Effects of "weak" localization and of electron-electron interaction in thin copper and silver films

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The temperature and field dependences of the resistance of thin copper and silver films with surface-square resistances $R_\square \approx 3$–$400\,\Omega$ were investigated at temperaures $T = 1.4$–100 K and in magnetic fields $H = 0$–70 kOe parallel as well as perpendicular to the film plane. The observed $R(T)$ and $R(H)$ dependences can be explained within the framework of a theory that takes into account the joint manifestations of the effects of "weak" localization and electron-electron interaction in two-dimensional disordered systems. A comparison of the experimental data with the theory yields information on the energy and spin relaxation of the conduction electrons.

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1. INTRODUCTION

The recent interest in disordered systems with metallic conductivity is due to the prediction of two effects, "weak" electron localization (WEL)[1] and enhancement of electron-electron interaction (EEI) in the presence of impurity scattering.[2] For disordered metallic systems having finite values of the parameter $k_F l$ (k_F is the wave vector of the electron on the Fermi surface and l is the electron mean free path) allowance for these quantum effects leads to the appearance of corrections to the conductivity of the system, calculated on the basis of classical transport theory. The appearance of a localization contribution to the conductivity is due to the fact that at low temperatures the coherence of the electron wave function is preserved over distances $L_\varphi = (D\tau_\varphi)^{1/2}$ (D is the diffusion coefficient and τ_φ is the time of loss of phase coherence of the electron wave function) much larger than the electron mean free path, and become substantial interference effects for an electron moving along self-intersecting trajectories.[3] The limitation of the electron mean free path because of disorder of the system influences also the EEI, causing it to be stronger. The interelectron interaction at finite values of l leads to the appearance of corrections to the state density of the electrons on the Fermi surface and to the system conductivity.[2] The characteristic scale in the EEI theory is the coherence length in the normal metal $L_T = (\hbar D/kT)^{1/2}$.

An important feature of the contributions made to the resistance of disordered systems by the WEL and EEI is their anomalous dependence on the temperature, on the magnetic field, and on a numer of other experimental actions. The actual manifestations of these effects turnout to depend on the action of various mechanisms of conduction-electron energy and spin relaxation. An experimental study of the WEL and EEI effects is therefore not only of independent interest, but can also yield important information on electron scattering.

The results of a theoretical analysis of the WEL and EEI effects turn out to depend on the ratios of the dimensions of the disordered system to the characteristic lengths involved in the analysis of these effects. These lengths are L_φ, L_T, as well as a number of other characteristic lengths that appear, e.g., account is taken of spin-orbit interaction in elastic scattering of the electrons (L_φ^*) and of the external magnetic field (L_H) (see below). The most widely studied in experiments were two-dimensional systems (for which the thickness $d \ll L_\varphi, L_T$), with thin metallic films and inversion semiconductor layers as the examples. This interest is due, on the one hand, to the striking theoretical results obtained for such systems, and on the other to the practical importance of such objects. By now data were obtained on various manifestations of the WEL and EEI in thin films of Li (Ref. 4), Mg (Ref. 5), Cu (Refs. 6–8), Ag (Ref. 9), Pt (Refs. 10 and 11), Bi (Refs. 12 and 13), and a number of other materials.

In the present paper, the anomalous behavior of the conductivity of two-dimensional disordered systems as a function of the magnetic field and of the temperature is investigated with thin copper and silver films as examples. Preliminary results of the investigations were published earlier.[8,9] The choice of noble-metal films for the research was governed by the fact that, first, silver and copper are not superconductors and second, the free-electron model is well applicable to them.[1)] This simplifies the comparison of the theory with experiment and makes it possible to separate unambiguously the observed manifestations of the WEL and EEI effects, which turn out to be complicated in a number of cases. The results offer evidence that the low-temperature behavior of the conductivity of thin metallic films is the result of the joint manifestation of WEL and EEI and is adequately described by the existing theory of these effects. Thus, study of two-dimensional systems, on the one hand, permits verification of the main conclusion of the theory of the WEL and EEI effects, and on the other hand yields important information on the energy and spin relaxation of conduction electrons.

The article is organized in the following manner. We first specify the theoretical premises needed for the subsequent discussion of the results under presently realizable experimental conditions. We next describe the experimental procedure. The experimental result and their discussion are dealt with in the succeeding sections.

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2. THEORETICAL QUESTIONS

Let us specify the theoretical results as applied to the experiment described below, namely, consider the case of a two-dimensional metal film to which the free-electron model is applicable, and also the region of magnetic fields that satisfy the condition $H \ll kT/g\mu_B$ (g is the Landé factor of the conduction electron and μ_B is the Bohr magneton). (When the last condition is violated it is necessary to take into account the influence of the Zeeman splitting of the levels of the electrons on the corrections to the conductivity, necessitated by the WEL[14] and by the interelectron interaction.[15])

The temperature dependence of the resistance of two-dimensional disordered systems, due to the joint manifestation of the WEL and EEI effects and with account taken of the spin-orbit interaction under the condition $k_F l \gg 1$ and in the absence of a magnetic field, is given by[16]

$$\frac{R(T_1)-R(T_2)}{R(T_1)} = \frac{e^2 R_\square}{2\pi^2\hbar}\left\{\frac{3}{2}\ln\frac{\tau_\varphi^*(T_1)}{\tau_\varphi^*(T_2)} - \frac{1}{2}\ln\frac{\tau_\varphi(T_1)}{\tau_\varphi(T_2)} + [(1-F)+g(T)]\ln\frac{T_2}{T_1}\right\}, \quad (1)$$

where $R_\square$ is the resistance of a square of the surface of the film, F is the parameter connected with the Hartree correction to the exchange interaction, and $g(T)$ is the interelectron-interaction constant at a small combined momentum. The temperature dependence of the localized contribution, described by the first and second terms in the right-hand side of (1), is determined by the temperature dependence of the times[17]

$$\tau_\varphi^{-1} = \tau_\varepsilon^{-1} + 2\tau_S^{-1}, \quad \tau_\varphi^{*-1} = \tau_\varepsilon^{-1} + {}^2/_3\tau_S^{-1} + {}^4/_3\tau_{SO}^{-1}, \quad (2)$$

where τ_ε is the time of energy relaxation of the electrons, τ_S is the time of the elastic scattering with spin flip by paramagnetic impurities, and τ_{SO} is the time of spin relaxation due to spin-orbit interaction in elastic scattering of the electrons. Expressions (2) are valid for the case of isotropic spin scattering. When considering the interelectron interaction, an important role is played by the interaction between electrons that differ little in energy and momentum (the so-called diffusion channel) and between electrons that differ little in energy and have a small total momentum (the so-called Cooper channel). The contributions of these channels to the resistance correspond to the coefficient $(1-F)$ and $g(T)$ of the third term in (1), which take into account the EEI. We note that in the case of a power-law dependence of τ_φ or τ_φ^* on T the contribution made to the resistance by the WEL and EEI effects should depend logarithmically on the temperature.

When an electron moves in a magnetic field, the phase of its wave function can advance, and this can lead to violation of the interference effects and suppression of the WEL in classically weak magnetic fields, for which $\omega_c\tau \ll 1$ ($\omega_c = eH/mc$ is the cyclotron frequency and τ is the momentum relaxation time). As for the EEI, in the region of weak magnetic fields interaction in the Cooper channel is suppressed (on account of violation of the symmetry with respect to the time-reversal operation), whereas the interaction in the diffusion channel should be strongly influenced only by magnetic fields that satisfy the condition $H \gtrsim mc/e\tau, kT/g\mu_B$. The anomalous magnetoresistance (AMR) of disordered systems, due to these mechanisms, should be anisotropic in the two-dimensional case with respect to the mutual orientation of the magnetic field and the plane of the film, and should not depend on the orientation of the current relative to H. In a magnetic field perpendicular to the plane of the film ($H_\perp$) and satisfying the conditions $H \ll kT/g\mu_B$ and $\Phi_0/2\pi l^2$ (Φ_0 is the magnetic-flux quantum), the function $R(H_\perp)$ is of the form[16]

$$\frac{R(0)-R(H_\perp)}{R(0)} = \frac{e^2 R_\square}{2\pi^2\hbar}\left\{\frac{3}{2}f\left(2\frac{L_\varphi^{*2}}{L_{H_\perp}^2}\right) - \frac{1}{2}f\left(2\frac{L_\varphi^2}{L_{H_\perp}^2}\right) + g(T)\,\varphi\left(\frac{L_T^2}{\pi L_{H_\perp}^2}\right)\right\}, \quad (3)$$

where $f(x) = \ln x + \psi(\frac{1}{2} + 1/x)$, $\psi(y)$ is the logarithmic derivative of the Γ function,

$$\varphi(x) = \begin{cases} \ln x, & x \gg 1 \\ \zeta(3)x^2/4, & x \ll 1 \end{cases},$$

$\zeta(3) \approx 1.2$, $L_\varphi^* = (D\tau_\varphi^*)^{1/2}$, and $L_H = (\Phi_0/2\pi H)^{1/2}$ is the magnetic length of a particle which charge $2e$. We note that in the field region $H \gg \Phi_0/4\pi L_\varphi^{*2}(T)$, $\Phi_0/2L_T^2$ the resistance of a disordered two-dimensional system should depend logarithmically on H. The sign of the magnetoresistance due to suppression of the interaction in the Cooper channel (the third term in (3)) depends on the character of this interaction: $g(T) > 0$ for attraction between the electrons and $g(T) < 0$ for repulsion. The AMR due to the suppression of the localization is negative in the field region $H \gtrsim \Phi_0/4\pi L_\varphi^{*2}(T)$ and its sign at $H \lesssim \Phi_0/4\pi L_\varphi^{*2}(T)$ depends on the relation between τ_φ and τ_φ^*.

A disordered system, regarded as two-dimensional in the WEL and EEI theory, can be three-dimensional from the point of view of the classical theory of transport processes, and can in particular be characterized by a three-dimensional diffusion coefficient $D = \frac{1}{3}lv_F$ (v_F is the Fermi velocity). For such quasi-two-dimensional systems, whose thickness satisfies the condition $\lambda \ll d \ll L_\varphi, L_T$, where λ is the electron wavelength, the AMR should be observed in a magnetic field parallel to the plane of the system ($H_\parallel$).[18] At $H_\parallel \ll kT/g\mu_B$, $\Phi_0/2\pi l^2$, and $\Phi_0/2\pi d^2$ the $R(H_\parallel)$ dependence is of the form

$$\frac{R(0)-R(H_\parallel)}{R(0)} = \frac{e^2 R_\square}{2\pi^2\hbar}\left\{\frac{3}{2}\ln\left(1+\frac{d^2 L_\varphi^{*2}}{12L_{H_\parallel}^4}\right) - \frac{1}{2}\ln\left(1+\frac{d^2 L_\varphi^2}{12L_{H_\parallel}^4}\right) + g(T)\ln\left(1+\frac{d^2 L_T^2}{12L_{H_\parallel}^4}\right)\right\}. \quad (4)$$

We note that for quasi-two-dimensional disordered systems in a magnetic field $H \gtrsim \Phi_0/2\pi d^2$ a transition should be observed to the three-dimensional case characterized by isotropic AMR.[18]

3. EXPERIMENTAL PROCEDURES

The objects of the investigation were thin films of copper and silver, obtained by high-frequency sputtering, in an argon plasma, from corresponding bulk materials 99.999% pure in the case of copper and 99.99% pure in the case of

silver. Preliminary evacuation to a pressure $P \approx 2 \times 10^{-6}$ mbar was effected with a turbomolecular pump, the argon pressure was 1×10^{-2} mbar in the sputtering of the copper and 3×10^{-3} mbar in the sputtering of silver. The rate of deposition of the films at such argon pressures, at a voltage 3 kV between the electrodes, at a current density in the discharge ~ 5 mA/cm^2 amounted to ~ 730 Å/min for copper and ~ 850 Å/min for silver. (The deposition rate was calibrated with an interference microscope.) The films were deposited on glass substrates at room temperature. The averaged (weight) thickness of the films (d^w) was determined from the sputtering time, and the effective thickness of the films was determined with the aid of the relation

$$d = \rho_{ph} (R_{\square}^{300} - R_{\square}^{\min})^{-1}, \quad (5)$$

where ρ_{ph} is the resistivity of the pure metal at $T = 300$ K (ρ_{ph}(Cu) $= 1.55 \cdot 10^{-6}$ Ω-cm, ρ_{ph}(Ag) $= 1.49 \cdot 10^{-6}$ Ω-cm), $R_{\square}^{300}$ and $R_{\square}^{\min}$ are the resistances referred to a square of the film surface and measured at 300 K and at the temperature corresponding to the minimum of the temperature dependence of the resistance (see below). The determined values of d and d^w practically coincided at $d^W \gtrsim 50$ Å, and started to differ when the continuity limit was approached, thus, for films with $d^w = 30$–40 Å the values of d obtained from Eq. (5) were 5–10 Å.

The mean free path of the electrons was calculated with allowance for the fact that the specularity coefficient for polycrystalline copper and silver films is close to zero[19]:

$$l = l_b (1 + {}^3/_8 l_b/d)^{-1}, \quad l_b = l_{ph} (R^{300}/R^{\min} - 1), \quad (6)$$

where l_{ph}(Cu) $= 430$ Å and l_{ph}(Ag) $= 575$ Å (Ref. 19). The values of d, d^w, and l for a number of investigated samples are listed in Table I. The proximity of the values of d and l indicate that the elastic scattering of the electrons takes place predominantly by the surface of the film and is of diffuse character. The value of $R_{\square}$ turns out to be in this case proportional to d^{-2}.

The electron diffusion coefficient was determined from the formula $D = \frac{1}{3} l v_F$(v_F(Cu) $= 1.57 \cdot 10^8$ cm/sec, v_F(Ag) $= 1.4 \cdot 10^8$ cm/sec (Ref. 20)), which is strictly valid only for the case of three-dimensional diffusion. In the investigated films, the values of d and l were close in magnitude, so that determination of D with the aid of this formula should lead to systematic errors whose relative value, however, does not exceed 20%.

The resistance of the samples was measured with direct current. The theoretical dependences (1), (3), and (4) show that the relative change of the sample resistance should be proportional to $R_{\square}$. The minimum values of $R_{\square}$ of the investigated films, amounting to ~ 3 Ω, correspond to a required resistance-measurement accuracy $\sim 10^6$. When measuring the voltage across the sample at an accuracy $\Delta U = 10^{-7}$ V, as read with a digital voltmeter with a dynamic range 80 dB, the voltage drop across the sample, to ensure the required relative accuracy of the measurements, should amount to $U \gtrsim 0.1$ V. At the given value of U, to decrease heating of the film, and also overheating of the electrons in the electron field, it is necessary to increase the length of the sample and decrease its width. In the experiment we used samples with various geometries, obtained by photolithography and subsequent plasma etching. The longest samples were strips 10 μm wide, bent in form of a meander and having a total length 20 cm. In the measurements the field intensity in the films did not exceed 0.1 V/cm, and the corresponding measuring currents amounted to 1–10 μA.[2)] The upper limit of $R_{\square}$ of the investigated films is due to the fact that at $R_{\square} \gtrsim 500$ Ω even films coated with a photoresist layer and stored at room temperature age very rapidly.

The measurements were carried out in the temperature interval 1.4–300 K and the magentoresistance of the films was determined at temperatures from 1.4 to 100 K in magnetic fields 0–70 kOe produced by a superconducting solenoid. Certain difficulties are raised in the experimental study of R ($H_{\parallel}$) and its correct comparison with the corresponding theoretical relation (4). These are caused by the fact that the characteristic fields $H_{\perp}$ and $H_{\parallel}$ at which the localization-induced AMR begins to manifest itself are related as $H_{\parallel}/H_{\perp} \sim L_{\varphi}/d$, so that for thin films ($d \ll L_{\varphi}$) a negligible deviation from parallelism between the magnetic-field intensity vector and the plane of the film can cause the observed AMR to be the result of either the parallel or the perpendicular H component relative to the film of the plane. The restriction on the angle γ between H and the plane film can be obtained from the expression for L_{φ} in the presence of a magnetic field[18]:

$$L_{\varphi}^{-2}(H) = L_{\varphi}^{-2}(0) + L_{H_{\perp}}^{-2} + d^2/12 L_{H_{\parallel}}^4. \quad (7)$$

The influence of $H_{\perp}$ can be neglected by imposing on the value of γ the condition

$$\gamma \leqslant \arcsin [d/2\sqrt{3} L_{\varphi}(0)]. \quad (8)$$

TABLE I.

Sample No.	$R_{\square}^{min}$, Ω	d^w, Å	d, Å	l, Å	τ_{φ} (4 K), 10^{-11} sec	τ_S, 10^{-11} sec	τ_{SO}, 10^{-11} sec
Cu 1	4.03	100	111	96	8.0	30	1.1
Cu 2	9.85	73	69	64	1.5	5.0	1.0
Cu 3	12.6	73	50	59	1.1	3.0	0.80
Cu 4	28.0	60	42	38	1.6	4.6	0.84
Cu 5	132	24	20	17	0.63	1.6	0.46
Ag 1	6.16	80	79	93	11	50	2.3
Ag 2	7.07	80	72	87	7.1	70	2.1
Ag 3	17.3	58	57	56	12	$\geqslant$150	1.7
Ag 4	17.9	58	59	51	8.1	65	2.0
Ag 5	34.6	42	29	39	6.7	$\geqslant$100	0.94
Ag 6	229	35	5.2	11	3.9	14	1.0

In a number of cases these values of γ turn out to be small. Thus, for example, for the investigated silver films with $d \approx 50$ Å at $T = 2$ K the corresponding value of γ should not exceed 0.1°. We note that the restriction on γ becomes less stringent when L_φ is decreased (T is increased), and also for thicker films. In the measurements in the parallel field we were unable to set the plane of the sample parallel to the field with the accuracy required by condition (8). For a correct comparison of the experiment with the theory[18] we used therefore the following procedure. A Hall pickup was placed exactly in the plane of the sample and used to determine the value of $H_\perp$. This made it possible, knowing the value of H, determined γ (as a rule the values of γ were in the range 0–0.5°). When the value of γ did not satisfy the condition (8), we subtracted from the experimentally obtained AMR the AMR due to the action of H_φ (in this case $L_\varphi(H)$ was determined from (7)). We note that to carry out calculations with the aid of expression (4) it is necessary to use the effective value of d determined, e.g., with the aid of Eq. (5). The employed value of the weight thickness, determined as a rule with the aid of a quartz thickness meter, can be, in the case of almost solid films, the cause of the discrepancy between theory and experiment.

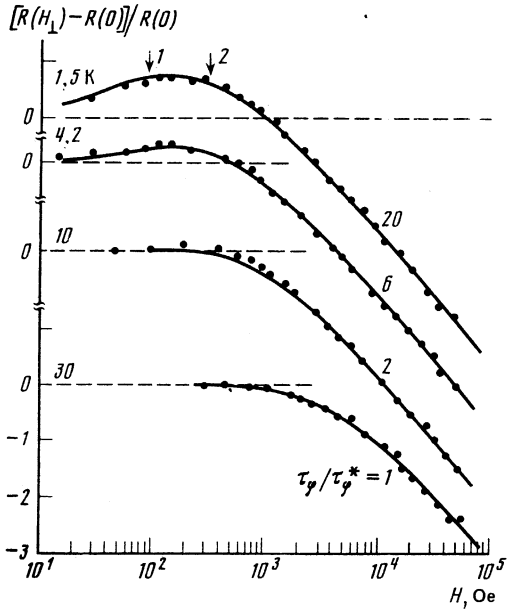

FIG. 1. Plots of $R(H_\perp)$ in units of $e^2 R_\square / 2\pi^2\hbar$, obtained for the sample Ag 4 at different T. Solid lines—theoretical relations (3) calculated at $g(T) = 0$ and at values of τ_φ and τ_φ^* shown in Fig. 2a. The arrows mark the values of the magnetic field for which $L_H = L_\varphi^*(1)$ and $L_H = L_T(2)$ are satisfied at $T = 1.5$ K.

4. RESULTS OF INVESTIGATION OF MAGNETORESISTANCE

We begin the description of the experimental results with consideration of the data on the magnetoresistance in a field perpendicular to the plane of the film. An investigation of the $R(H_\perp)$ dependence at different T makes it possible to determine uniquely all the characteristic lengths and relaxation times of interest to us, and the information on them will be used in the discussion of the other experimental results.

Figure 1 shows plots of $R(H_\perp)$ obtained for samples Ag 4 at different T (the samples are numbered in accord with Table I). The solid lines show the theoretical plots of (3), where the fit parameters were the times τ_φ and τ_φ^*, while $g(T)$ was assumed equal to zero. For all the investigated samples, the experimental $R(H_\perp)$ dependences are equally well described by the WEL theory.[16] (Exceptions are silver films with $R_\square \gtrsim 200\,\Omega$, for which the experimental and theoretical $R(H_\perp)$ differed at $H \gtrsim 20$ kOe, a field region in which their resistance became practically independent of H.) We note immediately that the agreement between a theory that does not take the EEI into account and the experimental data obtained in fields $H > \pi c k T / 2eD$ for copper and silver films is evidence of a small interaction constant at a small total momentum in these metals. This circumstance makes it possible, in the subsequent comparison of the theory with experiment, to disregard effects due to interelectron interaction in the Cooper channel, and also to disregard the mechanism of electron scattering by superconducting fluctuations, a mechanism present even in the case of effective repulsion between the electrons and leading to the same $R(H_\perp)$ depen-

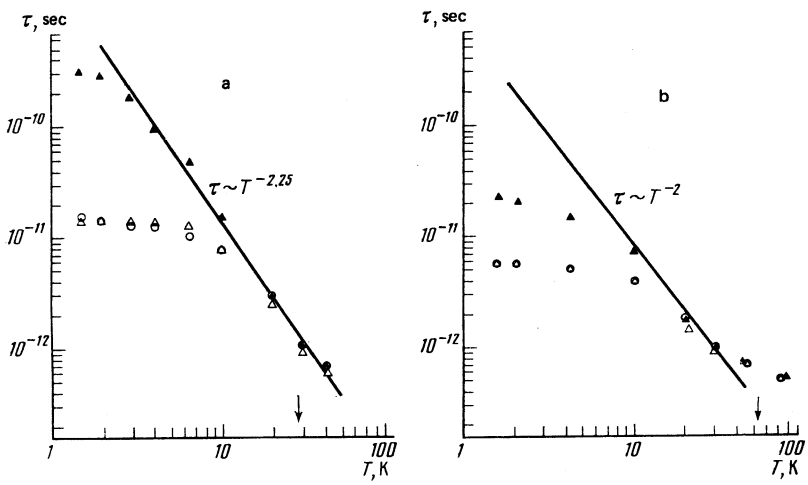

FIG. 2. Plots of $\tau_\varphi(T)$ and $\tau_\varphi^*(T)$ for the samples Ag 4 (a) and Cu 4 (b). ▲ and ○—experimentally obtained values of τ_φ and τ_φ^*, respectively, △—values of τ_φ^* calculated with the aid of τ_φ and the values of τ_S and τ_{SO} listed in the table. The arrow marks $\lambda_T \approx d$.

dence as the WEL (without allowance for the spin-orbit interaction), but with an additional factor $\beta(T) = -g^2(T)\pi^2/6(|g(T)|<1)$ (Ref. 21).

Figure 1 illustrates the observed transition, when the temperature is lowest, from single-parameter $R(H_\perp)$ ($\tau_\varphi = \tau_\varphi^*$), dependences obtained for the sample Ag 4 at $T \gtrsim 10$ K, to a two-parameter ($\tau_\varphi \neq \tau_\varphi^*$) dependence at $T < 10$ K. This behavior is due to the transition from the case with $\tau_\varphi \ll \tau_{SO}$ to $\tau_\varphi \gg \tau_{SO}$. Figures 2a and 2b show typical temperature dependences of τ_φ and τ_φ^*, obtained for samples Ag 4 and Cu 4. Both for Ag films and Cu films having different values of d, the $\tau_\varphi(T)$ and $\tau_\varphi^*(T)$ plots were similar in form; thinner films had a tendency to have smaller absolute values of τ_φ and τ_φ^* (see Table I).[3)] In the temperature interval $3 \lesssim T \lesssim 40$ K the values of τ_φ obtained experimentally for Ag are well described by the relation $\tau_\varphi \sim T^{-p}$, where $p \approx 2$. A similar temperature dependence of τ_φ is observed at $4.5 \lesssim T \lesssim 20$ K in Mg films.[5] For copper films, the temperature interval in which $\tau_\varphi \sim T^{-2}$ turns out to be narrower.

Measurements in a parallel field, performed in accordance with the procedure described in Sec. 3, yielded $R(H_\parallel)$ dependences that agreed well with the theoretical (4) ($g(T) = 0$), calculated for the values of τ_φ and τ_φ^* independently determined from measurements of $R(H_\perp)$. The change in the form of the $R(H_\parallel)$ dependences with temperatue for the sample Ag 3 is shown in Fig. 3. Figure 4 shows plots of $R(H_\perp)$ and $R(H_\parallel)$ obtained for sample Cu 2 at $T = 10$ K. The experimentally obtained value $\gamma \approx 0.4°$ satisfies at this temperature the condition (8), so that the fact that the field and the film plane are not exactly parallel can be disregarded. It can be seen that the experimental plots are well described by the theoretical expressions (3) and (4) with $g(T) = 0$ and the same values of τ_φ and τ_φ^*. In strong magnetic fields the $R(H_\perp)$ and $R(H_\parallel)$ curves come closer together, owing to the approach to the field region defined by the condition $L_H \lesssim d$, in which the sample becomes three-dimensional and is characterized by an isotropic magnetoresistance.

Let us analyze the experimentally obtained tempera-

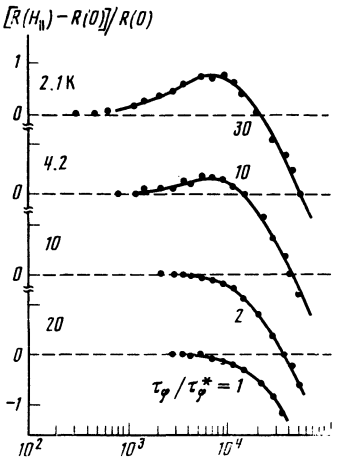

FIG. 3. Plots of $R(H_\parallel)$ in units of $e^2R_\square/2\pi^2\hbar$, obtained for the sample Ag 3 at different T. Solid curves—theoretical plots of (4) calculated at $g(T) = 0$.

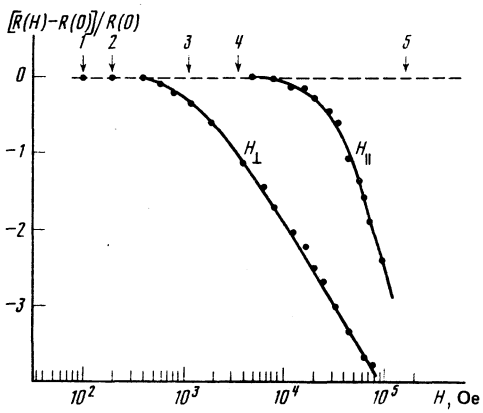

FIG. 4. Plots of $R(H_\perp)$ and $R(H_\parallel)$, in units of $e^2R_\square/2\pi^2\hbar$, obtained for the sample Cu 2 at $T = 10$ K. Solid lines—theoretical plots of (3) and (4) calculated for the values $\tau_\varphi = 9\cdot10^{-12}$ sec and $\tau_\varphi^* = 4.5\cdot10^{-12}$ sec. The arrows mark the values of H at which the relations $L_H = L_\varphi(10\ \text{K})$ (1), $L_H = L_\varphi^*(10\ \text{K})$ (2), $L_H = L_T$ (3), $L_H = (dL_\varphi^*(10\ \text{K})^{1/2}$ (4), $L_H = d$ (5) are satisfied.

ture dependence of τ_φ. The processes that lead to loss of phase coherence of the wave function of the electron and determine the value of τ_φ can, according to (2), be inelastic scattering and elastic scattering with spin flip from magnetic impurities. The clearly pronounced temperature dependence of τ_φ, observed for Ag films at $T \gtrsim 3$ K and for Cu films at $T \gtrsim 8$ K is evidence that in this temperature region the principal mechanism of the loss of phase coherence is inelastic scattering.

In principle, electrons can be elastically scattered both by phonons (τ_ε^{ph}) and by electrons (τ_ε^e). An analysis of the interelectron interaction with allowance for impurity scattering in the two-dimensional case ($d \ll L_T$) leads to the following expressionf or τ_ε^e (Ref. 22):

$$(\tau_\varepsilon^e)^{-1} = \frac{3\pi kT \ln[(k_F l)(k_F d)]}{2\hbar(k_F l)(k_F d)} + \frac{\pi(kT)^2}{8\hbar E_F}, \qquad (9)$$

where E_F is the Fermi energy, $k_F(\text{Ag}) = 1.2\cdot10^8$ cm^{-1} and $k_F(\text{Cu}) = 1.36\cdot10^8$ cm^{-1} (Ref. 20). The first and second terms in the right-hand side of (9) reflect the contributions corresponding to scattering with small and large (compared with h/l) momentum transfers. Estimates show that for the investigated films the values of τ_ε^e are determined by scattering with small momentum transfers and $\tau_\varepsilon^e \sim T^{-1}$. According to (9) for the same Ag 4 at $T = 10$ K we have $\tau_\varepsilon^e \approx 9\cdot10^{-11}$ sec. This exceeds by almost one order of magnitude the experimental value. The discrepancy between the theoretical estimates of τ_ε^e and the values of τ_φ obtained in experiment increase with increasing temperature and with increasing film thickness, and turn out to be small at $T \lesssim 4$ K for almost solid films.

The rate of energy relaxation due to inelastic electron-phonon interactions depends on the relation between the values of l, d, and the wavelength λ_T of the thermal phonons. For the investigated films at $T \lesssim hs/kd \approx 20$–$100$ K is the speed of sound) the condition $\lambda_T \gtrsim l,d$ is satisfied (the limits of the corresponding temperature interval are shown by arrows in Figs. 2a and 2b). In the "dirty" case ($\lambda_T \gg l$) the

phonon interacts with an electron that diffuses in the field of the impurities, and this leads to a change (compared with the "clean" case) in the expression for the characteristic time of the electron-phonon interaction (see, e.g., Ref. 23):

$$\tau_\varepsilon^{ph} = \frac{k_F l}{2\pi^2 a} \frac{\hbar}{k\Theta_D} \left(\frac{\Theta_D}{T}\right)^2, \tag{10}$$

where a is a constant of the order of unity, Θ_D is the Debye temperature, $\Theta_D(\text{Ag}) = 215$ K, $\Theta_D(\text{Cu}) = 315$ K (Ref. 20). When the condition $d \lesssim \lambda_T$ is satisfied, the phonon distribution function can be two-dimensional as well as three-dimensional, depending on the coefficient of phonon reflection from the film–substrate interface. For two-dimensional phonons in the case $\lambda_T \lesssim l$ the value of τ_ε^{ph} can be estimated by using an expression obtained for the three-dimensional case (see, e.g., Ref. 24) and by recognizing that in a direction perpendicular to the plane of the film the limit of the wave vectors of the phonons is the value $2\pi/d$. The expression obtained for τ_ε^{ph} in this manner is

$$\tau_\varepsilon^{ph} = \frac{1}{12\pi^2 \xi(3)\lambda} \frac{d}{s} \left(\frac{\Theta_D}{T}\right)^2, \tag{11}$$

where λ is the electron-phonon interaction constant ($\lambda(\text{Cu}) \approx 0.14$ $\lambda(\text{Ag}) \approx 0.13$). Estimates of τ_ε^{ph} obtained with the aid of (10) and (11) for the investigated films ($d \approx l$) are practically equal and turn out to be close in value to the experimentally determined values of τ_φ in the temperature where $\tau_\varphi \sim T^{-2}$. Thus, for the sample Ag 4, whose $\tau_\varphi(T)$ plot is shown in Fig. 2a, the value of τ_ε^{ph} obtained with the aid of Eqs. (10) and (11) at $T = 10$ K turns out to be 4×10^{-11} sec. Thus, both the absolute value and the temperature dependence of τ_φ in the indicated temperature range are in a reasonable agreement with the estimates of τ_ε^{ph}. It should be noted that the linear dependence of $\tau_\varepsilon^{ph}(d)$ ($d \approx l$), predicted by expressions (10) and (11), also agrees better with the experimental data on τ_φ than the relation $\tau_\varepsilon^e \sim d^2$ that follows from (9). The foregoing arguments seem to offer evidence that it is inelastic scattering by phonons which determines the temperature dependence of τ_φ for both copper and silver films.

The slowing down of $\tau_\varphi(T)$ at $T \lesssim 3$ K for silver and $T \lesssim 8$ K for copper may be due either to a change in the mechanism of the elastic scattering or to the "inclusion" of scattering with spin flip by paramagnetic impurities at $\tau_S < \tau_\varepsilon(T)$. Since the experimental $\tau_\varphi(T)$ dependences for a number of samples flatten out in practice at $T \lesssim 2$ K, the observed behavior of $\tau_\varphi(T)$ is apparently due to the restriction of τ_φ to the temperature-independent scattering time of the electron with spin flip, in which case $\tau_\varphi \approx \tau_S/2$. The fact that the copper films have smaller values of τ_S than silver films is possibly due to the appearance of a paramagnetic oxide on the copper-film surface.[25]

At $T \gtrsim hs/kd$ the phonon distribution function is close to three-dimensional independently of the degree of commensurability of the film with the substrate. Simultaneously, the relation $\lambda_T \lesssim l$ is satisfied and it becomes possible to use for τ_ε^{ph} the expression obtained in the "clean" case. For the investigated films, this transition takes place at temperatures amounting to 0.2–0.5 of Θ_D of the corresponding metals. At $T \gtrsim \Theta_D$ and $\lambda_T < l,d$ the relation $(\tau_\varepsilon^{ph})^{-1}(T)$ should be close to linear and this apparently explains the slowing down of the decrease of τ_φ at $T \gtrsim 50$ K.

An independent determination of $\tau_\varphi(T)$ and $\tau_\varphi^*(T)$ yields the times of scattering with spin flip on account of spin-orbit interaction in elastic scattering. The simplest is the procedure of determining τ_{SO} of samples for which $\tau_\varphi \approx \tau_S/2$ at low temperatures — in this case

$$(\tau_\varphi^*)^{-1} = {}^2/_3\tau_S^{-1} + {}^4/_3\tau_{SO}^{-1}.$$

Figures 2a and 2b show, besides the values of τ_φ^* determined from experiment, the values of τ_φ^* calculated with the aid of the values of τ_{SO} (which do not depend on T) obtained in this manner and the measured values of $\tau_\varphi(T)$. For pure films the values of τ_{SO} (see Table I) increase with increasing d and amount to $(0.4\text{–}1.3) \times 10^{-11}$ sec for copper films and $(1\text{–}2.5) \times 10^{-11}$ sec for silver films. The $R(H_\perp)$ dependences obtained in Refs. 6 and 7 for copper films also offer evidence of values of τ_{SO} close to those determined by us. At predominantly surface scattering, which is realized in the investigated films, and in the absence of impurities, the following relation should hold

$$\tau/\tau_{SO} \approx (\alpha Z)^4, \tag{12}$$

where $\alpha = e^2/\hbar c$ and Z is the charge of the nucleus of the atoms making up the films (see, e.g., Refs. 26 and 27). In experiment, however, for silver and copper whose Z differ by almost a factor of two, practically the same values of τ_{SO} were obtained. The apparent reason is that a layer containing impurity atoms was formed on the film surface in contact with the air, in which case the value of Z in (12) should be the difference between the charges of the impurity-atom nuclei and the film-material atom nuclei. This assumption is confirmed by the fact that contamination of the film surface with impurities having large values of Z (e.g., film sputtering in a setup in which lead was previously sputtered) leads to a considerable decrease of τ_{SO} while τ_φ remains constant. (Under controllable conditions, a decrease of τ_{SO} was obtained by Bergmann[5] in the precipitation of thin gold layers on a magnesium film.)

5. TEMPERATURE DEPENDENCE OF THE RESISTANCE

A study of the temperature dependences of the resistance of silver films and copper films has shown that for most investigated samples the decrease of the resistance with decreasing T, due to the "freezing out" of the phonons, slows down starting with a certain temperature and gives way to a growth of the resitsance. Exceptions are silver films for which the $\tau_\varphi \propto T^{-2}$ dependence can be tracked down to the lowest temperatures — for these samples the resistance flattens out with decreasing T. At a fixed value of $R_\square$ the value of $T^{\min}$ corresponding to the minimum of $R(T)$ turns out to be less for silver films, this being due to the relation $\Theta_D(\text{Ag}) < \Theta_D(\text{Cu})$. For copper films, the form of the $R(T)$ dependences at $T < T^{\min}$ is close to logarithmic; the coefficients A of the logarithms in the relations $\Delta R/R = A \ln(T_2/T_1)$ (in units of $e^2 R_\square/2\pi^2\hbar$) amount to 0.9–1.3. For silver films, approximation of $R(T)$ by a logarithmic dependence

led to values of A that ranged from ~ 0 to ~ 0.7. When the $R(T)$ dependences are measured in a magnetic field $H \gg \Phi_0/4\pi L_\varphi^2(T^{\min})$, the coefficients A for copper films remain practically unchanged, whereas for all the sample films a logarithmic increase of R appears and is characterized by values $A \approx 1$–1.2.

The observed behavior of the $R(T)$ dependences can be explained within the framework of the WEL and EEI theory by resorting to relaxation-time data obtained from an analysis of the magnetoresistance of the films. For power-law dependences of $\tau_\varphi(T)$ and $\tau_\varphi^*(T)$, the WEL and EEI theory (1) predicts at $H = 0$ a logarithmic $R(T)$ dependence with a prelogarithm coefficient A equal, in units of $e^2 R_\square/2\pi^2\hbar$, to

$$A = 1 - F + g(T) + {}^3/_2 p^* - {}^1/_2 p, \qquad (13)$$

where p and p^* are the exponents of the relations $\tau_\varphi \sim T^{-p}$ and $\tau_\varphi^* \sim T^{-p^*}$. For all the investigated films, an increase of the resistance is observed in the temperature region where $\tau_\varphi^*(T)$ reaches a plateau, therefore $p^* \approx 0$. In addition, data on the magnetoresistance point to values $g(T) \approx 0$. Therefore at $H = 0$ and for a power-law dependence of $\tau_\varphi(T)$ one should expect for the investigated films $A \approx 1 - F - \frac{1}{2}p$. At $H \gg \Phi_0/4\pi L_\varphi^2(T^{\min})$ the temperature dependence of the localization contribution to the resistance and an increase of A to $1 - F$ should occur.

The change of $R(T)$ with increasing H for the case $\tau_\varphi \propto T^{-2}$ is illustrated schematically in Fig. 5. Figure 5a shows the temperature dependence of the resistance, due to the joint manifestation of "weak" localization and the interelectron interaction in the diffusion channel ($F = 0$). The magnetic fields leads to cutoff of the logarithmic divergence of the localization contribution in a temperature region where $L_H \ll L_\varphi(T)$. Figure 5b shows, for different values of

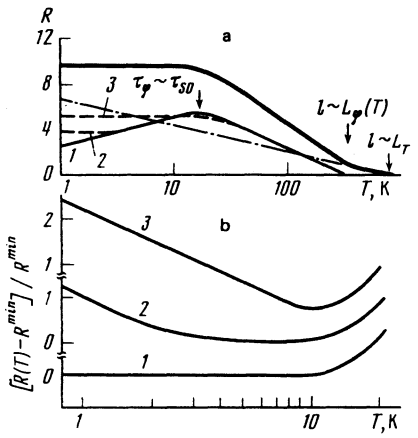

FIG. 5. a) Temperature dependences of the contributions made to the resistance, in units of $e^2R_0/2\pi^2\hbar$, by the EEI (dash-dot line) and WEL (thin solid line) when the conditions $F = 0$ and $\tau_\varphi \propto T^{-2}$ are satisfied. (The curves were plotted for l and τ_{SO} close to those realized in experiment). The thick solid line shows the sum of these contributions at $H = 0$. The dashed lines show the change of the temperature dependence of the localization contribution with increasing magnetic field: *1*—$L_H \gg L_\varphi$(1 K), *2*—$L_H \approx L_\varphi$(5 K), *3*—$L_H \approx L_\varphi$(20 K). b) Change, with increasing H, of the dependence of $(R(T) - R^{\min})/R^{\min}$, in units of $e^2R_\square/2\pi^2\hbar$, with account taken, besides the WEL and EEI effects, of scattering by phonons (which manifest itself at $T \gtrsim 10$ K). Curves 1, 2, and 3 correspond to cases 1, 2, and 3 in Fig. 5a.

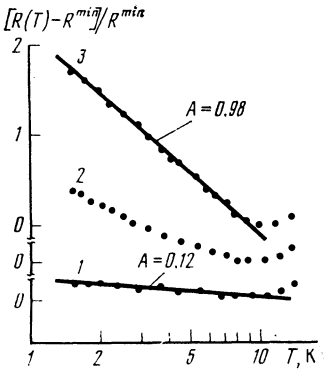

FIG. 6. Plots of $[R(T) - R^{\min}]/R^{\min}$ in units of $e^2R_\square/2\pi^2\hbar$ for the sample Ag 5, obtained for different values of H: *1*—$H = 0$, *2*—$H = 0.2$ kOe ($L_H \approx L_\varphi$(10 K), *3*—$H = 10$ kOe ($L_H \ll L_\varphi(T^{\min})$).

H, the $R(T)$ dependences with account taken of scattering by phonons. Thus, under the assumptions made, at low temperatures one should observe, with increasing H, a transition from the residual resistance regime to logarithmic growth of R with a coefficient $A \approx 1$ at $H \gg \Phi_0/4\pi L_\varphi^2(T^{\min})$. This is precisely the behavior observed for silver samples for which $\tau_\varphi(T)$ does not slow down its growth at low temperatures (see Fig. 6). For silver samples whose τ_φ at low temperatures is limited by scattering with spin flip, a situation that can be roughly treated as a decrease of p, the changes of the $R(T)$ dependence with increasing field turn out to be less pronounced. Thus, for example, for the sample Ag 4 whose $\tau_\varphi(T)$ dependence is shown in Fig. 2a the form of the $R(T)$ dependence at $H = 0$ is close to logarithmic with $A \approx 0.7$, while at $H \gg \Phi_0/4\pi L_\varphi^2(T^{\min})$ the value of A increases to ~ 1.2. Finally, for copper films the temperature dependence of τ_φ at $T < T^{\min}$ turns out to be suppressed by "turning-on" of temperature independent spin-relaxation processes. Therefore the magnetic field has practically no effect on the $R(T)$ dependence determined by the interelectron interaction in the diffusion channel. (Values $A \approx 1$ that do not depend on H were obtained also in Ref. 6 for granulated copper films.) Values $A \approx 1$ obtained at $H \gg \Phi_0/4\pi L_\varphi^2$ for silver and copper films point to a small screening parameter F in the investigated films. One should note in this connection that the estimate of F obtained on the basis of the free-electron model under the condition $k_F l \to \infty$ (see, e.g., Ref. 7) are apparently not applicable in the case of thin disordered films. The same conclusion can be reached by analyzing the experimental results obtained in Refs. 6 and 11–13.

6. CONCLUSION

Our investigation of the low-temperature behavior of the resistance of thin copper and silver films has shown that the anomalous temperature dependence of the resistance and the anomalous magnetoresistance of the investigated films are well described by the existing theory of the WEL and EEI effects in two-dimensional disordered systems. A comparison of the experimental magnetoresistance data obtained in a wide range of temperatures makes it possible to determine the temperature dependence and the absolute value of a number of relaxation times that characterize the pro-

cesses of electron scattering with a change of energy and spin, and also such interelectron interaction parameters as $g(T)$ and F.

By virtue of the small interaction constants at a small total momentum, the magnetoresistance of Ag and Cu films turns out to be governed exclusively by the suppression of the localization contribution to the resistance by the magnetic field. The temperature dependence of the resistance of the investigated films is determined by the joint manifestation of localization and interelectron interaction in the diffusion channel, and, depending on the relation between the times $\tau_\varepsilon(T)$, τ_S, and τ_{SO}, different situations are realized in experiment—from complete cancellation of the temperature dependences of the contributions of the WEL and EEI, and the assumption of $R(T)$ of its residual value, to the vanishing of the temperature dependence of the localization contribution and observation of a logarithmic growth of the resistance only as a result of interaction with the diffusion channel. (A substantial influence on the behavior of $R(T)$ is exerted by the form of the $\tau_\varphi(T)$ dependence.) At arbitrary relations between τ_ε, τ_S and τ_{SO}, the study of $R(T)$ in a magnetic field $H \gg \Phi_0/4\pi L_\varphi^{*2}(T^{\min})$ makes it possible to observe the temperature dependence of the resistance, which is governed only by the interaction in the diffusion channel.

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[1)]The last circumstance is important also because in this case the classical magnetoresistance of the investigated metals turns out to be small.
[2)]At these electric field intensities Ohm's law is satisfied for all the investigated samples.
[3)]In the table of Ref. 8 the values of τ and τ_{SO} for the first three samples are in error and must be multiplied by ten.

[1]E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
[2]B. L. Altshuler, A. G. Aronov, and P. A. Lee, *ibid.* **44**, 1288 (1980).
[3]L. P. Gor'kov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 228 (1979)].
[4]B. L. Al'tshuler, A. G. Aronov, B. Z. Spivak, D. Yu. Sharvin, and Yu. V. Sharvin, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 476 (1982) [JETP Lett. **35**, 588 (1982)].
[5]G. Bergmann, Phys. Rev. B **25**, 2037 (1982); Phys. Rev. Lett. **48**, 1046 (1962).
[6]F. Komori, S. Konahashi, Y. Ootuka, and W. Sasaki, J. Phys. Soc. Japan **50**, 1051 (1981).
[7]L. Van den Dries, C. Van Haesendonck, Y. Bruynseraede, and G. Deutscher, Phys. Rev. Lett. **46**, 565 (1981); Phys. Rev. B **25**, 5090 (1982).
[8]M. E. Gershenson and V. N. Gubankov, Sol. St. Commun. **41**, 33 (1982).
[9]M. E. Gershenzon, V. N. Gubankov, and Yu. E. Zhuravlev, Pis'ma Zh. Eskp. Teor. Fiz. **35**, 467 (1982) [JETP Lett. **35**, 576 (1982)].
[10]R. S. Markiewics and L. A. Harris, Phys. Rev. Lett. **46**, 1140 (1981).
[11]H. Hoffmann, F. Hofmann, and W. Schoepe, Phys. Rev. B **25**, 5563 (1982).
[12]A. K. Savchenko, V. N. Lutskiĭ, and A. S. Rylik, Pis'ma Zh. Eksp. Teor. Fiz. **34**, 367 (1981) [JETP Lett. **34**, 349 (1981)].
[13]Yu. F. Komnik, E. I. Bukhshtab, and Yu. V. Nikitin, Fiz. Nizk. Temp. **7**, 1350 (1981) [Sov. J. Low Temp. Phys. **7**, 656 (1981)].
[14]S. Maekawa and H. Fukuyama, J. Phys. Soc. Japan **50**, 2516 (1981).
[15]P. A. Lee and T. V. Ramakrishnan, Phys. Rev. (in press).
[16]B. L. Al'tshuler, A. G. Aronov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Zh. Eksp. Teor. Fiz. **81**, 768 (1981) [Sov. Phys. JETP **54**, 411 (1981)].
[17]S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980).
[18]V. L. Al'tshuler and A. G. Aronov, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 515 (1981) [JETP Lett. **33**, 499 (1981)].
[19]K. L. Chopra, Thin Film Phenomena, McGraw, 1969, Chap. 2.
[20]C. Kittel, Introduction to Solid State Phys., Wiley, 1956, Chap. 10.
[21]A. I. Larkin, Pis'ma Zh. Eksp. Teor. Fiz. **31**, 239 (1980) [JETP Lett. **31**, 219 (1980)].
[22]B. L. Altshuler, A. G. Aronov, and D. E. Khmelnitskii, J. Phys. C (1981), in press.
[23]H. Takayama, Z. Phys. **263**, 329 (1973).
[24]C. J. Pethick and H. Smith, Ann. Phys. (N.Y.) **119**, 133 (1979).
[25]S. Kobayashi, T. Takahashi, and W. Sasaki, J. Phys. Soc. Japan **32**, 1234 (1972).
[26]A. A. Abrikosov and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **42**, 1088 (1962) [Sov. Phys. JETP **15**, 752 (1962)].
[27]R. Meservey and P. M. Tedrow, Phys. Rev. Lett. **41**, 805 (1978).

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