Relaxation in spin glasses above the transition point

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(Submitted 18 July 1982)
Zh. Eksp. Teor. Fiz. **84**, 380–387 (January 1983)

A dynamic spin-glass model that is a natural generalization of the Sherrington-Kirkpatrick model is considered. An exact solution of this model above the vitrification point is obtained.

PACS numbers: 75.40.Bw

1. INTRODUCTION

Much progress was made recently in the understanding of the static properties of spin glasses. The description of the dynamic properties, however, remains somewhat contradictory. For example, numerical experiments[1] predict a nonexponential relaxation of spin glass even above the vitrification point T_c, whereas theoretical papers[2] predict, within the framework of the mean-field theory, an exponential relaxation above the transition point, with a time

$$\langle SS\rangle \infty \exp(-t/\tau), \quad \tau \sim (T-T_c)^{-1/2}.$$

A very useful model for the study of the static properties of spin glass is that of Sherrington and Kirkpatrick (SK),[3] which was found[4–6] to be solvable at all temperatures. In this model each spin interacts with all others, which corresponds to an infinite interaction radius, i.e., to the mean-field theory. The SK mean-field model admits of a natural dynamic generalization—to the so-called "soft" SK model. There is no exact solution analogous to that obtained in Refs. 4–6 for the dynamic SK model. In this paper the dynamic SK model is solved above T_c. It is shown that the order parameter of the model is the susceptibility, as was indicated in Refs. 7 and 8, and an integral equation is obtained for the order parameter. This integral equation can be solved both at high temperatures and near the transition point. The susceptibility near the transition point is of the form

$$G(t)=\int_{\gamma_1}^{\infty} d\gamma e^{-\gamma t}\tilde{\rho}(\gamma), \qquad \gamma_1 \sim (T-T_c)^4, \quad \tilde{\rho}(\gamma)\sim\gamma^{1/4}, \quad 1\gg\gamma\gg\gamma_1. \tag{1}$$

It follows from (1) that it decreases like $t^{-5/4}$ at the transition point.

The plan of the article is the following: the model is described in Sec. 2, and its transformation into a one-point model with an interaction that is nonlocal in time is described together with the self-consistency condition. In Sec. 4 is constructed a statistical one-dimensional model whose transfer matrix constitutes the laws that describe the dynamics of the model from Sec. 3. The static problem is solved in Sec. 5, and in the last, 6th section, is investigated an integral equation obtained for the order parameter from the self-consistency condition and from the solution of the one-point problem.

2. DYNAMIC SHERRINGTON-KIRKPATRICK (SK) MODEL

In the static SK model the spin takes on a value ± 1, and the interaction energy of the spins located at the lattice sites i is equal to

$$H_{int}=\sum_{ij} J_{ij}S_iS_j,$$

where $\langle J_{ij}\rangle = 0$ and $\langle J_{ij}^2\rangle = J^2/N$. In addition, it is usually assumed that the distribution of J_{ij} is Gaussian and that there are no correlations whatever between integrals J_{ij} with different pairs of ij.

To write down the dynamic equations that describe the static model in the static limit, it is necessary to introduce a continuous variable S that has a potential energy H_0 in the form

$$H_0=\sum_i\left(-rS_i^2+\frac{1}{2}uS_i^4+hS_i\right), \qquad H=H_0+H_{int}. \tag{2}$$

In the limit $r=u\gg 1$ the variable S will almost always take on the value ± 1. This limit will in fact be implied throughout. The equation of motion for S will be assumed to be of the Langevin form:

$$\partial S/\partial t=\delta H/\delta S+\zeta(t). \tag{3}$$

Here $\zeta(t)$ is the thermal noise:

$$\langle\zeta(t)\zeta(t')\rangle={}^1/_2\delta(t-t').$$

The static properties of the model will be given by the Gibbs distribution $\exp(-H)$; they coincide with the properties of the static SK model. The physical justification of Eq. (3) can be the following picture: classical spins that can be rotated in a substance with large anisotropy by the action of the matrix phonons.

3. TRANSFORMATION OF THE MODEL INTO A ONE-POINT ONE

We describe now a transformation that enables us to go from Eq. (3) with an infinite number of variables to an equation that is nonlocal in time for a single variable.

We transform first Eq. (3) by the method proposed by de Dominicis.[9] We introduce an auxiliary boson field φ and two fermion fields $(\psi,\bar{\psi})$. The correlators of S can be calculated with the aid of the generating functional

 0038-5646/83/010220-05$04.00

$$F(l)=\int e^{L}\mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\varphi\mathcal{D}S,$$

$$L=\int\Big[\sum_i i\varphi_i\left(\frac{\partial S_i}{\partial t}-\frac{\delta H}{\delta S_i}\right)-\varphi_i^2+\bar{\psi}_i\left(\frac{\partial}{\partial t}+\frac{\delta^2 H}{\delta S_i\,\delta S_j}\right)\psi_j+lS\Big]\,dt. \tag{4}$$

The generating functional can already be averaged over random J_{ij}. This yields

$$\langle F(l)\rangle_J=\int e^{\tilde{L}}\mathcal{D}\bar{\psi},\mathcal{D}\psi,\mathcal{D}\varphi,\mathcal{D}S, \tag{5}$$

where

$$\tilde{L}=L_0+L_{int},$$

$$L_0=\int\sum_i\Big[-\varphi_i^2+i\varphi_i\left(\frac{\partial S_i}{\partial t}-\frac{\partial H_0}{\partial S_i}\right)+\bar{\psi}_i\left(\frac{\partial}{\partial t}+\frac{\delta^2 H_0}{\delta S_i^2}\right)\psi_i\Big]dt,$$

$$L_{int}=-\frac{J^2}{N}\int\sum_{ij}[\varphi_i(t)\varphi_i(t')S_j(t)S_j(t')+\varphi_i(t)S_i(t')\times\varphi_j(t')S_j(t)+2i\varphi_i(t)\bar{\psi}_i(t')S_j(t')\psi_j(t)+2iS_i(t)\bar{\psi}_i(t')\varphi_j(t)\psi_j(t')+\bar{\psi}_i(t)\bar{\psi}_i(t')\psi_j(t)\psi_j(t')-\bar{\psi}_i(t)\psi_i(t')\bar{\psi}_j(t')\psi_j(t)]\,dt. \tag{6}$$

The products in (6) that pertain to different points can be separated with the aid of the Hubbard-Stratanovich transformation. In the mean-field approximation, the additional Hubbard variables can be replaced by mean values which, to be sure, are now nonlocal in time. For example, the first term of the sum in (6) is transformed in the following manner:

$$\begin{aligned}&\exp\left\{-\frac{J^2}{N}\int\sum_{ij}\varphi_i(t)\varphi_i(t')S_j(t)S_j(t')\right\}\\&=\int\exp\left\{\frac{1}{N}\int\Big[C(t,t')D(t,t')-JC(t,t')\sum_i S_i(t)S_i(t')\right.\\&\left.-JD(t,t')\sum_i\varphi_i(t)\varphi_i(t')\Big]dt\,dt'\right\}\mathcal{D}C\mathcal{D}D\\&=\exp\left[-\int J\left(\sum_i\varphi_i(t)\varphi_i(t')D(t-t')+\sum_i S_i(t)S_i(t')C(t-t')\right)dt\,dt'\right],\end{aligned} \tag{7}$$

where we have introduced the correlators

$$D(t-t')=\langle S(t)S(t')\rangle,\quad C(t-t')=\langle\varphi(t)\varphi(t')\rangle. \tag{8}$$

All the remaining terms of the sum (6) are similarly transformed, and equations of the type (8) are the self-consistency equations. Above the transition point, the fermion mean values that do not conserve the fermion number and are produced in analogy with C and D can be set equal to zero. This assumption is justified by the fact that under its terms there exists a solution of Eqs. (8) above T_c. It is still questionable, however, whether this assumption is valid below T, when this solution ceases to exist. It seems more probable that this is not the case, because the corresponding susceptibility $\langle\bar{\psi}\varphi S\psi\rangle$ has at the transition point the same singularity as the correlator $\langle SS\rangle$, concerning which it is known that $\langle S\rangle\neq 0$ appears below T_c. We have

$$C=-\delta^2F(0)/\delta h^2=0,$$

therefore the term CSS can be discarded. In addition, by adding to H_0 the term αS^2 and differentiating $F(0)$ with respect to α we obtain the useful identity

$$\langle\varphi S\rangle=\langle\bar{\psi}\psi\rangle.$$

L_{int} assumes ultimately the rather simple form

$$L_{int}=J\sum_i\int[S_i(t)\varphi_i(t')G(t-t')+\varphi_i(t)\varphi_i(t')D(t-t')+\bar{\psi}_i(t)\psi_i(t')G(t-t')]\,dt\,dt',\quad G=\langle\varphi(t)S(t')\rangle=\langle\delta S'/\delta h\rangle. \tag{9}$$

The total Lagrangian is quadratic in φ, therefore Eq. (5) can be integrated with respect to φ, and the new Lagrangian takes the form

$$\begin{aligned}L=\int\Big\{&\left(\frac{\partial S}{\partial t}+\frac{\delta H_0}{\delta S}-J\int G(t-t'')S(t'')\right)\\&\times[\delta(t-t')+D(t-t')]^{-1}\\&\times\left(\frac{\partial S}{\partial t'}+\frac{\delta H_0}{\delta S}-J\int G(t'-t'')S(t'')\,dt''\right)\\&+\bar{\psi}\left(\frac{\partial}{\partial t}-\frac{\delta^2 H}{\delta S^2}-JG(t-t')\right)\psi\Big\}dt\,dt'.\end{aligned} \tag{10}$$

We now use the condition $r=u\gg 1$ of proximity to the Ising model. In this case the main contribution to the integral (5) are made by field configurations of the soliton type, with the average time between solitons $\propto\exp(-r)$, while the duration of each is r^{-1}. Consequently, the characteristic times $\langle\tau\rangle$ of the decrease of D and C are of the order of $\exp(-r)$. The expression in the round brackets in the Lagrangian (10) differ from zero only at the instant of passage of the soliton. The contribution made by the soliton to the action is $\sim r$; in this case the contribution of the terms containing J, i.e., the interaction of order J, is much less than the total action. The smallness of the terms that contain G is offset by their long-range action, whereas the operator $(1+D)^{-1}$ is not long-range; the term D can therefore be neglected in comparison with G.

Making now the inverse transformation from field theory with Lagrangian (10) to the Langevin equations, we obtain

$$\frac{\partial S}{\partial t}=-\frac{\partial H}{\delta S}+\zeta,\quad H=H_0-J\int_{-\infty}^{t}G(t-t')S(t')S(t)-h^eS. \tag{11}$$

Equations (11) describe one variable situated in a field

$$h_t=\int G(t-t')S(t')\,dt'+h^e,$$

which the variable itself produces in the preceding instants of time.

4. TRANSITION TO THE ONE-DIMENSIONAL STATIC MODEL

The Fokker-Planck equations corresponding to the Langevin equations (11) are equivalent to the equations of the transfer matrix of the static problem. Let us obtain the free energy of the problem and express the susceptibility of the dynamic problem (11) in terms of the static correlator. The static problem is define by the free energy of the spin systems S_i, namely

$$F=\frac{1}{2}\sum_{ij}S_iA_{ij}S_j+h_i^eB_{ij}S_j. \tag{12}$$

The probability of any static-problem spin distribution given

by the free energy (12) is equal to the probability of the corresponding time-dependent process for an individual spin. Therefore, by calculating the reaction to the external field he for the static problem, we obtain the susceptibility of the initial time-dependent model. The self-consistency condition relates thus the susceptibility of the static problem with the A_{ij} that enter in (12). We note that the subscript i numbers the points in "time," and subsequently the "distance" between neighboring must be made to tend to zero. In the operator A we separate two parts:

$$A=A_0+A_1,$$
$$A_0=J_0\gamma_0 \exp(-\gamma_0|i-j|), \quad J_0\gg 1\gg\gamma_0. \tag{13}$$

The operator A_0 ensures here the correct behavior of the variable without the field h; its interaction radius is much smaller than that of A_i, and its actual form is of no importance for the answer and was chosen in the form most convenient for later use, to satisfy the condition that the correlator $\langle SS\rangle$ of the dynamic model without any field h coincide with the correlator of the static one without the term A_1. The term A_1 is obtained by equating the spin-flip probabilities within the time Δt in the dynamic model (11), equal to $\Delta t \exp(-h)$, to the corresponding contribution to the partition function (12). We obtain

$$A_1=[(\Delta t-{}^1/_2 d/dt-\Delta t\hat{G})\hat{G}]_{t,t'}+[(\Delta t-{}^1/_2 d/dt-\Delta t\hat{G})\hat{G}]_{t',t}. \tag{14}$$

For B we have analogously

$$B=\theta(t)(\Delta t-{}^1/_2 d/dt-\Delta t\hat{G})\hat{G}.$$

From this we get an expression for the susceptibility in terms of the correlator of the static problem:

$$G=\theta(t-t')(\Delta t-{}^1/_2 d/dt-\Delta t\hat{G})\hat{D}, \quad D=\langle S(t)S(t')\rangle. \tag{15}$$

Equations (12)–(15) determine completely the static moment and the self-consistency equations that relate the spin interaction with their correlator.

5. SOLUTION OF STATIC PROBLEM

The static problem can be solved, i.e., the correlator $\langle SS\rangle$ expressed in terms of the interaction, by using a technique developed by Kac.[10] It will now be convenient to change over to the Laplace transforms of the operators A, G, and D, i.e.,

$$A_1=\int d\gamma\rho(\gamma)e^{-\gamma r}=\sum_k J_k e^{-\gamma_k r},$$
$$G=\int d\gamma\tilde{\rho}(\gamma)e^{-\gamma r}, \quad D=\int d\gamma d(\gamma)e^{-\gamma r}. \tag{16}$$

The condition (14) is then rewritten in the form

$$\rho(\gamma)=\Delta t\left(1-\gamma-\int\frac{\tilde{\rho}(\gamma')}{\gamma'+\gamma}\,d\gamma'\right)\tilde{\rho}(\gamma). \tag{17}$$

We use an identity transformation that eliminates the long-range action:

$$\langle SS\rangle=\sum_{\{S\}}S_iS_{i+r}e^{\mathcal{F}}=\int\sum_{\{S\}}S_iS_{i+r}\exp\left(\sum_{i,k}S_i x_i^k - \frac{1}{2}x_i^k B_{ij}^k x_j^k\right)dx_i^k=\int \mathrm{th}\left(\sum_k J_k^{1/2}x_i^k\right)\mathrm{th}\left(\sum_k J_k^{1/2}x_{i+r}^k\right)$$
$$\times\exp\left(-\int\left[\sum_k(\nabla x_i^k)^2+\frac{1}{4}\gamma_k^2(x^k)^2 - \ln\mathrm{ch}\sum_k J_k^{1/2}\gamma_k x^k\right]dr\right)dx_i^k, \tag{18}$$

where $B_{ij}^k=(A_{ij}^k)^{-1}$.

It is convenient to transform back from the partition function (18) to the transfer matrix, for the eigenfunctions of which we obtain a multidimensional Schrödinger equation with a potential

$$V=\frac{1}{4}\sum_k\gamma_k^2x_k^2-\ln\mathrm{ch}\sum_k J_k^{1/2}\gamma_k x_k. \tag{19}$$

We note that all the $\gamma_k(k\neq 0)\ll\gamma_0$, therefore the "motion" in the x_0 direction is much faster than along the others; this gives grounds for using the adiabatic approximation with respect to the coordinates x_k with $k\neq 0$. Along the coordinate x_0 there are then two close levels spaced $\Delta E=\gamma_0\times\exp(-J_0/2\gamma_0)$ apart and corresponding approximately to the symmetrical and antisymmetrical wave functions; the remaining levels are much higher, $\Delta E\sim\gamma_0$, and can be disregarded. The effective Hamiltonian that depends only on the slow variables becomes a 2×2 matrix:

$$H=\sum_k\left(-\frac{d^2}{dx_k^2}+\frac{1}{4}\gamma_k^2x_k^2\right)\begin{pmatrix}1&0\\0&1\end{pmatrix}+\begin{pmatrix}1&\sum J_k^{1/2}\gamma_k x_k\\ \sum J_k^{1/2}\gamma_k x_k&0\end{pmatrix}. \tag{20}$$

We recall that $J_k=\rho(\gamma_k)\Delta\gamma_k\to 0$, therefore the second term in (20) can be determined by perturbation theory, recalling however that the level spacing is small, $\sim\Delta\gamma_k$. It suffices therefore to take into account transitions between a state that is antisymmetric with respect to the fast coordinate and corresponding to the null level of the slow oscillators, and states that are symmetric with respect to the fast coordinate and correspond to the first excited levels of the slow oscillators. In final analysis, to calculate $\langle SS\rangle$ it is necessary to find the eigenfunctions that contribute to the matrix element

$$\left\langle\psi_i\left|\mathrm{th}\left(\sum_k J_k^{1/2}x^k\right)\right|\psi_0\right\rangle=\langle\psi_i|\mathrm{sgn}\,x^0|\psi_0\rangle=a_i, \tag{21}$$

and their eigenvalues E_i. The correlator $\langle SS\rangle$ is then equal to

$$\langle SS\rangle=\sum_i\exp(-E_i r)|a_i|^2. \tag{22}$$

The main contribution to (22) is made by the matrix elements with functions ψ_i that are perturbed states symmetric in x^0, and first excited states with respect to one of the x^k. The perturbation introduces into these wave functions an admixture of a wave function that antisymmetric in x^0, therefore the matrix element (21) is not equal to zero. When account is taken of only such transitions, the perturbation matrix has nonzero elements on the zeroth row and zeroth column, which correspond to the antisymmetric wave function, and

on the diagonal. It is therefore easily diagonalized and leads to an equation for the shift $E-\gamma_k$ in the form

$$2\sum\frac{J_k}{E-\gamma_k}=1-E. \tag{23}$$

The weight of the zeroth antisymmetric wave function in a state with energy E is equal to

$$a=\left[1+2\sum_k\frac{J_k}{(\gamma_k-E)^2}\right]^{-1}. \tag{24}$$

The sum over k in (23) and (24) must, to permit transition to the continuous limit in γ, be broken up into two parts: the contribution from the γ_k that are close to E, and from the remote ones. We obtain

$$\begin{aligned}\sum_k\frac{J_k}{E-\gamma_k}&=\pi\rho\operatorname{ctg}y+\int\frac{\rho(\gamma')d\gamma'}{\gamma'-\gamma},\\ \sum_k\frac{J_k}{(\gamma_k-E)^2}&=\pi^2\rho\sin^{-2}y,\end{aligned} \tag{25}$$

where $E=\gamma+\Delta\gamma(\frac{1}{2}+y)/\pi$. By eliminating y from (25) and substituting the expression obtained for a from (24) into Eq. (22) for the correlator, we obtain

$$d(\gamma)=2\rho(\gamma)\left\{(2\pi\rho)^2+\left[1-\gamma-2\int\frac{\rho(\gamma')d\gamma'}{\gamma'-\gamma}\right]^2\right\}^{-1}. \tag{26}$$

Equation (26) together with the self-consistency condition (17) forms a closed system of integral equations, sufficient for the determination of $\rho(\gamma)$. Equation (26) can be verified at a specific form of $\tilde\rho(\gamma)$, namely:

$$\tilde\rho=J\gamma\delta(\gamma-\gamma_0).$$

At small J we have

$$d(\gamma)=J\gamma\delta[\gamma-\gamma_0(1-J)].$$

On the other hand for $\tilde\rho$ in this form we can write down directly and solve the Fokker-Planck equations that corresponds to the Langevin equation. This leads to the same value of $d(\gamma)$.

6. INVESTIGATION OF THE INTEGRAL EQUATION

We note first that $\tilde\rho(\gamma)=0$ satisfies the integral equations (26) and (17) at those points where the expression in the square brackets of (26) does not vanish. It is natural to assume that $\tilde\rho\neq0$ on the segment (γ_1,γ_2) and $\tilde\rho=0$ outside it. Then Eqs. (26) and (17) can be rewritten in the form

$$\pi^2\rho^2+\left[1-\gamma-\int_{\gamma_1}^{\gamma_2}\frac{\rho d\gamma'}{\gamma'-\gamma}\right]^2=\left(\frac{J}{2}\right)^2\left[1+\gamma-\int_{\gamma_1}^{\gamma_2}\frac{\tilde\rho d\gamma'}{\gamma'+\gamma}\right]^2, \tag{27}$$

$$\rho(\gamma)=\frac{1}{2}\left(1+\gamma-\int_{\gamma_1}^{\gamma_2}\frac{\tilde\rho d\gamma'}{\gamma'+\gamma}\right)\tilde\rho(\gamma). \tag{28}$$

We shall investigate the solution of these integral equations in two limiting cases: $J\ll1$ (high temperatures) and $\gamma_1\ll1$, $J-1\ll1$.

At high temperatures $\tilde\rho\ll1$ and the region (γ_1,γ_2) is small and is concentrated near unity. The contribution of the integral of $\tilde\rho$ to (27) and (28) can be neglected; (27) is transformed into a closed integral equation for ρ, and its solution is

$$\tilde\rho=\rho=(2/\pi)\left[(\gamma-\gamma_1)(\gamma_2-\gamma)\right]^{1/2},\qquad \gamma_{1,2}=1\pm J. \tag{29}$$

We proceed to the case of temperatures near the transition point. We investigate first the analytic properties of solutions that are valid at all temperatures. To this end we continue the system (27) and (28) over the entire complex γ plane, using the identity

$$\int\frac{\rho\,d\gamma'}{\gamma'-\gamma}=\int\frac{\rho\,d\gamma'}{\gamma'-\gamma+i\varepsilon}+i\pi\rho(\gamma). \tag{30}$$

Substituting (30) in (27) we find that $\tilde\rho$ and ρ have no singularities on the complex plane, with the exception of the two cuts $(-\gamma_1,-\gamma_2)$ and (γ_1,γ_2), and possibly poles at the points where

$$1-\gamma=\int_{\gamma_1}^{\gamma_2}\frac{\rho(\gamma')d\gamma'}{\gamma'-\gamma+i\varepsilon}. \tag{31}$$

Let us calculate the jump on the cuts of the right- and left-hand sides of Eqs. (27) and (28). We find that $\Delta\tilde\rho=-2\tilde\rho$ on the right cut, therefore this singularity is eliminated by separating the factor $[(\gamma-\gamma_1)(\gamma_2-\gamma)]^{1/2}$.

At small $\tilde\rho$ the equations are simpler. We eliminate ρ from (27) and (28). To this end we express ρ on the basis of (27) and the known formulas of the theory of integral equations with singular kernels[11]:

$$\frac{1}{\pi}\left[(\gamma-\gamma_1)(\gamma_2-\gamma)\right]^{1/2}\left[\left(1+\frac{J}{\sqrt2}\right)+\frac{J}{\sqrt2}\int_{\gamma_1}^{\gamma_2}\frac{dt\tilde\rho(t)}{(t+\gamma)\left[(t+\gamma_1)(t+\gamma_2)\right]^{1/2}}\right]$$
$$=\left(1+\gamma-\int\frac{\tilde\rho d\gamma'}{\gamma'+\gamma}\right)\tilde\rho. \tag{32}$$

At the transition point we have $J=1$ and $\gamma_1=0$. The form of $\tilde\rho(\gamma)$ at $\gamma\ll1$ follows from (32):

$$\tilde\rho(\gamma)=\gamma^{1/4}\alpha, \tag{33}$$

where α is a number of the order of unity.

Assume that $\tilde\rho(\gamma)$ retains the same form also near the transition point at $\gamma\gg\gamma_1$, but $\gamma\ll1$. Let $\gamma\sim\gamma_1$; then $\tilde\rho$ is equal to

$$\tilde\rho=\left[(\gamma-\gamma_1)/\gamma_1\right]^{1/2}\alpha. \tag{34}$$

The transition from (33) to (34) takes place at $\gamma-\gamma_1\sim\gamma_1^{3/2}$. This makes it possible to express γ_1 through the same number α:

$$\gamma_1=(1-J)^4\alpha^{12}. \tag{35}$$

Equations (33)–(35) determine the susceptibility at temperatures close to T_c. Above T_c the asymptotic form of the susceptibility at the longest times $t>\tau^{3/2}$ takes the usual form $t^{-1/2}\exp(-t/\tau)$, but at shorter times $t>\tau$ the asymptotic form is different: $t^{-1/4}\exp(-t/\tau)$. At the transition point the susceptibility decreases very slowly, like $t^{-5/4}$.

In conclusion I wish to thank A. I. Larkin, I. E. Dzyaloshinskiĭ, and M. V. Feĭgel'man fore helpful discussions and interest in the work.

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Translated by J. G. Adashko