A rarefaction magnetosonic soliton and the generation of hf oscillations during the dynamical development of a Z-pinch

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(Submitted 10 April 1982)
Zh. Eksp. Teor. Fiz. 83, 2065–2072 (December 1982)

We show that when magnetic viscosity is taken into account rarefaction-type soliton solutions are possible even in the framework of single-fluid magnetohydrodynamics. When electron inertia is taken into account, hf oscillations approximately up to a level $\tilde{E}/B_0 \sim (m_e/m_i)^{1/2}$ ($\tilde{E}$ is the amplitude of the oscillations and B_0 the unperturbed magnetic field) must develop and this occurs apparently in Z-pinch experiments.

PACS numbers: 52.55.Ez, 52.30. + r, 52.35.Mw

§1. INTRODUCTION

Spectroscopic studies of plasma Z-pinches show[1] that at the time of the "second singularity" strong hf oscillations are observed in them with frequencies of the order of $\omega_0 = (4\pi ne^2)/m_e)^{1/2}$ and $\omega_{Be} + eB_0/m_e c$ and amplitudes $\tilde{E}/B_0 \leqslant (m_e/m_i)^{1/2}$. We try in the present paper to explain these oscillations using a one-dimensional planar model. In what follows we show (§2) that when account is taken of the magnetic viscosity due to the finite ion Larmor radus rarefaction soliton-type solutions (solitary waves) are possible even in the framework of single-fluid magnetohydrodynamics rarefaction soliton-type. We bear in mind that in the usual single-fluid magnetohydrodynamics the waves do not have a dispersion so that nonlinear soliton-type solutions do not occur and can be obtained only when we go over to a two-fluid model which takes the electron inertia into account.[2,3]

In our case (with a dissipationless magnetic viscosity) taking electron inertia into account leads (§3) to the build-up, at the leading front of the rarefaction soliton, of electrostatic oscillations $\tilde{E}$ with a frequency of the order of ω_0 and an amplitude (in the center of the rarefaction soliton) of the order $\tilde{E}/B_0 \sim (m_e/m_i)^{1/2}$, in qualitative agreement with observations in Z-pinches.

One should note that oscillations with such an amplitude are also observed in several other experiments, for instance, in transverse magnetosonic shock waves. Trushin and Sholin[4,5] were the first to attempt in a number of papers to explain these oscillations and the accompanying effects (such as a widening of the thickness of the front); however, they only considered condensation waves and compression solitons, rather than the rarefaction solitons we do. We emphasize that taking the magnetic viscosity into account in the single-fluid model leads just to rarefaction solitons, owing to the positive dispersion of the linear waves.[6] To complete the picture, however, we consider in conclusion (§4) also the two-fluid compression soliton studied earlier by Sagdeev[3] and we show that when the magnetic viscosity is taken into account (here as a correction factor) oscillations are also built up in this soliton but now in the range $\omega \gtrsim |\omega_{Be}\omega_{Bi}|^{1/2}$, where $\omega_{Be,Bi}$ are the electron and ion cyclotron frequencies. We thus show in the present paper that taking the (dissipationless!) magnetic viscosity into account appreciably affects the dynamics of the nonlinear solitary waves in a plasma.

§2. SINGLE-FLUID RAREFACTION SOLITON IN A PLASMA WITH MAGNETIC VISCOSITY

It has been shown earlier[6] that a correct approximation to describe a plasma in the collisionless limit is anisotropic magnetohydrodynamics with magnetic viscosity which takes into account the finite Larmor radius of the ions and which leads to a positive dispersion of the linear waves. For waves which propagate across the magnetic field the dispersion law has the form

$$\omega^2 = k_\perp^2[\tilde{c}_A^2 + (\eta_B k_\perp/\rho)^2], \quad \tilde{c}_A = c_A(1+\beta)^{1/2}, \quad \beta = 8\pi p_\perp/B^2, \tag{1}$$

where $c_A = B/(4\pi\rho)^{1/2}$ is the Alfvén speed, $\eta_B = p_\perp^i/2\omega_{Bi}$ the ion magnetic viscosity coefficient, $p_\perp^i$ the ion pressure, and $p_\perp$ and $\rho = m_i n$ the plasma pressure and density. The dispersion law (1) means that rarefaction soliton-type solutions may occur in the given, essentially single-fluid, model.

In the present section we consider the propagation of waves (1) with a finite amplitude in a one-dimensional planar model with a geometry similar to the one used in Ref. 6. Let the wave propagate in the direction $\mathbf{e}_x$, let the magnetic field $\mathbf{B}$ be parallel to $\mathbf{e}_y$, and let the perturbations depend solely on the variable $\xi = x - ut$ where $u > 0$ is the wave velocity. We find in that case from the single-fluid magnetohydrodynamics equations[6]

$$\begin{gathered} v_x = u(1-n_0/n), \quad p_\perp = p_0(n/n_0)^2, \quad B = B_0 n/n_0, \\ m_i n_0 u v_x = p_\perp - p_0 + (B^2 - B_0^2)/8\pi + \pi_{xx}, \\ \pi_{xx} = \eta_B dv_z/d\xi, \\ m_i n_0 u v_z = \pi_{xz} = -\eta_B dv_x/d\xi, \quad \eta_B = p_\perp/4\omega_{Bi}. \end{gathered} \tag{2}$$

Here $v_{x,z}$ are the plasma velocity components; the index zero denotes the unperturbed values of the various quantities; π_{xx} and π_{xz} are components of the magnetic viscosity tensor; we assume $p_\perp^i = p_\perp/2$.

One easily derives from the set (2) an equation for the plasma density. Writing $N(\xi) = n/n_0$ we have

$$2a_B^2 \frac{d}{Nd\xi}\left(\frac{dN}{Nd\xi}\right) = \frac{N-1}{N^3}[N^2+N-2M^2]. \tag{3}$$

Here $M = u/\tilde{c}_A$ is the Mach number evaluated using the velocity $\tilde{c}_A$ in the unperturbed plasma; $a_B = n_B^0/m_i n_0 \tilde{c}_A$ is a parameter with the dimensions of length and determines the size of the soliton.

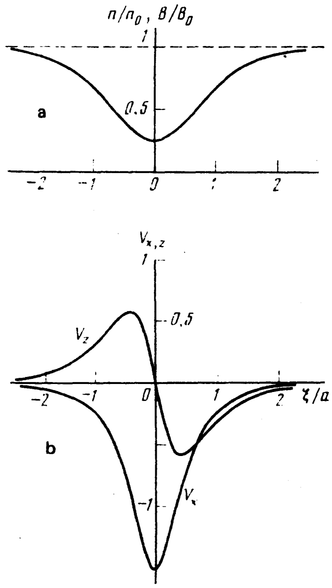

FIG. 1. Profiles of (a) the density and the magnetic field and (b) of the plasma velocity components $V_{x,z} = v_{x,z}/\tilde{c}_A$ for a single-fluid soliton for $M = 1/2$.

A solution of Eq. (3) satisfying the conditions $N = 1$ and $dN/d\xi = 0$ on the front of the wave is a rarefunction soliton (see Fig. 1):

$$N(\xi)=1-\left[\frac{2a_B/a}{\mathrm{ch}(\xi/a)}\right]^2,\quad a=\frac{2a_B}{(1-M^2)^{1/2}},\quad M^2<1,\qquad(4)$$

which can be large as $M\to 1$.

This solution differs somewhat from the Korteweg–de Vries solitons. Most strikingly this difference manifests itself in the fact that the density well (4) can be completely stationary since the momentum flux is finite, although the plasma velocity components have a singuarity when $M = 0$.

Because of the possible violation of the plasma quasineutrality and the freezing in of the magnetic field the single-fluid description is, strictly speaking, not correct and one should use therefore the exacter two-fluid description.[7] Moreover, such an approach leads to principally new results considered in the next section.

§3. BUILDUP OF LANGMUIR OSCILLATIONS AT THE FRONT OF A RAREFACTION SOLITON

3.1. The equations of two-fluid magnetohydrodynamics in their full form turn out to be too complicated for analysis. To simplify the situation we restrict ourselves to the case of cold electrons ($T_e = 0$) but retain the electron inertia (taking its influence into account turns out to be important as a matter of principle).

Under these assumptions the generalization of the set of single-fluid Eqs. (2) is

$$\begin{gathered}N_\alpha=(1-v_{x\alpha}/u)^{-1},\quad p_i=p_0N_i^2,\quad p_e=0,\quad E_z=\gamma B_0(1-b),\\ n_0um_\alpha v_{x\alpha}'=(d/d\xi)(p_\alpha+\eta_\alpha v_{z\alpha}')-e_\alpha n_0B_0N_\alpha(E-v_{z\alpha}b/c),\\ n_0um_\alpha v_{z\alpha}'=-(d/d\xi)\eta_\alpha v_{x\alpha}'-\gamma e_\alpha n_0B_0(N_\alpha-b),\\ (1-\gamma^2)b'=(4\pi en_0/cB_0)(N_iv_{zi}-N_ev_{ze}),\\ E'=(4\pi en_0/B_0)(N_i-N_e),\quad \eta_\alpha=p_\alpha/2\omega_{B\alpha},\quad \omega_{B\alpha}=e_\alpha B/m_\alpha c.\end{gathered}\qquad(5)$$

Here $\alpha = e$, i identifies the particles, $b = B/B_0$, $N_\alpha = N_\alpha/n_0$, $\gamma = u/c$, $E = E_x/B_0$, $E_{x,z}$ are the electrical field components, and a dash indicates differentiation with respect to the variable $\xi = x - ut$.

We introduce a convenient notation

$$\beta=8\pi p_0/B_0^2,\quad \nu=\beta/(\beta+1),\quad \varepsilon=c_A/c,\quad \alpha=m_e/m_i,\qquad(6)$$

and characteristic scales with dimensions of length:

$$a_B=\eta_{i,0}/m_in_0\tilde{c}_A,\quad a_D=c/\omega_0\quad (\omega_0=(4\pi n_0e^2/m_e)^{1/2}).\qquad(7)$$

The first of them, a_B, is determined by the ion magnetic viscosity and is analogous to the one introduced above for the single-fluid soliton (3), whereas the size a_D determines the scale connected with taking electron inertia into account.

In this notation the set (5) reduces to equations for the functions $b(\xi)$, $N = N_i(\xi)$, and $E(\xi)$ of the following form (we use the face that $\gamma^2 = u^2/c^2 \ll 1$):

$$2a_B\frac{N^2}{b}\frac{d}{d\xi}\left[a_B\frac{N'}{b}+\varepsilon E(1-\nu)^{1/2}\right]=\nu(N^2-1)+\frac{\nu}{2}N^2\frac{b-N_e}{b}+(1-\nu)(b^2-1-E^2)-2M^2\left(\frac{N-1}{N}+\alpha\frac{N_e-1}{N_e}\right),\qquad(8)$$

$$a_D\frac{d}{d\xi}\left\{\left[a_D\frac{b'}{N}+\left(\frac{\alpha}{1-\nu}\right)^{1/2}\left(a_B\frac{N'}{b}+\varepsilon E(1-\nu)^{1/2}\right)\right]\times(\alpha+N_e/N)^{-1}\right\}=b-N_e,\qquad(9)$$

$$\frac{(M\varepsilon a_D)^2}{1-\nu}E''=-EN_e^3\left(1+\frac{\varepsilon^2b}{\alpha+N_e/N}\right)+\frac{\alpha^{1/2}M^2\varepsilon a_D}{1-\nu}N'-\frac{\varepsilon}{\alpha^{1/2}}\frac{N_e^3}{N}\left[a_Dbb'+\left(\frac{\alpha}{1-\nu}\right)^{1/2}a_BNN'\right](\alpha+N_e/N)^{-1},\qquad(10)$$

where

$$N_e=N-(\varepsilon a_D/\alpha^{1/2})E'(\xi).\qquad(11)$$

We note that Eq. (10) of this set has the form of the equation for a nonlinear oscillator and describes Langmuir oscillations. These oscillations correspond to a scale a_L:

$$a_L=(1+\beta)^{1/2}M\varepsilon a_D=u/\omega_0\ll a_D.\qquad(12)$$

Trushin and Sholin[4] were the first to note the role of such oscillations in the dynamics of transverse magnetosonic waves.

The set (8) to (11) can be further simplified when the plasma quasineutrality is not violated too strongly since, using the fact that the mass ratio is small, $\alpha \ll 1$, we can put $N_e/N \gg \alpha$. Moreover, we assume for the sake of simplicity, where necessary, that $\varepsilon \ll 1$ and $\beta \ll 1$.

3.2. We consider limiting cases for the set (8) to (11). Going over to the single-fluid description corresponds formally to $a_D \to 0$. We then get from (8) to (11) the equation

$$\frac{2a_B^2N}{(1+\varepsilon^2N)^{1/2}}\frac{d}{d\xi}\left[\frac{N'/N}{(1+\varepsilon^2N)^{1/2}}\right]=N^2-1-2M^2\frac{N-1}{N},\qquad(13)$$

which differs from the corresponding single-fluid Eq. (3) by the presence of terms $\propto\varepsilon^2$. One checks easily that their appearance is due to the fact that we have taken the Hall effect into account. Nonetheless, when we take a finite ε^2 into account, Eq. (13) describes a rarefaction soliton and such a solution is possible when $M^2 < N(\xi) < 1$.

In the opposite case when the magnetic viscosity is assumed to be unimportant we put formally $a_B \to 0(\nu, \beta \to 0)$. In that case the set (8) to (11) goes over into one studied earlier[4] and describes a compression magnetosonic soliton when account is taken of small-scale Langmuir oscillations.

The limiting cases noted above assume neglect of small terms in combination with the leading derivative. However, in such situations the occurrence of other kinds of solutions is possible. It is thus important to study the set at finite values of the scale ratio. As the basic principle in our choice of solutions we then take boundedness, and considering only compact, finite-amplitude solitary waves.

3.3. We note first of all that for solutions of the set (8) to (11) with sizes of the order of the size of the single-fluid rarefaction soliton ($\xi \sim a_B$) the deviation from the freezing-in of the magnetic field ($b \neq N, N_e$) and from plasma quasineutrality ($N \neq N_e$) is determined, respectively, by the parameters

$$\tilde{\alpha}=4\alpha^{1/2}/\beta, \quad \tilde{\varepsilon}=4\varepsilon/\beta, \quad \alpha, \ \varepsilon \ll 1. \tag{14}$$

In a plasma with not too low a pressure we have $\tilde{\alpha}, \tilde{\varepsilon} \ll 1$ and we may expect that taking these factors into account has only a weak effect. The analysis of the role of the small parameters (14) is facililtated if as a first step we eliminate the Langmuir oscillations by formally putting $\alpha \to 0$ but retaining terms with $\tilde{\alpha}$. The corresponding study does not have any difficulties as far as principle is concerned and therefore we formulate here only qualitatively the final result. In fact, when there are no Langmuir oscillations the violation of the freezing-in of the magnetic field and of the plasma quasineutrality weakly distorts the rarefaction soliton if $\tilde{\alpha}, \tilde{\varepsilon} \ll 1$ and the Mach number of the wave is not too small.

3.4. We consider finally the role of the smallest scale $\xi_{\text{char}} \sim M\varepsilon a_D$ corresponding to the Langmuir oscillations. First of all we take into account oscillations in the linear approximation, putting in the set (8) to (11)

$$b=b_0+\tilde{b}, \quad N=N_0+\tilde{N}, \quad E=E_0+\tilde{E},$$

where the index zero indicates the solution of the set neglecting the oscillations (we showed above that one can take for this "zeroth" solution the single-fluid soliton (3)) and the tilde denotes a small correction to this "equilibrium" level. We restrict ourselves to the case $\mu = \varepsilon/\alpha^{1/2} \ll 1$ (in other words, $\omega_{Be} \ll \omega_0$). To simplify matters it is also convenient to introduce a new variable

$$y=\xi/a_D. \tag{15}$$

To evaluate the perturbations we use the fact that the scale of the Langmuir oscillations $y_{\text{char}} \sim \varepsilon$ is appreciably smaller than the other ones and we shall therefore neglect small terms $\propto (M \propto)^2$. We then find for the perturbations

$$\tilde{N}'=(N_0'/N_0)\tilde{b}, \quad \tilde{b}'=\mu\tilde{E}N_0+(N_0'/N_0)(\tilde{N}-\mu\tilde{E}'),$$
$$M^2\varepsilon^2\tilde{E}''+\tilde{E}(1+\mu^2N_0)N_0^3+\mu\tilde{b}(N_0'/N_0)N_0^3=0. \tag{16}$$

The dash here indicates differentiation with respect to y and in the derivation we have dropped terms $\propto \beta \ll 1$.

We find the solution of (16) assuming that the perturbations are proportional to

$$\exp\left(\int dy\, k(y)/M\varepsilon\right),$$

where $k(y)$ is a smooth function. Putting this representation into the set we get

$$k^2+\tilde{\varepsilon}k'(y)+N_0^3[1+\mu^2(N_0-N_0'^2/N_0^2)]=0.$$

Using the fact that $\tilde{\varepsilon}, \mu \ll 1$ we easily find the solution of this equation

$$k\approx\pm iN_0^{3/2}[1+{}^1/{}_2\mu^2(N_0-N_0'^2/N_0^2)]-{}^3/{}_2\tilde{\varepsilon}N_0'/N_0. \tag{17}$$

Equation (17) determines the spatial periodicity and growth rate of the oscillations. As for a rarefaction soliton we have at the trailing edge ($y \to -\infty$) $N_0' < 0$ and at the leading edge $N_0' > 0$ (see Fig. 1) the result (17) means a build-up of oscillations from the front of the soliton to its center. Therefore incipient oscillations are magnified from the front towards the center and the soliton is filled by oscillations (Fig. 2). This distinguishes a rarefaction soliton from a compression soliton.[4] In the latter the Langmuir oscillations grow from the center to the periphery, i.e., in the direction in which the wave propagates.[5]

3.5. We attempt to answer the question of the limiting amplitude of the growing oscillations. Let there be small phonon oscillations (here $N_0' \to 0$) in front of the soliton ($y \to \infty$). According to what we have said earlier, as they move into the depth of the soliton they are amplified (here $N_0' > 0$) and afterwards in the center we have $N_0' = 0$ and

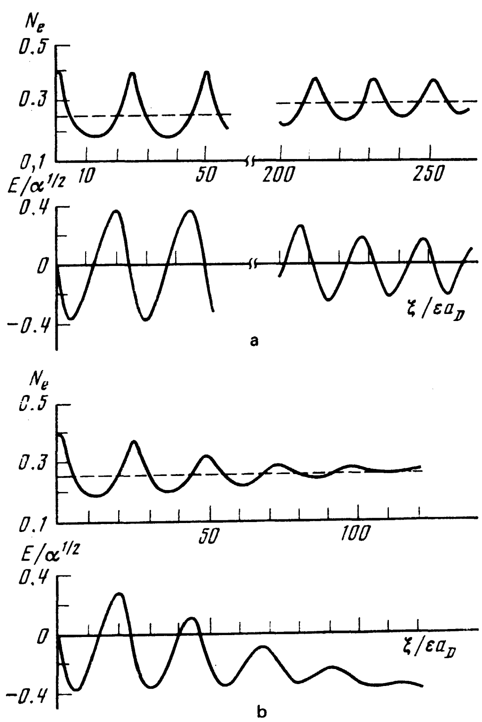

FIG. 2. Character of the Langmuir oscillations of the electrical field E and of the electron density N_e in the center of a rarefaction soliton $a_L/Ma = 10^{-3}$: a—$\varepsilon/\alpha^{1/2} = 0.05\beta$; b—$\varepsilon/\alpha^{1/2} = \beta$; dashed lines—ion density N.

there is no buildup, so that the oscillations saturate. The limiting level of the oscillations is thus determined by the solution of Eq. (10) at the center of the soliton:

$$M^2\varepsilon^2 d^2E/dy^2 = -(N_{\min} - \mu dE/dy)^3 E, \quad N_{\min} \approx M^2,$$
$$\mu = \varepsilon/\alpha^{1/2} = |\omega_{Be}/\omega_0| \ll 1. \tag{18}$$

Integrating Eq. (18) we find the relation

$$E^2(y) + \frac{M^2\varepsilon^2}{N_{\min}}\left[\frac{E'(y)}{N_{\min} - \mu E'(y)}\right]^2 = A^2, \tag{19}$$

where A is the amplitude of the oscillations. One easily concludes from this relation that a single-valued solution is only possible if the inequality

$$A^2 < \alpha(M^2/N_{\min}) \approx \alpha \equiv m_e/m_i \tag{20}$$

is satisfied. This then determines the amplitude of the Langmuir oscillations:

$$|E| = |E_x/B_0| \leqslant (m_e/m_i)^{1/2} \ll 1.$$

The two-fluid description thus leads to a qualitatively new result—it is possible to fill the single-fluid rarefaction soliton, determined by the ion magnetic viscosity, with small-scale Langmuir oscillations up to a level

$$E_x^2/8\pi \leqslant (m_e/m_i)(B_0^2/8\pi).$$

Finally, we note briefly possible applications of the results. In our opinion of most interest, when taking into account the characteristic spatial and temporal scales, are the rarefaction solitons which are caused by the magnetic viscosity for fast pinch discharges. For instance, in Ref. 1 the generation of Langmuir oscillations in a Z-pinch was observed. This fact may be connected with the generation of rarefaction solitons by the pinch; these, as we have just shown, amplify this kind of oscillations.

Moreover, the presence of a "well" in the magnetic field of a rarefaction/soliton is, possibly, of interest in the problem of particle acceleration in a Z-pinch and in the plasma focus across the strong magnetic field of a discharge. It was shown in Ref. 8 in the linear approximation that it is just rarefaction waves which are generated when the constriction of the pinch is terminated. It is clearly important in this scheme to generalize the results of the present paper to nonuniform (in particular, cylindrical) geometries.

§4. THE ROLE OF THE MAGNETIC VISCOSITY IN COMPRESSION-SOLITON DYNAMICS

4.1. The role of the magnetic viscosity turns out to be important also in the case when it is small. We shall show below that taking a small magnetic viscosity into account for the well known magnetosonic compression solitons[3] may lead to the buildup of oscillations with frequencies in the range $\omega \gtrsim |\omega_{Be}\omega_{Bi}|^{1/2}$.

We therefore assume that the scale (7) determined by the magnetic viscosity is small, i.e., $a_B/a_D \ll 1$. Moreover, we restrict ourselves solely to the case when charge separation is unimportant. We can obtain the corresponding equations from the set (8) to (11) putting formally $\varepsilon = 0$. The problem then reduces to two second order equations for the functions $b(\xi)$ and $N(\xi) \equiv N_e = N_i$:

$$2a_B^2\frac{N^2}{b}\frac{d}{d\xi}\frac{N'}{b} = \beta(N^2-1) + \frac{\beta}{2}N^2\frac{b-N}{b} + b^2 - 1 - 2M^2\frac{N-1}{N},$$
$$a_D^2\frac{d}{d\xi}\left[\frac{b'}{N} + \frac{\beta}{4}\frac{N}{b}\right] = b - N, \tag{21}$$

where we have dropped terms $\propto \alpha \ll 1$.

When there is no magnetic viscosity ($a_B = 0$) the system (21) describes in a zero pressure plasma ($\beta = 0$) a well known compression soliton determined by the finite electron inertia with a characteristic size $\xi_{\text{char}} \sim a_D = c/\omega_0$ and existing in a range of Mach numbers $1 < M < 2$ while the amplitude of the soliton equals $b_{\max} = 2M - 1$.[3] We further assume that the plasma pressure is not too low so that

$$\alpha \ll \beta \ll \alpha^{1/2}. \tag{22}$$

The ratio of the scales is then still small:

$$a_B/a_D \approx \beta/4\alpha^{1/2} \equiv 1/\tilde{\alpha} \ll 1,$$

and moreover, we can now drop in Eqs. (21) terms $\propto \beta$. Writing $y = \xi/a_D$ we then get

$$\frac{2}{\tilde{\alpha}^2}\frac{N^2}{b}\frac{d}{dy}\frac{N'}{b} = b^2 - 1 - 2M^2\frac{N-1}{N}, \tag{23}$$

$$\frac{d}{dy}\frac{b'}{N} = b - N. \tag{24}$$

We note that for fixed $b(y)$ Eq. (23) describes a nonlinear oscillator and when $b, N \sim 1$ the characteristic period of the oscillations will be $y_{\text{char}} \sim 1/M\tilde{\alpha} \ll 1$.

4.2. We consider the oscillations in the linear approximation putting in the set (23), (24)

$$b = b_0(y) + \tilde{b}(y), \quad N = N_0(y) + \tilde{N}(y),$$

where b_0 and N_0 are smooth functions determining the "equilibrium" level (i.e., the compression soliton) while the corrections $\tilde{b}$ and $\tilde{N}$ oscillate with the above indicated frequency and satisfy the equations ($\mu = 1/\tilde{\alpha}M \ll 1$)

$$\tilde{b}'(y) = b_0'\tilde{N}/N_0, \quad \mu^3\tilde{N}''' + \mu\tilde{N}'b_0^2/N_0^4 = \mu Q, \tag{25}$$

$$Q = \tilde{N}\frac{b_0^2}{N_0^4}\left[\frac{b_0'}{b_0}\left(1 + \frac{N_0 b_0^2}{M^2}\right) + 2\frac{N_0'}{N_0}\right] - 2\mu^2\tilde{N}''\left(\frac{N_0'}{N_0} - 2\frac{b_0'}{b_0}\right).$$

In deriving (25) we neglected small terms $\propto \mu^2$.

Putting in Eq. (25)

$$\tilde{N} \sim \exp\left(\frac{1}{\mu}\int k(y)\,dy\right),$$

where $k(y)$ is a smooth function we find approximately

$$k(y) \approx \pm i b_0/N_0^2 + \mu(N_0 b_0/M^2)\, b_0'; \tag{26}$$

clearly, the oscillations build up with increasing y when

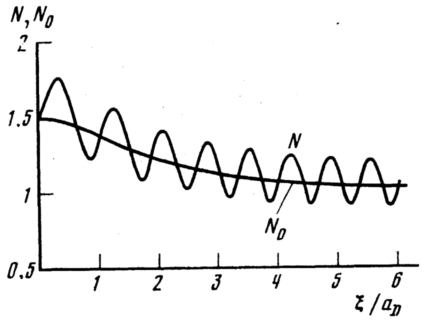

FIG. 3. Plasma density oscillations of a two-fluid compression soliton when a small magnetic viscosity is taken into account. $Ma_D/a_B = 10$, $M = 1.2$. N_0 is the density profile of the unperturbed soliton ($\beta = 0$).

$b_0' > 0$ and are damped if $b_0' < 0$. This means that the compression soliton is filled with oscillations of the indicated shape and it is just due to relation (26) that the amplitude grows from the periphery to the center of the soliton (Fig. 3).

We note that the spatial period in (26) corresponds to oscillations in time with a frequency (we assume b_0, N_0, $M \sim 1$)

$$\omega \sim 4\omega_{Bi}/\beta, \tag{27}$$

which when we use the conditions (22) on the quantity β gives the range

$$|\omega_{Be}\omega_{Bi}|^{1/2} \ll \omega \ll |\omega_{Be}| \ll \omega_0. \tag{28}$$

The authors are very grateful to S. A. Trushin, G. V. Sholin, and V. I. Petviashvili for discussing the present paper and for fruitful discussions.

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Translated by D.ter Haar