Temperature relaxation in a magnetized electron beam

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The velocity distribution of electrons in an intense beam shaped in an electrostatic accelerating device is strongly anisotropic: the electron longitudinal-velocity scatter is much less than that of the transverse velocities. When such a beam moves in a drift chamber, the longitudinal and transverse temperatures become equalized. This process slows down substantially when the beam is shaped and transported in a device with a longitudinal magnetic field. The present paper is devoted to an investigation of this effect.

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Intense low-temperature electron beams are of interest for a large number of various applications, such as low-noise electronics, etc. These beams have recently attracted interest also in the physics of charged-particle accelerators in view of the development of the electron-cooling method.[1] The efficiency of this method, namely the fall-off rate of the components of the phase volume of the cooled heavy particles, depends significantly on the particle temperature in the cooling electron beam.

Investigation of the properties of electron cooling has revealed a distinguishing feature of the electron velocity distribution in an intense beam accelerated in an electrostatic device (gun), namely, the electron-longitudinal-velocity scatter after the acceleration is much less than the transverse-velocity scatter.[2] Indeed, neglecting the interaction of the electrons with one another, we obtain from the energy conservation law and from the condition for the transformation of the longitudinal momentum component

$$T_{\parallel} = \Delta p_{\parallel}^2/2m = T_{\perp}^2/2\beta^2\gamma^2 mc^2 \longrightarrow T_{\perp}^2/4W, \tag{1}$$

where $\Delta p_{\parallel}$ is the electron-momentum scatter in the system in which the average electron velocity v is zero; m is the electron mass; $T_{\perp}$ is the kinetic energy of the electrons prior to acceleration; $\beta = v/c$, $\gamma = (1 - \beta^2)^{1/2}$, and $W = mv^2/2$ at $v \ll c$. This result is none other than a consequence of the Liouville theorem: conservation of the longitudinal phase volume of the beam leads, upon acceleration, to a decrease in the scatter of the longitudinal component of the momentum in proportion to the decreasing particle density. In the nonrelativistic case, to which we confine ourselves here, this yields

$$v\Delta v = \text{inv}, \quad \text{or} \quad W T_{\parallel} = \text{inv}. \tag{2}$$

The effect of such "oblateness" of the electron velocity distribution function is very strong: thus, for thermionic sources in which the electrons are accelerated to an energy of the order of 30 keV, we obtain respectively

$$T_{\perp} \approx 0.1 \text{ eV}, \quad T_{\parallel} \approx 10^{-7} \text{ eV} \approx 10^{-3} \text{ K}.$$

At such low longitudinal temperatures, however, one can no longer neglect the interaction of the electrons with one another and this imposes a limit on the minimum possible longitudinal temperature of the beam after acceleration.[3]

It was found that this feature of the distribution function makes it possible to increase strongly the effectiveness of electron cooling by an electron beam moving in a strong longitudinal magnetic field (magnetized stream).[3]

The efficiency of electron cooling increases linearly with the length of the cooling section, if the properties of the electron beam remain unchanged. This raises the natural question: how long is this oblateness of the distribution function preserved when the electron moves over a long drift section.

Batalin[4] attempted to obtain, using a thermodynamic approach, the change of the transverse emittance (transverse temperature) of a beam accelerated in an ideal Pierce gun as a result of (electrostatic) interaction of the electrons. This change, as shown in his paper, is quite weak. Unfortunately, he did not consider the changes of the longitudinal temperature, whereas for the problem of electron cooling (which was the subject of his paper), the longitudinal temperature plays the decisive role.[1]

In this paper we describe experiments on temperature relaxation in a magnetized electron stream. The experiments were performed with an electron beam whose energy ranged from 200 to 1000 eV and the electron density from 10^7 to 10^9 cm^{-3}, corresponding to a beam current from 50 μA to 10 mA.

1. INITIAL PREMISES

We examine the principal premises on which the organization of the experiment was based. They evolved to a considerable degree as a result of the development of the method of electron cooling and are described in greater detail in the review[3] and in the literature cited therein.

When an electron beam moves in a vacuum chamber, its longitudinal temperature increases because of the internal scattering of the electrons by one another, and the transfer of energy from transverse to longitudinal motion takes place until the longitudinal and transverse temperatures become equalized. In the case of a weak magnetic field (nonmagnetized beam), this relaxation can be easily calculated[5]:

$$dT_{\parallel}/dz = (\pi e^3 j Lk/W)(m/T_{\perp})^{1/2}, \tag{3}$$

where $z = vt$ is the longitudinal coordinate; j is the current density; L is the Coulomb logarithm; k is a numerical coeffi-

cient of the order of unity and depends on the type of the electron velocity distribution function. For a Maxwellian distribution and at $T_\parallel \ll T_\perp$ it is equal to 0.87. An increase of the longitudinal temperature is in this case quite rapid: thus, in an electron beam of energy 400 eV at a current density 0.5 A/cm^2 the temperatures become practically fully equalized over a length of 3 meters (the characteristic parameters of the described experiment).

In a longitudinal magnetic field, the energy transfer from the transverse degrees of freedom to the longitudinal is made difficult by the limited transverse displacement of the electrons, and decreases abruptly if

$$\rho_\perp \lesssim r_{min}, \quad \rho_\perp = (2T_\parallel mc^2)^{1/2}/eH, \tag{4}$$

where $\rho_\perp$ is the Larmor radius of the transverse rotation of the electrons in the magnetic field H, and r_{min} is the minimum distance between the scattered particles. In the case of a strong magnetic field, the motion of the electrons can be represented as the motion of Larmor circles with small longitudinal velocity and the energy transfer from transverse to longitudinal motion is due to violation of the adiabaticity of the collisions of the Larmor circles. At a low electron density, the minimum distance between the electrons is determined by the longitudinal temperature $r_{min} \approx e^2/T_\parallel$. Substituting this condition in (4) we obtain

$$\rho_\perp \lesssim e^2/T_\parallel. \tag{5}$$

In the case of a low longitudinal temperature and a high electron density, the minimum distance between them is determined by the interparticle distance, and condition (4) yields

$$\rho_\perp \lesssim n_e^{-1/3}. \tag{6}$$

Simultaneous application of inequalities (5) and (6) determines the necessary conditions for the appearance of transverse-longitudinal relaxation. For a beam with the same parameters as in the preceding example, and at $T_\perp \approx 0.1$ eV, these conditions call for $H \approx 1$ kG and $T_\parallel < 10^{-4}$ eV. Therefore, to decrease the relaxation in the beam it is necessary to have as low as possible a longitudinal temperature after the acceleration. Near the cathode, the electrons have a high density, a low translational-motion energy, and a high longitudinal temperature $T_\parallel$. In this case the conditions (5) and (6) are violated and the relaxation can proceed rapidly.

Another effect that leads to an increase of the longitudinal temperature of the beam is longitudinal-transverse relaxation. If the acceleration is rapid compared with the period of the plasma oscillations of the electrons, the relative distances between the electrons do not change during the acceleration time, and the initial state with the random disposition of the orbits that move slowly relative to one another is preserved; these orbits subsequently relax to a distribution with longitudinal velocity spread corresponding to the approximate equality

$$T_\parallel \approx T_\perp^2/4W + e^2 n_e^{1/3}. \tag{7}$$

The time of such a relaxation is short and agrees in order of magnitude with the period of the plasma oscillations. At $T_\perp$

≈ 0.1 eV (the cathode temperature) and $W = 400$ eV, both terms in (7) become equal already at $n_e \approx 10^5$ cm^{-3} ($j \approx 15$ μA/cm^2). With further increase of the current density, the contribution of the second term becomes decisive.

In the case of slow acceleration of the electrons, the plasma oscillations manage to intermix the density fluctuations, and the longitudinal temperature after the acceleration is determined by expression (1). The condition that the acceleration be adiabatic with respect to plasma oscillations takes the form

$$\lambda \equiv \frac{1}{\omega_p(z)T_\parallel} \frac{dT_\parallel}{dt} \ll 1, \tag{8}$$

where $\omega_p(z)$ is the plasma frequency at the point z. It is interesting that the adiabaticity condition is not satisfied even when electrons are accelerated in a gun operating in the "3/2 regime." Indeed, neglecting the interaction of the electrons we get from (2)

$$\frac{1}{T_\parallel} \frac{dT_\parallel}{dz} = -\frac{1}{W} \frac{dW}{dz}. \tag{9}$$

Substituting this relation in (7) and taking into account the $W(z)$ and $\omega_p(z)$ dependences we obtain for a Pierce gun ("the 3/2 law") $\lambda_P = 2^{3/2}$. In particular, upon acceleration to 400 eV, approximately half of the plasma oscillation takes place in the beam.

The change of the longitudinal temperature upon acceleration, with allowance for the internal scattering of the electrons (3) and neglecting the influence of the magnetic field, is described by the equation

$$\frac{dT_\parallel}{dz} = -\frac{T_\parallel}{W} \frac{dW}{dz} + \frac{\pi e^3 jkL}{W} \left(\frac{m}{T_\perp}\right)^{1/2}, \tag{10}$$

whose solution is

$$T_\parallel(z) = T_c^2/4W + \pi e^3 jLk(m/T_\perp)^{1/2}z/W, \tag{11}$$

where $T_c \equiv T_\parallel(0)$ is the cathode temperature.

At $j = 0.5$ A/cm^2, $W = 400$ eV and $z = 0.5$ cm, the second term in (11) is equal to 1.2×10^{-4} eV and is approximately ten times larger than the first. The magnetic field is capable of altering substantially this relation if the magnetization conditions (5) and (6) are satisfied. The conditions themselves, represented in the form of rough inequalities, make it possible to estimate the situation quite approximately, especially near the cathode, where the density and the longitudinal temperature of the beam are large. Therefore the question of whether a low longitudinal temperature of the beam can be reached after the acceleration calls for experimental study.

2. ORGANIZATION OF EXPERIMENT

The experiments were performed with an electron-cooling setup[6] in which the electron beam, shaped by a three-electrode gun, was transported in a longitudinal magnetic field of intensity up to 1.4 kG and, after passing through a three-meter drift space, entered the analyzer (Fig. 1). The residual-gas pressure in the vacuum chamber 4 was lower than 10^{-8} Torr.
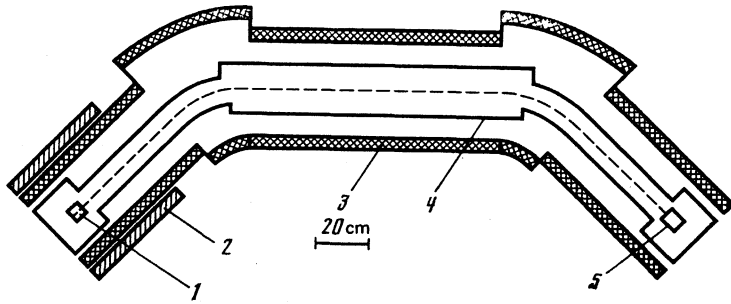
The gun cathode was thermionic, oxide coated, of 2 mm diameter, with a negative potential U_c that set the beam energy. It was able to regulate the potential of the first anode from U_c to $+3$ kV; this made it possible to control the beam current. The second anode was at zero potential (grounded).

The intensity of the longitudinal magnetic field in the gun region could be varied with an additional short solenoid 2 relative to the level of the field in the remaining part of the channel in the range from 0.5 to 4.5 kG.

At the entry to the analyzer (Fig. 2), the beam 1 entered a limiting diaphragm 3, with a central opening of 0.1 mm diameter. The diaphragm was under a positive potential of the order of 30 V. This made it possible, first, to suppress considerably the secondary electron emission from it and, second, to block the ions of the residual gas in the gun (see below). The position of the beam at the entrance to the analyzer was monitored by reading the current from two pairs of parallel wires 2 of 0.3 mm diameter. This device had a slight design defect: when the beam struck the wires they were heated and deformed, partially covering up the entrance hole. This defect was noted at beam currents higher than 1 mA, and explains the scatter in the measurements when the beam scanned the collector (see Fig. 3). It did not influence, however, the results of the measurements of the longitudinal temperature, carried out under stationary conditions; it more readily improved the situation with respect to the space charge in the cut-out beam (usually the current to the collector did not exceed $3\,\mu$A). The limited beam was slowed down in the longitudinal electric field of the analyzing diaphragm 5 and proceeded to the collector 6, which was grounded through a resistor $R_{coll} = 10$ kΩ–10 MΩ. The beam current could be determined by measuring the voltage drop across the resistor. The potential of the diaphragm 5 could be varied within specified limits in the range from

-30 to $+30$ V relative to the cathode potential. It was this which made it possible to analyze the limited beam in energy. The diaphragm potential was varied in response to commands from the computer through a digital-analog converter (DAC) "following up" on the cathode potential. The measured dependence of the collector current $I_{coll}(U)$ on the potential U of the analyzing diaphragm was reduced with a computer, which calculated, from the measured integral spectrum, the differential spectrum dI_{coll}/dU, its width δU, and the value of the potential ΔU of the analyzing diaphragm at the half-maximum level of the limited beam (the position of the center of gravity of the spectrum).

The systematic errors that appear in this method of analyzing the longitudinal beam temperature are due mainly to the sag of the potential inside the analyzing diaphragm and to the shift of the electron energy in the analyzed (limited) beam because of the space-charge field. Both errors were negligibly small in the described experiments. In addition, the beam can have a longitudinal-velocity scatter due to the "optical" perturbations in the beam (the influence of the anode apertures) and having no bearing on the investigated effect. This scatter has an "aperture" character (it increases with increasing distance from the system axis) and can be appreciable for diode or triode guns without an accompanying magnetic field. In guns immersed in a strong magnetic field, this scatter is substantially suppressed.[7] To eliminate completely the error connected with this scatter the beam carried out in each measurement run a control scanning of the input opening of the analyzer (Fig. 3), and the "unperturbed" region near the beam axis was chosen for the temperature measurement. The scanning was effected by introducing a transverse magnetic field in the drift section.

Typical $I_{coll}(U)$ plots are shown in Fig. 4. A unique demonstration of the sensitivity of the described method of

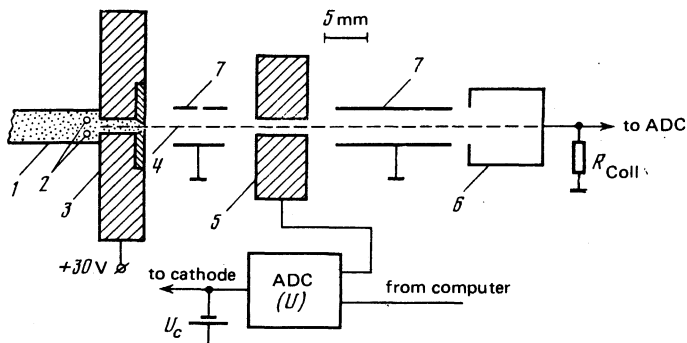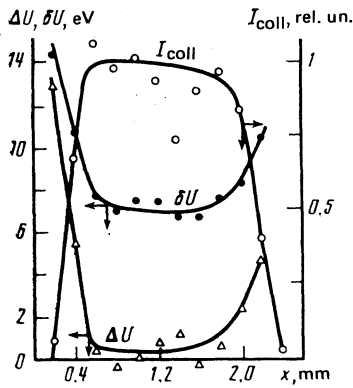FIG. 3. Results of beam scanning of analyzer entrance aperture: dependence of the collector current (of the beam current density), of the width δU of the differential spectrum, and of the value ΔU of its center of gravity on the transverse displacement x of the beam at the entrance to the analyzer.

FIG. 5. Influence of gun operating regime on the electron distribution in longitudinal velocity: 1—limited emission regime; 2—space-charge-limited current regime (3/2 regime). Beam current 90 μA, energy 400 eV, magnetic field 1 kG everywhere.

analysis of the longitudinal temperature is Fig. 5, which shows two differential spectra obtained for a beam with the same current $I = 90\mu$A, but shaped by a gun which operated in one case in the limited-emission regime and in the other in the 3/2 regime. In the latter case, owing to the formation of a potential minimum near the cathode, a minimum that reflected the "extra" current from the cathode and equal to[8]

$$U_{min} = (T_c/e) \ln (I_s/I), \qquad (12)$$

the average electron energy in the beam was higher. Here I_s is the saturation current of the cathode, I is the current in the beam. The longitudinal electron temperature near the cathode, calculated from (12) using the results of Fig. 5, is approximately 0.2 eV.

The width of the differential spectrum is connected with the longitudinal temperature at the entrance to the analyzer by a relation that is the inverse of (1):

$$e\delta U = 2(WT_{\parallel})^{1/2}. \qquad (13)$$

3. RESULTS OF EXPERIMENTS

1. The first and basic result of the experiments is the substantial dependence of the longitudinal temperature of

the beam, after passing through the drift gap, on the values of the magnetic fields in the apparatus. The arguments presented above allow us to assume that in a nonmagnetized beam the width of the spectrum δU can increase with the current density like $j^{1/2}$—this follows from a comparison of Eqs. (11) and (13), which were used to plot curve 1 of Fig. 6. The experimental curves shown in Fig. 6 have a distinctive form: at low currents there exists a plateau whose length depends on the values of the fields, and at sufficiently large currents the curve approaches the asymptote $I^{1/2}$, corresponding to heating of the beam in the gun and in the drift interval, see (3) and (11). The presence of a plateau on the experimental curves indicates a strong influence of the longitudinal magnetic field, which suppresses the transverse-longitudinal temperature relaxation. The length of the plateau increases with increasing field (curves 2 and 3). The minimum values of the width of the spectrum registered in experiment were 0.2 V, which is close to $T_c e$.
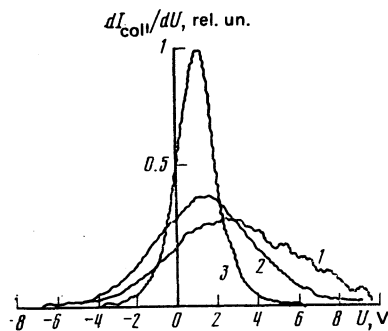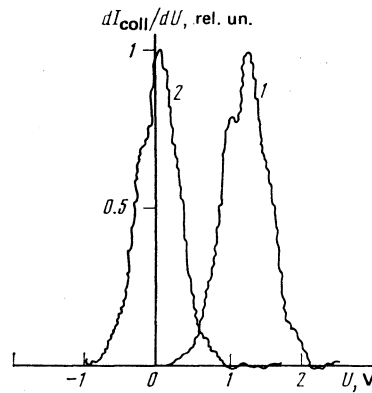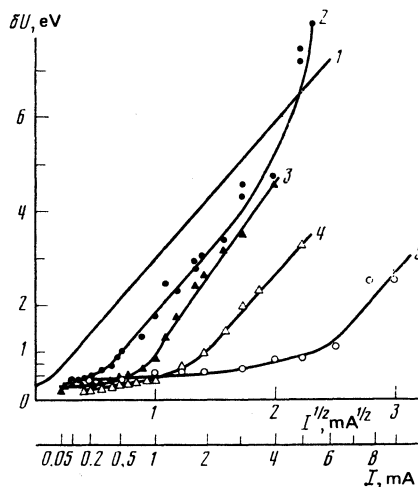

FIG. 4. Dependence of dI_{coll}/dU on the potential of the analyzing diaphragm for a system magnetic field 1 kG in the drift section and for different values of the field in the gun region: 1) 0.6 kG, $\delta U = 4.3$ eV; 2) 1 kG, $\delta U = 3.1$ eV; 3) 3.4 kG, $\delta U = 1.3$ eV. Beam current 2.4 mA, electron energy 400 eV.

FIG. 6. Dependence of the spectrum width δU on the parameters of the electron beam in the installation. 1) Calculation by Eq. (11); δU for different values of the magnetic field (in kG) on the drift section and in the gun region (first and second number, respectively); 2) 1; 1; 3) 1.4; 1.4; 4) 1.2; 3.85; 5) 1; 3.2. Electron energies 400 (1–4) and 1200 (5) eV.

2. The possibility of independently regulating the magnetic field in the gun region and in the drift gap has revealed a radical influence of the former on the relaxation process. The results shown in Fig. 4 and on curves 4 and 5 of Fig. 6 demonstrate clearly the essential role of increasing the magnetic field H_c in the gun. The length of the plateau in this case is much larger than for curves 2 and 3. This lengthening of the plateau with increasing magnetic field in the gun is due to the action of the following two effects, whose relative contributions is difficult to separate. The first is the suppression of the internal scattering of the electrons upon acceleration: in the gun region the conditions for the satisfaction of inequalities (5) and (6) are very stringent, and with increasing magnetic field in the gun they "begin to operate." The second is the adiabatic expansion of the beam on going from the region of the strong magnetic field H_c in the gun (at the cathode) into the region of the weaker field H in the drift gap. In this case the beam density decreases in proportion to H/H_c and $\rho_\perp$ increases in proportion to $(H_c/H)^{1/2}$, and as a result the condition (6) is satisfied on the drift section only up to

$$I = I_{cr} \approx (\pi e^4 r_c^2 / 2m^2 c^3)\,(WHH_c^5/T_c^3)^{1/2}. \tag{14}$$

If the first effect is not produced for some reason, then I_{cr} in (14) determines the lengths of the plateaus on the curves of Fig. 6. Here R_c and T_c are the radius and temperature of the beam at the cathode.

3. At a field in the gun weaker than 1 kG, it was observed that the $\delta U(I)$ dependence ceases to be monotonic. Maxima and minima appear on the curve and their positions vary when scanned over the transverse cross section of the beam, while the depth increases with increasing current (Fig. 7). These oscillations do not disappear when the field in the drift gap is increased, provided the field in the gun remains weak. The onset of these oscillations is apparently due to the large transverse electron velocities produced because of the

nonideal optics of the gun, which manifest itself ever more noticeably with decreasing magnetic field in the gun.[7]

4. By applying on the limiting diaphragm of the analyzer and on the special electrode at the exit from the gun a positive potential relative to the walls of the vacuum chamber, it is possible to block in the gun the ions produced upon ionization of the residual gas by the beam. This eliminates the beam space-charge electric field and the change it produces in the longitudinal velocity with changing radius. In this case, as shown by the experiments, the width of the spectrum decreases somewhat in the case of large currents. This can be attributed to the fact that with increasing compensation of the beam the radial gradient of the longitudinal electron velocities decreases.

5. The influence of the magnetic field on the relaxation is very clearly illustrated by the obtained dependence of the width of the spectrum on the particle energy (see Fig. 6). For classical diffusion the width of the spectrum, according to (13) and (11), does not depend on the particle energy and is determined only by the current density in the beam and by the length of the drift gap. It was observed in the experiment that when the magnetic fields in the apparatus remain unchanged, the length of the plateau on the curves of Fig. 6 increases with increasing electron energy (curves 4 and 5); this is due to the decrease of the longitudinal temperature and of the density when the electrons are accelerated to high energy.

In the case of a weak beam current and a strong magnetic field, the longitudinal-transverse relaxation is strongly suppressed and the longitudinal temperature in the electron beam is determined by the longitudinal-longitudinal relaxation. Figure 8 shows the dependence of the width of the spectrum on the energy at different magnetic fields in the setup. Curve 2 corresponds to a strong magnetic field and agrees well with the calculated curve 3 plotted in accordance with Eqs. (7) and (13). Curve 1 corresponds to a weak magnetic field in the apparatus. In this case the electrons acquire large transverse velocities in the gun, leading to an increase of the width of the differential spectrum because of the finite entrance aperture of the analyzer. The minimum on curve 1 is
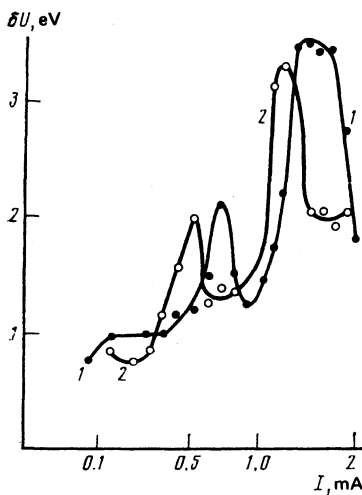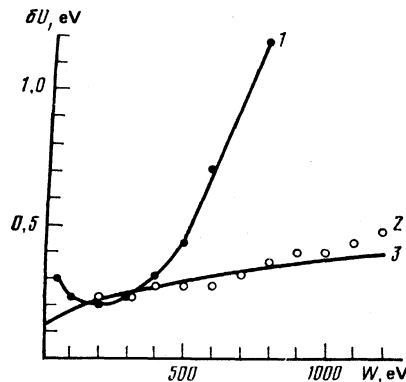

FIG. 7. Dependence of the spectrum width on the electron-gun current at a decreased value of the magnetic field in the gun region. Curves 1 and 2 are drawn for different positions of the beam on the analyzing diaphragm. Field 0.8 kG in the gun region and 1.2 kG in the drift section. Electron energy 400 eV.

FIG. 8. Dependence of spectrum width on the electron energy at low beam currents: 1) 50 μA; 2) 100 μA; 3) calculation by Eqs. (7) and (13). The magnetic fields on the drift section and in the gun region amount to 1 kG (curve 1) and 1 and 4.3 kG respectively (curve 2).

due to the decrease of the transverse velocities on account of the "resonant optics" effect.[7]

CONCLUSION

The experiments have shown that in electron-gun devices a longitudinal magnetic field suppresses substantially the longitudinal-transverse temperature relaxation, making it possible to shape intense electron beams with quite small scatter of the longitudinal velocities and with a correspondingly high degree of ordering of the particles in the beam. Such beams can be transported in a longitudinal magnetic field without distorting their characteristics. The decisive role is played here by the longitudinal temperature of the beam at the exit from the source (gun). If the temperature is low, the relaxation process on the drift gap is substantially suppressed.

Several recommendations on the shaping of such beams can be formulated:

The distribution of the electric field in the near-cathode region of the gun should obey the 3/2 law (Pierce gun) and, if possible, the acceleration of the particles past this region should be adiabatically slow relative to the frequency of the plasma oscillations of the electrons.

A sufficiently strong magnetic field satisfying the inequality (5) should be applied along the entire channel. For a given field in the drift gap, the critical current density at which the magnetization of the beam "works" increases with increasing field in the gun region.

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