/2}=\frac{c}{c_e}\left(\frac{c_e\omega}{2\sigma_e}\right)^{1/2}(1-i), \tag{27}$$

of which the approximate expressions correspond to very low frequencies satisfying the conditions $Kl\ll 1$ and $Hl\ll 1$.

Introducing the notation

$$f_\perp(z,t)=\frac{1}{S}\int f_z(\mathbf{r},t)\,dS, \tag{28}$$

where the integration is carried out over a transverse cross section $z=\text{const}$, we can use Eq. (5) to obtain in the one-dimensional problem

$$\overline{f_\perp(z,t)f_\perp(z',t')}=\frac{\Theta(z)}{S}\delta(z-z')\delta(t-t') \tag{29}$$

and then Eq. (25) gives, on the basis of Eq. (16), the required spectral intensity

$$S_v(\omega)=\frac{S_R(\omega)}{R^2}$$
$$=\frac{4k\gamma^2}{Sl^2}\left[\frac{1}{\sigma}\int_0^{l/2}|\Gamma(z)|^2dz+\frac{1}{\sigma_e}\int_{l/2}^{\infty}|\Gamma(z)|^2dz\right], \tag{30}$$

where we have assumed that the sample and the external circuit are at the same temperature T.

Substitution of the approximate expressions from Eq. (26) into the integrals of Eq. (30) gives

$$S_v(\omega)=\frac{k\gamma^2}{S}\left(\frac{l}{6\sigma}+\frac{1}{(2c_e\sigma_e\omega)^{1/2}}\right), \tag{31}$$

where the first term in the parentheses represents the contribution of random fluxes in the sample itself and the second the corresponding fluxes in the external circuit; if the external circuit is finite, it then follows from Eq. (32) that in the limit $\omega\to 0$ the second term is also finite. We can see that the law $1/\omega$ is not obtained under the conditions $Kl\ll 1$ and $Hl\ll 1$.

It is appropriate to make here the following critical comment. Van Vliet, van der Ziel, and Schmidt[6] used the same initial assumptions and considered a one-dimensional system shown in Fig. 2b and corresponding to a film on a substrate. If at $z=0$ in Fig. 2b we substitute the condition $j=0$ and assume that $L=\infty$, such a system is equivalent to that in Fig. 2a, which is investigated above. However, in the calculation of fluctuations due to bulk sources (i.e., random fluxes to which our treatment is limited) Van Vliet, van der Ziel, and Schmidt[6] proceeded incorrectly, namely at the boundary between two media they assumed the condition of continuity of u and $\sigma\partial u/\partial z$, and not of u and j_z, as they should have done; exactly the same way at the boundary impermeable to heat the condition $j_n=0$ and not $\partial u/\partial n=0$ should be satisfied. It is interesting to note that after correction of this error the final formulas for the spectrum $S_v(\omega)$ are obtained much more simply; in the case of an external circuit of finite width the expressions of Eq. (26) become

$$\Gamma(z)=-\frac{\sin Kz}{K\cos(Kl/2)+H\sin(Kl/2)}$$

$$\text{for} \quad |z|<\frac{l}{2}, \quad H=\frac{c}{c_e}K_e\operatorname{ctg}\frac{K_el_e}{2}$$

$$\Gamma(z)=-\Gamma(-z)$$
$$=-\frac{\sin(Kl/2)}{K\cos(Kl/2)+H\sin(Kl/2)}\frac{\sin K_e(L/2-z)}{\sin(K_el_e/2)}$$
$$\text{for} \quad \frac{l}{2}<z<\frac{L}{2}, \quad L=l+l_e, \tag{32}$$

and in the second integral of Eq. (30) the upper limit should be taken as $L/2$.

3. FLICKER NOISE IN A ONE-DIMENSIONAL SYSTEM

The spectral intensity $S_R(\omega)$ obtained in the preceding section gives, as expected, an integral (3) which converges at $\omega=0$. For this reason the law $1/\omega$ cannot be obtained at very low frequencies.

We shall now assume that the system satisfies the conditions

$$Kl\ll 1, \quad Hl\gg 1, \tag{33}$$

which is possible if the following parameter is small:

$$\xi=|K/H|^2=c_e\sigma_e/c\sigma\ll 1. \tag{34}$$

This means that heat exchange with the external circuit is difficult because of the low thermal conductivity of this circuit. Under the conditions of Eq. (33) the function $\Gamma(z)$ can be replaced by approximate expressions

$$\Gamma(z)=-\frac{2z}{Hl} \quad \text{for} \quad |z|<\frac{l}{2},$$
$$\Gamma(z)=-\frac{1}{H}\exp\left\{iK_e\left(z-\frac{l}{2}\right)\right\} \quad \text{for} \quad z>\frac{l}{2}. \tag{35}$$

Neglecting the second integral in Eq. (30), which determines the contribution of an infinite external circuit, we obtain the expression

$$S_R(\omega)/R^2=2k\gamma^2\xi/3C\omega, \tag{36}$$

describing a noise with the $1/\omega$ spectrum in the one-dimensional system under consideration.

If we assume that the expression (36) is valid in the frequency range $\omega_1<\omega<\omega_2$, we find that the total intensity of the flicker noise is

$$\frac{1}{\pi}\int_{\omega_1}^{\omega_2}\frac{S_R(\omega)}{R^2}d\omega=\frac{k\gamma^2}{C}\frac{2\xi}{3\pi}\ln\frac{\omega_2}{\omega_1} \tag{37}$$

and a comparison with Eqs. (2) and (3) leads to the inequality

$$\frac{2\xi}{3\pi}\ln\frac{\omega_2}{\omega_1}<1, \tag{38}$$

which limits the possible values of the ratio ω_2/ω_1. If ω_2 is defined by the condition $|K|l=1$ and ω_1 by the condition $|H|l=1$, we obtain

$$\omega_2=\sigma/cl^2, \quad \omega_1=\xi\omega_2 \tag{39}$$

and the inequality (38) becomes strong, i.e., the noise with the $1/\omega$ spectrum represents only a small proportion of the total fluctuation intensity.

The expression (36) is obtained by dropping the second integral (30) which has a different frequency dependence and is much greater than the first. In considering this problem, we shall introduce a dimensionless frequency ν and a dimensionless spectral intensity $s(\nu)$ satisfying the condition

$$\int_0^\infty s(\nu)\,d\nu=1 \tag{40}$$

in accordance with the formulas

$$\nu=\omega/4\omega_1, \quad s(\nu)=4\omega_1 CS_R(\omega)/\pi k\gamma^2R^2. \tag{41}$$

Representing $s(\nu)$ in the form

$$s(\nu)=s_i(\nu)+s_e(\nu), \tag{42}$$

where $s_i(\nu)$ is governed by the sources in the sample and $s_e(\nu)$ by those in the external circuit, we can describe $\Gamma(z)$ by the exact expressions in Eq. (26), assume that $H=-iK/\xi^{1/2}$, and calculate the integrals of Eq. (30) without any approximations. In this way we obtain

$$s_i(\nu)=\frac{4\xi^2}{\pi}\frac{\operatorname{sh}\mu-\sin\mu}{\mu^3N}, \quad s_e(\nu)=\frac{4\xi^{1/2}}{\pi}\frac{\operatorname{ch}\mu-\cos\mu}{\mu^3N}, \tag{43}$$

where

$$\mu=(\omega/2\omega_2)^{1/2}=(2\xi\nu)^{1/2},$$
$$N=(1+\xi)\operatorname{ch}\mu-(1-\xi)\cos\mu+2\xi^{1/2}\operatorname{sh}\mu, \tag{44}$$

and, in particular, if $\xi=1$, then

$$\int_0^\infty s_i(\nu)\,d\nu=\frac{2}{\pi}\int_0^\infty\frac{\operatorname{sh}\mu-\sin\mu}{\mu^2}e^{-\mu}\,d\mu=\frac{1}{2}-\frac{1}{\pi}\ln 2=0.28, \tag{45}$$

i.e., only 28% of the mean square of the fluctuations is due to random sources located in the selected part of the homogeneous system. In this case the function $s_i(\nu)$ cannot be proportional to $1/\nu$.

If $\mu\ll1$, the expressions in Eq. (43) become

$$s_i(\nu)=\frac{2\xi}{3\pi}\frac{1}{1+(2\nu)^{1/2}+\nu}, \quad s_e(\nu)=\frac{1}{\pi}\left(\frac{2}{\nu}\right)^{1/2}\frac{1}{1+(2\nu)^{1/2}+\nu}. \tag{46}$$

Hence, it follows that in the case of small values of ξ and μ we always have $s_i(\nu)\ll s_e(\nu)$ so that a simple one-dimensional system is not described by the law $1/\omega$ corresponding to Eq. (36) or to the dimensionless formulas

$$s_i(\nu)=2\xi/3\pi\nu \quad (\nu\gg1). \tag{47}$$

However, it is clear that the usual idea that the spectral intensity of any random process should have a zero derivative of $\omega=0$ (i.e., a horizontal asymptote if the logarithm of the frequency is plotted along the abscissa) does not apply to the present case.

We shall now consider the physical meaning of the expresions (46) and (47) for $s_i(\nu)$, and also of the condition $\xi\ll1$.

We can easily show that the reflection of a propagating temperature wave by the boundaries of a sample at $z=\pm l/2$ is represented by the coefficient

$$\mathcal{R}=-\frac{1-\xi^{1/2}}{1+\xi^{1/2}}\approx-1 \quad \text{if} \quad \xi\ll1. \tag{48}$$

This coefficient applies to j so that the heat flux reaching a boundary is almost completely turned back and this is why temperature perturbations accumulate giving rise to a characteristic resonance at zero frequency with a resonance denominator $1+(2\nu)^{1/2}+\nu$. A monotonic rise of $s_i(\nu)$ on reduction in ν is explained by the fact that damping of temperature waves decreases on reduction in the frequency. It is important to note that the reflection of temperature waves is not total, because in the case of total reflection (i.e., in the case when $\xi=0$ the value of ΔT does not vary with time and there are no fluctuations. The parameter ξ represents heat exchange between a sample and the external circuit, and $1/\omega_2$ is of the order of the time needed for equalization of the temperature in an isolated sample.

The additional factor $(2/\nu)^{1/2}$ in the formula for $s_e(\nu)$ is due to the fact that the damping coefficient $\operatorname{Im}K_e$ of the temperature waves in the external circuit is proportional to $\omega^{1/2}$ so that fluctuations in the sample at a frequency ω are created by those sources in the external circuit which are located at a distance from the sample of the order of $1/\operatorname{Im}K_e$ or closer. If the expressions in Eq. (32) are used to allow for the finite dimensions of the external circuit, the behavior of the spectral intensity in the limit $\omega\to0$ agrees with the usual ideas (see above). However, it is not quite clear which model—with an infinite or a finite external circuit—corresponds closer to reality, because in the case of lowering of the frequency (and in some experimental investigations, frequencies of the order of 10^{-6} Hz have been reached) the important channels of heat exchange between a given system and the external world are those which can be ignored at higher frequencies.

4. THREE-DIMENSIONAL PROBLEM

We can now go over to the three-dimensional problem and allow both for different transverse dimensions of a sample and the external circuit (Fig. 1), and for their transverse inhomogeneity, i.e., for the dependences of c and σ on the transverse coordinates x and y. As before, we shall regard the side surface of the whole system impermeable to heat and, moreover, we shall assume that the product of the complex wave number of temperature waves and the transverse size is small in the absolute sense, and the transverse dimensions themselves are small compared with the length of the sample l. Under these conditions the three-dimensional problem under discussion can be reduced to the one-dimensional form, exactly as it is done in the theory of transmission lines obeying the telegraph equations (see, for example, Ref. 7). Using the expressions in Eq. (19), we integrate over the transverse cross section $z=$ const to introduce the quantitites

$$J=\int\tilde{j}_z\,dS, \quad F=\int\tilde{f}_z\,dS, \quad U=\hat{c}^{-1}\int c\tilde{u}\,dS,$$
$$\hat{c}=\int c\,dS, \quad \hat{\sigma}=\int\sigma\,dS, \tag{49}$$

which depend only on z ($\hat{c}$ and $\hat{\sigma}$ change abruptly at $z = \pm l/2$ assuming the values of $\hat{c}_e = c_e S_e$ and $\hat{\sigma}_e = \sigma_e S_e$ for the homogeneous external circuit). Assuming that at each cross section $z =$ const the function $\tilde{u}$ is approximately constant, and replacing $\tilde{u}$ with U, we obtain the equations

$$J=-\hat{\sigma}\frac{dU}{dz}+F, \quad i\hat{c}\omega U=\frac{dJ}{dz}, \tag{50}$$

analogous to the system of equations (22) and solvable in the same way; the boundary conditions at $z = \pm l/2$ reduce to the continuity of U and J.

In view of this analogy, assuming that

$$K=(i\hat{c}\omega/\hat{\sigma})^{1/2}, \quad K_e=(i\hat{c}_e\omega/\hat{\sigma}_e)^{1/2}, \quad H=-iK/\xi^{1/2}, \quad \xi=\hat{c}_e\hat{\sigma}_e/\hat{c}\hat{\sigma}, \tag{51}$$

we eventually obtain the same formulas (43), (46), and (47) in which we have to introduce now the coefficient $C/\hat{c}l$ and assume that

$$\omega_2=\hat{\sigma}/\hat{c}l^2, \quad \omega_1=\xi\omega_2, \quad \nu=\omega/4\omega_1\ll 1/\xi, \tag{52}$$

where the parameter ξ occurring in the expressions for $s_i(\nu)$ and ω_1 determines the rate of heat exchange between the sample and the external circuit. In fact, the ratio U/J for a temperature wave traveling along the z axis of a homogeneous system with the parameters $\hat{c}$ and $\hat{\sigma}$ is given by $\pm Z$, where

$$Z=(i/\hat{c}\hat{\sigma}\omega)^{1/2} \tag{53}$$

can be called the wave impedance of the system; the quantity

$$\xi^{1/2}=Z/Z_e \tag{54}$$

determines, in accordance with Eq. (48), the coefficient of reflection of a wave from the boundary between a sample and the external circuit. The sample may consist, for example, of a conducting film (parameters c and σ, transverse cross section S, heat capacity $C = cSl$) on an insulating substrate (parameters c_d and σ_d, cross section S_d); then,

$$\hat{c}=cS+c_dS_d; \quad \hat{\sigma}=\sigma S+\sigma_d S_d. \tag{55}$$

According to Eq. (47), the law $1/\omega$ may be obeyed by the assumed models only in that part of the spectral intensity which is due to the sources in a sample, and only for small values of the parameter ξ, i.e., when the exchange of heat between the sample and the external circuit is slow. However, under these conditions the spectral density due to the external sources of fluctuations predominates and has a different frequency dependence. If we attempt to attribute the $1/\omega$ dependence to equilibrium temperature fluctuations, we then encounter generally two problems: we have to find the reason why the effects of external sources become weaker (they are "frozen" or "screened") and the reason for the enhancement of the internal sources (in powder resistors this behavior is exhibited by contracts between powder grains, which conduct heat poorly, as can be confirmed by calculations). It is very difficult to find these reasons.

Another difficulty is associated with the fact that at $l \sim 1$ cm the upper frequency for the $1/\omega$ law in accordance with the first formula in Eq. (39) is found to be less than 1 Hz. Since the $1/\omega$ law is observed also at much higher frequencies, it can only be due to temperature fluctuations in fine and very fine structures whose very existence is in doubt.

Recently published experimental results[8] indicate that the $1/\omega$ noise in metal films (Au, Bi, and Cr) is such that the separate parts of the system fluctuate practically independently (the mutual spectral intensity vanishes at low frequencies at which temperature flucutations result in an almost uniform temperature distribution) and much more strongly than one would expect of temperature fluctuations (though a calculation of the latter is carried out in a not very convincing manner). Hence, it follows that in these experiments there is an additional and a stronger mechanism of fluctuations which is of local nature. However, in other experiments (see, for example, Ref. 9) a strong spatial correlation has been observed.

CONCLUSIONS

In the simplest electric circuit consisting of a source of a current with a constant emf $\mathscr{E}$ and a resistance R, the current I is given by the relationship $\mathscr{E} = RI$. If we consider the noise of physical origin in such a system we find that fluctuations of $\mathscr{E}$ are usually due to the thermal noise, spontaneous fluctuations of I give rise to the shot noise, and fluctuations of R are referred to as the flicker noise using the term to indicate the slowness of this process and its nonelectrical origin. Therefore, the flicker noise is an essential element of a trio which determines the unavoidable electrical fluctuations in any physical system.

In the absence of a current, when a system is in a state of thermodynamic equilibrium, its resistance R can only depend on thermodynamic parameters (temperature, volume and number of particles of different kinds) and R can change because of fluctuations in these parameters. This is also true during the passage of a current that does not alter significantly a thermodynamic equilibrium.

We have carried out a theoretical investigation of fluctuations of the resistance due to equilibrium fluctuations of temperature (average over the volume of a cylindrical sample), giving rise to fluctuations of the voltage and current in the electric circuit containing a sample. Since temperature fluctuations described by Eq. (1) are due to heat exchange between a sample and the ambient medium, the frequency distribution of the intensity of these fluctuations depends strongly on the nature of heat exchange. Our aim has been to determine under which conditions the spectral intensity of such fluctuations is proportional to $1/\omega$. The results indicate that such a spectrum appears under conditions of hindered (slow) heat exchange, when temperature waves which appear inside a sample or near it because of thermal motion hardly leak outside and result in accumulation of temperature perturbations in the interior of a sample. However, additional calculations indicate that temperature waves excited in the external circuit far from the sample contribute a dominant term in the formula for the spectral sensitivity, which has a different frequency dependence. It follows that temperature fluctuations do not give rise to the $1/\omega$ noise in its pure form (at least within the framework of the adopted models) and they are usually masked by a more intense noise of different origin.

The practical importance of the flicker noise is associated not only with the fact that it predominates at frequencies below 5 or 10 kHz, but also because it determines the frequency instability of oscillators operating at much higher frequencies. Vakman[10] carried out a rigorous calculation for triode oscillators, but the results can easily be extended also to other types of oscillator (semiconductor, laser, etc.).

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