Restructuring of polariton and phonon spectra of a semiconductor in the presence of a strong electromagnetic wave

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(Submitted 30 July 1982)
Zh. Eksp. Teor. Fiz. **84**, 404–421 (January 1982)

We investigate the propagation of a strong electromagnetic wave in a direct-band semiconductor following resonant excitation of excitons. It is assumed that the given polariton wave attenuates because of scattering by longitudinal acoustic phonons, and the case is analyzed when the scattering due to the absorption of the phonon wave by the excitons greatly exceeds the scattering at which the wave excitons emit phonons. Using a diagram technique for nonequilibrium processes, a system of equations is obtained for the description of the behavior of the initial polariton wave, of the scattered electrons, and of the scattering phonons. The polariton- and phonon-spectrum restructuring accompanied by an abrupt decrease of the electromagnetic-wave absorption coefficient is considered within the framework of the model indicated.

PACS numbers: 71.35. + z, 71.36. + c

The interaction of electromagnetic radiation with excitons in direct-band semiconductors has been attracting considerable interest both theoretically and experimentally. As a rule, the problems investigated are connected with the luminescence and kinetics of exciton systems[1–6] and with the description of the propagation of electromagnetic waves in resonant excitation of excitons.[7–12] For a correct treatment of these problems, the polariton concept is introduced[13] because in direct-band semi-conductors at low temperatures there is realized a strong exciton-photon coupling that leads to a mixing of the exciton and photon states and to a restructuring of the photon and exciton spectra.

Among the problems of luminescence and kinetics of exciton systems is included the investigation of thermalization of strongly excited exciton systems, and in particular the consideration of the possibility of Bose condensation of excitons both on the upper polariton branch[5,6] and on the lower one.[1–4] It is appropriate to note that one of the principal difficulties of experimentally observing Bose condensation of excitons in the course of their thermalization is as a rule the long lifetime of the excitons in the semiconductor.

The problem of propagation of electromagnetic radiation in resonant excitation of excitons, a problem connected with the calculation of the absorption coefficient of the electromagnetic wave,[7,8] with the experimental confirmation of the polariton character of the electromagnetic waves in the semiconductors,[9,10,12] and with others, has also a bearing on the manifestation of Bose condensation of excitons. In fact, an electromagnetic wave $\mathbf{k}_0$ produced by an external source and propagating in a crystal has a finite amplitude, and thus the mode $\mathbf{k}_0$ is macroscopically filled in the usual sense:

$$\langle \hat{a}_{\mathbf{k}_0}^{+} \hat{a}_{\mathbf{k}_0} \rangle \propto \langle \hat{b}_{\mathbf{k}_0}^{+} b_{\mathbf{k}_0} \rangle \propto V,$$

where $\hat{a}_{\mathbf{k}_0}^{+}$ and $\hat{b}_{\mathbf{k}_0}^{+}$ are the photon and exciton creation operators in the mode $\mathbf{k}_0$ and V is the volume of the crystal. In this case there can take place a relatively short-time thermalization with possible formation of a Bose condensate of nonequilibrium-excited excitons, as is proposed in the first group of problems, and a much faster scattering of the excitons from the initial wave and establishment of quasiequilibrium in such an exciton system with a condensate in the mode $\mathbf{k}_0$.

The purpose of the present article is a consistent description of the propagation of a macroscopically filled polariton wave $\mathbf{k}_0$ with allowance for the specific damping mechanism. Just as in Refs. 7 and 8, we shall consider the case when the cause of the damping of the wave is scattering of the exciton component by a longitudinal acoustic phonons. In addition, we propose that the scattering as a result of absorption of an acoustic-phonon wave by the exciton is much larger than the scattering due to the emission of a phonon wave by the exciton. This assumption makes it possible, on the one hand, to investigate a number of singularities in the propagation of a high-power polariton wave with the indicated damping mechanism, and on the other hand apply the results to the phenomenon of ordinary equilibrium Bose condensation of an ideal Bose gas in a phonon thermostat into the mode $\mathbf{k}_0 = 0$. We shall take into account hereafter only scattering connected with absorption of acoustic phonons, and defer the analysis of the indicated assumptions and the extent to which the model is realistic to the end of the article.

In the general case the problem consists of a joint examination of a macroscopically filled wave $\mathbf{k}_0$, scattered excitons $\mathbf{k}$, and scattering acoustic phonons $\mathbf{k}$–$\mathbf{k}_0$. With the aid of the diagram technique for nonequilibrium processes[14] used in Ref. 15 for the case when an explicit account is taken of the finite lifetime of interacting quasiparticles, we obtain kinetic equations for the distribution functions of the scattered excitons $N_k(\mathbf{r}t)$, for the scattering phonons $n_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t)$, as well as the restructuring of the spectra of these quasiparticles, and describe the propagation of the initial wave. The latter will depend substantially on the state of the wave $\mathbf{k}_0$. Namely, if the wave $\mathbf{k}_0$ is in a Glauber state, i.e., coherent, it is described by the Maxwell and Schrödinger field equations with respect to the mean values of the corresponding field operators, while if the wave $\mathbf{k}_0$ is completely incoherent, i.e., a noise, it is described by a kinetic equation with respect to

 0038-5646/83/010234-11$04.00

$N_0(\mathbf{r}t)$—the density of the excitons $\mathbf{k}_0$ produced by the wave. The density $N_0(\mathbf{r}t)$ is connected with the intensity of the polariton wave $I_0(\mathbf{r}t)$ by relation (78) below. In the general case it is necessary to consider simultaneously in the indicated manner the coherent and noise components of the wave.

An important parameter of the problem is the quantity $\Gamma(\mathbf{k},\mathbf{k}_0)$—the characteristic energy uncertainty in the elementary act of scattering of the exciton by the initial wave. We shall obtain the following expression for this quantity:

$$\Gamma(\mathbf{k},\ \mathbf{k}_0)\approx\{[\gamma(\mathbf{k})+\gamma_{\text{ph}}^{A}(\mathbf{k}-\mathbf{k}_0)]^2+4N_0m_{\mathbf{k}-\mathbf{k}_0}\}^{1/2}, \tag{1}$$

where $\gamma(\mathbf{k})=1/\tau$ is the reciprocal lifetime of the scattered exciton $\mathbf{k}$ and is connected with the possibility of further scattering by equilibrium acoustic phonons, $\gamma_{\text{ph}}^{A}(\mathbf{k}-\mathbf{k}_0)=1/\tau_{\text{ph}}^{A}$ is the reciprocal lifetime of the scattering phonon $\mathbf{k}-\mathbf{k}_0$ because of the lattice anharmonicity of the crystal, and $m_{\mathbf{k}-\mathbf{k}_0}$ is defined by (17) below. The quantity $\Gamma(\mathbf{k},\mathbf{k}_0)$ determines the number of polariton modes $\mathbf{k}$ in which scattering of excitons by the initial wave takes place, and also the number of the phonon-subsystem $\mathbf{k}-\mathbf{k}_0$ modes whose phonons participate most effectively in the scattering process. Depending on the intensity of the polariton wave, we can separate different cases.

Low intensities:

$$N_0<\gamma(k_0)\,\gamma_{\text{ph}}^{A}(k_0)/m_{k_0}. \tag{2}$$

These low intensities correspond to the linear theory[8]: the absorption coefficient of the wave σ_{sp} is proportional to the semiconductor temperature T and is independent of the wave intensity.

Medium intensities:

$$\gamma(k_0)\,\gamma_{\text{ph}}^{A}(k_0)/m_{k_0}<N_0<[\gamma(k_0)+\gamma_{\text{ph}}^{A}(k_0)]^2/4m_{k_0}. \tag{3}$$

This case corresponds to nonliner-wave propagation, and the absorption coefficient $\sigma_{\text{sp}}\propto 1/N_0$ and can reach values that are smaller by several orders than for low intensities. At low temperatures, the inequality $\gamma_{\text{ph}}^{A}(k_0)\ll\gamma(k_0)$ is satisfied, and in this case we have phonon nonlinearity, wherein the phonon subsystem is greatly depleted in the course of scattering of the initial wave: the act of exciton scattering into the mode $\mathbf{k}$ is annihilation of the phonon $\mathbf{k}-\mathbf{k}_0$. Such a strong decrease of the occupation numbers of the modes of the phonon subsystem is naturally accompanied by a decrease of the wave absorption coefficient. At high temperatures $\gamma(k_0)\ll\gamma_{\text{ph}}^{A}(k_0)$, and the nonlinear decrease of the absorption can be attributed to nonequilibrium Bose condensation. This phenomenon consists in the possibility of filling the polariton modes $\mathbf{k}$, in which the excitons are scattered up to values of the equilibrium phonon occupation numbers of the modes $\mathbf{k}-\mathbf{k}_0$. The scattering of the initial wave is then effectively suppressed and the absorption is determined exclusively by the further scattering of the excitons $\mathbf{k}$.

High intensities:

$$[\gamma(k_0)+\gamma_{\text{ph}}^{A}(k_0)]^2/4m_{k_0}<N_0. \tag{4}$$

At such high intensities, a restructuring of the polariton and phonon spectra takes place (see Fig. 1). It consists of unification and splitting of the indicated terms, in analogy with the formation of the polariton dispersion curve from the exciton and photon spectra. The restructured spectrum corresponds to a new excitation consisting of polariton and phonon components, which we shall call phonoriton for short. The wave absorption coefficient is nonliner, as before, and is proportional to $N_0^{-1/2}$. The inequality (4) is the condition that the splitting of the spectrum $2(m_{k0}N_0)^{1/2}$ exceed the reciprocal lifetime of the obtained excitation $[\gamma(k_0)+\gamma_{\text{ph}}^{A}(k_0)]$, i.e., the indicated restructuring of the spectrum can be observed. On the other hand, this condition corresponds to the case when the quantity $\Gamma(\mathbf{k},\mathbf{k}_0)$ is determined principally by the contribution of the last term under the square root in expression (1), i.e., the action of a macroscopically filled mode $\mathbf{k}_0$. This means that the scattered excitons returns with high probability to the initial mode $\mathbf{k}_0$, this being a consequence of the macroscopic filling of the mode $\mathbf{k}_0$. Such a possibility of exciton scattering followed by return to the wave, accompanied by the absorption of a phonon $\mathbf{k}-\mathbf{k}_0$ with its subsequent emission, corresponds precisely to the restructuring of the polariton and phonon spectra. We shall show below that for the restructured spectrum the characteristic uncertainty of the energy in the elementary scattering act will in fact be independent of the intensity of the initial wave. Moreover, a consistent analysis of the problem presupposes the use of the restructured spectrum at any wave intensity, in particular at intensities defined by inequalities (2) and (3). We note that condition (4) is satisfied at $T=4.2$ K for $N_0\sim 10^{15}-10^{17}$ cm^{-3}, depending on the type of semiconductor and on its quality.

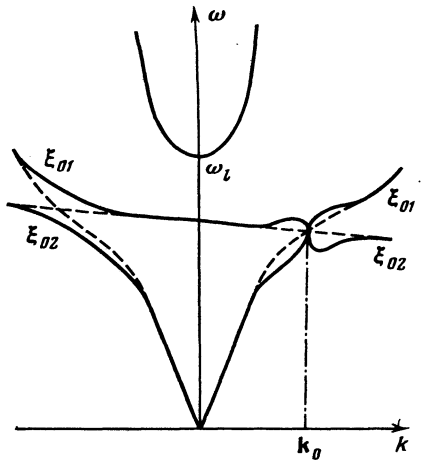

FIG. 1.

We begin the analysis of the formulated problem with the case when the initial wave $\mathbf{k}_0$ is fully incoherent. For a consistent solution, we need introduce the retarded $(G^{(R)},D^{R})$, advanced (G^{A},D^{A}), and statistical (G^{+},D^{+}) Green's functions[14] of the excitons and phonons, respectively. If there is no interaction between the quasiparticles, these functions take the form

$$G_0^{R,A}(k)=\frac{1}{\omega-\omega_{\mathbf{k}}\pm i\delta},\qquad G_0^{+}(k)=-2\pi iN_{\mathbf{k}}\delta(\omega-\omega_{\mathbf{k}}); \tag{5}$$

$$D_0^{R,A}(k)=\left[\frac{1}{\omega-\omega_{\mathbf{k}}^{\mathrm{ph}}\pm i\delta}-\frac{1}{\omega+\omega_{\mathbf{k}}^{\mathrm{ph}}\pm i\delta}\right],$$
$$D_0^{+}(k)=-2\pi i[(1+n_{\mathbf{k}})\delta(\omega+\omega_{\mathbf{k}}^{\mathrm{ph}})+n_{\mathbf{k}}\delta(\omega-\omega_{\mathbf{k}}^{\mathrm{ph}})]; \quad (6)$$
$$k=(\omega,\mathbf{k}).$$

Here $\hbar\omega_k^{\mathrm{ph}}=\hbar uk$ (u is the speed of sound in the crystal) is the energy of the acoustic phonon $\mathbf{k}$ and ω_k is the energy of the exciton $\mathbf{k}$ and is determined by the polariton dispersion curve. Since the mode k_0 is macroscopically filled, the statistical Green's function (5) takes at $\mathbf{k}=\mathbf{k}_0$ form

$$G_0^{+}(k_0)=-2\pi iN_0V\delta(\varepsilon-\omega_{\mathbf{k}_0}),$$
$$N_{\mathbf{k}_0}=(2\pi)^3N_0\delta(\mathbf{k}-\mathbf{k}_0), \quad k_0=(\varepsilon,\mathbf{k}_0). \quad (7)$$

We note that the exciton Green's functions represent in fact the exciton part of the Green's function of the polariton, and should therefore be multiplied by a factor [see (79) below] that depends on $\mathbf{k}$. However, as will be noted below, this factor, in the interesting region of values of $\mathbf{k}$ near the polariton splitting of the spectrum, is equal to unity with high accuracy, therefore the polariton character of the excitons can be taken into account only by assuming $\omega_{\mathbf{k}}$ to be the polariton energy. In addition, to simplify the description we shall consider the case when the propagating wave $\mathbf{k}_0$ is linearly polarized, and the dipole moment of the exciton transition $\mathbf{d}$ has a preferred direction. These assumptions allow us to disregard the tensor character of the Green's functions of the excitons and photons.

In this case, when quasiparticle interaction takes place, the Green's functions are determined by the corresponding Dyson equations and we can make the following assumption concerning their form[15]:

$$G^R(k)=(\omega-\tilde{\omega}_{\mathbf{k}}+i\tilde{\gamma})^{-1}, \quad G^{+}(k)=-2\pi iN_{\mathbf{k}\omega}\tilde{\delta}[\tilde{\gamma}|\omega-\tilde{\omega}_{\mathbf{k}}]. \quad (8)$$

Here

$$\tilde{\delta}[\tilde{\gamma}|\omega-\tilde{\omega}_{\mathbf{k}}]=\tilde{\gamma}(k)/\pi[(\omega-\tilde{\omega}_{\mathbf{k}})^2+\tilde{\gamma}^2(k)] \quad (9)$$

is the energy difference smeared by a δ function, and the generalized occupation numbers $N_{\mathbf{k}\omega}$ are connected with the usual numbers $N_{\mathbf{k}}$ by the relation

$$N_{\mathbf{k}}=\int N_{\mathbf{k}\omega}\tilde{\delta}[\tilde{\gamma}|\omega-\tilde{\omega}_{\mathbf{k}}]\,d\omega. \quad (10)$$

The smearing $\tilde{\gamma}(k)$ of the δ function and the changed frequencies $\tilde{\omega}_{\mathbf{k}}$ are connected with the self-energy part $\Sigma_f^R(\omega,\mathbf{k})$ in the following manner:

$$\tilde{\omega}_{\mathbf{k}}=\omega_{\mathbf{k}}+\mathrm{Re}\,\Sigma_f^R(\omega_{\mathbf{k}},\mathbf{k}), \quad (11)$$
$$\tilde{\gamma}(k)=-\mathrm{Im}\,\Sigma_f^R(\omega,\mathbf{k}). \quad (12)$$

Similar relations hold for the phonon Green's functions.

To describe the scattered excitons, the scattering phonons, and the excitons of the initial wave we shall separate explicitly only the first stage of the scattering, which consists of absorption of one acoustic phonon, by the exciton of the wave or of emission of an acoustic phonon by the scattered exciton with transition of this exciton into the mode k_0. In addition, as already noted, the scattered excitons and scattering phonons have characteristic lifetimes τ and τ_{ph}^A respectively. The self-energy parts in the Dyson equations investigated below will be defined by an integral convolution of the corresponding complete Green's functions, i.e., we assume the vertex part to be equal to unity. In this approximation there are no anomalous Green's functions, for in this case they enter into the Dyson equations together with a factor $m_{\mathbf{k}=0}=0$ for the acoustic (and also optical) phonons [for a definition of $m_{\mathbf{k}}$ see (17)].

We consider first the dynamic part of the problem, namely questions connected with the shift or restructuring of the quasiparticle spectra and with the change of the characteristic lifetimes, i.e., of the widths of the energy levels, when the interaction is turned on. As indicated, this information is provided by the Dyson equations for the advanced or retarded Green's functions of the corresponding quasiparticles. For scattered excitons we have

$$G^R(k)=G_0^R(k)+G_0^R(k)\Sigma_f^R(k)G^R(k), \quad (13)$$
$$G^R(k)=[\omega-\omega_{\mathbf{k}}+i\gamma(\mathbf{k})-\Sigma^R(k)]^{-1}. \quad (14)$$

The self-energy part $\Sigma_f^R(k)$ consists of two terms, the first of which $\Sigma^R(k)$ is connected with the first stage of the scattering, and the second term $\Sigma_\tau^R(k)$ is the self-energy part connected with the probable further scattering of the exciton $\mathbf{k}$ not into the mode $\mathbf{k}_0$. These self-energy parts are determined by the relations

$$\Sigma_\tau^R(k)=-i/\tau=-i\gamma(\mathbf{k}), \quad (15)$$
$$\Sigma^R(k)=i\int[D^R(k-q)G^R(q)+D^{+}(k-q)G^R(q)+D^R(k-q)G^{+}(q)]m_{\mathbf{k}-\mathbf{k}_0}\frac{d^4q}{(2\pi)^4}=\frac{N_0m_{\mathbf{k}-\mathbf{k}_0}}{\omega-\tilde{\omega}_{\mathbf{k}-\mathbf{k}_0}^{\mathrm{ph}}-\tilde{\omega}_{\mathbf{k}_0}+i(\tilde{\gamma}_{N_0}+\tilde{\gamma}_{\mathrm{ph}})}. \quad (16)$$

The factor $m_{\mathbf{k}}$ is connected here with the value of the matrix element $M_{\mathbf{k}}$ of exciton scattering by a phonon in the following manner:

$$m_{\mathbf{k}}=\frac{2\pi}{\hbar}V|M_{\mathbf{k}}|^2. \quad (17)$$

From the relations (12), (15), and (16) we can obtain an expression for the width of the energy level of the scattered exciton:

$$\tilde{\gamma}(k)\equiv\tilde{\gamma}(\omega,\mathbf{k})=\gamma(\mathbf{k})+\pi N_0m_{\mathbf{k}-\mathbf{k}_0}\tilde{\delta}[\tilde{\gamma}_{\mathrm{ph}}+\tilde{\gamma}_{N_0}|\omega-\tilde{\omega}_{\mathbf{k}-\mathbf{k}_0}^{\mathrm{ph}}-\tilde{\omega}_{\mathbf{k}_0}]. \quad (18)$$

Similarly, for the scattering phonons $\mathbf{k}-\mathbf{k}_0$ we have

$$D^R(k-k_0)=[(\omega-\varepsilon)-\omega_{\mathbf{k}-\mathbf{k}_0}^{\mathrm{ph}}+i\gamma_{\mathrm{ph}}^A(\mathbf{k}-\mathbf{k}_0)-\Sigma^R(k-k_0)]^{-1}, \quad (19)$$

$$\Sigma_{\tau_{\mathrm{ph}}^A}^R(k-k_0)=-i\frac{1}{\tau_{\mathrm{ph}}^A}=-i\gamma_{\mathrm{ph}}^A(\mathbf{k}-\mathbf{k}_0),$$
$$\Sigma^R(k-k_0)=\frac{N_0m_{\mathbf{k}-\mathbf{k}_0}}{(\omega-\varepsilon)-(\tilde{\omega}_{\mathbf{k}}-\tilde{\omega}_{\mathbf{k}_0})+i(\tilde{\gamma}_{N_0}+\tilde{\gamma})} \quad (20)$$

$$\tilde{\gamma}_{\mathrm{ph}}(k-k_0)=\gamma_{\mathrm{ph}}^A(\mathbf{k}-\mathbf{k}_0)+\pi N_0m_{\mathbf{k}-\mathbf{k}_0}\tilde{\delta}[\tilde{\gamma}_{N_0}+\tilde{\gamma}|(\omega-\varepsilon)-(\tilde{\omega}_{\mathbf{k}}-\tilde{\omega}_{\mathbf{k}_0})]. \quad (21)$$

The last relation connects the width of the energy level of the scattering phonon $\mathbf{k}-\mathbf{k}_0$ with the corresponding values for the scattered excitons and the excitons of the initial mode $\mathbf{k}_0$. An analysis of the Dyson equation for the retarded Green's function of the excitons $G^{(R)}(k_0)$ of a macroscopically filled mode yields the remaining relations needed to obtain the closed system of equations (18), (21), and (23) with respect to the widths $\tilde{\gamma}$, $\tilde{\gamma}_{N_0}$ and $\tilde{\gamma}_{\text{ph}}$:

$$\Sigma^R(k_0)=\int m_{\mathbf{k}-\mathbf{k}_0}[n_{\mathbf{k}-\mathbf{k}_0,\omega-\varepsilon}-N_{\mathbf{k},\omega}] \times\frac{1}{\tilde{\omega}_{\mathbf{k}}-\varepsilon-\tilde{\omega}^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}+i(\tilde{\gamma}+\tilde{\gamma}_{\text{ph}})}\frac{d^3k}{(2\pi)^3}, \tag{22}$$

$$\tilde{\gamma}_{N_0}(k_0)=\pi\int m_{\mathbf{k}-\mathbf{k}_0}[n_{\mathbf{k}-\mathbf{k}_0,\omega-\varepsilon}-N_{\mathbf{k},\omega}] \times\tilde{\delta}[\tilde{\gamma}+\tilde{\gamma}_{\text{ph}}|\tilde{\omega}_{\mathbf{k}}-\varepsilon-\tilde{\omega}^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}]\frac{d^3k}{(2\pi)^3}. \tag{23}$$

A consistent treatment of the problem within the framework of the assumption (8) concerning the form of the Green's function shows that the characteristic energy uncertainty in the act of the scattering of the excitons of the initial wave $\mathbf{k}_0$ is determined by the sum of the widths of the levels of all the quasiparticles that take part in the scattering:

$$\Gamma(\mathbf{k},\mathbf{k}_0)=\tilde{\gamma}_{N_0}(\tilde{\omega}_{\mathbf{k}_0},\mathbf{k}_0)+\tilde{\gamma}(\tilde{\omega}_{\mathbf{k}},\mathbf{k})+\tilde{\gamma}^{\text{ph}}_{\text{ph}}(\tilde{\omega}_{\mathbf{k}-\mathbf{k}_0},\mathbf{k}-\mathbf{k}_0), \tag{24}$$

After solving the indicated system of equations relative to the widths of the energy levels and substituting them in (24), we obtain for $\Gamma(\mathbf{k},\mathbf{k}_0)$ the approximate relation (1). As indicated earlier, the physically unjustified strong dependence of the level widths, of the frequency shifts, and also of the value of $\Gamma(\mathbf{k},\mathbf{k}_0)$ on the intensity of the wave $\mathbf{k}_0$ is eliminated in practice by introducing a renormalization of the phonon and polariton spectra. Namely, taking into consideration the relations (16) and (20) for the self-energy parts, the expressions for the retarded Green's functions of the scattered excitons and of the scattering phonons can be transformed into

$$G^R(k)=\frac{\omega-\tilde{\omega}^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}-\tilde{\omega}_{\mathbf{k}_0}+i\gamma^A_{\text{ph}}}{(\omega-\omega_{\mathbf{k}}+i\gamma)(\omega-\omega^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}-\tilde{\omega}_{\mathbf{k}_0}+i\gamma^A_{\text{ph}})-N_0m_{\mathbf{k}-\mathbf{k}_0}}, \tag{25}$$

$$D^R(k-k_0)=\frac{(\omega-\varepsilon)-(\omega_k-\tilde{\omega}_{k_0})+i\gamma}{[(\omega-\varepsilon)-\omega^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}+i\gamma^A_{\text{ph}}][(\omega-\varepsilon)-(\omega_{\mathbf{k}}-\tilde{\omega}_{\mathbf{k}_0})+i\gamma]-N_0m_{\mathbf{k}-\mathbf{k}_0}}. \tag{26}$$

It is known that the poles of the Green's functions yield information on the spectrum and damping of the considered excitation, therefore the dispersion curves of the restructured spectrum are determined by the equation

$$(\omega-\omega_{\mathbf{k}}+i\gamma)(\omega-\tilde{\omega}^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}-\tilde{\omega}_{\mathbf{k}_0}+i\gamma^A_{\text{ph}})-N_0m_{\mathbf{k}-\mathbf{k}_0}=0, \tag{27}$$

which have the following roots:

$$\xi_1=\xi_{01}-i\Gamma_1,$$

$$\xi_{01}={}^1/_2\{(\omega_{\mathbf{k}}+\tilde{\omega}^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}+\tilde{\omega}_{\mathbf{k}_0})+[(\omega_{\mathbf{k}}-\tilde{\omega}^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}-\tilde{\omega}_{\mathbf{k}_0})^2+4N_0m_{\mathbf{k}-\mathbf{k}_0}]^{1/2}\},$$

$$2\Gamma_1=\gamma+\gamma^A_{\text{ph}}+(\gamma-\gamma^A_{\text{ph}})\frac{\omega_{\mathbf{k}}-\tilde{\omega}^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}-\tilde{\omega}_{\mathbf{k}_0}}{[(\omega_{\mathbf{k}}-\omega^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}-\tilde{\omega}_{\mathbf{k}_0})^2+4N_0m_{\mathbf{k}-\mathbf{k}_0}]^{1/2}}; \tag{28}$$

$$\xi_2=\xi_{02}-i\Gamma_2,$$

$$\xi_{02}={}^1/_2\{(\omega_{\mathbf{k}}+\tilde{\omega}^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}+\tilde{\omega}_{\mathbf{k}_0})-[(\omega_{\mathbf{k}}-\tilde{\omega}^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}-\tilde{\omega}_{\mathbf{k}_0})^2+4N_0m_{\mathbf{k}-\mathbf{k}_0}]^{1/2}\},$$

$$2\Gamma_2=\gamma+\gamma^A_{\text{ph}}-(\gamma-\gamma^A_{\text{ph}})\frac{\omega_{\mathbf{k}}-\tilde{\omega}^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}-\tilde{\omega}_{\mathbf{k}_0}}{[(\omega_{\mathbf{k}}-\omega^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}-\tilde{\omega}_{\mathbf{k}_0})^2+4N_0m_{\mathbf{k}-\mathbf{k}_0}]^{1/2}}.$$

Here, as well as hereafter, we neglect terms of order $(\gamma^A_{\text{ph}})^2/N_0m_{\mathbf{k}-\mathbf{k}_0}$ and $\gamma^2/N_0m_{\mathbf{k}-\mathbf{k}_0}$ in the calculations. The dispersion curves $\xi_{01}(k)$ and $\xi_{02}(k)$ shown in Fig. 1 are the restructured phonon and polariton spectra. Their deviation from the original ones is most substantial near values of $\mathbf{k}$ defined by the expression

$$\omega_{\mathbf{k}}-\tilde{\omega}^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}-\tilde{\omega}_{\mathbf{k}_0}=0, \tag{29}$$

i.e., for a given scattering direction near two points of intersection of the unperturbed terms. In the vicinity of one of them, a characteristic term splitting $\Delta\xi=\xi_{01}-\xi_{02}=2(N_0m_{\mathbf{k}-\mathbf{k}_0})^{1/2}$, takes place, and in the vicinity of the point $\mathbf{k}_0$, at values of $\mathbf{k}$ satisfying the condition

$$|\mathbf{k}-\mathbf{k}_0|<4N_0m_{\mathbf{k}-\mathbf{k}_0}/k_0(u+V^p_{\mathbf{k}_0})^2 \tag{30}$$

($V^p_{\mathbf{k}_0}$ is the polariton group velocity of the wave $\mathbf{k}_0$) the spectrum restructuring is such that $\xi_{i0}\propto|\mathbf{k}_0-\mathbf{k}|^{1/2}$. The final expressions for the Green's functions (25) and (26) can be written in the form

$$G^R(k)=\frac{\varphi_1{}^R(k)}{\omega-\xi_1}+\frac{\varphi_2{}^R(k)}{\omega-\xi_2}, \tag{31}$$

$$D^R(k-k_0)=\frac{\psi_1{}^R(k-k_0)}{(\omega-\varepsilon)-(\xi_1-\tilde{\omega}_{k_0})}+\frac{\psi_2{}^R(k-k_0)}{(\omega-\varepsilon)-(\xi_2-\tilde{\omega}_{k_0})}, \tag{32}$$

where the functions $\varphi^R_i(k)$ and $\psi^R_i(k-k_0)$ are given by

$$\varphi^R_{1,2}(k)=\pm\frac{\omega-\tilde{\omega}^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}-\tilde{\omega}_{\mathbf{k}_0}+i\gamma^A_{\text{ph}}(\mathbf{k}-\mathbf{k}_0)}{[(\omega_{\mathbf{k}}-\omega^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}-\tilde{\omega}_{\mathbf{k}_0})^2+4N_0m_{\mathbf{k}-\mathbf{k}_0}]^{1/2}}, \tag{33}$$

$$\psi^R_{1,2}(k-k_0)=\pm\frac{(\omega-\varepsilon)-(\omega_{\mathbf{k}}-\tilde{\omega}_{\mathbf{k}_0})+i\gamma(\mathbf{k})}{[(\omega_{\mathbf{k}}-\omega^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0}-\tilde{\omega}_{\mathbf{k}_0})^2+4N_0m_{\mathbf{k}-\mathbf{k}_0}]^{1/2}}. \tag{34}$$

The energy $\tilde{\omega}_{k_0}$ of the excitons of the mode $\mathbf{k}_0$ is given by Eq. (11), where $\Sigma^R(k_0)$ is given by expression (55) below.

We proceed now to obtain and analyze the kinetic equations for scattered excitons that scatter phonons and excitons of the mode $\mathbf{k}_0$. The kinetic equations are derived from the Dyson equations for the corresponding statistical Green's functions by a known method,[14] but in this case the derivation contains a number of peculiarities, and we shall therefore carry it out in sufficient detail.

The Dyson equation for the Green's function of the scattered excitons if of the form

$$G^+(\tilde{x},x')=G_0^+(\tilde{x},x')+\int G_0^+(\tilde{x},x_2)\Sigma_f^A(x_2,x_1)G^A(x_1,x')\,d^4x_2\,d^4x_1+\int G_0^R(\tilde{x},x_2)[\Sigma_f^R(x_2,x_1)G^+(x_1,x')+\Sigma_f^+(x_2,x_1)G^A(x_1,x')]\,d^4x_2\,d^4x_1, \tag{35}$$

where $x_i=(t_i,\mathbf{r}_i)$.

From this we obtain

$$\mathcal{G}_0^{-1}(\tilde{x})G^+(\tilde{x},x')=\int[\Sigma_f^R(\tilde{x},x_1)G^+(x_1,x')+\Sigma_f^+(\tilde{x},x_1)G^A(x_1,x')]\,d^4x_1, \tag{36}$$

$$[\mathcal{G}_0^{-1}(x')]\cdot G^+(\tilde{x},x')=\int[G^+(\tilde{x},x_1)\Sigma^A(x_1,x')+G^R(\tilde{x},x_1)\Sigma^+(x_1,x')]\,d^4x_1.$$

We now take the difference of these two equations, make the change of variables $x=\frac{1}{2}(x'+\tilde{x}),\eta=\tilde{x}-x'$ and take the total Fourier transform[5] with respect to η of both sides of the obtained equations. After the foregoing operations the left-hand side of the equation takes the form

$$-\sum_{i=1}^{2}\left[\frac{\partial}{\partial t}+\mathbf{V}_{\mathbf{k}}^{(i)}\nabla_r\right]N_{\mathbf{k}}^{(i)}(\mathbf{r}t), \tag{37}$$

where $\mathbf{V}_{\mathbf{k}}^{(i)}=\partial\xi_{0i}/\partial\mathbf{k}$. The distribution function of the scattered excitons $N_{\mathbf{k}}(\mathbf{r}t)$ breaks up into two components $N_{\mathbf{k}}^{(i)}(\mathbf{r}t)$, each of which is a distribution function of excitons with a corresponding restructured spectrum $\omega=\xi_{0i}(\mathbf{k})$, with the index i numbering the branch of the restructured spectrum. Such a breakdown is formally connected with the definition of the distribution fuction in terms of the statistical Green's function:

$$N_{\mathbf{k}}(\mathbf{r}t)=i\int G^+(x,k)\frac{d\omega}{2\pi}, \tag{38}$$

since the function $G^+\propto G^R-G^A$ and according to (31) it contains two terms proportional to $\tilde{\delta}[\Gamma_i|\omega-\xi_{0i}]$.

The right-hand side of the equation assumes after the indicated transformations the form

$$\int\frac{d\omega}{2\pi}\{[\Sigma^+(x,k)+\Sigma_\tau(x,k)][G^R(x,k)-G^A(x,k)]-G^+(x,k)[\Sigma^R(x,k)+\Sigma_\tau^R(x,k)-\Sigma^A(x,k)-\Sigma_\tau^A(x,k)]\}. \tag{39}$$

The self-energy parts are defined in the following manner:

$$\Sigma_{(k)}^R=\sum_{i=1}^{2}\Sigma_i^R(k)=\psi_1^R(k-k_0)\frac{N_0m_{\mathbf{k}-\mathbf{k}_0}}{\omega-\xi_1}+\psi_2^R(k-k_0)\frac{N_0m_{\mathbf{k}-\mathbf{k}_0}}{\omega-\xi_2}, \tag{40}$$

$$\Sigma^+(k)=i\int m_{\mathbf{k}-\mathbf{q}}D^+(k-q)G^+(q)\frac{d^4q}{(2\pi)^4}=\sum_{i=1}^{2}n_{\mathbf{k}-\mathbf{k}_0}^{(i)}[\Sigma_i^R(k)-\Sigma_i^A(k)], \tag{41}$$

$$\Sigma_\tau^R(x,k)=-i\frac{1}{\tau},\quad \Sigma_\tau^A(x,k)=i\frac{1}{\tau},\quad \Sigma_\tau^+(x,k)=-i\frac{2N_{\mathbf{k}}^0}{\tau}=0. \tag{42}$$

Here $N_{\mathbf{k}}^0$ are the equilibrium exciton occupation numbers, equal to zero, since we assume complete absence of excitons when there is no polariton wave in the crystal. Relation (40) is obtained from (16) when (32) is taken into account. After certain transformations of (39) using (40)–(42) we obtain the right-hand side of the equation

$$\sum_{i=1}^{2}\Big\{-\frac{2N_{\mathbf{k}}^{(i)}(\mathbf{r}t)}{\tau}+N_0m_{\mathbf{k}-\mathbf{k}_0}[\varphi_i(\mathbf{k})n_{\mathbf{k}-\mathbf{k}_0}^{(i)}(\mathbf{r}t)-\psi_i(\mathbf{k}-\mathbf{k}_0)N_{\mathbf{k}}^{(i)}(\mathbf{r}t)]\frac{1}{\Gamma_i(\mathbf{k})}+N_0m_{\mathbf{k}-\mathbf{k}_0}[\varphi_j(\mathbf{k})n_{\mathbf{k}-\mathbf{k}_0}^{(i)}(\mathbf{r}t)-\psi_j(\mathbf{k}-\mathbf{k}_0)N_{\mathbf{k}}^{(i)}(\mathbf{r}t)]_{i\neq j}\times 2\pi\tilde{\delta}[\Gamma_1+\Gamma_2|\xi_{01}-\xi_{02}]\}. \tag{43}$$

The distribution functions of the phonons of the corresponding branches of the restructured spectrum $n_{\mathbf{k}-\mathbf{k}_0}^{(i)}(\mathbf{r}t)$ are determined by a formula similar to (38), and the factors $\varphi_i(\mathbf{k})$ and $\varphi_i(\mathbf{k}-\mathbf{k}_0)$, which can be called the weighting factors, satisfy the following equations:

$$\psi_1(\mathbf{k}-\mathbf{k}_0)=\varphi_2(\mathbf{k})=\frac{1}{2}[\varphi_2^R(\omega=\xi_{02},\mathbf{k})+\varphi_2^{R^*}(\omega=\xi_{02},\mathbf{k})]=\frac{(\omega_{\mathbf{k}-\mathbf{k}_0}^{\mathrm{ph}}+\tilde{\omega}_{\mathbf{k}_0}-\omega_{\mathbf{k}})+[(\omega_{\mathbf{k}-\mathbf{k}_0}^{\mathrm{ph}}+\tilde{\omega}_{\mathbf{k}_0}-\omega_{\mathbf{k}})^2+4N_0m_{\mathbf{k}-\mathbf{k}_0}]^{1/2}}{2[(\omega_{\mathbf{k}-\mathbf{k}_0}^{\mathrm{ph}}+\tilde{\omega}_{\mathbf{k}_0}-\omega_{\mathbf{k}})^2+4N_0m_{\mathbf{k}-\mathbf{k}_0}]^{1/2}}, \tag{44}$$

$$\psi_2(\mathbf{k}-\mathbf{k}_0)=1-\varphi_2(\mathbf{k})=\varphi_1(\mathbf{k})=\frac{1}{2}[\varphi_1^R(\omega=\xi_{01},\mathbf{k})+\varphi_1^{R^*}(\omega=\xi_{01},\mathbf{k})]$$

$$= \frac{(\omega_{\mathbf{k}} - \overset{\text{ph}}{\omega}_{\mathbf{k}-\mathbf{k}_0} - \tilde{\omega}_{\mathbf{k}_0}) + [(\omega_{\mathbf{k}} - \overset{\text{ph}}{\omega}_{\mathbf{k}-\mathbf{k}_0} - \tilde{\omega}_{\mathbf{k}_0})^2 + 4N_0 m_{\mathbf{k}-\mathbf{k}_0}]^{1/2}}{2[(\omega^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0} + \tilde{\omega}_{\mathbf{k}_0} - \omega_{\mathbf{k}})^2 + 4N_0 m_{\mathbf{k}-\mathbf{k}_0}]^{1/2}}. \tag{45}$$

From (37) and (43) we obtain kinetic equations for the functions of the scattered excitons $N^0_{\mathbf{k}}(\mathbf{r}t)$:

$$\left[\frac{\partial}{\partial t} + \mathbf{V}^{(i)}_{\mathbf{k}} \nabla_{\mathbf{r}}\right] N^{(i)}_{\mathbf{k}}(\mathbf{r}t) = -\frac{2N^{(i)}_{\mathbf{k}}(\mathbf{r}t)}{\tau} + N_0 m_{\mathbf{k}-\mathbf{k}_0}$$
$$\times \{[\varphi_i(\mathbf{k}) n^{(i)}_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t) - \psi_i(\mathbf{k}-\mathbf{k}_0) N^{(i)}_{\mathbf{k}}(\mathbf{r}t)] \frac{1}{\Gamma_i}$$
$$+ 2\pi\tilde{\delta}[\Gamma_i + \Gamma_j | \xi_{0i} - \xi_{0j}] [\varphi_j(\mathbf{k}) n^{(i)}_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t) - \psi_j(\mathbf{k}-\mathbf{k}_0) N^{(i)}_{\mathbf{k}}(\mathbf{r}t)]_{i \neq j}\}. \tag{46}$$

Similarly we derive kinetic equations for the scattering phonons, the only difference being that now the self-energy part $\Sigma^{+A}_{\text{rph}}(x, k - k_0)$ differs from zero and is given by

$$\Sigma^{+}_{\tau_{\text{ph}}^{A}}(x, k - k_0) = -i \frac{2}{\tau^{A}_{\text{ph}}} n_0(\mathbf{k} - \mathbf{k}_0), \tag{47}$$

where

$$n_0(\mathbf{k}-\mathbf{k}_0) = \left[\exp\left(\frac{\hbar \overset{\text{ph}}{\omega}_{\mathbf{k}-\mathbf{k}_0}}{T}\right) - 1\right]^{-1}$$

are the occupations numbers of the unperturbed phonon thermostat. In the kinetic equations this self-energy part constitutes a temperature source of phonons in the phonon subsystem, owing to the presence of the thermostat. These equations take the form

$$\left[\frac{\partial}{\partial t} + \mathbf{V}^{(i)}_{\mathbf{k}} \nabla_{\mathbf{r}}\right] n^{(i)}_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t) = \frac{2}{\tau^{A}_{\text{ph}}} [\psi_i(\mathbf{k}-\mathbf{k}_0) n_0(\mathbf{k}-\mathbf{k}_0) - n^{(i)}_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t)]$$
$$- N_0 m_{\mathbf{k}-\mathbf{k}_0} \Big\{ [\varphi_i(\mathbf{k}) n^{(i)}_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t) - \psi_i(\mathbf{k}-\mathbf{k}_0) N^{(i)}_{\mathbf{k}}(\mathbf{r}t)] \frac{1}{\Gamma_i}$$
$$+ 2\pi\tilde{\delta}[\Gamma_1 + \Gamma_2 | \xi_{01} - \xi_{02}]$$
$$\times [\varphi_j(\mathbf{k}) n^{(i)}_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t) - \psi_j(\mathbf{k}-\mathbf{k}_0) N^{(i)}_{\mathbf{k}}(\mathbf{r}t)]_{i \neq j} \Big\}. \tag{48}$$

The kinetic equations (46) and (48) must be supplemented by conditions on the occupation numbers of the phonons and excitons in the absence of an initial wave ($N_0 = 0$); these conditions are the initial conditions of the problem in the case of slow growth of the initial-wave amplitude:

$$n^{(i=1)}_{\mathbf{k}-\mathbf{k}_0} = \begin{cases} 0, & \omega_{\mathbf{k}} - \omega_{\mathbf{k}_0} - \overset{\text{ph}}{\omega}_{\mathbf{k}-\mathbf{k}_0} > 0 \\ n_0(\mathbf{k}-\mathbf{k}_0), & \omega_{\mathbf{k}} - \omega_{\mathbf{k}_0} - \omega^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0} < 0 \end{cases},$$
$$n^{(i=2)}_{\mathbf{k}-\mathbf{k}_0} = \begin{cases} n_0(\mathbf{k}-\mathbf{k}_0), & \omega_{\mathbf{k}} - \omega_{\mathbf{k}_0} - \overset{\text{ph}}{\omega}_{\mathbf{k}-\mathbf{k}_0} > 0 \\ 0, & \omega_{\mathbf{k}} - \omega_{\mathbf{k}_0} - \omega^{\text{ph}}_{\mathbf{k}-\mathbf{k}_0} < 0 \end{cases}; \quad N^{(i)}_{k} = 0. \tag{49}$$

In the derivation of (46) and (48) we used the following approximate relations:

$$\int N_{k,u} \tilde{\delta}[\Gamma_i | (\varepsilon - \omega) - (\tilde{\omega}_{\mathbf{k}_0} - u)] \tilde{\delta}[\Gamma_j | u - \xi_{0j}] du$$
$$\approx N_{\mathbf{k}} \tilde{\delta}[\Gamma_i + \Gamma_j | (\omega - \varepsilon) - (\xi_{0j} - \tilde{\omega}_{\mathbf{k}_0})], \tag{50}$$
$$\int n_{\mathbf{k}-\mathbf{k}_0,u} \tilde{\delta}[\Gamma_i | \omega - u - \tilde{\omega}_{\mathbf{k}_0}] \tilde{\delta}[\Gamma_j | u - \xi_{0j}] du$$
$$\approx n_{\mathbf{k}-\mathbf{k}_0} \tilde{\delta}[\Gamma_i + \Gamma_j | \omega - \tilde{\omega}_{\mathbf{k}_0} - \xi_{0j}].$$

Let us explain the physical meaning of the kinetic equations (46) and (48). The left-hand side of the equations has the usual form of Aboltzmann operator acting on the distribution function. The first term of the right-hand side of the equations describes the relaxation of the distribution functions to the corresponding equilibrium values, while the second term in the curly brackets describes roughly speaking the tendency of the phonon and exciton components of the given branch of the restructured spectrum to become balanced. This second term, in turn, consists of two parts, the first describing the aforementioned tendency for that branch of the spectrum whose distribution functions are determined by the given equation, while the second corresponds to be second branch of the spectrum. The second part makes a substantial contribution to the corresponding kinetic equation only in the case when the reciprocal lifetime of the phonoriton excitation $\Gamma = \Gamma_1 + \Gamma_2 = \gamma(\mathbf{k}) + \gamma^{A}_{\text{ph}}(\mathbf{k} - \mathbf{k}_0)$ exceeds the spectral splitting $2(N_0 m_{\mathbf{k}-\mathbf{k}_0})^{1/2}$. In this case the obtained kinetic equations can be substantially simplified by changing over to phonon and exciton distribution functions with a non-restructured spectrum[16]:

$$\left[\frac{\partial}{\partial t} + \mathbf{V}_{\mathbf{k}^p} \nabla_{\mathbf{r}}\right] N_{\mathbf{k}}(\mathbf{r}t) = -\frac{2N_{\mathbf{k}}(\mathbf{r}t)}{\tau}$$
$$+ N_0 m_{\mathbf{k}-\mathbf{k}_0} [n_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t) - N_{\mathbf{k}}(\mathbf{r}t)]$$
$$\times \tilde{\delta}[\gamma(\mathbf{k}) + \gamma^{A}_{\text{ph}}(\mathbf{k}-\mathbf{k}_0) | \omega_{\mathbf{k}} - \tilde{\omega}_{\mathbf{k}_0} - \overset{\text{ph}}{\omega}_{\mathbf{k}-\mathbf{k}_0}], \tag{51}$$
$$\left[\frac{\partial}{\partial t} + \mathbf{u}_{\mathbf{k}-\mathbf{k}_0} \nabla_{\mathbf{r}}\right] n_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t) = \frac{2}{\tau^{A}_{\text{ph}}} [n_0(\mathbf{k}-\mathbf{k}_0) - n_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t)]$$
$$- N_0 m_{\mathbf{k}-\mathbf{k}_0} [n_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t) - N_{\mathbf{k}}(\mathbf{r}t)]$$
$$\times \tilde{\delta}[\gamma(\mathbf{k}) + \gamma^{A}_{\text{ph}}(\mathbf{k}-\mathbf{k}_0) | \omega_{\mathbf{k}} - \tilde{\omega}_{\mathbf{k}_0} - \overset{\text{ph}}{\omega}_{\mathbf{k}-\mathbf{k}_0}].$$

In the case

$$\gamma(\mathbf{k}) + \gamma^{A}_{\text{ph}}(\mathbf{k}-\mathbf{k}_0) < 2(N_0 m_{\mathbf{k}-\mathbf{k}_0})^{1/2}$$

it follows from the system of quasilinear kinetic equations (46) and (48), under the condition that the quantity $N_0(\mathbf{r}t)$ varies slowly and with allowance for the initial conditions (49), that the reciprocal of the characteristic lifetimes of the phonons and excitons having the restructured spectrum is determined by Γ_i for the i-phonoriton branch, and from these equations it is possible to obtain equations of the form

$$\left[\frac{\partial}{\partial t} + \mathbf{V}^{(i)}_{\mathbf{k}} \nabla_{\mathbf{r}}\right] F^{(i)}_{\mathbf{k}}(\mathbf{r}t) = 2\Gamma_i(\mathbf{k}) [F^{(i)}_0(\mathbf{r}t) - F^{(i)}_{\mathbf{k}}(\mathbf{r}t)],$$
$$F^{(i)}_0(\mathbf{r}t) = \varphi_i(\mathbf{k}) \psi_i(\mathbf{k}-\mathbf{k}_0) n_0(\mathbf{k}-\mathbf{k}_0) \tag{52}$$

relative to the phonoriton distribution function of the corresponding branch:

$$F_{\mathbf{k}}^{(i)}(\mathbf{r}t)=\psi_i(\mathbf{k}-\mathbf{k}_0)N_{\mathbf{k}}^{(i)}(\mathbf{r}t)+\varphi_i(\mathbf{k})n_{\mathbf{k}-\mathbf{k}_0}^{(i)}(\mathbf{r}t). \quad (53)$$

We note that, for a given branch i of the phonoriton spectrum, the homogeneous solutions of the two equations (46) and (48) at constant N_0 have generally speaking two different characteristic damping times, owing to the initial conditions (49), however, the case actually realized will be the indicated one, i.e., that of Eq. (52) and the damping Γ_i. Moreover, to a certain degree the two equations (52) with allowance for (53) turn out to be equivalent to the four kinetic equations (46), (48) with allowance for (49). The latter, however, are more lucid and also follow directly from the diagram technique.

A kinetic equation for the excitons of a macroscopically filled mode $\mathbf{k}_0$ is obtained from the Dyson equation

$$G^+(k_0)=G_0^+(k_0)[1+\Sigma^A(k_0)G^A(k_0)]+G_0^R(k_0)\Sigma^R(k_0)G^+(k_0), \quad (54)$$

where $\Sigma^R(k_0)$ is defined by the relation

$$\Sigma^R(k_0)=i\sum_{i=1}^{2}\int\frac{d^3k}{(2\pi)^4}m_{\mathbf{k}-\mathbf{k}_0}\times\left\{[\psi_i(\mathbf{k}-\mathbf{k}_0)N_{\mathbf{k}}^{(i)}-\varphi_i(\mathbf{k})n_{\mathbf{k}-\mathbf{k}_0}^{(i)}]\frac{\pi}{\Gamma_i(\mathbf{k})}+[\psi_j(\mathbf{k}-\mathbf{k}_0)N_{\mathbf{k}}^{(i)}-\varphi_j(\mathbf{k})n_{\mathbf{k}-\mathbf{k}_0}^{(i)}]_{i\neq j}\frac{2\pi i}{(\xi_{01}-\xi_{02})+i(\Gamma_1(\mathbf{k})+\Gamma_2(\mathbf{k}))}\right\}. \quad (55)$$

From (54) and (55) we obtain in the manner considered above a kinetic equation for the excitons of the macroscopically filled $\mathbf{k}_0$:

$$\left[\frac{\partial}{\partial t}+\mathbf{V}_{\mathbf{k}_0}^{p}\nabla_{\mathbf{r}}\right]N_0(\mathbf{r}t)=-N_0\sum_{i=1}^{2}\int\frac{d^3k}{(2\pi)^3}m_{\mathbf{k}-\mathbf{k}_0}\times\left\{[\varphi_i(\mathbf{k})n_{\mathbf{k}-\mathbf{k}_0}^{(i)}(\mathbf{r}t)-\psi_i(\mathbf{k}-\mathbf{k}_0)N_{\mathbf{k}}^{(i)}(\mathbf{r}t)]\times\frac{1}{\Gamma_i(\mathbf{k})}+2\pi\delta[\Gamma_1+\Gamma_2|\xi_{01}-\xi_{02}]\times[\varphi_j(\mathbf{k})n_{\mathbf{k}-\mathbf{k}_0}^{(i)}(\mathbf{r}t)-\psi_j(\mathbf{k}-\mathbf{k}_0)N_{\mathbf{k}}^{(i)}(\mathbf{r}t)]_{i\neq j}\right\}, \quad (56)$$

where $\mathbf{V}_{\mathbf{k}Q}^{p}$ is the polariton group velocity.

Thus, the system of five equations (46), (48), and (56) for the five variables $n_{\mathbf{k}-\mathbf{k}_0}^{(i)}(\mathbf{r}t)$, $N_{\mathbf{k}}^{(i)}(\mathbf{r}t)$ and $N_0(\mathbf{r}t)$ is the sought system of kinetic equations. We note that it was obtained in the approximation of a weak exciton-phonon interaction and for slow variation of the amplitude of the initial polariton wave $\mathbf{k}_0$.

In the case of a quasistationary spatially homogeneous problem, the indicated system of equations can be solved, and for the coefficient of the temporal damping $\sigma(\mathbf{k}_0)$ of the polariton wave we have the approximate formula

$$\frac{\sigma(\mathbf{k}_0)}{2\gamma(\mathbf{k}_0)}\approx\left[1+\frac{2(N_0m_{\mathbf{k}_0})^{1/2}}{\gamma_{\text{ph}}^{A}(\mathbf{k}_0)+\gamma(\mathbf{k}_0)}\right]\left[1+\frac{N_0m_{\mathbf{k}_0}}{\gamma(\mathbf{k}_0)\gamma_{\text{ph}}^{A}(\mathbf{k}_0)}\right]^{-1} \quad (57)$$

According to Ref. 17, the reciprocal lifetime of the longitudinal acoustic phonons $\mathbf{k}-\mathbf{k}_0$ is given by

$$\gamma_{\text{ph}}^{A}(\mathbf{k}-\mathbf{k}_0)=\frac{1}{\tau_{\text{ph}}^{A}(\mathbf{k}-\mathbf{k}_0)}=\frac{\chi_0T^4|\mathbf{k}-\mathbf{k}_0|}{\hbar^3\rho u^4}, \quad (58)$$

where ρ is the crystal density and χ_0 is a dimensionless constant of the order of unity. As for $m_{\mathbf{k}}$, it is defined in accordance with (17), and for acoustic phonons we have

$$M_{\mathbf{k}}=[2kC^2/9\rho uV]^{1/2}, \quad (59)$$

where C is the deformation potential of the semiconducting crystal.[8]

Analyzing (57) with account taken of (58) and (59), we can distinguish, as indicated above, between three regions of the behavior of the coefficient of the temporal damping $\sigma(\mathbf{k}_0)$, depending on the polariton-wave intensity.

1. Weak intensities (linear theory):

$$N_0<\frac{1}{m_{\mathbf{k}_0}}\gamma(\mathbf{k}_0)\gamma_{\text{ph}}^{A}(\mathbf{k}_0)\approx\frac{k_0^2T^5\chi_0}{2\pi^2\hbar^4\rho u^5u_{\mathbf{k}_0}}, \quad (60)$$

$$\sigma(\mathbf{k}_0)=2\gamma(\mathbf{k}_0)\approx\frac{2C^2(Tk_0^2)}{9\pi^2u^2u_{\mathbf{k}_0}\rho\hbar^2}. \quad (61)$$

Here $u_{\mathbf{k}0}$ is the average value of the phonoriton velocity $\mathbf{V}_{\mathbf{k}}^{(i)}$ in the vicinity of the phonoriton splitting of the dispersion curves:

$$u<u_{\mathbf{k}_0}<V_{\mathbf{k}_0}^{p}. \quad (62)$$

2. Medium intensities:

$$\frac{1}{m_{\mathbf{k}_0}}\gamma(\mathbf{k}_0)\gamma_{\text{ph}}^{A}(\mathbf{k}_0)<N_0<\frac{1}{4m_{\mathbf{k}_0}}[\gamma(\mathbf{k}_0)+\gamma_{\text{ph}}^{A}(\mathbf{k}_0)]^2, \quad (63)$$

$$\sigma\approx\frac{C^2\chi_0}{9\pi^4u^7u_{\mathbf{k}_0}^2\hbar^6\rho^2}\left(\frac{T^6k_0^4}{N_0}\right)\sim\frac{1}{N_0}. \quad (64)$$

Depending on the temperature and consequently on the ratio of $\gamma(\mathbf{k}_0)$ and $\gamma_{\text{ph}}^{A}(\mathbf{k}_0)$, Eq. (64) corresponds to different nonlinearity mechanisms. For low temperatures, when

$$T<[2C^2u^2\hbar k_0/9\pi^2u_{\mathbf{k}_0}\chi_0]^{1/3}\equiv T_c, \quad (65)$$

the phonon nonlinearity mechanism comes into play and can be realized even at sufficiently low intensities. Thus, e.g., at $T=4.2$ K, phonon nonlinearity can set in at $N_0\gtrsim10^{10}$ cm^{-3}. High temperatures correspond to nonequilibrium Bose condensation whose meaning was explained at the beginning of the article.

3. High intensities (phonoriton restructuring of the spectra):

$$N_0>\frac{1}{4m_{\mathbf{k}_0}}[\gamma(\mathbf{k}_0)+\gamma_{\text{ph}}^{A}(\mathbf{k}_0)]^2, \quad (66)$$

$$\sigma(\mathbf{k}_0)\approx\frac{2}{3}\frac{C\chi_0}{\pi^{5/2}u^{11/2}u_{\mathbf{k}_0}\rho^{3/2}\hbar^{9/2}}\left(\frac{T^5k_0^{5/2}}{N_0^{1/2}}\right)\sim\frac{1}{N_0^{1/2}},\quad T<T_c, \quad (67)$$

$$\sigma(\mathbf{k}_0)\approx\frac{4}{27}\frac{C^3}{\pi^{9/2}u^{7/2}u_{\mathbf{k}_0}^2\rho^{3/2}\hbar^{7/2}}\left(\frac{T^2k_0^{7/2}}{N_0^{1/2}}\right)\sim\frac{1}{N_0^{1/2}},\quad T>T_c.$$

In this case, which is accompanied by restructuring of the phonon and polariton spectra, the damping coefficient (67) takes on values smaller by several orders of magnitude than in the linear theory. We note that such a decrease of the damping coefficient at high intensities of the wave $\mathbf{k}_0$ is determined in the general case simultaneously also by the phonon nonlinearity, i.e., by the depletion of the phonon subsystem that participates in the scattering, and by the nonequilibrium Bose condensation, which manifests itself in this case in an active return of the scattered excitons to the initial mode.

The conversion to the spatial absorption coefficient $\sigma_{sp}(\mathbf{k}_0)$ of the polariton wave $\mathbf{k}_0$ is by means of the formula

$$\sigma_{sp}(\mathbf{k}_0)\approx\sigma(\mathbf{k}_0)/V_{\mathbf{k}_0}^{p}, \tag{68}$$

in which case Eq. (57) goes over into

$$\frac{\sigma_{sp}(\mathbf{k}_0, N_0)}{\sigma_{sp}(\mathbf{k}_0, N_0\to 0)}$$

$$\approx\left[1+\frac{2(N_0 m_{\mathbf{k}_0})^{1/2}}{\gamma_{ph}^{A}(\mathbf{k}_0)+\gamma(\mathbf{k}_0)}\right]\left[1+\frac{N_0 m_{\mathbf{k}_0}}{\gamma(\mathbf{k}_0)\gamma_{ph}^{A}(\mathbf{k}_0)}\right]^{-1}$$

$$\times\frac{V_{\mathbf{k}_0}^{p}(N_0)}{V_{\mathbf{k}_0}^{p}(N_0\to 0)}. \tag{69}$$

We have used here the notation $\mathbf{V}_{\mathbf{k}_0}^{p}(N_0)$ for the polariton group velocity $V_{\mathbf{k}_0}^{p}$, since this velocity will be shown below to be substantially dependent on the wave intensity.

We proceed now to the case when the initial wave $\mathbf{k}_0$ is coherent. Now, as already noted, it will be described by the Maxwell and Schrödinger equations relative to the mean values of the operator of the positive-frequency part of the electromagnetic field $\langle\hat{E}(\mathbf{r}t)\rangle = E(x)$ and the operator of the exciton field $\langle\hat{\Phi}(\mathbf{r}t)\rangle = \Phi(x)$, respectively. To obtain the indicated equations by the diagram technique for the nonequilibrium processes we introduce the following quantities

$$\Phi_+(x)=\Phi(x)=\langle\hat{S}^{-1}T[\hat{\Phi}_0(x)\hat{S}]\rangle,$$

$$E_+(x)=E(x)=\langle\hat{S}^{-1}T[\hat{E}_0(x)\hat{S}]\rangle, \tag{70}$$

$$\Phi_-(x)=\langle[\bar{T}\hat{S}^{-1}\hat{\Phi}_0(x)]\hat{S}\rangle,$$

$$E_-(x)=\langle[\bar{T}\hat{S}^{-1}\hat{E}_0(x)]\hat{S}\rangle,$$

where $\hat{E}_0(x)$ and $\hat{\Phi}_0(x)$ are operators in the interaction representation,

$$\hat{S}=\exp\left[-i\int_{-\infty}^{+\infty}\hat{H}_{int}(t)\,dt\right]$$

is the evolution operator, and $\hat{H}_{\text{int}}(t)$ is the operator of the exciton-photon and exciton-phonon interactions in the interaction representation. We recall that we are considering exciton-phonon interaction in the dipole approximation.

The Dyson equation for the quantities $E_{\pm}(x)$ and $\Phi_{\pm}(x)$ are of the form

$$E_+(x)=E_+^{0}(x)+\int D_0^{+}(x,x_1)\Phi_-(x_1)\,dd^4x_1$$

$$-\int D_0^{c}(x,x_1)\Phi_+(x_1)\,dd^4x_1,$$

$$E_-(x)=E_-^{0}(x)+\int D_0^{-}(x,x_1)\Phi_+(x_1)\,dd^4x_1$$

$$-\int\tilde{D}_0^{c}(x,x_1)\Phi_-(x_1)\,dd^4x_1,$$

$$\Phi_+(x)=\Phi_+^{0}(x)+\int[G_0^{+}(x,x_1)E_-(x_1)-G_0^{c}(x,x_1)E_+(x_1)]$$

$$\times dd^4x_1+\int G_0^{c}(x,x_1)[\Sigma^{c}(x_1,x_2)\Phi_+(x_2)$$

$$-\Sigma^{+}(x_1,x_2)\Phi_-(x_2)]d^4x_1\,d^4x_2$$

$$+\int G_0^{+}(x,x_1)[\tilde{\Sigma}^{c}(x_1,x_2)\Phi_-(x_2)-\Sigma^{-}(x_1,x_2)\Phi_+(x_2)]d^4x_1d^4x_2,$$

$$\Phi_-(x)=\Phi_-^{0}(x)+\int[G_0^{-}(x,x_1)E_+(x_1)-\tilde{G}_0^{c}(x,x_1)E_-(x_1)]$$

$$\times dd^4x_1+\int\tilde{G}_0^{c}(x,x_1)[\tilde{\Sigma}^{c}(x_1,x_2)\Phi_-(x_2)$$

$$-\Sigma^{-}(x_1,x_2)\Phi_+(x_2)]d^4x_1\,d^4x_2$$

$$+\int G_0^{-}(x,x_1)[\Sigma^{c}(x_1,x_2)\Phi_+(x_2)-\Sigma^{+}(x_1,x_2)\Phi_-(x_2)]d^4x_1d^4x_2. \tag{71}$$

Here all the Green's functions $G(x,x_1)$ and $D(x,x_1)$, the self-energy parts $\Sigma(x,x_1)$, as well as their connections with the corresponding retarded and advanced functions are defined in accordance with Ref. 14. From these equations we obtain a system for $\Phi_{\pm}(x)$ and $E_{\pm}(x)$:

$$\left[i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\nabla_r^2-\hbar\omega_t\right]\Phi_+(x)=-dE_+(x)+\int[\Sigma^{c}(x,x_1)$$

$$\times\Phi_+(x_1)-\Sigma^{+}(x,x_1)\Phi_-(x_1)]d^4x_1,$$

$$\left[i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\nabla_r^2-\hbar\omega_t\right]\Phi_-(x)=-dE_-(x)+\int[\Sigma^{-}(x,x_1)\Phi_+(x_1)$$

$$-\tilde{\Sigma}^{c}(x,x_1)\Phi_-(x_1)]d^4x_1, \tag{72}$$

$$\left[-\frac{\varepsilon_0}{c^2}\frac{\partial^2}{\partial t^2}-\nabla_r^2\right]E_{\pm}(x)=-\frac{4\pi}{c^2}d\frac{\partial^2}{\partial t^2}\Phi_{\pm}(x),$$

where ω_t is the position of the exciton level relative to the valence band, ε_0 is the background dielectric constant of the semiconductor, and m is the translational mass of the exciton. It can be easily shown that only one homogeneous solution with respect to $\Phi_{\pm}(x)$ and $E_{\pm}(x)$ of the equations of the system (72) gives a physically plausible result $\Phi_+(x)=\Phi_-(x)=\Phi(x)$ with a damping determined by the self-energy part $\Sigma^{R}(x,x_1)$ and $E_+(x)=E_-(x)=E(x)$. In this case the system (72) assumes the simpler form

$$\left[i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\nabla_r^2-\hbar\omega_t\right]\Phi(\mathbf{r}t)$$

$$=-\mathbf{dE}(\mathbf{r}t)+\int\Sigma^R(\mathbf{r}t,\mathbf{r}_1t_1)\Phi(\mathbf{r}_1t_1)\,d^3r_1\,dt_1, \tag{73}$$

$$\left[\frac{\varepsilon_0}{c^2}\frac{\partial}{\partial t^2}-\nabla_r^2\right]\mathbf{E}(\mathbf{r}t)=-\frac{4\pi}{c^2}\frac{\mathbf{d}}{v_0}\frac{\partial^2\Phi(\mathbf{r}t)}{\partial t^2}.$$

In these equations we transformed to the dimensionless variable $\Phi(\mathbf{r}t)$, and v_0 is the volume of the unit cell of the crystal.

As for the behavior of the scattered excitons and the scattering phonons, they are subject to all the conclusions of the preceding case both with respect to the restructuring of the spectra and with respect to the form of the kinetic equations (46) and (48). In all these relations, which pertain to scattered excitons and scattering phonons, it is necessary to make the substitution

$$N_0(\mathbf{r}t)\to|\Phi(\mathbf{r}t)|^2/v_0=|\Phi_{\mathbf{k}_0}(\mathbf{r}t)|^2/v_0. \tag{74}$$

If now we take into account the explicit form of $\Sigma^R(x,x_1)$ in accord with (55), the system (73) assumes its final form

$$\left[i\hbar\frac{\partial}{\partial t}+\hbar\tilde{\omega}_{\mathbf{k}_0}-\hbar\tilde{\omega}_t(\mathbf{k}_0)\right]\Phi_{\mathbf{k}_0}(\mathbf{r}t)+\frac{\hbar^2}{2m}[i(\mathbf{k}_0\nabla_r)-k_0^2]\Phi_{\mathbf{k}_0}(\mathbf{r}t)$$

$$=-\mathbf{dE}_{\mathbf{k}_0}(\mathbf{r}t)-i\hbar\Phi_{\mathbf{k}_0}(\mathbf{r}t)\sum_{t=1}^{2}\int\frac{d^3k}{(2\pi)^3}\frac{m_{\mathbf{k}-\mathbf{k}_0}}{2}$$

$$\{[\varphi_t(\mathbf{k})n^t_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t)-\psi_t(\mathbf{k}-\mathbf{k}_0)$$

$$\times N_{\mathbf{k}}^{(t)}(\mathbf{r}t)]\frac{1}{\Gamma_t(\mathbf{k})}+2\pi\delta[\Gamma_1+\Gamma_2|\xi_{01}-\xi_{02}][\varphi_j(\mathbf{k})n^{(t)}_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t)$$

$$-\psi_j(\mathbf{k}-\mathbf{k}_0)N_{\mathbf{k}}^{(t)}(\mathbf{r}t)]_{t\neq j}\}, \tag{75}$$

$$\frac{\varepsilon_0}{c^2}\tilde{\omega}_{\mathbf{k}_0}\left[i\frac{\partial}{\partial t}+\tilde{\omega}_{\mathbf{k}_0}\right]\mathbf{E}_{\mathbf{k}_0}(\mathbf{r}t)+[i(\mathbf{k}_0\nabla_r)-k_0^2]\mathbf{E}_{\mathbf{k}_0}(\mathbf{r}t)$$

$$=-\frac{4\pi}{c^2}\frac{\mathbf{d}}{v_0}\tilde{\omega}_{\mathbf{k}_0}\left[i\frac{\partial}{\partial t}+\tilde{\omega}_{\mathbf{k}_0}\right]\Phi_{\mathbf{k}_0}(\mathbf{r}t).$$

A transition was carried out here to slow envelopes of the polariton wave $\mathbf{k}_0$:

$$\begin{aligned}\Phi(\mathbf{r}t)&=\Phi_{\mathbf{k}_0}(\mathbf{r}t)\,e^{-i\tilde{\omega}_{\mathbf{k}_0}t+i\mathbf{k}_0\mathbf{r}},\\ \mathbf{E}(\mathbf{r}t)&=\mathbf{E}_{\mathbf{k}_0}(\mathbf{r}t)\,e^{-i\tilde{\omega}_{\mathbf{k}_0}t+i\mathbf{k}_0\mathbf{r}}\end{aligned} \tag{76}$$

and second-order derivatives were neglected. The displaced position of the exciton level $\tilde{\omega}_t(\mathbf{k}_0)$ is determined from (55) by means of Eq. (11), while $\tilde{\omega}_{\mathbf{k}_0}$ is determined directly from the dispersion polariton equation (75). We have thus obtained a system of nonlinear equations (46), (48), and (75) relative to $\Phi_{\mathbf{k}_0}(\mathbf{r}t)$, $E_{\mathbf{k}0}(\mathbf{r}t)$, $n^{(t)}_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t)$ and $N^{(t)}_{\mathbf{k}}(\mathbf{r}t)$. The dispersion equation obtained from the polariton equations (75) takes the same form as in the preceding noise-wave case, where it was determined by the denominator of a retarded Green's function; in particular, the absorption of the coherent wave $\mathbf{k}_0$ was determined by the imaginary part of the same self-energy part Σ^R as in the case of the noise wave. This fact greatly facilitates the analysis of the obtained equations; e.g., it is possible to treat in the same manner the spatially homogeneous problem and obtain the results (57)–(69) subject only to the difference connected with the substitution (74).

We dwell now briefly on the general case, when the initial wave k_0 is partially coherent. The general system of equations for the distribution functions $n^{(t)}_{\mathbf{k}-\mathbf{k}_0}(\mathbf{r}t)$, and $N^{(t)}_{\mathbf{k}_0}(\mathbf{r}t)$, for the noise part of the wave $N_0(\mathbf{r}t)$, and for its coherent parts $\Phi_{\mathbf{k}_0}(\mathbf{r}t)$ and $E_{\mathbf{k}_0}(\mathbf{r}t)$ can also be obtained with the aid of the diagram technique. This closed system will consist of the equations (46), (48), (56), and (75), and in all the relations, including the kinetic equations (46) and (48) pertaining to scattered excitons and to scattering phonons, it is necessary to make the substitution

$$N_0(\mathbf{r}t)\to N_0(\mathbf{r}t)+|\Phi_{\mathbf{k}_0}(\mathbf{r}t)|^2/v_0, \tag{77}$$

where the last expression is the total density of the excitons of the mode $\mathbf{k}_0$. From an analysis of this system it can be seen that the coherent and noise components of the initial wave $\mathbf{k}_0$ retain their relative shares in the total intensity in the wave propagation process,, and formulas (57)–(69) are again valid if the substitution (77) is made in them. We note that in this paper we define total coherence in the sense

$$\langle\hat{\Phi}(x)\rangle=\Phi(x),$$

$$G_{\text{св}}{}^+(x,x')=G^+(x,x')-\langle\hat{\Phi}^+(x)\rangle\langle\hat{\Phi}(x')\rangle=0$$

and do not consider at all the higher-order Green's functiosn.

The results can be applied to the phenomenon of equilibrium Bose condensation into the mode $\mathbf{k}_0=0$ of an ideal Bose gas in a phonon thermostat; in this case, however, the kinetic and field equations are actually not necessary, since the Bose gas is at equilibrium and the entire useful information is contained in the corresponding retarded Green's functions. The possibility of quasiparticle-spectrum restructuring in such a system, in the case of Bose condensation of dipole inactive excitons into $\mathbf{k}_0=0$, was indicated also in Ref. 5.

The connection between the total intensity $I_0(\mathbf{r}t)$ of the polariton wave with the concentration of the excitons of the wave $N_0(\mathbf{r}t)$, i.e., with the intensity of the exciton part of the polariton wave, is determined by the relation

$$N_0(\mathbf{r}t)=\alpha(\mathbf{k}_0)\frac{I_0(\mathbf{r}t)}{\hbar\tilde{\omega}_{\mathbf{k}_0}V_{\mathbf{k}_0}{}^pS}\approx\frac{W(\mathbf{r}t)}{\hbar\tilde{\omega}_{\mathbf{k}_0}V_{\mathbf{k}_0}{}^p}, \tag{78}$$

where S is the cross-section area, $W=I_0/S$ is the power of the flux of the initial wave $\mathbf{k}_0$, and the weighting factor $\alpha(\mathbf{k}_0)$, which is analogous to the factors $\varphi_i(\mathbf{k}_0)$ and $\psi_i(\mathbf{k}-\mathbf{k}_0)$ considered above for the case of polariton-phonon splitting of the spectra, is defined by the formula

$$\alpha(\mathbf{k}_0)=\frac{(c'k_0-\omega_t)+[(c'k_0-\omega_t)^2+4\pi\beta\omega_t{}^2]^{1/2}}{2[(c'k_0-\omega_t)^2+4\pi\beta\omega_t{}^2]^{1/2}}, \tag{79}$$

where

$$4\pi\beta=2\omega_{lt}\varepsilon_0/\omega_t, \qquad c'=c/\varepsilon_0^{1/2}.$$

This factor is equal, with high degree of accuracy, to unity in the region of the polariton dispersion curve of interest to us and discussed below.

We consider now the dependence of the group velocity $V^p_{\mathbf{k}_0}=V^p_{\mathbf{k}_0}(N_0)$ of the initial wave $\mathbf{k}_0$ on its intensity. Since, on the one hand, the propagation of the $\mathbf{k}_0$ wave is described by nonlinear equations, and on the other hand the absorption $\sigma_{sp}(\mathbf{k}_0)$ for weak intensities can reach large values, the group velocity cannot be defined in the usual manner in the form of a derivative of a dispersion function. A more general definition of the group velocity is

$$\mathbf{V}_{\mathbf{k}_0 p}=\langle \mathbf{s}\rangle/Q, \tag{80}$$

where $\langle \mathbf{s}\rangle$ is the Poynting vector averaged over the period of the wave and Q is the polariton-wave energy density averaged over the period. With the aid of the Poynting theorem and the definition (80), the following expression was obtained in Ref. 7 for the group velocity in the case of a given frequency of the initial wave

$$V_{\mathbf{k}_0 p}(N_0)\approx\sigma(\mathbf{k}_0)\,[k_0\sigma(\mathbf{k}_0)/\tilde{\omega}_{\mathbf{k}_0}+2k_0']^{-1}. \tag{81}$$

Here k_0 and k_0' are respectively the real and imaginary parts of the wave vector and are determined from the equation $\omega=\tilde{\omega}_{\mathbf{k}_0}$, while the dependence of the group velocity on the wave intensity is contained in the previously obtained dependence of $\sigma(\mathbf{k}_0)$ on the density $N_0(\mathbf{r}t)$. It follows from an analysis of (81) that the values of the velocity $V^p_{\mathbf{k}_0}$ lie between the velocity of light c' in the medium and the polariton velocity, which is determined as $\sigma(\mathbf{k}_0)\to 0$ in the usual manner and which can reach values smaller by several orders of magnitude than the velocity of light in the medium. At high intensities (3), (4) of the initial wave $\mathbf{k}_0$, an abrupt decrease of the damping coefficient $\sigma(\mathbf{k}_0)$ takes place, and consequently a strong decrease of the polariton group velocity. This can be physically attributed to the fact that at such a decrease of the wave damping the exciton-photon interaction becomes stronger than the exciton-phonon interaction, and this enhances the polariton character of the initial wave, and, in particular, to decreases its group velocity.

We now examine how realistic is the assumed neglect of the polariton-wave absorption due to the emission of acoustic phonon wave by the excitons. This assumption holds if, following absorption of the phonon, the exciton of the initial wave lands on the essentially exciton-like part of the polariton dispersion curve, where the exciton state density is large, and when a phonon is emitted it lands on the essentially photon-like part, where the state density is low. These state densities are inversely proportional to the group velocities at the corresponding points of the dispersion curve and consequently the ratio of the state densities of the exciton-like part of the dispersion curve and of the photon-like part can reach values $\sim 10^3$. This can be realized in semiconductors in which the longitudinal-transverse splitting satisfies the condition

$$\omega_{lt}<2(u/c)\,\varepsilon_0^{1/2}\omega_t, \tag{82}$$

and the case of the ground energy state of the excitons is considered. This condition is satisfied, e.g., for the semiconductors GaSe, GaAs, Cu_2O, CdTe. In addition, to exclude the possible accumulation of excitons scattered from the wave $\mathbf{k}_0$ with emission of phonons, an accumulation that can lead to stimulated phonon emission by the excitons $\mathbf{k}_0$ and to an abrupt increase of this absorption channel, it is necessary that these scattered electrons leave the region of the wave within a time shorter than $\tau(\mathbf{k})$. This means that the transverse dimensions of the wave should be limited to a diameter $l\lesssim(c/\varepsilon_0^{1/2})\tau\sim 1$–$0.1$ mm, i.e., should actually be a rather narrow beam.

For a possible experimental observation of the considered phenomena, besides the indicated conditions, it is necessary that the frequency of the polariton wave $\mathbf{k}_0$, equal to $\omega=\tilde{\omega}_{\mathbf{k}_0}$, be located on the inflection of the lower polariton dispersion wave for best realization of condition (82), neglecting scattering due to phonon emission. In this case the wave intensity (78) should be high enough in accord with (3) and (4). If pulsed emission $\mathbf{k}_0$ is considered, of intensity (3) or (4), experimental observation of this decrease of the absorption coefficient calls for a pulse duration τ_p that satisfies the condition

$$\tau_p\gg\tau_{eff}=[(\gamma(\mathbf{k}_0)+\gamma^A_{ph}(\mathbf{k}_0))^2+4N_0m_{\mathbf{k}_0}]^{1/2}/N_0m_{\mathbf{k}_0}. \tag{83}$$

The spectral width of the pulse $\Delta\omega_p$ should in turn satisfy the inequalities

$$\Delta\omega_p<\omega_{lt}, \qquad \Delta\omega_p<\gamma(\mathbf{k}_0)+\gamma^A_{ph}(\mathbf{k}_0), \tag{84}$$

which are the conditions for the applicability of the obtained kinetic and field equations. This imposes an additional limitation on the pulse duration: $\tau_p>1/\Delta\omega_p$.

Within the framework of the indicated mode, it is necessary in a general analysis to take into account the damping of the polariton wave on account of phonon emission. In this case the proposed description must be supplemented by kinetic and field (if the initial $\mathbf{k}_0$ wave is coherent) equations for the scattered excitons and for the emitted phonons, and also introduce the corresponding terms in expressions (56) and (75), which describe the initial wave. Such transitions of excitons with emission of phonons will also be accompanied by a restructuring of the spectra, and the corresponding system of equations actually reflects processes of stimulated Brillouin scattering.

Excitons of the initial wave can also be scattered by impurities as a result of exciton-exciton interaction, etc., therefore in the general case the results obtained above on the decrease of the absorption coefficient of the $\mathbf{k}_0$ wave should be regarded as a suppression of the contribution of one of the scattering mechanisms, namely scattering as a result of absorption of acoustic phonons by the wave excitons. Even in this case, however, when the condition (4) is satisfied, where $\gamma(\mathbf{k})$ is determined by all the possible exciton-scattering mechanisms, a restructuring of the phonon and polariton spectra is also possible. Moreover, it appears that a similar restructuring of the spectra of the scattered waves will take place for any anti-Stokes scattering, when the initial $\mathbf{k}_0$ wave is intense enough. We note case of simultaneous presence in the semiconductor of several microscopically

filled polariton waves, a restructuring of the phonon and polariton spectra, much more complicated in structure than considered in the present paper, is possible.

The authors thank V. S. Dneprovskiĭ and S. S. Fanchenko for a discussion of a number of questions touched upon in the article.

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Translated by J. G. Adashko