

Quantum theory of acoustic and electromagnetic nonresonant absorption in glasses

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A theory is developed of nonresonant sound and electromagnetic absorption in dielectric glasses at low temperatures T in the frequency range $\hbar\omega \gtrsim T$. It is shown that at $\hbar\omega \gtrsim T$ the nonresonant absorptances are proportional to ω^3 and are independent of temperature. The existence of a characteristic energy E_c is established, a typical value of which for glasses is 10–20 K. For level spacings exceeding E_c the conventional definition of a two-level system becomes meaningless, since the energy uncertainty due to interaction with the phonons exceeds the level spacing. Nor does perturbation theory hold for the calculation of the absorption at frequencies $\hbar\omega \gtrsim E_c$ or temperatures $T \gtrsim E_c$.

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§1. INTRODUCTION

The purpose of the present paper is to construct a theory of low-temperature nonresonant electromagnetic and ultrasonic absorption in glasses. We are primarily interested in the quantum region of frequencies ω satisfying the condition

$$\hbar\omega \gtrsim T \quad (1.1)$$

(T is the temperature in energy units). It is assumed that the absorption is due to interaction of ultrasound (or electromagnetic) waves with so-called two-level systems. The existence of such systems in glasses was suggested independently by Anderson, Halperin, and Varma¹ and by Phillips.²

The physics of acoustic and electromagnetic absorption in glasses are quite similar. We shall therefore carry out the entire exposition with acoustic absorption as the example, and present in §7 simply the result for electromagnetic absorption.

Next, so as not to clutter up the notation, we present first the results for the so-called isotropic model of two-level systems. In this model the deformation-potential tensors that characterize the interaction of these systems with the deformation (see below) degenerate into scalars, and the two-level systems themselves interact with longitudinal acoustic phonons. Later, however, we shall lift this restriction and consider the general case.

For high-frequency sound of sufficiently low intensity, the main contribution to the absorption is resonant. Its physics consists in the fact that the wave of frequency ω “chooses” for itself two-level systems with energy (level spacing) $E = \hbar\omega$, which are the ones responsible for the absorption. The obtained expression for the resonant absorption coefficient $\Gamma^{(r)}$ is^{1–3}

$$\Gamma^{(r)} = \frac{\pi\omega n(\hbar\omega)}{\rho v^3} \langle M^2 \rangle_{n\omega} \text{th} \frac{\hbar\omega}{2T}. \quad (1.2)$$

It is proportional to the quantity $\text{tanh}(\hbar\omega/2T)$, which is the equilibrium population difference between the lower and upper levels; $n(E)$ is the state density of the two-level system, and the expression for $\Gamma^{(r)}$ contains naturally $n(E)$ at $E = \hbar\omega$.

In general, however, according to the experimental data, $n(E)$ is a practically constant quantity at least in the energy interval E from several dozen mK to several K; M denotes the so-called deformation-potential constant that characterizes the amplitude of the probability of a transition between two levels under the influence of the deformation. The symbol $\langle \dots \rangle_E$ denotes averaging over all the two-level systems with specified value of E . From the experimental data one can apparently conclude that the quantity $\langle M^2 \rangle_E$ also depends quite little on E . Finally, v is the speed of sound and ρ is the density of the glass.

Resonant absorption decreases with increasing T , inasmuch as in this case the level populations become equal. The equalization of the populations takes place also when the sound intensity J increases. The characteristic intensity J_c at which this becomes noticeable is in fact quite low. For example, in experiment on sound absorption in SiO₂ glass at frequencies $\omega/2\pi \approx 0.5$ GHz, the value of J_c was $\approx 10^{-7}$ W/cm² at $T = 23$ mK (Ref. 4).

If the sound intensity exceeds J_c noticeably, the resonant absorption is suppressed and we are left with the so-called nonresonant or relaxation absorption (see, e.g., the review³). The reason for this designation is that the contribution from one arbitrary two-level system to this absorption is proportional to a quantity determined by a relaxation formula of the Debye or of the Mandel'shtam-Leontovich type; as applied to glasses, it was obtained by Jäckle⁵:

$$D^2 \frac{\omega^2 \tau}{1 + (\omega\tau)^2} \left(-\frac{\partial f_0}{\partial E} \right), \quad f_0 = \frac{1}{e^{E/T} + 1}. \quad (1.3)$$

Physically, such a relaxation contribution is due to the periodic (with frequency ω) level-spacing oscillation produced under the influence of the sound. Naturally, in this case the answer is expressed in terms of the diagonal constant of the deformation potential D , which characterizes the ratio of the separation of the levels under the influence of the deformation to the value of the deformation itself. As a result of variation of the level spacing, the distribution function f (which characterizes the occupation of the upper levels) lags the equilibrium value f_0 . The measure of the lag is deter-

mined by the relation between the frequency ω and the relaxation time $\tau(E)$, the latter given by^{3,5}

$$\frac{1}{\tau(E)} = \frac{M^2 E^3}{2\pi\rho\hbar^4 v^5} \operatorname{cth} \frac{E}{2T}. \quad (1.4)$$

This expression is proportional to the square of the matrix element M that characterizes the probability of transition accompanied by emission or absorption of a phonon of energy E . At $T = 0$ expression (1.4) yields simply the lifetime of the two-level system in the excited state relative to the spontaneous emission of a phonon of energy E . To obtain the total nonresonant absorption, an expression of the type (1.3) must be summed over all the two-level systems.

We shall be interested below in the high-frequency edge of the nonresonant absorption. The corresponding contribution is obtained from (1.3) if it is assumed that for most systems that take part in the absorption the following inequality is

$$\omega\tau(E) \gg 1. \quad (1.5)$$

In the classical frequency region, when $\hbar\omega \ll T$, the characteristic energy E which is the argument of τ in this inequality, is of the order of T . Substituting for this case (1.4) in (1.3) and summing over all the two-level systems, we arrive at the following result for the nonresonant absorption coefficient $\Gamma_i^{(nr)}$, obtained by Jäckle⁵:

$$\Gamma_i^{(nr)} = \frac{\pi^3 n_0 \langle M^2 D^2 \rangle}{32\rho^2 v^8 \hbar^4} T^3. \quad (1.6)$$

This result was obtained for the isotropic model. In addition, it was assumed in its derivation that mean values of the type $\langle M^2 D^2 \rangle_E$ are practically independent of E — it is precisely in this case that $\Gamma^{(nr)}$ is proportional to T^3 . Favoring this assumption is the fact that the $\Gamma^{(nr)} \sim T^3$ dependence is observed in experiment.⁶⁻⁸

The model in which both $n(E)$ and the mean values of the products of the quantities of the type M and D do not depend on E in the significant energy region will be called for brevity the E -independent model.

Our task is to obtain an expression for $\Gamma^{(nr)}$ in the quantum region of frequency, where the inequality (1.1) holds. To this end it is natural to use quantum-mechanical perturbation theory. Its first-order approximations should yield the correct answer both in the classical and in the quantum frequency regions provided that the inequality (1.5) is satisfied. We shall now formulate and discuss the results. Their derivation will be given in the subsequent sections.

In the lowest (second) perturbation-theory approximation, the nonresonant absorption is described by processes in which, besides the acoustic quantum $\hbar\omega$, one more phonon takes part. We assume that in this absorption process the very state of the two-level system changes, i.e., it goes over from the ground state 1 to the excited state 2 or vice versa. There are also possible processes in which the initial and final states of the two-level system coincide. In this approximation they describe elastic scattering of sound with conservation of its frequency. We shall not discuss these processes here. At the end of the introduction we shall examine briefly

how to distinguish in experiment between the absorption and scattering processes.

The general expression for the coefficient of nonresonant absorption, obtained in the isotropic model, takes the following form:

$$\Gamma_i^{(nr)} = \frac{\operatorname{sh}(\hbar\omega/2T)}{8\pi\rho^2 v^8 \hbar^3 \omega} \int_0^\infty dE n(E) \frac{E^2 \langle M^2 D^2 \rangle_E}{\operatorname{ch}(E/2T)} \times \left(\frac{\hbar\omega + E}{\operatorname{sh} \frac{\hbar\omega + E}{2T}} + \frac{\hbar\omega - E}{\operatorname{sh} \frac{\hbar\omega - E}{2T}} \right). \quad (1.7)$$

The factor of $n(E)$ in the integrand describes the contribution¹⁾ made to the absorption by two-level systems with energy E . If both $n(E)$ and $\langle M^2 D^2 \rangle_E$ are smooth functions of E , then the important role in the integral (1.7) at $\hbar\omega \ll T$ is played by values of E of the order of T , and this expression goes over into

$$\Gamma_i^{(nr)} = \frac{1}{4\pi\rho^2 v^8 \hbar^3 T} \int_0^\infty dE n(E) \frac{E^3 \langle M^2 D^2 \rangle_E}{\operatorname{sh}(E/T)}. \quad (1.8)$$

In the E -independent model, this formula goes over into (1.6).

In the opposite limiting case, $\hbar\omega \gg T$, we obtain from (1.7)

$$\Gamma_i^{(nr)} = \frac{1}{4\pi\rho^2 v^8 \hbar^3 \omega} \int_0^{\hbar\omega} dE n(E) \langle M^2 D^2 \rangle_E E^2 (\hbar\omega - E). \quad (1.9)$$

In the E -independent isotropic model it is easy to write down an expression for the case of an arbitrary relation between $\hbar\omega$ and T :

$$\Gamma_i^{(nr)} = \frac{\pi^3 n_0 \langle M^2 D^2 \rangle T^4}{16\rho^2 \hbar^3 v^8 \omega} \times \operatorname{th} \frac{\hbar\omega}{2T} \left[1 + \frac{16}{3\pi^2} \left(\frac{\hbar\omega}{2T} \right)^2 + \frac{16}{3\pi^4} \left(\frac{\hbar\omega}{2T} \right)^4 \right]. \quad (1.10)$$

From this we obtain at $\hbar\omega \gg T$

$$\Gamma_i^{(nr)} = \frac{n_0 \langle M^2 D^2 \rangle \omega^3}{48\pi\rho^2 \hbar v^8}. \quad (1.11)$$

The relation $\Gamma^{(nr)} \propto \omega^3$ can be easily interpreted physically. In second-order perturbation theory, absorption of a quantum $\hbar\omega$ is accompanied by excitation of the two-level system with energy E and by emission of a phonon of energy $\hbar\Omega = \hbar\omega - E$. The density of the phonon states is proportional to ω^2 . From among the two-level systems, the only ones that take part in absorption are those having $E \ll \hbar\omega$. The total number of such systems is also proportional to ω . In the upshot we obtain the proportionality $\propto \omega^3$.

It is quite difficult to satisfy the inequality $\hbar\omega \gg T$ in experiment for sound, since temperatures of the order of 1 K correspond to frequencies $\omega/2\pi$ in the 20 GHz range. We know of only one experiment in which the frequency dependence of the phonon mean free path l was measured in the frequency range $\omega/2\pi$ from 100 to 300 GHz at $T \sim 1$ K in SiO_2 . The result¹⁰ was $l^{-1} \sim \omega^{2.9 \pm 0.3}$. The measurements were performed at a total power level $(3-300) \times 10^{-4}$ W/cm².

We discuss in this article (§§5, 6) also the fundamental problem of the general structure of the theory of interaction of two-level systems with a phonon field, and of the applicability of perturbation theory to the description of this interaction. It is evident from (1.4) that this interaction increases rapidly with E , since perturbation theory should no longer be valid at a certain value E_c . This energy turns out to be of the order of

$$E_c \approx (\rho \hbar^3 v^3)^{1/2} / M \approx (mv^2 \Theta^3)^{1/2} / M, \quad (1.12)$$

where m is the average mass of the atoms making up the glass and Θ is the Debye temperature of the glass. In estimates of this kind we do not distinguish between the constants D and M , whose experimental values are of the same order, 1–2 eV. The characteristic energy E_c amounts then to 10–20 K, i.e., it is much lower than the Debye energy Θ . The quantity $\hbar/\tau(E)$ (the energy uncertainty of the two-level system) is rewritten in terms of E_c in order of magnitude in the form

$$\frac{\hbar}{\tau(E)} \approx \frac{E^3}{E_c^2} \operatorname{cth} \frac{E}{2T}.$$

This form explains the physical reason why perturbation theory is not valid: at $E \gtrsim E_c$ the energy uncertainty of the levels should exceed the distance between the levels.

The foregoing leads to the following conclusion that at $E \gtrsim E_c$ the concept of two-level systems in its traditional form^{1,2} becomes meaningless. It is tempting to compare this conclusion with the known experimental facts that definite anomalies are observed in the thermal conductivity of glasses and in their acoustic properties at $T \sim 10$ –20 K.

The condition that allows us to confine ourselves to the second-order perturbation-theory approximation in the description of nonresonant absorption takes at $\hbar\omega \gtrsim T$ the form

$$\omega \ll \omega_c \equiv E_c / \hbar, \quad (1.13)$$

while the perturbation-theory expansion is in powers of the parameter $(\omega/\omega_c)^2$. It can be assumed that for the inverse inequality the absorption is due to multiphonon processes. The development of a theory for absorption in this frequency region is one of the most interesting as yet unsolved problems.

The more complicated structure of the perturbation-theory series arises in the classical region of the frequencies $\hbar\omega \ll T$, where the expansion should be carried out in terms of two parameters, $1/\omega\tau(T)$ and $\hbar\omega/T$. It is usually more convenient, however, retaining the first parameter, to choose as the second the product

$$\frac{1}{\omega\tau(T)} \frac{\hbar\omega}{T} \approx \left(\frac{T}{E_c} \right)^2.$$

The twofold requirement: 1) applicability of perturbation theory and 2) a classical frequency interval, leads to the following double inequality

$$1 \ll \omega\tau(T) \ll (E_c/T)^2. \quad (1.14)$$

The upper bound on $\omega\tau(T)$ is none other than the condition $\hbar\omega \ll T$.

As a result, the following unusual situation arises in the classical frequency region (1.14). The resonant absorption (in

the linear regime) turns out to be proportional to the small parameter $\hbar\omega/T$, whereas the nonresonant absorption is proportional to an entirely different small parameter $1/\omega\tau(T)$. It turns out in final analysis that in the classical region there exists a certain frequency interval where the second parameter is larger than the first, i.e., where nonresonant absorption prevails over the resonant. This interval is given by the double inequality

$$T^3/E_c^2 \ll \hbar\omega \ll T^2/E_c; \quad (1.15)$$

it exists at $T \ll E_c$.

As for the case $T \gtrsim E_c$, we see no reason for assuming that the existing theories can describe the contribution of two-level systems in absorption in this temperature region, since it corresponds (as already mentioned) to the strong-coupling case.

In this paper we also take into account the anisotropy of the interaction of deformation with an individual two-level system. Detailed expressions for the absorption coefficients are given in §§3, 4, and 7. Here we wish only to note the following circumstances. The frequency and the temperature dependence of the total absorption coefficient in the E -independent model (with allowance for the anisotropy) can be represented by the following formula:

$$\Gamma \equiv \Gamma / \operatorname{th} \frac{\hbar\omega}{2T} = d\omega + a\omega^{-1}T^4 + b\omega T^2 + c\omega^3. \quad (1.16)$$

Here a , b , c , and d are constants; the first term in (1.16) describes the contribution of the resonant absorption; the three remaining ones describe the contribution of the nonresonant absorption.

Consider the difference $\tilde{\Gamma}(\omega, T_1) - \tilde{\Gamma}(\omega, T_2)$ at two different temperatures, T_1 and T_2 and at equal frequency ω . We have

$$\tilde{\Gamma}(\omega, T_1) - \tilde{\Gamma}(\omega, T_2) = (T_1^2 - T_2^2) [a\omega^{-1}(T_1^2 + T_2^2) + b\omega].$$

This difference depends thus on the frequency ω in nonmonotonic fashion: at relatively low temperatures it decreases, and then begins to increase. The minimum value is reached at the frequency

$$\omega_0 = [ab^{-1}(T_1^2 + T_2^2)]^{1/2}.$$

If $\operatorname{tanh}(\hbar\omega/2T)$ can be replaced with sufficient accuracy by unity at this frequency and at the temperatures T_1 and T_2 , this expression is simply the difference between the absorption coefficients at the temperatures T_1 and T_2 . This difference was investigated in experiment^{11,12} and it was found that it has a nonmonotonic dependence on ω . It is possible that the results of our theory are capable of explaining this behavior of this difference without invoking for this purpose other additional hypotheses.

So far, all the foregoing pertained to dielectric glasses. It is of interest, however, that the results can lead to an important conclusion with respect to the behavior of the coefficient of nonresonant absorption of sound in metallic glasses. It is well known (see, e.g., Ref. 13) that this absorption is usually due to the interaction of two-level systems with the conduction electrons. Owing to the high state density of the conduction electrons, the corresponding relaxation time of a

two-level system turns out to be shorter by several orders of magnitude than in interaction with phonons. The latter, however, decreases rapidly with the characteristic energy E of the two-level system, and as a result its role becomes more and more pronounced with increasing T or ω . This should mean, in particular, that at sufficiently high frequencies ω the phonon contribution to nonresonant sound absorption in metallic glasses may turn out to be larger than the electron contribution.

There are two types of experiment that permit observation of nonresonant absorption. In the first one observes the damping of traveling waves propagating through the glass. The problem in this case is how to separate in experiment the true sound absorption from its damping due to elastic scattering by the two-level systems. Increasing the intensity of the incident signal it is possible to suppress the resonant damping of the sound. The latter, in fact, is not absorption in the true sense of the word, but resonant scattering: all the absorbed waves are reradiated in various directions with their frequency unchanged.

It is still, however, not quite clear what happens with the scattering upon deviation from resonance when the sound intensity is increased. Such nonresonant scattering should also cause a certain damping of the traveling waves. The appropriate analysis calls for the development of a nonlinear theory of sound scattering, and this is beyond the scope of the present paper.

The second type of experiment consists in exciting acoustic oscillations in a resonator whose dimensions are much smaller than the mean free path of the sound waves relative to their nonresonant absorption. This can be a resonator of complicated shape, such as was used, e.g., by Ganapol'skiĭ *et al.*¹⁴ in experiments where inversion of acoustic waves took place and as a result it was possible to observe their true absorption. This can be also a system consisting of many resonators, such as glass powder. If it becomes possible to excite an electroacoustic echo in such a system, the nonresonant absorption can also be determined from the time dependence of the damping of the echo signals (see, e.g., Ref. 15).

§2. DIAGRAM TECHNIQUE. INTERACTION OF TWO-LEVEL SYSTEMS WITH PHONONS

The Hamiltonian of one two-level system interacting with the phonon field takes the form

$$H = H_0 + H', \quad H_0 = \frac{E}{2} \sigma_3 + H_{ph}, \quad H' = \sum_{i=1,3} K_{\alpha\beta}^i u_{\alpha\beta} \sigma_i. \quad (2.1)$$

Here H_{ph} is the Hamiltonian of the free field of the phonons, $\sigma_1 \equiv \sigma_x$ and $\sigma_3 \equiv \sigma_z$ are Pauli matrix elements acting in the space of the wave functions of the two-level systems, $u_{\alpha\beta}(\mathbf{r}_s)$ is the operator of the strain tensor of the glass vibrations and is expressed in standard fashion in terms of the phonon creation and annihilation operators; the vector \mathbf{r}_s defines the position of the given two-level system in space. To abbreviate the notation in the expressions that follow, we have introduced a single notation for the constants of the deformation potential tensor, putting $K_{\alpha\beta}^1 \equiv M_{\alpha\beta}$ and $K_{\alpha\beta}^3 \equiv D_{\alpha\beta}/2$.

To construct a diagram technique with the operators σ_i we use a method proposed by Abrikosov.¹⁶ We introduce the Fermi-field creation and annihilation operators a_M and a_M^+ and express in their terms the operators σ_i :

$$\sigma_i = \sum_{M, M'} a_{M'}^+ (\sigma_i)_{M'M} a_M,$$

where $(\sigma_i)_{MM'}$ ($M, M' = 1, 2$) are the elements of corresponding Pauli matrices.

The definitions of the phonon (or photon) Green's functions contain, besides the standard operations, also the averaging operation

$$\langle \dots \rangle = \text{Sp}' (e^{-H/T} \dots) / \text{Sp}' e^{-H/T}.$$

The prime denotes here that the trace is taken only over the physical states. For a single two-level system these are the states $|0,1\rangle$ and $|1,0\rangle$, and the unphysical states $|0,0\rangle$ and $|1,1\rangle$ are excluded.¹⁶ For the Fermi operators a_M and a_M^+ the physical states do not constitute a complete set. This makes the formulation of the diagram technique difficult. To get around this difficulty, Abrikosov proposed to carry out in the initial Hamiltonian an energy substitution

$$-(E_s/2) (-1)^M \rightarrow -(E_s/2) (-1)^M + \lambda$$

and to take into account in the calculation of the mean values all the states, and after calculating the mean values let λ go to $+\infty$, which corresponds physically to introduction of an infinitely large negative chemical potential. After this operation²⁾ a contribution to the mean value is made only by the physical state—this contribution is proportional to $\exp(-\lambda/T)$, whereas the unphysical state of the type $|1,1\rangle$ make an infinitely small contribution, proportional to $\exp(-2\lambda/T)$, compared with the first.

To eliminate the contribution of the unphysical state of the type $|0,0\rangle$ from those two-level systems whose operators are not located under the trace sign, it suffices to normalize the obtained expressions in suitable fashion.¹⁶ To this end, each diagram for the phonon Green's function is set in correspondence with a product of factors

$$\frac{e^{N_s/T}}{\mathcal{N}_s}, \quad \mathcal{N}_s = 2 \text{ch} \frac{E_s}{2T},$$

which extends over all the two-level systems (s in the number of the system) that contribute to the diagram.

As for the fermion Green's functions, the averaging in them is carried out over all the states.

The phonon damping, i.e., the sound absorption, is determined by the poles of the retarded Green's function. To find the latter it is necessary to continue analytically the temperature diagrams. This procedure was carried out by Maleev.¹⁷ We write down the main results obtained thereby. We consider the interaction of phonons with a single two-level system, and then sum over all such systems. In other words, we confine ourselves to the linear approximation in their density. The corresponding parameter is $K^2 n_0 / \rho v^2$, which amounts to several thousandths in typical cases.

The (retarded) Green's functions of the long-wave phonons are given by

$$d_j = \frac{1}{\rho} \left[\omega^2 - v_j^2 q^2 - \frac{1}{\rho} \chi_j(\omega) q^2 \right]^{-1}, \quad (2.2)$$

where the index j takes on two values: l and t (longitudinal and transverse phonons). The last term in the square brackets, due to the interaction of the phonons with the two-level systems, is determined by the following expressions (summation over the repeated symbols i and m is implied):

$$\chi_l(\omega) = \frac{1}{15} \sum_s \Pi_s^{im}(\omega) [2 \text{Sp } K_s^i K_s^m + \text{Sp } K_s^i \text{Sp } K_s^m], \quad (2.3)$$

$$\chi_t(\omega) = \frac{1}{30} \sum_s \Pi_s^{im}(\omega) [3 \text{Sp } K_s^i K_s^m - \text{Sp } K_s^i \text{Sp } K_s^m]. \quad (2.4)$$

Here $\Pi_s(\omega)$ is the polarization operator of the s -th two-level system and is determined by the sum of diagrams in Fig. 1. To each vertex of type 1 or 3 corresponds a Pauli matrix σ_1 or σ_3 . The solid lines, which denote the retarded Green's function of the fermions, corresponds to the matrix $g^{(0)}(\epsilon)$ with elements

$$g_{MM'}^{(0)}(\epsilon) = \frac{\delta_{MM'}}{\epsilon + i/2E(-1)^{M-\lambda} + i0}. \quad (2.5)$$

The wavy lines corresponds to the quantity

$$\frac{N(\omega) \omega^3 \Lambda(\omega)}{60\pi^2 \rho v_l^5} L_{im}, \quad (2.6)$$

which is the product of the spectral factor of the phonons and the matrix

$$L_{im} = \xi_1 \text{Sp } K^i K^m - \xi_2 \text{Sp } K^i \text{Sp } K^m, \quad (2.7)$$

that characterizes the interaction of the phonons with the two-level systems. Here

$$\xi_1 = 3 + 2(v_l/v_t)^5, \quad \xi_2 = 1 - (v_l/v_t)^5,$$

$N(\omega)$ is the Planck function, and $\Lambda(\omega)$ is a cutoff factor. In the simplest case it is equal to unity at $\omega < \omega_D$ and falls off rapidly to zero at $\omega > \omega_D$, where ω_D is the characteristic Debye frequency.

Finally, according to Maleev¹⁷

$$\begin{aligned} \Pi^{im}(\omega) = & \frac{e^{\lambda/T}}{2\pi i \mathcal{N}^2} \int_{-\infty}^{+\infty} d\epsilon e^{-\epsilon/T} \text{Sp } \sigma_i [g(\epsilon + \omega) \Gamma_{++}^m(\epsilon + \omega, \epsilon) g(\epsilon) \\ & - e^{-\omega/T} g^*(\epsilon + \omega) \Gamma_{--}^m(\epsilon + \omega, \epsilon) g^*(\epsilon) \\ & - (1 - e^{-\omega/T}) g(\epsilon + \omega) \Gamma_{+-}^m(\epsilon + \omega, \epsilon) g^*(\epsilon)]. \end{aligned} \quad (2.8)$$

The plus and minus signs of the vertex Γ^m indicate the signs of the infinitely small parts of the corresponding arguments:

$$\Gamma_{+-}^m(\epsilon, \epsilon') = \Gamma^m(\epsilon + i0, \epsilon' - i0).$$

We have left out of (2.8) the index s and will do so hereafter whenever it does not lead to misunderstanding.

The sound absorption coefficient, as is evident from (2.2) is

$$\Gamma_j = \frac{\omega}{\rho v_j^3} \text{Im } \chi_j(\omega). \quad (2.9)$$

In the calculation of (2.9) we shall find it convenient to use the following expression for the imaginary part of the polar-

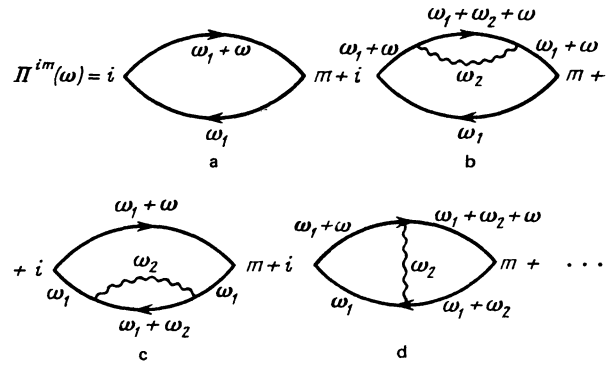


FIG. 1

ization operator, an expression that follows directly from (2.8):

$$\begin{aligned} \text{Im } \Pi^{im}(\omega) = & \frac{e^{\lambda/T}}{2\pi \mathcal{N}^2} (1 - e^{-\omega/T}) \text{Re} \int_{-\infty}^{+\infty} d\epsilon e^{-\epsilon/T} \\ & \times \text{Sp } \sigma_i [g(\epsilon + \omega) \Gamma_{+-}^m(\epsilon + \omega, \epsilon) g^*(\epsilon) \\ & - g(\epsilon + \omega) \Gamma_{++}^m(\epsilon + \omega, \epsilon) g(\epsilon)]. \end{aligned} \quad (2.10)$$

It can be seen that the contribution from each diagram in $\text{Im } \Pi(\omega)$ is determined by the imaginary part of the product of the Green's functions (2.5) of the upper arc of the loop of the polarization operator by the imaginary part of the product of the Green's functions of the lower arc.

We shall calculate $\text{Im } \Pi(\omega)$ by perturbation theory (the conditions for its applicability will be given in §5). The lowest approximation corresponds to diagram a of Fig. 1; it describes resonant absorption, which will be considered in the next section.

§3. RESONANT ABSORPTION

Substituting for Γ^m in (2.10) the nonrenormalized vertex σ_m , we obtain

$$\text{Im } \Pi^{im}(\omega) = \pi \text{th}(\hbar\omega/2T) [\delta(\hbar\omega - E) + \delta(\hbar\omega + E)] \delta_{ii} \delta_{m1}. \quad (3.1)$$

Substituting this expression in (2.9) and taking (2.3) and (2.4) into account, introducing the state density $n(E)$ of the two-level systems and integrating with respect to E , we obtain the following well known result^{1,18}:

$$\Gamma_j^{(r)} = \frac{\pi n(\hbar\omega) \omega}{\rho v_j^3} \langle M_j^2 \rangle_{\hbar\omega} \text{th} \frac{\hbar\omega}{2T}. \quad (3.2)$$

Here, by definition,

$$\langle M_i^2 \rangle_E = \frac{1}{15} \langle (\text{Sp } M)^2 + 2 \text{Sp } M^2 \rangle_E, \quad (3.3)$$

$$\langle M_t^2 \rangle_E = \frac{1}{30} \langle 3 \text{Sp } M^2 - (\text{Sp } M)^2 \rangle_E.$$

We recall that the angle brackets denote averaging over the ensemble of two-level systems with energy E .

It can be seen that the quantities (3.3) are non-negative and satisfy the inequality

$$\langle M_t^2 \rangle_E \geq \frac{4}{3} \langle M_i^2 \rangle_E. \quad (3.4)$$

It agrees with most experimental data. For example, in SiO_2 at the frequency $\omega/2\pi = 0.5$ GHz we have $\langle M_i^2 \rangle / \langle M_t^2 \rangle$

= 2.2 (Ref. 4). In a metallic glass such as Ni-P, this ratio is equal to 1.5 (Ref. 19).

§4. NONRESONANT ABSORPTION

The nonresonant absorption is calculated in the next order in the interaction with the phonons. In this order $\text{Im } \Pi(\omega)$ is determined by the sum of the diagrams with a single wavy line (diagrams b, c, and d of Fig. 1). After rather cumbersome calculations we obtain

$$\begin{aligned} \text{Im } \Pi^{11}(\omega) &= \frac{L_{33}}{30\pi\rho v_i^3 \hbar^3} \frac{\text{sh}(\hbar\omega/2T)}{\text{ch}(E/2T)} \\ &\times \left[\frac{(\hbar\omega+E)\Lambda(\hbar\omega+E)}{\text{sh} \frac{\hbar\omega+E}{2T}} + \frac{(\hbar\omega-E)\Lambda(\hbar\omega-E)}{\text{sh} \frac{\hbar\omega-E}{2T}} \right], \\ \text{Im } \Pi^{33}(\omega) &= \frac{L_{11}}{30\pi\rho v_i^3 \hbar^3 \omega^2} \frac{\text{sh}(\hbar\omega/2T)}{\text{ch}(E/2T)} \\ &\times \left[\frac{(\hbar\omega+E)^3 \Lambda(\hbar\omega+E)}{\text{sh} \frac{\hbar\omega+E}{2T}} + \frac{(\hbar\omega-E)^3 \Lambda(\hbar\omega-E)}{\text{sh} \frac{\hbar\omega-E}{2T}} \right], \\ \text{Im } \Pi^{13}(\omega) = \text{Im } \Pi^{31}(\omega) &= -\frac{L_{13}}{30\pi\rho v_i^3 \hbar^4 \omega} \frac{\text{sh}(\hbar\omega/2T)}{\text{ch}(E/2T)} \\ &\times \left[\frac{(\hbar\omega+E)^2 \Lambda(\hbar\omega+E)}{\text{sh} \frac{\hbar\omega+E}{2T}} + \frac{(\hbar\omega-E)^2 \Lambda(\hbar\omega-E)}{\text{sh} \frac{\hbar\omega-E}{2T}} \right]. \end{aligned} \quad (4.1)$$

The nonresonant character of these expressions can be seen directly.

To find the contribution made to the absorption coefficient by second-order processes we must substitute the expressions for $\text{Im } \Pi(\omega)$ in (2.3) and (2.4) and sum over s , i.e., over all the two-level systems. We shall perform the summation operation in two steps. We first sum over all the two-level systems whose energies E lie in the range from E to $E + dE$. This results in mean values of the products of the components of the deformation potential, of the type

$$\begin{aligned} f_{impn}(a, b) &= \frac{1}{225} \langle (a \text{ Sp } K^i K^m + b \text{ Sp } K^i \text{ Sp } K^m) \\ &\times (\xi_1 \text{ Sp } K^p K^n - \xi_2 \text{ Sp } K^p \text{ Sp } K^n) \rangle_E, \end{aligned} \quad (4.2)$$

where i, m, p and n are the indices 1 and 3, while a and b are numbers. These mean values are functions of the energy E . In the second stage it is necessary to multiply the expressions obtained by the state density $n(E)$ and integrate with respect to E . Since the characteristic energies that contribute to such integral do not exceed the larger of the quantities $\hbar\omega$ or T , which is much less than the Debye energy, we can put $\Lambda(\omega) = 1$. In the upshot we obtain the following general expression for the coefficient of the nonresonant absorption $\Gamma^{(nr)}$:

$$\begin{aligned} \Gamma_i^{(nr)} &= \frac{\omega \text{sh}(\hbar\omega/2T)}{2\pi\rho^2 v_i^3 \hbar^3 \omega^3} \int_0^\infty dE \frac{n(E)}{\text{ch}(E/2T)} \\ &\times \left\{ \frac{\hbar\omega - E}{\text{sh} \frac{\hbar\omega - E}{2T}} \left[k_i^{(1)} \left(1 + \frac{E}{\hbar\omega} \right)^2 \right. \right. \\ &\left. \left. - 2k_i^{(3)} \left(1 + \frac{E}{\hbar\omega} \right) + k_i^{(2)} \right] \right. \\ &\left. + \text{terms obtained by the} \right. \\ &\left. \text{substitution } E \rightarrow -E \right\}. \end{aligned} \quad (4.3)$$

We have introduced here the notation

$$\begin{aligned} k_i^{(1)} &= f_{3311}(2, 1), & k_i^{(4)} &= 1/2 f_{3311}(3, -1), \\ k_i^{(2)} &= f_{1133}(2, 1), & k_i^{(2)} &= 1/2 f_{1133}(3, -1), \\ k_i^{(3)} &= f_{1313}(2, 1), & k_i^{(3)} &= 1/2 f_{1313}(3, -1). \end{aligned} \quad (4.4)$$

The coefficient of $n(E)$ in the integrand of (4.3) describes the contribution made to the absorption by two-level systems with energy E . In particular, at $E = 0$, for degenerate systems, we obtain the result of Ivanov and Fishman²⁰ and of Maleev¹⁷, i.e., $\Gamma^{(nr)} \propto \omega^2$. In the case of the isotropic model ($k^{(1)} = k^{(2)} = k^{(3)}$), however, the absorption coefficient vanishes (at $E = 0$).

The integral (4.3) can be easily calculated for the E -independent model of glass. We have

$$\Gamma_j^{(nr)} = \frac{\pi^3 n_0 T^4}{16\rho^2 v_j^3 \hbar^3 \omega^3} \text{th} \frac{\hbar\omega}{2T} \left[A_j + B_j \left(\frac{\hbar\omega}{T} \right)^2 + C_j \left(\frac{\hbar\omega}{T} \right)^4 \right], \quad (4.5)$$

where the constant A, B , and C are given for (for any j)

$$A = 4k^{(1)}, \quad B = 8\pi^{-2} (k^{(1)} + k^{(2)} - 1/3 k^{(3)}), \quad C = \pi^{-2} B - \pi^{-4} A.$$

It can be verified that these constants are not negative and satisfy the inequalities

$$B_j > \frac{10}{9\pi^2} A_j, \quad (4.6)$$

$$B_i < \frac{3}{\pi^2} \left[\left(\frac{v_i}{v_t} \right)^5 + \frac{44}{27} \right] A_i, \quad (4.7)$$

$$A_i \geq \frac{4}{3} A_t, \quad B_i \geq \frac{4}{3} B_t; \quad (4.8)$$

$$A_i \geq \frac{\pi^2}{3} B_t, \quad B_i \geq \frac{16}{9\pi^2} A_t. \quad (4.9)$$

In the case when the tensors $M_{\alpha\beta}$ and $D_{\alpha\beta}$ degenerate into scalars (isotropic model), Eq. (4.5) goes over into (1.10).

In the limiting case $\hbar\omega \ll T$ we obtain from (4.5) a formula of the type (1.6). The only difference, however, is that the mean value $\langle M^2 D^2 \rangle_E$ in it is replaced in it by a quantity A_j , which is a mean value of much more complicated form. In the extreme quantum frequency region $\hbar\omega \gg T$ we obtain the relation $\Gamma^{(nr)} \propto \omega^3$, where, however, the proportionality coefficient $\langle M^2 D^2 \rangle$ is replaced by C_j .

§5. CONDITION FOR APPLICABILITY OF PERTURBATION THEORY

We start with an analysis of the simplest case $\hbar\omega \gg T$ (this is equivalent to $T = 0$). The order of magnitude of the

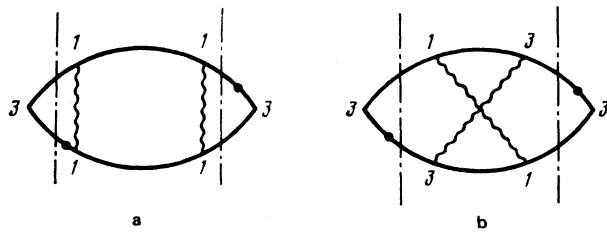


FIG. 2.

perturbation-theory parameter is

$$\Gamma^{(nr)}/\Gamma^{(r)} \approx (\omega/\omega_c)^2, \quad (5.1)$$

where $\hbar\omega_c = E_c$ [Eq. (1.13)]. The same parameter is obtained from a comparison of the two-phonon perturbation-theory approximations for $\text{Im } \Pi(\omega)$ (graphs a and b in Fig. 2) with the single-phonon graphs (b, c, and d of Fig. 1).

More complicated, as already stated in the Introduction, is the structure of the perturbation-theory series in the classical region $\hbar\omega \ll T$. To analyze this case we use expression (2.10) for the imaginary part of the polarization operator. It is evident from it that the integral

$$\mathcal{J} = \int_{-\infty}^{+\infty} \frac{dE}{\text{ch}(E/2T)} \text{Im } \Pi_E(\omega), \quad (5.2)$$

which determines the absorption of the sound, is dimensionless. It should be small compared with unity at least with respect to one of the parameters indicated in the introduction, $\hbar\omega/T$ or $1/\omega\tau(T)$. The contribution made to \mathcal{J} by the first-order diagrams (Fig. 1a) was calculated §3. Its order of magnitude is $\hbar\omega/T \ll 1$.

In next order, we have diagrams b, c, and d of Fig. 1. The principle contribution from them to \mathcal{J} at $\hbar\omega \ll T$ is of the order of

$$\mathcal{J} \approx K^2 T^3 / \varepsilon_0^4 \hbar\omega, \quad (5.3)$$

where $\varepsilon_0 \equiv (\rho \hbar^3 v^5)^{1/4} \approx (mv^2 \Theta^3)^{1/4}$ [see (1.13)]; for fused quartz it amounts to 500 K, for example. Expression (5.3) is none other than the small parameter $1/\omega\tau(T)$.

We can now compare in the classical region the nonlinear resonant contribution and the nonresonant lowest-order contribution. Since the former is determined by the small parameter $\hbar\omega/T$ and the latter by the small parameter $1/\omega\tau(T)$, there exists in the classical region a frequency and temperature interval where the second parameter is larger than the first, i.e., where the nonresonant absorption exceeds the resonant. This interval is determined by the inequalities (1.15).

Let us analyze the contribution from the higher-order diagrams. To each diagram there corresponds, for the polarization operator $\Pi_E(\omega)$, a factor $(K/\varepsilon_0^2)^{2n} \equiv E_c^{-2n}$, where n is the number of the phonon lines. We note furthermore that at $\hbar\omega \ll T$, the largest contribution from among all the diagrams of the given order is made by those containing the highest power of ω in the denominator. To determine this power, we introduce the concept of dangerous cross sections. A cross section is regarded by definition as dangerous if: 1) it intersects on the diagram two "fermion" lines whose

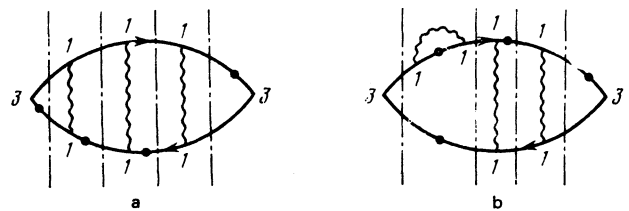


FIG. 3.

frequency arguments differ by ω ; 2) to both lines there corresponds either the ground or the excited state of the two-level systems; 3) the imaginary part is taken of one such line and the real of the other. The imaginary part of the propagator will be designated on the diagram by a point.

A diagram containing n phonon lines and m dangerous sections is of the order of

$$\frac{\hbar\omega}{T} \frac{T^{2n+m}}{E_c^{2n} (\hbar\omega)^m} = \frac{T^{2n+m-1}}{E_c^{2n} (\hbar\omega)^{m-1}}.$$

By way of example we indicate the following two diagrams, which contain two phonon lines each, see Fig. 2. The dangerous sections are indicated by a vertical dash-dot lines. The numbers 1 and 3 designate the types of vertices. To acquire the largest number of dangerous sections, both outermost vertices should belong to type 3. The indicated two diagrams yield, in order of magnitude,

$$\frac{T^5}{\hbar\omega E_c^4} \approx \frac{1}{\omega\tau} \left(\frac{T}{E_c} \right)^2.$$

Thus, the two-phonon nonresonant diagrams are related to the one-phonon diagrams of type b, c, and d of Fig. 1 as $(T/T_c)^2$.

In the next order, diagrams appear with four dangerous sections (Fig. 3). They yield

$$T^9 / (\hbar\omega)^3 E_c^6 \approx 1 / (\omega\tau)^2.$$

We can draw from the foregoing the following conclusion. The perturbation-theory series can be constructed in such a way that at any fixed power of $1/\omega\tau$ there appears as a factor a series in powers of T/T_c . This means, in particular, that it is possible to guarantee validity of the known relaxation formulas^{3,5} of the type (1.3) for the absorption coefficient only if $T \ll T_c$.

§6. INTERACTION OF TWO-LEVEL SYSTEMS WITH ZERO-POINT LATTICE VIBRATIONS

So far we have discussed the structure of the perturbation-theory series for real processes. In the next section we shall discuss the role of virtual processes, primarily the interaction of two-level systems with zero-point lattice vibrations. We shall show how this interaction renormalizes the bare level spacing E_0 .

The renormalized value of the energy E is determined by the difference between the two roots of the secular equation

$$\|g_{MM'}^{(0)-1}(\varepsilon, E_0) - \text{Re } \Sigma_{MM'}(\varepsilon, E_0)\| = 0, \quad (6.1)$$

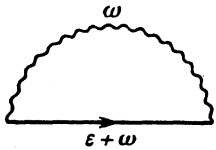


FIG. 4.

where Σ_{MM} is the total self-energy part of the two-level system. To consider an interaction with zero-point vibrations it suffices to analyze the case $T = 0$.

We consider by way of an illustrative and simple example the first-order approximation of perturbation theory, which is given by the diagram of Fig. 4. We have

$$\text{Re } \Sigma_{11,22} = \frac{\hbar}{60\pi^2 \rho v_i^3} \left[L_{11}^{(0)} \int_0^\infty \frac{\omega^3 \Lambda(\omega) d\omega}{\varepsilon - \hbar\omega \pm E_0/2} + L_{33}^{(0)} \int_0^\infty \frac{\omega^3 \Lambda(\omega) d\omega}{\varepsilon - \hbar\omega \mp E_0/2} \right],$$

where $L^{(0)}$ denotes an interaction corresponding to the unrenormalized vertex, the upper sign in front of $E_0/2$ pertains to Σ_{11} , and the lower to Σ_{22} . The main contribution in the integrals is made by ω of order ω_D . The characteristic energies ε likewise turn out to be of order $\hbar\omega_D$, as can be shown from (6.1). Expanding the denominators of the integrands in terms of the small ratio $E_0/\hbar\omega_D$, we obtain

$$\text{Re } \Sigma_{11,22} = -a_0(\varepsilon) \pm^{1/2} E_0 b_0(\varepsilon). \quad (6.2)$$

We can similarly find that

$$\text{Re } \Sigma_{12} = \frac{\hbar E_0}{60\pi^2 \rho v_i^3} L_{13}^{(0)} \int_0^\infty \frac{\omega^3 \Lambda(\omega) d\omega}{(\varepsilon - \hbar\omega)^2 -^{1/4} E_0^2},$$

from which it follows, at sufficiently small E_0 , that

$$\text{Re } \Sigma_{12} = c_0(\varepsilon) E_0. \quad (6.3)$$

For the coefficients $b_0(\varepsilon)$ and $c_0(\varepsilon)$ the following order-of-magnitude estimates are valid:

$$b_0 \approx c_0 \approx (K^{(0)})^2 \frac{(\hbar\omega_D)^2}{\varepsilon_0^4} \approx \left(\frac{K^{(0)}}{K} \right)^2 \left(\frac{\omega_D}{\omega_c} \right)^2. \quad (6.4)$$

In these estimates we took into account the fact that the definition of the energy E includes the renormalized vertex K , whereas the expression for the first-order perturbation-theory approximation of the mass operator contains the unrenormalized vertices $K^{(0)}$ (and in order-of-magnitude estimates we did not distinguish between the different unrenormalized vertices).

The coefficient b_0 or c_0 is in fact the perturbation-theory parameter in terms of which the expansion is made. The ratio $(\omega_D/\omega_c)^2$ is of the order of 10^2 , and there is every reason for assuming that perturbation theory cannot be used to describe the interaction with the zero-point oscillations of the lattice and it is necessary to analyze the exact self-energy parts. To this end we use the following relations:

$$\text{Re } \Sigma_{11}(\varepsilon, -E_0) = \text{Re } \Sigma_{22}(\varepsilon, E_0), \quad (6.5)$$

$$\text{Re } \Sigma_{12}(\varepsilon, -E_0) = -\text{Re } \Sigma_{12}(\varepsilon, E_0).$$

They are valid not only in the lowest-order perturbation theory but, as can be shown by analyzing the diagrams for Σ , also in the general case.

Expanding $\text{Re } \Sigma$ in terms of E_0 , we obtain relations of the type (6.2) and (6.3):

$$\text{Re } \Sigma_{11,22} = -a(\varepsilon) \pm^{1/2} E_0 b(\varepsilon), \quad \text{Re } \Sigma_{12} = c(\varepsilon) E_0, \quad (6.6)$$

where the coefficients a , b , and c are determined by the sum of all the diagrams. It is important, however, that the solution of Eq. (6.1), expressed in terms of these coefficients, is of the form

$$E = \frac{\{[1+b(\varepsilon_1)]^2 + 4c^2(\varepsilon_1)\}^{1/2}}{|1+a'(\varepsilon_1)|} E_0, \quad (6.7)$$

where ε_1 is the root of the equation

$$\varepsilon + a(\varepsilon) = 0, \quad (6.8)$$

and $a'(\varepsilon_1)$ is the derivative of $a(\varepsilon)$ with respect to ε at the point $\varepsilon = \varepsilon_1$. Thus, if we make the natural, in our opinion, assumption that the quantities $a'(\varepsilon_1)$, $b(\varepsilon_1)$ and $c(\varepsilon_1)$ are of the same order, and the quantity $|1+a'(\varepsilon_1)|$ is not anomalously small (we see no physical grounds for this), the renormalization factor turns out to be of the order of unity at any coupling force with the phonons. The other result is that because of the interaction with the phonons there is no additive increment to the energy E_0 . This statement follows directly from the symmetry of the initial theory.

Allowance for the finite temperatures leads to the appearance of temperature corrections to the renormalization coefficient in terms of the parameter T/T_c .

As for the vertices, they can become strongly renormalized by the interaction with the zero-point lattice vibrations. In our opinion, the natural point of view is to regard the quantities K' introduced in §2 as already renormalized on account of this interaction. It is precisely such quantities that are known from experiment, since they enter in the real physical effect; it is precisely these quantities which enter in the expression for the characteristic energy E_c . Thus, the initial Hamiltonian that describes the interaction of phonons with two-level systems includes two-level systems as well as vertex parts renormalized on account of the interaction with the zero-point oscillations.

In regard to the phonon spectrum, its renormalization on account of the interaction with the two-level systems is small at $\hbar\omega \ll E_c$ and $T \ll E_c$, being proportional to the small parameter $n_0 K^2/\rho v^2$.

§7. ELECTROMAGNETIC NONRESONANT ABSORPTION

The Hamiltonian of the interaction of an alternating electromagnetic field E with a two-level system is of the form²¹

$$\sum_{i=1,3} \mu^{(i)} \sigma_i E.$$

We have introduced here an identical notation for the dipole-moment constants of the system, putting $\mu^{(1)} = \mu'$ and $\mu^{(3)} = \mu/2$. Proceeding as before, we obtain for the absorption coefficient of the electromagnetic radiation, in the approximation linear in the center density, the following gen-

eral formula (the electromagnetic-absorption coefficient will be designated by the symbol Γ without a subscript):

$$\Gamma = \frac{4\pi\omega}{3c\sqrt{\epsilon}} \sum_s \mu_s^{(i)} \mu_s^{(h)} \text{Im} \Pi_s^{ih}(\omega). \quad (7.1)$$

Here c is the speed of light and ϵ is a dielectric constant of the glass. From this, using (3.1), we obtain for the resonant-absorption coefficient

$$\Gamma^{(r)} = \frac{4\pi^2\omega n(\hbar\omega)}{3c\sqrt{\epsilon}} \langle (\mu')^2 \rangle_{\hbar\omega} \text{th} \frac{\hbar\omega}{2T}. \quad (7.2)$$

The nonresonant absorption is due to second-order processes which proceed with absorption of a photon, emission or absorption of a phonon,²² and a change in the state of the two-level system. For the nonresonant absorption coefficient, using (4.1), we obtain in the E -independent, anisotropic model

$$\Gamma^{(nr)} = \frac{\pi^4 n_0 T^4}{4c\epsilon^{3/2} \rho v_i^5 \hbar^5 \omega} \text{th} \frac{\hbar\omega}{2T} \left[A + B \left(\frac{\hbar\omega}{T} \right)^2 + C \left(\frac{\hbar\omega}{T} \right)^4 \right]. \quad (7.3)$$

Here

$$A = 4\hbar h_{3311}, \quad B = 8\pi^{-2} (\hbar h_{3311} + \hbar_{1133} - \frac{1}{3}\hbar_{1313}), \quad (7.4)$$

$$C = \pi^{-2} B - \pi^{-4} A,$$

where

$$\hbar_{ikpn} = \frac{1}{45} \langle \mu^{(i)} \mu^{(h)} (\xi_1 \text{Sp} K^p K^n - \xi_2 \text{Sp} K^p \text{Sp} K^n) \rangle. \quad (7.5)$$

All three coefficients A , B , and C are positive. The first two of them satisfy the inequality (4.6).

At $\hbar\omega \ll T$, expression (7.3) does not depend on ω and is proportional to T^3 (Ref. 23). In the opposite case, $\hbar\omega \gg T$, the coefficient of nonresonant electromagnetic absorption should be proportional to ω^3 . A cubic frequency dependence of the absorption coefficient was observed in experiment in Refs. 12, 24 and 25.

The authors are sincerely grateful to S. V. Maleev for a very interesting discussion of the work and for a large number of helpful hints.

¹¹Compare with the result of Böttger and Bryksin⁹ for the contribution from two-center configurations to the hopping conductivity of a disordered semiconductor.

²¹As noted by Abrikosov, this procedure is not obligatory for spin 1/2, since the Pauli matrices σ_i , acting on unphysical states, yield 0. We, however, will use this procedure, since it simplifies all the calculations and, particularly, as follows from Maleev's paper,¹⁷ it facilitates the analytic continuation.

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