Self-sustaining waveguide channels in a plasma

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(Submitted 3 May 1982)
Zh. Eksp. Teor. Fiz. **84**, 48–59 (January 1983)

We investigate the structure of homogeneous (along the wave-propagation direction) TM channels in a plasma with interaction of the striction and ionization type. In the case of striction nonlinearity, a new class of solutions is obtained and describes, in a plasma that is supercritical in the linear approximation, channels whose production calls for maximum energy expenditure. For a plasma with ionization type of nonlinearity, localized self-consistent distributions are obtained for the concentration and for the field, and make up in the simplest case a dense plasma layer along which a slowed-down TM surface wave propagates.

PACS numbers: 52.40.Fd, 52.35.Mw

INTRODUCTION

Waveguide propagation of waves of finite amplitude in an infinite homogeneous medium is one of the most interesting nonlinear phenomena. The possibility of such propagation of electromagnetic waves in an isotropic plasma was considered in a rather large number of papers. The question of the structure of the waveguides for the simplest case of a TE wave can by now be regarded as completely answered.[1–4]

At the same time the problem of the TM-waveguide structure, wherein the specific features of a plasma medium are most pronounced in view of the possible strong growth of the field in the plasma-resonance region, is much more complicated and calls for additional investigations. An approach to the solution of this problem was formulated in Refs. 3 and 4, and a particular class of solutions was obtained for the case of a plasma with striction-type nonlinearity.

In this paper we investigate the structure of homogeneous (in the wave-propagation direction) TM channels in a plasma with both striction and ionization nonlinearity. The solutions obtained can be used also in the analysis of the propagation of a strong electromagnetic wave in a smoothly inhomogeneous medium and to solve the problem of incidence of such a wave on a plasma layer having an abrupt boundary.

§1. BASIC EQUATIONS

We seek the solution of Maxwell's equations in the form of a monochromatic wave traveling along the x axis and standing in the z direction:

$$\mathbf{E}=[\mathbf{x}^0E_x(z)-i\mathbf{z}^0E_z(z)]\exp(i\omega t-ihx),\qquad \mathbf{H}=i\mathbf{y}^0H(z)\exp(i\omega t-ihx). \tag{1}$$

The system of equations for the field components is

$$\frac{d^2E_x}{dz^2}+h\frac{dE_z}{dz}+\varepsilon E_x=0,\qquad (k_0^2\varepsilon-h^2)E_z-h\frac{dE_x}{dz}=0. \tag{2}$$

In Eq. (2) $k_0=\omega/c$ and E_x and E_z are real quantities [the constant phase shift between the components, which ensures the absence of an energy flux in the z direction, is taken into account in (1)].

The dielectric constant ε in (2) is assumed to be a local function of the modulus of the electric field, $\varepsilon=\varepsilon(|\mathbf{E}|^2)$. We confine ourselves for simplicity to the case of cubic nonlinearity

$$\varepsilon=\varepsilon_0+\delta|\mathbf{E}|^2/E_c^2. \tag{3}$$

It can be shown that the obtained solutions remain qualitatively the same for any monotonic behavior of $\varepsilon(|\mathbf{E}|^2)$. In (3), $\varepsilon_0=1-\omega_p{}^2/\omega^2$ is the unperturbed dielectric constant of the plasma, the parameter δ takes on values ± 1 depending on the type of nonlinearity, and E_c is the field amplitude characteristic of the given nonlinearity.

The order of the system of Eq. (2) can be lowered. The system of second-order equations equivalent to (2), written in the dimensionless variables $\xi=k_0z|\varepsilon_0|^{1/2}$, $e=E/E_c|\varepsilon_0|^{1/2}$, $\gamma=h/k_0|\varepsilon_0|^{1/2}$ takes the form[4]

$$\frac{de_z}{d\xi}=-\frac{\gamma^2\varepsilon+2\delta e_z^2(\varepsilon-\gamma^2)}{\gamma(\varepsilon+2\delta e_z^2)}e_x,\qquad \frac{de_x}{d\xi}=\frac{\varepsilon-\gamma^2}{\gamma}e_z, \tag{4}$$

$$\varepsilon=\varepsilon_0/|\varepsilon_0|+\delta(e_x^2+e_z^2). \tag{5}$$

The system (4), (5) has a first integral[4]

$$\gamma^2\mathscr{H}=\varepsilon^2e_z^2-2\gamma^2\varepsilon e_z^2+\gamma^2\varepsilon^2/2\delta. \tag{6}$$

This means that in principle the order of the system can be lowered once more, and its solutions can therefore be expressed in quadratures. This investigation procedure, however, is exceedingly cumbersome and offers few advantages.

A complete qualitative analysis of the system of differential equations (4) and (5) can be carried out on the phase plane of the variables e_x and e_z.

§2. WAVEGUIDE CHANNELS IN A PLASMA WITH STRICTION NONLINEARITY

We consider first a plasma whose nonlinear properties are determined by striction ($\delta=+1$). In this case the specific features of TM waves are realized only in a plasma that is opaque in the linear approximation ($\varepsilon_0<0$), in which the striction nonlinearity leads to "bleaching" of the medium in a certain region.

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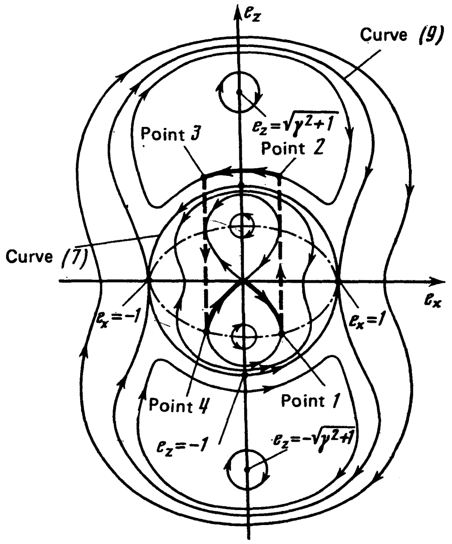

FIG. 1. Phase plane of the alternating components e_x and e_z of the electric field of a nonlinear TM wave. In the absence of the wave the plasma is supercritical ($\varepsilon_0 < 0$) and the nonlinearity is of the striction type ($\delta = +1$). The thick (solid and dashed) lines single out a solution in the form of a waveguide channel of the simplest type.

The (e_x, e_z) plane for the case $\delta = +1$ and $\varepsilon_0 < 0$ is shown in Fig. 1.

We make a few preliminary remarks concerning some typical phase trajectories.

We note first the circle

$$e_x^2+e_z^2=1. \tag{7}$$

It describes the solution of the system (4), (5) in the form of a potential wave[1)] propagating in a medium with a constant dielectric constant $\varepsilon = 0$. This integral curve is the boundary, on the phase plane, between the region corresponding to a supercritical plasma $\varepsilon < 0$ (the interior of the circle) and the region corresponding to a less dense plasma with $\varepsilon > 0$ (outside the circle $\varepsilon = 0$).

At the origin ($\varepsilon = \varepsilon_0 < 0$) there is an equilibrium state of the "saddle" type, a small vicinity of which describes linear (as $e_x, e_z \to 0$) solutions of Eqs. (4) and (5) in the form of increasing and decreasing exponentials. Two other equilibrium states (at the points $e_x = 0$, $e_z = \pm(\gamma^2+1)^{1/2}$ are equilibrium states of the "center" type and correspond to a homogeneous (in the z direction) purely transverse wave propagating in a homogeneous plasma whose dielectric constant is $\varepsilon = \gamma^2$.

Two points on the abscissa axis,

$$e_z=0, \quad e_x=\pm 1 \tag{8}$$

not being equilibrium points, are singular points of Eqs. (4) and (5). At these points two phase trajectories corresponding to the value $\mathscr{H} = 0$ [see Eq. (6)] are tangent to each other. One of them is the curve $\varepsilon = 0$ described above, and the equation for the other is obtained from (6) and is

$$\varepsilon e_z^2-2\gamma^2 e_z^2+{}^1/_2\gamma^2\varepsilon=0. \tag{9}$$

The main singularity that characterizes the TM polarization is the existence of singular line

$$\varepsilon+2e_z^2=0, \tag{10}$$

located inside the circle $\varepsilon = 0$.

The plot of (10) (the dash-dot ellipse in Fig. 1) is not integral. It is the vertical-tangent isocline, on which the derivative de_z/de_x becomes infinite, but in contrast to ordinary isoclines of vertical tangents, what changes sign on it is not the derivative $de_x/d\xi$, but the derivative $de_z/d\xi$. As a result, when the phase trajectories cross the curve (1) the direction of motion along them is reversed.[2)] It is therefore impossible to construct for Eqs. (4) and (5) continuous solutions that include a region where $\varepsilon < 0$. Only discontinuous solutions of this type exist. The phase trajectory that corresponds to them should contain a "jump" of the representative point from one integral curve to another.

2. We are interested in this paper in solutions that are localized in ξ and satisfy the conditions

$$e_x, e_z \to 0 \quad \text{as} \quad \xi \to \infty.$$

It follows, first, that the integral curve corresponding to such a solution should include part of the separatrix

$$\varepsilon^2 e_z^2-2\gamma^2\varepsilon e_z^2+\gamma^2(\varepsilon^2-1)/2=0, \tag{11}$$

which starts out from an equilibrium state of the saddle type. Second, from the discussion in the preceding subsection it follows that the localized solution must of necessity be discontinuous.

It must be emphasized here that the location of the "jump" in the stationary problem depends on the prior history of the stationary state.[6] It can be shown that in the situation considered by us, when the field amplitude varies smoothly with time, the jump occurs in a place where the height of the "step" on the density profile is a minimum. On the phase plane of the variables (e_x, e_z) this corresponds to a jump of the representative point from the spinode curve (10). We confine ourselves in the present paper to an analysis of just such stationary distributions, which set in when the field is "turned on" slowly.

3. We now have all the necessary information for the construction of the solution of Eqs. (4) and (5), which describes a waveguide channel of the TM type.

Corresponding to it on the phase plane (Fig. 1) is, first, the section of the separatrix from the origin to the curve (10). When the representative point moves from the saddle at the origin to the jump point, namely the intersection of the separatrix and curve (10), the electric-field components e_x and e_z increase monotonically and the dielectric constant ε approaches zero in absolute value. The values ε_1, e_{z1}, and e_{x1} of these quantities ahead of the jump are solutions of the system of equations (10) and (11), and consequently satisfy the relations[4]

$$|\varepsilon_1|^3+3\gamma^2\varepsilon_1^2-\gamma^2=0, \tag{12}$$

$$e_{z1}^2=-\varepsilon_1/2, \quad e_{x1}=1+3\varepsilon_1/2. \tag{13}$$

To find the changes of ε, e_x, and e_z on the jump it is necessary to use boundary conditions. If plasma wave generation is neglected, these conditions are continuity of the tangential components of the electric and magnetic fields and are written in the form

$$e_{x1}=e_{x2}, \quad \varepsilon_1 e_{z1}=\varepsilon_2 e_{z2}. \tag{14}$$

The subscript 2 labels values after the jump.

From this we obtain the following relation on the jump (naturally, they coincide with those determined in the quasistatic problem[6]):

$$\varepsilon_2=-\varepsilon_1/2, \quad e_{z2}=-2e_{z1}=\pm(2|\varepsilon_1|)^{1/2}. \tag{15}$$

The + and − signs in the expression for e_{z2} correspond to two possible solutions of Eqs. (4) and (5), in which the distributions of $e_x(\xi)$ coincide, while the distributions of $e_z(\xi)$ [and of the magnetic field $b(\xi)=\varepsilon e_z/\gamma$] are shifted in phase by π. In the analysis that follows we use as an example a solution with $e_{z1}<0$ and $e_{z2}>0$, which includes a piece of the separatrix (11) in the fourth quadrant of the phase plane.

It follows from the conditions (15) that the dielectric constant reverses sign on the jump. The distributions of the quantities $e_x(\xi)$ and $e_z(\xi)$ directly in the channel in which the dielectric constant has become positive under the influence of the pressure of the RF field depend on the parameter γ that determines the degree of slowing down of the nonlinear TM wave relative to the homogeneous TEM wave propagating in a linear medium with a dielectric constant $\varepsilon=|\varepsilon_0|$. It can be shown that at $\gamma^2<1/\sqrt{20}\approx 0.22$ the jump is into a transparent plasma, i.e., $\varepsilon_2>\gamma^2$. If, however $\gamma^2<8\sqrt{3}/135\approx 0.1$ the phase trajectory that describes the solution in the region $\varepsilon>0$ lies even farther from the circle $\varepsilon=0$, outside the curve (9).

The solutions that are realized at $\gamma^2<0.1$ were determined with a computer in Ref. 4. Since, however, the (e_x, e_z) phase plane was not fully investigated in Ref. 4, a number of solutions that are of interest at $\gamma^2>1/\sqrt{20}$ were left out. The entire analysis that follows will deal precisely with that case.

For waves with $\gamma^2>1/\sqrt{20}$ the plasma can remain opaque even in a channel where $\varepsilon>0$. The phase trajectory that describes the solution in this region is singled out in Fig. 1. The representative point lands on this trajectory following a jump from point 1 to point 2 (with coordinates e_{x2}, e_{z2}), after which it can rotate around an equilibrium point of the "center" type for an arbitrarily "long" time, inasmuch as in the region $\varepsilon>0$ the system of equations (4) and (5) has no singularities such as spinode curve (10). At the same time, to construct a solution in the form of a waveguide channel it is necessary to "throw" the representative point into the region $\varepsilon<0$ on the separatrix (11). The phase plane of Eqs. (4) and (5) admits of an infinite manifold of such "throws." We shall stop to investigate the simplest symmetrical mode, which can be produced at a minimum energy. It corresponds to a return jump to the phase plane from the point 3 (in the region $\varepsilon>0$) to the point of intersection of the separatrix (11) and the spinode curve in the third quadrant, and to motion along the separatrix into the equilibrium state at the origin.

Our solution can be called the first symmetrical soliton mode[3] of Eqs. (4) and (5). We obtain the higher modes if we "allow" the representative point first to execute the corresponding number of revolutions around the center and only then to jump over from the point 3 to the point 4.

It is precisely the second soliton mode that was calculated in Ref. 4. For comparison, Fig. 2 shows the structure of the field and of the dielectric constant in the first and second symmetrical soliton modes at $\gamma=3$.

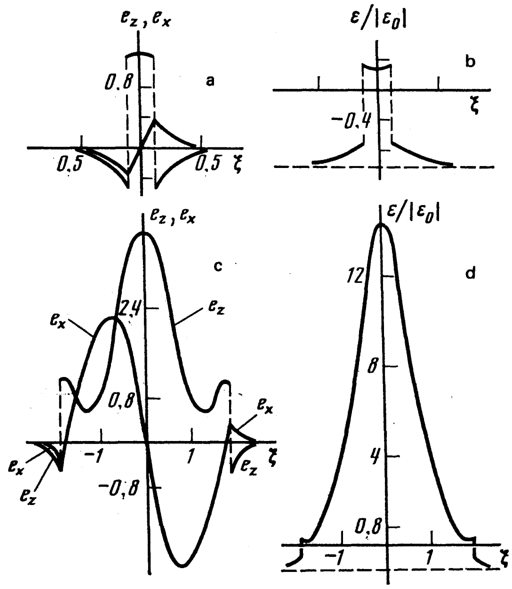

FIG. 2. Distributions of the field and of the dielectric constant in a self-sustaining waveguide channel for the first (a, b) and second (c, d) symmetric TM modes at $\gamma=3$.

It can be seen that even at this relatively small γ the amplitude of the second mode is noticeably (about ~4 times) larger than that of the first.

With increasing slowdown, the ratio of the field amplitudes in the second and first modes will increase. To verify this we obtain analytic expressions for the amplitudes of the first and second modes on the channel axis. This can be done in the limiting case $\gamma\gg 1$, when the wave becomes quasipotential. The field amplitude of such a wave is given in the first mode by

$$E_{1\,max}=[(2+\sqrt{31})|\varepsilon_0|/6]^{1/2}E_p\approx 1{,}12|\varepsilon_0|^{1/2}E_p, \tag{16}$$

and in the second by

$$E_{2\,max}=(3/2)^{1/2}hE_p/k_0. \tag{17}$$

Here $E_p=(16\pi NT)^{1/2}$ is the characteristic field of the striction nonlinearity. It can be seen that in the case of greatly slowed-down waves the amplitude of the second and all higher modes is γ times larger than the amplitude of the first mode. We note that the region of validity of Eq. (17) is limited, since the cubic-nonlinearity approximation (3) used by us is valid only at $|\mathbf{E}|^2<E_p^2$, i.e., at not too large h,

$$h/k_0<(2/3)^{1/2}. \tag{18}$$

At large amplitudes of the fields in the channel, account must be taken of the nonlinearity saturation. As noted in Sec. 2, the results above remain qualitatively in force for any $\varepsilon(|E|^2)$ dependence (although the quantitative relations can become quite unwieldy).

We emphasize that expression (16) for the amplitude of the first mode remains valid for all $h/k_0\gg(|\varepsilon_0|)^{1/2}$.

It is also of interest to compare the power channeled in the first and second modes and their characteristic scales.

We define the channeled power, as usual, as

$$P=\frac{c}{8\pi}\operatorname{Re}\int_{-\infty}^{\infty}[\mathbf{E}\times\mathbf{H}^*]dz, \tag{19}$$

and the effective width of the channel as

$$a=\left[\int_{-\infty}^{\infty}z^2|\mathbf{E}|^2dz\Big/\int_{-\infty}^{\infty}|\mathbf{E}|^2dz\right]^{1/2}. \tag{20}$$

The dependence of P and a on γ can be obtained analytically in the case of sufficiently strong slowdown $\gamma\ll 1$.

For the first mode we obtain from (20) and (19)[4]

$$P_1=\frac{cE_p^2}{8\pi k_0}\left(\frac{k_0|\varepsilon_0|}{h}\right)^2p_1,\qquad p_1=\int_{-\infty}^{\infty}\varepsilon(\zeta)e_z^2(\zeta)d\zeta\approx 0.095, \tag{21}$$

$$a_1=\frac{\alpha_1}{h},\qquad \alpha_1=\left[\int_{-\infty}^{\infty}\zeta^2e^2(\zeta)d\zeta\Big/\int_{-\infty}^{\infty}e^2(\zeta)d\zeta\right]^{1/2}\approx 0.48. \tag{22}$$

Here the dimensionless integration variable is $\zeta=hz$, $e^2=e_x^{\,2}+e_z^{\,2}$, and the integrals p_1 and α_1 along the phase trajectories that describe the first mode were obtained numerically. It is seen therefore that $P_1\to 0$ as $\varepsilon_0\to 0$. Indeed, it can be shown that the first soliton mode exists only in an initially supercritical plasma, at $\varepsilon_0<0$.

Numerical computer calculations show that the asymptotic expressions (21) and (22) are valid already at $\gamma\gtrsim 1$ for the function $a_1(\gamma)$ and at $\gamma>10$ for $P_1(\gamma)$. It follows from (22) that the characteristic scales of the field variation in the first mode are of the same order along and across the channel.

In the second mode at $\gamma\gg 1$ the channeled power ceases to depend on ε_0 and its dependence on the slowdown of γ reduces to a dependence on the longitudinal wave number h. The expressions obtained for P_2 and a_2 in Ref. 4 are of the form[5]

$$P_2=\frac{cE_p^2}{8\pi k_0}\frac{h^2}{k_0^2}p_2,\qquad p_2=\int_{-\infty}^{\infty}\varepsilon G^2\,d\zeta\approx 5.06, \tag{23}$$

$$a_2=\alpha_2/h,\qquad \alpha_2=\left[\int_{-\infty}^{\infty}\zeta^2G^2\,d\zeta\Big/\int_{-\infty}^{\infty}G^2\,d\zeta\right]^{1/2}\approx 2.6. \tag{24}$$

The p_2 and α_2 in (23) and (24) are taken along the phase trajectories that correspond to the second mode,

$$G=Ek_0/E_ph,\quad G^2(\zeta)=G_x^2(\zeta)+G_z^2(\zeta).$$

Comparing expressions (21) and (23) we find that at $(h/k_0)^2\gg|\varepsilon_0|$ (i.e., $\gamma^2\gg 1$) the ratio

$$P_2/P_1\sim(h/k_0)^4\varepsilon_0^{-2}$$

increases with increasing h/k_0. The ratio of the characteristic scales is here independent of h and is equal to

$$a_2/a_1=\alpha_2/\alpha_1\approx 5.4.$$

§3. PLASMA WAVEGUIDES IN A MEDIUM WITH IONIZATION TYPE NONLINEARITY

1. We consider waveguide TM channels in a plasma with nonlinearity of the ionization type. We have in mind a situation wherein the plasma density is higher in the region where the electron-field amplitude is larger.

The propagation of surface wave along the interface between a dielectric and a plasma with ionization nonlinearity was investigated in Refs. 7 and 8. It was furthermore indicated in Ref. 7 that in such a plasma there can exist field and density distributions in the form of wave channels, but the appropriate solutions were not obtained.

In the case of ionization nonlinearity, the change of the plasma density under the influence of the field is the result of "creation" of charged particles upon ionization of the neutral molecules and atoms, and their loss by sticking, recombination, or diffusion. We analyze here the case when the diffusion is negligible and the connection between the plasma density and the field amplitude can be regarded as local.

It is known that electric breakdown of a neutral gas is characterized by a threshold value of the field amplitude. In weak fields, the plasma density does not depend on the field, E_a increases rapidly with increasing field and when the field amplitude exceeds the threshold value. The simplest approximation of such a dependence of the plasma density N on the field amplitude is

$$\begin{aligned}&N=N_0+\theta N_c(\mathscr{E}^2-1),\quad \mathscr{E}^2\geqslant 1,\\&N=N_0,\qquad\qquad \mathscr{E}^2\leqslant 1.\end{aligned} \tag{25}$$

Here N_0 is the "priming" plasma density in the absence of a field, $N_c=\omega^2m/4\pi e^2$ is the critical density for a wave of frequency ω, the field is measured in threshold units $\mathscr{E}=E/E_a$, and the parameter θ characterizes the "rate" of growth of the plasma density when the field exceeds the threshold value E_a. It is convenient to normalize the dielectric constant as before against the quantity $\varepsilon_0=(1-N_0/N_c)>0$. The expressions for ε corresponding to (25) take the form

$$\varepsilon=(1-N/N_c)\varepsilon_0^{-1}=1-\beta(\mathscr{E}^2-1),\quad \mathscr{E}^2\geqslant 1, \tag{26}$$

$$\varepsilon=1,\qquad \mathscr{E}^2\leqslant 1, \tag{27}$$

where $\beta=\theta/\varepsilon_0$. In the limiting case of a weak threshold field[6] we obtain for ε from (26) and (17), following the field transformation $\beta\mathscr{E}^2=e^2$, expression (3) with $\delta=-1$.

It can be shown that self-focusing of TE waves and formation of TE waveguides are impossible for an ionization nonlinearity, and the TM waveguides of interest to us exist only at $\gamma^2>1$ and $\varepsilon_0>0$.

A complete qualitative analysis of all the solutions of the system (4), (26), and (27) can be carried out as before on the phase plane.

In this case the phase plane $(\mathscr{E}_x,\mathscr{E}_z)$ must consist of two regions separated by the circle:

$$\mathscr{E}_x^2+\mathscr{E}_z^2=1. \tag{28}$$

Inside this circle the behavior of the system is described by linear equations obtained from (4) at $\delta=0$ and $\varepsilon=1$. At $\gamma^2>1$ the "linear" part of the phase plane has at the origin an equilibrium state of the saddle type, with a separatrix described by the relation

$$\mathscr{E}_z=\pm\frac{\gamma}{(\gamma^2-1)^{1/2}}\mathscr{E}_x. \tag{29}$$

The region outside the circle (28) corresponds to a plasma waveguide "broached" through the gas. The structure of the field and the density in it are determined by the solution of Eqs. (4), in which the function $\varepsilon(\mathscr{E}^2)$ is given by (16), and the substitutions $e\to\mathscr{E}$ and $\delta\to-\beta$ were made. As before, the regions corresponding to supercritical and subcritical plasma are separated by an integral curve $\varepsilon=0$ whose radius depends on the parameter β:

$$\mathscr{E}_x^2+\mathscr{E}_z^2=1+\beta^{-1}. \tag{30}$$

The behavior of the phase trajectories in the ring between the circles (28) and (30), where $\varepsilon>0$, is determined by the parameter $\tilde{\gamma}^2=\gamma^2/(1+\beta)$. In particular, this region contains (in its entirety or in part, depending on $\tilde{\gamma}^2$) a singular spinode curve

$$3\mathscr{E}_z^2+\mathscr{E}_x^2=1+\beta^{-1}. \tag{31}$$

It can be shown, however, that the behavior of the trajectories in this ring does not influence the soliton solution that describes the localized mode.

We shall be interested as before in symmetric stationary distributions that result from a smooth increase of the wave field. An analysis of the phase plane makes it possible to construct such solutions in analogy with the procedure used in Sec. 3 of the preceding paragraph for the case of striction nonlinearity.

The phase trajectories that describe plasma waveguide in a medium with ionization nonlinearity are shown in Fig. 3. The distribution of the field and of the density in the waveguides depends significantly on the parameter β. At not too large β,

$$\beta<\beta^*=(2\gamma^2-1)/2\gamma^2, \tag{32}$$

the plasma density increases smoothly from the interface between the neutral gas and the plasma towards the waveguide axis, and the discontinuities in the distribution of the z component of the field and of the density is produced in a rather dense plasma. The corresponding trajectory of the representative point on the phase plane is shown in Fig. 3a. For a strongly slowed-down wave with $\gamma^2\gg1$ it becomes possible to obtain the dependence of the field amplitude $\mathscr{E}_m$ in the waveguide on the value of the parameter β:

$$\mathscr{E}_m^2=[2(1+\beta)+(16\beta^2+62\beta+31)^{1/2}](6\beta)^{-1}. \tag{33}$$

In the limiting case $\beta\to0$, as expected, we obtain from (33) for the field amplitude in the channel the expression (16) previously obtained for the zero-threshold striction mechanism of the nonlinearity.

If condition (31) is violated, the jump of the dielectric constant takes place already at the waveguide boundary, so that the plasma density exceeds the critical value everywhere inside the waveguide. The trajectory of the representative point on the phase plane is shown for this case in Fig. 3b.

For a strongly slowed-down wave propagating in a plasma with large β,

$$\gamma^2\gg\beta\gg1,$$

the coordinate dependences of the field and of the dielectric constant can be obtained analytically and are of the form

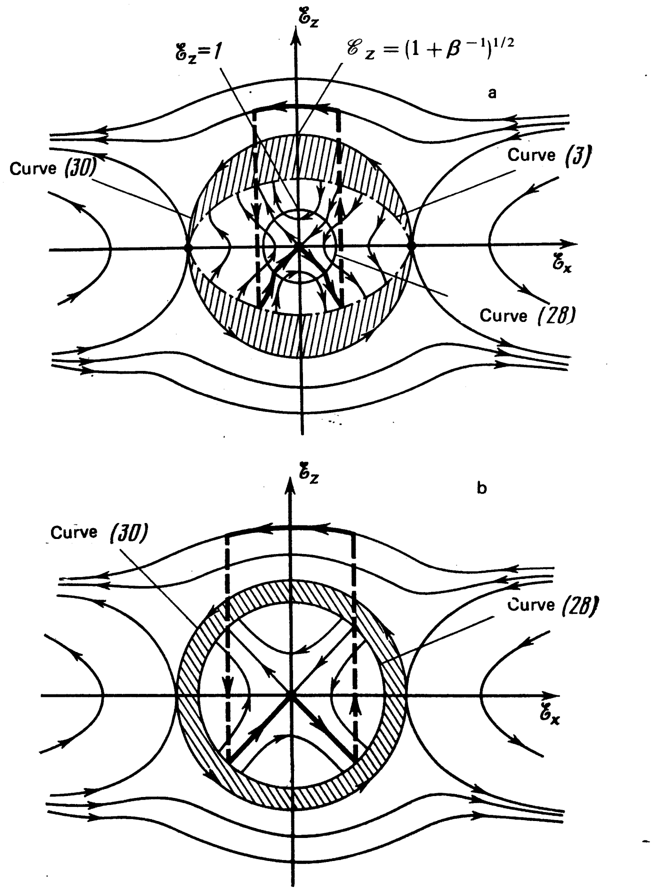

FIG. 3. Phase plane of the variables $(\mathscr{E}_x, \mathscr{E}_z)$ for a plasma with ionization nonlinearity in the case of a nonzero threshold field $E_a^2>0$ for different values of the parameter β: a) $\beta<\beta^*$, b) $\beta>\beta^*$. In the shaded region the behavior of the integral curves, which depends on the parameter $\tilde{\gamma}^2$, does not influence the plasma-waveguide solution outlined in the figure by a thick (solid and dashed) line.

$$E_z=\begin{cases}(-1/\sqrt{2})E_a\exp(-|hz|+\pi/4-5/6\beta), & |hz|>\pi/4-5/6\beta\\ E_a\cos hz\left(1+\dfrac{2-\cos^2 hz+8\cos^4 hz}{12\beta\cos^4 hz}\right), & |hz|<\pi/4-5/6\beta\end{cases}; \tag{34}$$

$$E_x=\begin{cases}(\operatorname{sign} hz/\sqrt{2})E_a\exp(-hz+\pi/4-5/6\beta), & |hz|\geqslant\pi/4-5/6\beta\\ -E_a\sin hz[1+(1+8\cos^2 hz)/12\beta\cos^2 hz], & |hz|\leqslant\pi/4-5/6\beta\end{cases}; \tag{35}$$

$$\varepsilon=\begin{cases}\varepsilon_0, & |hz|>\pi/4-5/6\beta\\ -\varepsilon_0/2\cos^2 hz+O(\beta^{-1}), & |hz|<\pi/4-5/6\beta\end{cases}. \tag{36}$$

The functions (34)–(36)[7)] are shown in Fig. 4. They can be used to determine the power P_β channeled by this waveguide, as well as its effective width a_β. In the limiting case $\beta\to\infty$ they are expressed by the formulas

$$P_\beta=-\frac{cE_a^2}{8\pi k_0}\frac{k_0^2\varepsilon_0}{h^2}\left(\frac{\pi}{8}-\frac{1}{4}\right), \tag{37}$$

$$a_\beta=\frac{\pi^3+6\pi^2+24\pi+48}{48(2+\pi)h}\approx\frac{0.87}{h}. \tag{38}$$

Numerical computer calculations show that for a quasipo-

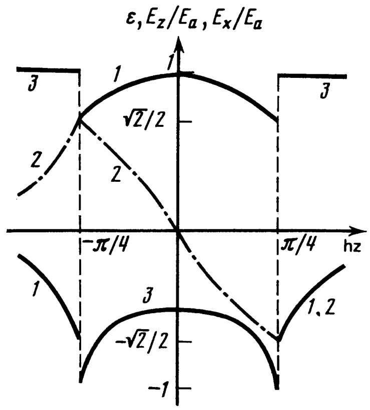

FIG. 4. Distribution of the electric-field components E_x (curve 1) and E_z (curve 2), and of the dielectric constant ε (curve 3) in a self-sustaining plasma waveguide in a medium with ionization nonlinearity at $\gamma^2 \gg \beta \gg 1$.

tential wave with $\gamma^2 \gg 1$ the functions $P_\beta(\beta)$ and $a_\beta(\beta)$ reach the asymptotic values (37) and (38) already at $\beta \gtrsim 1$.

Let us dwell on the singularities connected with the ionization character of the nonlinearity.

First, the phase trajectories located outside the circle $\varepsilon = 0$ are not closed. There are therefore no higher soliton modes. The first mode constructed above is always slowed down: $\gamma^2 > 1$.

Second, the power channeled by this mode is negative, $P_\beta < 0$. Consequently in the case of ionization nonlinearity the TM waveguide is produced by a backward wave.

Finally, a "priming" plasma is mandatory for the production of a channel in the case of ionization nonlinearity: A self-consistent structure in the form of a layer of supercritical plasma with a TM wave traveling along it can arise also in an initially neutral gas.

2. We conclude this section with a few words on the parameters of a plasma waveguide in a medium with a zero threshold field: $E_a = 0$. This situation is described by Eqs. (4) and (5) with $\delta = -1$ and $\varepsilon_0 > 0$, and is naturally a particular case of the more general problem considered in the preceding subsection of this section. The distinctive feature of the model with $E_a = 0$ is that it describes not only a medium with an ionization nonlinearity, but also a plasma whose nonlinear properties are determined by striction and moves (in contrast to the case considered in Sec. 2) with supersonic velocity. The last circumstance is typical of a plasma corona produced when laser radiation acts on a solid target.

A qualitative analysis of the solutions of the system (4) and (5) at $\delta = -1$ and $\varepsilon_0 > 0$ is similar to that in Sec. 2. Naturally, the characteristic features of the solution that describes the localized mode are the same in this case as the aforementioned general properties of plasma TM waveguides in media with ionization instability. The quantitative characteristics of this mode, viz., the field amplitude E_m in the channel, the channeled power P, and the effective width a can be obtained analytically in the case of strong slowing down $\gamma^2 \gg 1$. The values of E_m and are determined, just as in the case of striction nonlinearity, by expressions (16) and (22), and the channeled power is negative: $P = -P_1$, where P_1 is determined by Eq. (21).

§4. ESTIMATE OF WAVE DAMPING

We estimate now the dissipation of the electromagnetic energy in the TM waveguides considered above. We do this using as an example a plasma with striction nonlinearity. It is clear that the losses will primarily affect the structure of the first soliton mode. We confine ourselves to the case of a collisionless plasma, in which the dissipation is determined by two effects. First is the conversion of the electromagnetic wave into a Langmuir wave at the dielectric-constant jump. In addition, the electromagnetic wave can become attenuated by resonant interaction with the plasma electrons.

Since generation of the longitudinal (Langmuir) wave is due to a small vicinity of the jump, the absorption can be calculated using the results of a quasistatic analysis of the effects in the region of the plasma resonance.[10] Comparing the energy flux into the plasma wave

$$S_p = 0.63\sqrt{3}\, v_{Te} D_c E_p / 8,$$

obtained in Ref. 10, with the power (21) transmitted along the channel, and determining with the aid of (12) and (13) the induction $D_c = \varepsilon_1 E_{z1}$ of the electric field at the jump, we obtain in the limiting case $\gamma^2 \gg 1$ the condition under which the losses influence little the field structure in the channel

$$h/k_0 \ll c/v_{Te}. \qquad (39)$$

Here v_{Te} is the electron thermal velocity. The condition (39) is very "soft" and is always compatible with the condition $h/k_0 \gg |\varepsilon_0|^{1/2}$ for a strong slowdown of the wave. In addition, (39) is simultaneously the condition for small resonant damping by the electrons. Indeed, after substituting $c = \omega/k_0$ in (39) this condition takes the form

$$v_{ph} \gg v_{Te}.$$

Here $v_{ph} = \omega/h$ is the phase velocity of the wave along the layer as well, by virtue of (22), the minimum phase velocity of the greater part of the spatial spectrum of the field wave across the channel. For a collisionless plasma the inequality (39) is therefore a universal condition for low-energy losses in the channel.

We note in conclusion that to determine the realizability of stationary structures it is necessary to investigate their stability. Up to now the instability had been discussed only for TE waves.[4,11]

A rigorous investigation of the stability of the stationary solutions for TM waves is a most complicated problem. We formulate briefly only the results of a qualitative analysis of the stability of the localized distributions obtained above.

This analysis is similar to the qualitative treatment of the stability of localized distributions in the theories of self-focusing[12] and of Langmuir collapse,[13] and is based on an investigation of the character of the dependence of the integrals of motion of the nonstationary problem on the effective width of the distribution. The results of the investigation allow us to expect the following: 1) The waveguide channel in a supercritical plasma with striction nonlinearity is stable to spreading in the transverse direction, but unstable to collapse. 2) A plasma waveguide in a medium with ionization nonlinearity is stable both to spreading and to collapse.

The authors thank V. A. Mironov and A. V. Kim for helpful discussions.

1)This wave is a superposition of two longitudinal waves propagating in mutually perpendicular directions at an angle $\alpha = \pi/4$ to the x axis.

2)The first to encounter a similar behavior of phase trajectories were Andronov and Vitt in an analysis of differential equations that simulates a system with lumped parameters.[5]

3)Here and below we have in mind those symmetric modes whose structure contains the smallest number of jumps—two. There exist, in addition, multijump modes that are apparently less stable.

4)The channeled power can also be expressed in terms of the field strength on the channel axis. From (16) and (21) we obtain $P_1 = 0.1cE_{1\max}\, p_1/\pi k_0\gamma$.

5)Expressions (23) and (24) for P_2 and a_2 are valid only if the condition (18) is satisfied. Therefore, if the cubic nonlinearity is neglected, the maximum ratio P_2/P_1 is limited to $P_2/P_1 \lesssim 23.7|\varepsilon_0|^{-2}$. At larger amplitudes of the second mode it is necessary to take the nonlinearity saturation into account.

6)To go to the limit $E_a \to 0$ we must let $\beta \to 0$ and leave $\beta\mathscr{E}^2$ constant at the same time.

7)We note that expressions (34) and (35) for the field (without allowance for the small corrections $\sim 1/\beta$) in expression (36) for the dielectric constant can be obtained also on the basis of a method proposed in Ref. 9 for the case $\beta \to \infty$, where stationary structures of a high-frequency discharge are deduced directly from the condition $|\mathbf{E}| = E_a =$ const.

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Translated by J. G. Adashko