Theory of generation of a strong field in a semiconductor laser

V. M. Dubovik, V. D. Popov, and V. P. Yakovlev

Engineering-Physics Institute, Moscow

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A kinetic theory of the generation of a strong field in a semiconductor laser is generalized to the case of spatially inhomogeneous fields of the standing wave type. The Winger representation is used to obtain rate equations for the density matrix of electrons in a semiconductor, which interact with the generated field. It is shown that under conditions of a high Q factor the saturation of such generation occurs in fields higher than in the homogeneous case. Under low-Q conditions the spatial inhomogeneity of the field has practically no effect on the maximum possible field. An estimate is obtained of the contribution of the elastic impurity scattering to the limiting value of the generation field in the case of a semiconductor laser characterized by band–band electron transitions. The dependence of the limiting field on the concentration of an impurity in a semiconductor is obtained for a semiconductor laser exhibiting impurity–band transitions. The results obtained are in qualitative agreement with the published experimental data.

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In the kientic theory of generation of a strong field in semiconductor lasers developed by Galitskiĭ and Elesin[1] it is shown that there is a limiting lasing field in the steady-state single-mode regime. The effect is related directly to the appearance in the electron excitation spectrum of a gap[2] of width 2λ, where $\lambda = \mathbf{d}\cdot\mathbf{E}$ is the frequency of field-induced interband transitions which, under strong field conditions defined by $\lambda\tau_{ph} \gg 1$, exceeds the frequencies of collisions of electrons with phonons, other electrons, etc. The theory of the effect is in qualitative agreement with the experimental results.[3–6]

The case of a traveling wave is considered in Refs. 1 and 7 and this gives rise to spatially homogeneous solutions of the rate equations describing lasing. However, in practice one frequently encounters a situation when the field in a laser is a standing wave (or, more generally, a superposition of standing and traveling waves).

Since the wavelength of the emitted radiation is usually considerably less than the linear dimensions of the active zone, $kL \gg 1$, the spatial inhomogeneity of the field becomes significant.

As shown in Ref. 8, there is no gap in the spectrum of quasiparticle excitations in the field of a standing wave. Therefore, it is not a priori evident how the nature of the lasing equations then changes.

We shall generalize the kinetic theory of lasing to the case of spatially inhomogeneous fields of the standing wave type. We shall consider the main physical aspects.

The interaction of electrons with the generated field is allowed for by going over to quasi particles employing the unitary (u,v) transformation. The transformation itself, and the dispersion law of quasiparticles

$$\varepsilon(\mathbf{r}, \mathbf{p}) = (\xi_p^2 + |\lambda(\mathbf{r})|^2)^{1/2}, \quad \xi_p = (p^2 - p_0^2)/2m, \quad \lambda(\mathbf{r}) = \mathbf{d}\mathbf{E}(\mathbf{r})$$

now depend on the local value of the field. In the case of a homogeneous field this results in exact diagonalization of the Hamiltonian[2] whereas in the present case it is diagonalized in the quasiclassical approximation which is valid because of the smallness of the wave vector of the field compared with the characteristic electron momenta ($k \ll p \sim p_0$).

A strong field saturates direct transitions between the bands and the amplification (or absorption) of the field is due to indirect transitions involving phonons. According to Ref. 1, under steady-state conditions (at $T = 0$) the rate of creation of strong-field photons is determined by the rate of annihilation of quasiparticles accompanied by phonon creation. The kinematics of this process imposes an upper limit on the phonon frequency $\omega_{\mathbf{q}} \leqslant \omega_{ph} = 2p_0 s$, where s is the velocity of sound, and it determines the regions of space $|\lambda(\mathbf{r})| \leqslant \omega_{ph}/2$ which make contribution to the annihilation process. We can readily see that in a homogeneous field we have $\lambda = \omega_{ph}/2$ (Ref. 1). In a standing wave $\lambda(\mathbf{r}) = \lambda \cos \mathbf{kr}$, when there is no gap in the spectrum, the amplitude of the field may exceed $\omega_{ph}/2$. However, an incrase in the field reduces the spatial volume of the annihilation region and the annihilation rate decreases. As soon as this rate becomes comparable with the loss rate in the resonator $\sim \tau_0^{-1}$, saturation of lasing takes place. This corresponds to the case of a high Q factor ($\eta \sim \tau_0/\tau_{ph} \gg 1$). It is shown in §2 that in this case the limiting value of the field is $\lambda_0 = \omega_{ph}\eta^{1/3}$. We shall discuss here the process of saturation of the generated field, which is in the form of a superposition of standing and traveling waves.

In the low-Q case (§3) the spatial inhomogeneity of the generated field has practically no influence on its limiting value.

The results obtained are generalized to the case when the active material of a semiconductor laser is a doped semiconductor.

§1. RATE EQUATIONS

In the resonance approximation the Hamiltonian of electrons in a semiconductor interacting with a strong electromagnetic field

$$\mathbf{E}_1 \cos(\mathbf{kr} - \omega t) + \mathbf{E}_2 \cos(\mathbf{kr} + \omega t)$$

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and with phonons has the form

$$H_e = \int d^3r \Psi^+ [H_0\sigma_3 + \lambda(\mathbf{r})\sigma_+ + \lambda^*(\mathbf{r})\sigma_-]\Psi, \quad (1)$$

$$H_{eph} = g\int d^3r \Psi^+ \varphi \Psi, \quad \hbar = c = 1, \quad (2)$$

where $\Psi(\mathbf{r},t)$ is a two-component electron field operator, $\varphi(\mathbf{r})$ is the phonon field,

$$H_0 = \nabla^2/2m - \mu_0, \quad \mu_0 = p_0^2/2m = (\omega - E_g)/2,$$
$$\lambda(\mathbf{r}) = (\lambda_1 e^{i\mathbf{kr}} + \lambda_2 e^{-i\mathbf{kr}})/2, \quad \lambda_i = \mathbf{d}\mathbf{E}_i,$$

d is the matrix element of the interband transition, σ are the Pauli matrices, and V is the volume. We shall use a model of two symmetric energy bands with identical masses and a quadratic dispersion law. The frequency ω of the strong field is close to the band gap E_g.

We shall introduce the density matrix ρ and the correlation function y in the Wigner representation:

$$\left\langle \Psi^+\left(\mathbf{r}+\frac{\mathbf{r}'}{2},t\right)\Psi\left(\mathbf{r}-\frac{\mathbf{r}'}{2},t\right)a_j^+ e^{i\mathbf{b}_j\mathbf{r}}\right\rangle$$
$$= \int \frac{d^3r}{(2\pi)^3} e^{-i\mathbf{pr}'}\chi_j(\mathbf{r},\mathbf{p},\mathbf{b}_j,t), \quad (3)$$
$$\chi_j(\mathbf{r},\mathbf{p},\mathbf{b}_j,t) = \{\rho(\mathbf{r},\mathbf{p},t),\ y(\mathbf{r},\mathbf{p},\mathbf{q},t)\},$$

$$\mathbf{b}_j = \{0,\mathbf{q}\}, \quad a_j^+ = \{1, c_\mathbf{q}^+\}, \quad (4)$$

and $c_\mathbf{q}^+$ is the phonon creation operator.

Standard procedures[9,10] allow us to use the weakness of the interaction between electrons and phonons to obtain the following equations for ρ and y:

$$\left(-i\frac{\partial}{\partial t} - \Omega_j + D_1 + D_2 + D_3\right)\chi_j(\mathbf{r},\mathbf{p},\mathbf{b}_j,t) = 0, \quad (5)$$

where

$$D_1\chi_j = \left[\frac{1}{8m}(\nabla + i\mathbf{b}_j)^2 - \xi_p\right](\sigma_3\chi_j - \chi_j\sigma_3)$$
$$+ \frac{-i\mathbf{p}(\nabla + i\mathbf{b}_j)}{2m}(\sigma_3\chi_j + \chi_j\sigma_3) + [\lambda(\mathbf{r})(\chi_j\sigma_- - \sigma_-\chi_j) - \text{H.c.}],$$
$$D_2\chi_j = \delta_{j,2}g(\mathbf{q})\Big[(1+N_\mathbf{q})\chi_1\left(\mathbf{r},\mathbf{p}+\frac{\mathbf{q}}{2},t\right)$$
$$-N_\mathbf{q}\chi_1\left(\mathbf{r},\mathbf{p}-\frac{\mathbf{q}}{2},t\right) - \chi_1\left(\mathbf{r},\mathbf{p}+\frac{\mathbf{q}}{2},t\right)\chi_1\left(\mathbf{r},\mathbf{p}-\frac{\mathbf{q}}{2},t\right)\Big],$$
$$D_3\chi_j = \sum_\mathbf{q}\delta_{j,1}g(\mathbf{q})$$
$$\times\left\{\left[\chi_2\left(\mathbf{r},\mathbf{p}+\frac{\mathbf{q}}{2},\mathbf{q},t\right) - \chi_2\left(\mathbf{r},\mathbf{p}-\frac{\mathbf{q}}{2},\mathbf{q},t\right)\right] - \text{H.c.}\right\}$$
$$N_\mathbf{q} = \langle c_\mathbf{q}^+ c_\mathbf{q}\rangle, \quad \Omega_j = (0,\omega_\mathbf{q}), \quad g(\mathbf{q}) = g/(2V\omega_\mathbf{q})^{1/2}$$

[there is no summation over repeated indices in Eqs. (4) and (5)]. In the system (5) we have neglected the recoil of electrons in the course of emission or absorption of a strong-field photon, because the wave vector **k** is considerably less than the characteristic values of the electron and phonon momenta ($(k \ll p$, $q \sim p_0$). Moreover, the smallness of k means that the spatial inhomogeneity of the field is weak. The distance transversed by an electron during the time between collisions with phonons $v_0\tau_{ph}$ is less than the wavelength of the field:

$$kv_0\tau_{ph} \ll 1. \quad (6)$$

Then, the gradient terms in Eq. (5) are small and can be neglected. Consequently, in this approximation the relaxation processes associated with the electron–phonon collisions are determined by the local value of the field $\lambda(\mathbf{r})$, and the functions ρ and y depend on $\mathbf{r}$ as a parameter. In the subsequent analysis it is convenient to adopt the quasiparticle representation:

$$\tilde{\chi}_j = U(\mathbf{r},\mathbf{p}-\mathbf{b}_j/2)\chi_j U^{-1}(\mathbf{r},\mathbf{p}+\mathbf{b}_j/2) \quad (7)$$

employing the unitary transformation

$$U(\mathbf{r},\mathbf{p}) = \begin{pmatrix} u(\mathbf{r},\mathbf{p}), & -v(\mathbf{r},\mathbf{p})\dfrac{\lambda(\mathbf{r})}{|\lambda(\mathbf{r})|} \\ v(\mathbf{r},\mathbf{p})\dfrac{\lambda^*(\mathbf{r})}{|\lambda(\mathbf{r})|}, & u(\mathbf{r},\mathbf{p}) \end{pmatrix}, \quad (8)$$

where $u^2(\mathbf{r},\mathbf{p})$ and $v^2(\mathbf{r},\mathbf{p}) = [1 \pm \xi_p/\varepsilon(\mathbf{r},\mathbf{p})]/2$, the latter depending on the local value of the field and, therefore, varying considerably over distances of the order of one wavelength. The spatial derivatives of $U(\mathbf{r},\mathbf{p})$ are also neglected on the strength of the condition (6). The dispersion law of quasiparticles is of the form

$$\varepsilon(\mathbf{r},\mathbf{p}) = (\xi_p^2 + |\lambda(\mathbf{r})|^2)^{1/2}. \quad (9)$$

In a homogeneous field [$\lambda(\mathbf{r}) = \text{const}$] the transformation of Eq. (7) ensures exact diagonalization[2] of the Hamiltonian H_e of Eq. (1). In a spatially inhomogeneous field the Hamiltonian H_e is diagonalized approximately with the quasiclassical precision (based on the parameter k/p_0). It should be pointed out that the density of states calculated using the dispersion law of Eq. (9) are in exact agreement with the results of Refs. 8 and 11. In particular, in the case of a standing wave there is no gap in the spectrum,[8] whereas in the more general case of two opposite waves with different amplitudes[11] there should be a gap in the density of states and its width should be $|\lambda_1 - \lambda_2|$. We shall confine ourselves to a discussion of the kinetic processes in a semiconductor at absolute zero $T = 0$.

In the quasisteady-state approximation,[9,10] we eliminate $\tilde{y}$ from a system of the resultant equations, and obtain an equation for the density matrix $\tilde{\rho}$ and seek its solution in the form of an expansion in terms of the parameter

$$1/\varepsilon\tau_{ph} \ll 1. \quad (10)$$

In the zeroth approximation this gives the following equation for the distribution function of quasiparticles $n(\mathbf{r},\mathbf{p}) = \tilde{\rho}_{11}(\mathbf{r},\mathbf{p})$, where $1 - \tilde{\rho}_{22}(\mathbf{r},\mathbf{p})$:

$$\frac{\partial}{\partial t}n = (1-n)S^+ - nS^- - nS^A + (1-n)J_p, \quad (11)$$

where

$$\begin{bmatrix} S^+(\mathbf{r},\mathbf{p}) \\ S^-(\mathbf{r},\mathbf{p}) \\ S^A(\mathbf{r},\mathbf{p}) \end{bmatrix} = 2\pi\sum_{\mathbf{p}',\mathbf{q}} g^2(\mathbf{q})\,\delta(\mathbf{p}-\mathbf{p}'+\mathbf{q})$$
$$\times\begin{bmatrix} n'(uu'+vv')^2\delta(\varepsilon-\varepsilon'+\omega_\mathbf{q}) \\ (1-n')(uu'+vv')^2\delta(\varepsilon'-\varepsilon+\omega_\mathbf{q}) \\ n'(uv'-vu')^2\delta(\varepsilon+\varepsilon'-\omega_\mathbf{q}) \end{bmatrix}, \quad (12)$$

$$n', u', v', \varepsilon' \equiv n(\mathbf{r}, \mathbf{p}'), \quad u(\mathbf{r}, \mathbf{p}'), \quad v(\mathbf{r}, \mathbf{p}'), \quad \varepsilon(\mathbf{r}, \mathbf{p}');$$

J_p is a spatially homogeneous source of quasiparticles.

The nondiagonal elements of the density matrix which determine, in particular, the absorption coefficient (gain) of a strong field are expressed in terms of the diagonal elements in the first approximation with respect to the parameter (10). These expressions differ from the corresponding formulas of Ref. 1 by a parametric dependence on the coordinate.

In homogeneous case, Eq. (10) is identical with the strong field condition $\lambda\tau_{ph} \gg 1$. In an inhomogeneous field (for example, in a standing wave) the condition (10) may not be obeyed in small regions near the wave nodes and at $p \approx p_0$. However, these small regions of the phase space play no significant role in the process of generation of a strong field under discussion.

We shall supplement Eq. (11) with the rate equation for the number of photons in a resonator:

$$(\partial/\partial t + 1/\tau_0)N = Q, \qquad N = E^2 V/8\pi\omega, \tag{13}$$

where τ_0 is the lifetime of a photon in the resonator, V is the volume of the system, and

$$Q = \int d\Gamma \frac{\xi_p}{\varepsilon(\mathbf{r}, \mathbf{p})}[nS^A + nS^- - (1-n)S^+], \qquad d\Gamma = \frac{d^3r\, d^3p}{(2\pi)^3}. \tag{14}$$

In Eq. (14) the quantity Q is the number of photons created in the semiconductor per unit time. We can show that in the steady-state case (for a source localized in a region $\xi_0 \gg \xi, \lambda$), Eqs. (11) and (13) yield equations relating the rate of annihilation of quasiparticles, on the one hand, to the rate of their creation by a source and, on the other, to the rate of loss of photons from the resonator:

$$N/\tau_0 = \int d\Gamma n S^A = \int d\Gamma (1-n) J. \tag{15}$$

Therefore, the steady-state value of the generated field is determined by the rate of annihilation of quasiparticles. If it is represented in the form

$$\int d\Gamma n S^A = \frac{2p_0 m}{\pi\tau_{ph}\omega_{ph}} \int d^3r \int d\xi \int d\xi' (uv' - vu')^2 nn', \tag{16}$$

where

$$2|\lambda(\mathbf{r})| \leqslant \varepsilon + \varepsilon' \leqslant \omega_{ph}, \qquad \frac{1}{\tau_{ph}} = \frac{mV}{\pi} g_0^2, \qquad g(\mathbf{q}) = \frac{g_0^2}{q},$$

it can readily be shown that this quantity is a nonmonotonic and bounded function of the field amplitude λ. It then follows from Eq. (15) that there exists a limiting lasing field whose magnitude depends in a self-consistent manner on the nature of the quasiparticle distribution function.

We note that the annihilation rate is influenced by several factors: the laws of conservation of the momentum and energy of quasiparticles and phonons, the nature of the distribution function n, and the coherence of the interaction.

§2. HIGH-Q REGIME

In the high-Q regime ($\eta \sim \tau_0/\tau_{ph} \gg 1$) the photon lifetime in the resonator is long. We shall consider the specific case of a standing wave $\lambda(\mathbf{r}) = \lambda \cos kx$. We shall show later that the field amplitude may reach high values ($\lambda_0 \gg \omega_{ph}$). Then, spatial restrictions appear and these are imposed by the kinematics of the annihilation process. It follows from the laws of conservation that the contribution to the annihilation is made only by those particles which satisfy the condition $|\lambda(x)| \leqslant \omega_{ph}/2$. The width of the corresponding spatial regions near the standing-wave nodes is of the order of $k\Delta x \sim \omega_{ph}/\lambda \ll 1$, so that an increase in the field reduces the annihilation rate. We shall find an approximate solution of the steady-state rate equation for quasiparticles (11). In these regions of space the nature of the distribution function is governed by the scattering integrals $(1-n)S^+ - nS^- \approx 0$, since the source is localized when $\xi_0 \gg \lambda$ and the annihilation collision integral S^A is small. As a result, we obtain a Fermi function with a chemical potential μ which is found from the second equation of the system (15). The chemical potential is close to ω_{ph}: $\omega_{ph} - \mu \ll \omega_{ph}$. Therefore, we shall substitute in Eq. (16) the distribution function $n(x,\mathbf{p}) \approx 1$. This gives rise to an error which is small in respect of the parameter $(\omega_{ph} - \mu)/\omega_{ph} \ll 1$. Then, the annihilation rate of Eq. (16) is given by the following expression

$$\int d\Gamma n S^A = V p_0 m \omega_{ph}/2\pi^2\lambda\tau_{ph}. \tag{17}$$

In the calculation we shall assume that $g^2(\mathbf{q}) = g_0^2/q$. For a different dependence of the matrix element, for example when $g^2(\mathbf{q}) \propto q$, only a numerical coeffcient of the order of unity changes in Eq. (17). Substituting in Eq. (17) the first of the equations of the system (15), we find the limiting value of the generated field:

$$\lambda_0 = \omega_{ph}\eta^{1/3}, \qquad \eta = r_0\frac{\tau_0}{\tau_{ph}}, \qquad r_0 = \frac{\omega\, d^2 m}{\pi s}. \tag{18}$$

It is higher than the corresponding limit for a homogeneous field $\lambda_0 = \omega_{ph}/2$ and, moreover, it depends on the photon lifetime in the resonator as $\tau_0^{1/3}$. This difference is due to the different behavior of the annihilation rate in the homogeneous and inhomogeneous cases when the field amplitude is large. In the former case it tends to zero as $(\omega_{ph} - 2\lambda)^2$, whereas in the latter case as λ^{-1}. The limiting field of Eq. (18) corresponds to the critical value of the pump current $I_{cr} = \omega_{ph}\eta^{2/3}/4\pi\omega d^2\tau_0 \sim \tau_0^{-1/3}$.

The steady-state lasing regime breaks down in the range $I > I_{cr}$. The results obtained can be generalized to the case when the lasing field represents a superposition of two opposite waves:

$$[\lambda_1 e^{ikx} + \lambda_2 e^{-ikx}]/2.$$

In the high-Q case the amplitude of these waves are similar. To be specific, we shall assume that the reflection coefficient of one of the mirrors is equal to unity and that of the other is close to unity, i.e., $\lambda_1 = \lambda$, $\lambda_2 = R\lambda$, $R \lesssim 1$.

In the presence of a gap in the spectrum

$$\varepsilon(x, \mathbf{p}) = (\xi_p^2 + \Delta^2 + \Lambda^2 \cos^2 kx)^{1/2},$$
$$\Delta = (\lambda_1 - \lambda_2)/2, \qquad \Lambda = (\lambda_1\lambda_2)^{1/2}$$

gives rise to an additional reduction in the volume of the phase space that determines the annihilation rate.

The equation for the limiting field can be represented, with the aid of Eqs. (15) and (16), in the following convenient form

$$\eta(1-R)^3 f(z)=1, \quad z=\omega_{ph}/\lambda_0(1-R)$$
$$f(z)=0, \qquad z<1,$$
$$f(z)=\sim(z-1)^{5/2}, \quad z\sim 1, \qquad (19)$$
$$f(z)=\sim z^3, \qquad z\gg 1.$$

If $\eta^{1/3}(1-R)\ll 1$, the gap is small ($2\Delta\ll\omega_{ph}$) and has no influence on the limiting field, so that the result is identical with that obtained in the standing wave case: $\lambda_1\approx\lambda_2\approx\omega_{ph}\eta^{1/3}$.

In the opposite limit $\eta^{1/3}(1-R)\gg 1$ the gap is close to ω_{ph}, i.e., $2\Delta\approx\omega_{ph}$, which determines the maximum value of the field:

$$\lambda_1\approx\lambda_2\approx\omega_{ph}(1-R)\ll\omega_{ph}\eta^{1/3}. \qquad (20)$$

We shall conclude by specifying more precisely the conditions under which the approximations (6) and (10) can be used. Since the energies of the quasiparticles participating in the annihilation process are of the order of ω_{ph}, the condition (10) assumes the form $\omega_{ph}\tau_{ph}\gg 1$, which is satisfied for typical values $\omega_{ph}\sim 10^{13}$ sec^{-1} and $\tau_{ph}\sim 10^{-12}$ sec. In the inequality (6) the wavelength k^{-1} should be replaced with the characteristic spatial size of the annihilation region $\omega_{ph}/\lambda_0 k$. The condition $\eta\ll(kv_0\tau_{ph})^{-3}$ is then obtained. In view of the smallness of the wave vector, this inequality imposes in practice no upper limit on the Q factor.

§3. SATURATION OF LASING IN THE LOW-Q REGIME

In the case when the Q factor is low because of the short photon lifetime in the resonator ($\tau_0\ll\tau_{ph}$) the saturation of lasing occurs in fields corresponding to ($\lambda_0\ll\omega_{ph}$). Therefore, the kinematics imposes no significant restrictions on the volume of the space that determines the contribution to the annihilation process.

The coherent factor is $(uv'-vu')^2\sim 1$ in the range of strong mixing of electron states by the wave field [$\xi\sim\xi'\sim\lambda(x)$], but this energy interval gives on the whole a small contribution to the integral (16) ($\sim\lambda^2/\omega_{ph}^2$). At high energies ($\xi\sim\xi'\sim\omega_{ph}$) the coherent factor itself is small ($\sim\lambda^2/\omega_{ph}^2$). Consequently, the main contribution to Eq. (16) is made by the momentum range $\xi\sim\lambda(x)$, $\xi'\sim\omega_{ph}$ (or vice versa) and the annihilation rate is proportional to λ. It can be calculated by finding the distribution function of quasiparticles in a wide range of values of ξ (from $\xi\sim\lambda$ to $\xi\sim\omega_{ph}$). We shall use the relevant expressions from Ref. 1 except that the quantities, such as the quasiparticle energy, distribution functions, etc. will now depend on x as a parameter. However, in the low-Q regime [$\lambda(x)\leqslant\lambda\ll\omega_{ph}$] this dependence appears usually only in small terms of the order of λ/ω_{ph}. In the general case of superposition of traveling and standing waves the distribution function is of the form

$$n(x,\mathbf{p})=u^2(x,\mathbf{p}), \quad -\infty\leqslant\xi\leqslant a(x)\omega_{ph}$$
$$n(x,\mathbf{p})=0, \qquad \xi>a(x)\omega_{ph}, \qquad (21)$$

where

$$a(x)=\int_0^{\omega_{ph}} d\xi\,\frac{n(\xi,x)}{\omega_{ph}}=1-o\left(\frac{\lambda(x)}{\omega_{ph}}\right).$$

Using Eqs. (16) and (21), we shall now find the limiting value of the lasing field

$$\lambda_0=2\pi\omega_{ph}\eta\chi(R), \qquad (22)$$

where

$$\chi(R)=\frac{1+R}{1+R^2}E\left(\frac{2R^{1/2}}{1+R}\right)$$

and E is complete elliptic integral.

The traveling wave limit corresponds to $R=0$ and we then obtain the result of Ref. 1. The function $\chi(R)$ varies little within the limits $1\leqslant\chi(R)\leqslant\pi/2$ when R is reduced from 1 (standing wave case) to zero. Therefore, in general the limiting field

$$\lambda_0\approx\eta\omega_{ph}, \quad \eta\ll 1 \qquad (23)$$

differs from the result given in Ref. 1 by a numerical factor ~ 1.

It follows that inhomogeneity does not alter qualitatively the limiting field in the low-Q regime.

§4. SEMICONDUCTOR LASER UTILIZING DOPED ACTIVE MATERIALS

Real semiconductors used in lasers contain impurities. Moreover, depending on the actual experimental and application conditions, deliberate doping of semiconductors is used. Therefore, the process of impurity scattering of electrons should be allowed for in the kinetic theory of generation of a strong field in a semiconductor laser.

The results obtained in §§2 and 3 can be generalized to a doped semiconductor if the main relaxation process is the electron–phonon interaction, i.e.,

$$\tau_{ph}\ll\tau_a, \qquad (24)$$

where τ_a is the impurity relaxation time. For example, in the case of GaAs, the relaxation times are $\tau_a\sim 10^{-11}$ and $\tau_{ph}\sim 10^{-12}$ sec, respectively.

The process of impurity scattering does not contribute to the annihilation rate and the rate equations (15) are unaffected. It is shown in Ref. 12 that the impurity scattering destroys effectively the gap and if $\lambda\tau_a\sim 1$ there is no gap in the density of states. This means that in the spatially inhomogeneous field of a standing wave in the intervals

$$k\Delta x\sim(\lambda_0\tau_a)^{-1} \qquad (25)$$

near the wave nodes the density of electron states changes and in the remaining spatial intervals the gap is renormalized and is given by the expression

$$\Delta(\mathbf{r})=\lambda(\mathbf{r})\,[1-(\lambda(\mathbf{r})\tau_a)^{-2/3}]^{3/2}. \qquad (26)$$

Moreover, the impurity scattering affects the distribution function of quasiparticles.

In the high-Q regime (τ_0, $\tau_a\gg\tau_{ph}$) the distribution function is determined by the phonon relaxation mechanism and is still unity in the range of energies of importance to us. Therefore, the impurity scattering affects only the density of states. We shall estimate the change in the annihilation rate (and, consequently, in the upper limit to the generated field) because of a change in the density of electron states in spatial intervals described by Eq. (25). This change is proportional to a small quantity $(\omega_{ph}\tau_a)^{-1}$, i.e., the impurity scattering has practically no effect on the upper limit of the field generated in a semiconductor laser operating in the high-Q regime.

In the low-Q regime ($\tau_a \gg \tau_{ph} \gg \tau_a$) the electron distribution function is still in the form of a unit step with a dip of depth 1/2 at $\xi_p = 0$ (this is known as the overpopulation burning effect[1]). However, in the spatial intervals given by Eq. (25) near the wave nodes this dip is liquidated by the collapse of the gap due to impurity scattering. In other spatial intervals the width of the dip is governed by the normalized gap Δ (**r**). We shall now estimate the change in the upper limit of the field generated in a semiconductor laser because of a change in the distribution function caused by the impurity scattering. This change is proportional to a small quantity $(\omega_{ph}\eta\tau_a)^{-2/3}$ and it is of the same order of magnitude as the terms (proportional to η) omitted in the calculation of the upper limit of the field generated in a semiconductor laser made of a pure active material and operating in the low-Q regime (§3). An allowance for the influence of the impurity scattering on the density of states gives rise to additional terms in the upper limit for the lasing field but these terms are of even higher orders of smallness. Therefore, in the low-Q case once again the impurity scattering does not affect the upper limit of the field which is created.

We shall now draw attention to the fact that the concentration of an impurity in a semiconductor has practically no influence (via τ_a) on the upper limit of the field generated in a semiconductor laser operating on the basis of band–band electron transitions. Experimental and theoretical investigations have shown that in the case of a semiconductor laser utilizing a doped active material we can expect electron transitions between the conduction band and a discrete impurity (acceptor) level.

Using the representation of an impurity band[13] formed as a result of delocalization of impurity electrons undergoing transitions between this band and another impurity under the action of a strong field, we can reformulate the rate equations for a semiconductor laser made of a doped material. If the inequalities

$$\mu_0 > \tilde{\lambda} > \tau_a^{-1} \tag{27}$$

are satisfied, the impurity scattering of electrons is unimportant and the rate equations (15) can again be used in the case of doped semiconductors provided the following substitutions are made:

$$m \to m_{c,v} = \frac{2m_c m_v}{m_c + m_v}, \quad \lambda \to \tilde{\lambda} = \lambda \left(\frac{\pi N a_B^3}{V}\right)^{1/2}, \tag{28}$$

where a_B is the state of an electron bound to an impurity, N is the number of randomly distributed impurities, V is the volume of a crystal, and m_i is the mass of an electron in the conduction (c) and impurity (v) bands.

Following the same procedure as in §§2 and 3, we find that the limiting field generated in a semiconductor laser operating in the high- and low-Q regimes is given respectively by the following

$$\lambda_0 = \omega_{ph}\eta^{1/3}\delta^{-1/6}, \tag{29}$$

$$\lambda_0 = \omega_{ph}\eta\delta^{1/2}, \tag{30}$$

where

$$\delta = \pi N a_B^3 / V.$$

It follows that in the case of a semiconductor laser exhibiting impurity–band transitions the upper limit on the field depends both on the impurity concentration (N/V) and on the impurity used in the active zone (a_B). This can be utilized experimentally to identify electron transitions occurring in a semiconductor laser.

It should be noted that impurity scattering may alter the results (29) and (30), but only in the case when

$$\mu_0\tau_a \sim 1, \quad \mu_0 \sim \tilde{\lambda}, \tag{31}$$

which would have altered the inequality (24), i.e., which would have required a different approach to an analysis of the kinetic theory of generation of a strong field.

§5. EXPERIMENTAL RESULTS

Experimental studies of saturation of the field generated in a semiconductor laser in the case when the radiation intensity was $\sim 10^6$ W/cm^2 were reported in Refs. 6 and 14. Measurements indicated that the limiting power and its dependence on τ_0 were in agreement with the conclusions deduced from the kinetic theory developed for the case of a homogeneous field. A calculation of the Q factor for the experimental conditions gave $\eta \sim 0.1$ (Ref. 14), i.e., the semiconductor laser operated in the low-Q regime when, as shown above, the limiting field and its dependence on τ_0 were practically identical for the cases of a homogeneous field and a field of the standing-wave type.

The results of Ref. 14 also confirmed qualitatively the conclusions reached in Ref. 15 that the limiting field should increase on increase in the impurity concentration in the active material of an injection laser.

The results of the present study allow us to conclude that in the case of semiconductor lasers employed in these experimental investigations the electron transitions giving rise to the optical radiation were of the impurity–band type. The observed increase in the limiting field on increase in the impurity concentration[14,15] in the case of semiconductor lasers operating in the low-Q regime is in qualitative agreement with the results obtained in our study.

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