Superconductivity in a system of interacting localized and delocalized electrons

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It is shown that, owing to the interaction of electrons localized near the plane of a defect with free electron, superconducting states that penetrate deeply into the interior of the superconductor are produced at a temperature exceeding the critical temperature of a homogeneous superconductor. The diagram of coexistence of the normal and superconducting phases as a function of the angle between the direction of the magnetic field and the plane of the defect is constructed. The appearance of characteristic kinks on this diagram is predicted. The possible connection of the theory with the experiments of Khaĭkin and Khlyustnikov is discussed, as is also the possibility of a Kosterlitz-Thouless transition in such a system.

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1. INTRODUCTION

Inhomogeneous superconductors are now the subject of intensive research. The inhomogeneity may be due to the presence in the metal of strains (dislocations), surfaces, twin boundaries, composition inhomogeneities (layer of one metal on the surface of another, periodic intermetallic structures).[1–3] (Here and elsewhere we consider only inhomogeneous systems of the metal-metal type.)

Naturally, the inhomogneity affects all the quantities of importance in superconductivity: the electron and phonon spectra and the electron-phonon interactons. Within the framework of the weak-coupling model this should lead both to a change of the Debye frequency and to inhomogeneity of the electron-electron interaction constant. In most cases it is qualitatively correct to take into account the inhomogeneity of the electron-electron interaction. Indeed, since the critical temperature $T_c \sim \exp(-1/g)$, a relatively small change of $g(\mathbf{r})$ leads to substantial effects. In this approximation there appear the so-called proximity effects, wherein regions with good superconducting properties induce superconductivity in regions with poor superconducting properties, while regions with poor superconducting properties suppress the superconductivity. Since the characteristic quantity upon onset of superconductivity is the coherence length ξ, the averaging of the superconducting properties can likewise take place over such distances.

This model of the phenomenon served as the basis for the description of metallic inhomogeneity in Refs. 4–10. There exists, however, a situation when such a decription cannot be used.

If electron states localized in a certain region are produced in a crystal, the Green's function of such states, and with it also the kernels of the integral equations that determine the critical characteristics of the superconductor, will decrease over a distance $D \ll \xi$, where D is the characteristic length of the localization of the state. An example of such systems can be Tamm states near the surface of a twin boundary in a single crystal, states whose importance was pointed out by Khaĭkin and Khlyustnikov[11] and to which they have attributed the relations observed by them; electron states near dislocations are a similar example.

The ordinary proximity effects may be absent in these cases. If, for example, the superconducting properties of a localized subsystem are "better" than that of the bulk subsystem, the superconducting transition will take place in the localized subsystem, in analogy with its ocurrence in a thin film on the surface of a dielectric[12] (or in a system of Tamm electrons on a dielectric substrate[13]), and its critical temperature will not be lowered by the volume electrons. In contrast to states on the surface of a dielectric, however, deeply penetrating superconducting states will be induced in the interior of the superconductor. Application of a magnetic field H to such a system gives rise to effects due to the fact that the field acts differently on the localized and bulk superconducting states.

Our paper is devoted to a description of the superconducting transition within the framework of the two-band model with a zone of localized and volume electrons that interact with each other.

2. BASIC EQUATIONS

We write the Hamiltonian of the electron system in the form ($\hbar = k_B = 1$)

$$\hat{\mathscr{H}} = \hat{\mathscr{H}}_0 + \hat{\mathscr{H}}_{int},$$

$$\hat{\mathscr{H}}_0 = \sum_{\alpha=1}^{2} \int \psi_\alpha^+(\mathbf{r}) (\varepsilon_\alpha(p) - \mu) \psi_\alpha(\mathbf{r}) d^3\mathbf{r},$$

$$\hat{\mathscr{H}}_{int} = \sum_{i,j,k,m=1}^{2} \int \Lambda_{ijkm} \psi_{\sigma,i}^{+}(\mathbf{r}) \psi_{\sigma',j}(\mathbf{r}) \psi_{\sigma',k}^{+}(\mathbf{r}) \psi_{\sigma,m}(\mathbf{r}) d^3\mathbf{r}, \tag{1}$$

$$\varepsilon_1(p) = \frac{p^2}{2m_1}, \quad \varepsilon_2(p) = -\varepsilon_0 + \frac{p_\parallel^2}{2m_2}, \quad p = -i\nabla - \frac{2e}{c}\mathbf{A}.$$

Here and elsewhere the indices 1 and 2 pertain to the volume and localized electron zones, respectively, m_i is the effective mass, Λ_{ijkm} is the set of electron-electron interaction constants, $\Psi^+\sigma i(\mathbf{r})$ and $\Psi\sigma, i(\mathbf{r})$ are the creation and annihilation operators of an electron with spin σ in the i-th band with the usual commutation relations for the Fermi operators, $\mathbf{A}$ is the magnetic-field vector potential, e is the electron charge, $\hbar$ is Planck's constant, k_B is Boltzmann's constant, ε_0 is the energy of the electron bound state, $p_{\|}$ is the momentum of the electron along the localization plane.

The normal and anomalous Green's functions are defined in the usual fashion[14]:

$$G_{ij\sigma\sigma'}(x,x')=-\langle T_\tau\{\psi^M_{i\sigma 0}(x)\psi^{+M}_{j\sigma' 0}(x')S\}\rangle/\langle S\rangle,$$

$$F^+_{ij\sigma\sigma'}(x,x')=-\langle T_\tau\{\psi^{+M}_{i\sigma 0}(x)\psi^{+M}_{j\sigma' 0}(x')S\}\rangle/\langle S\rangle, \qquad (2)$$

$$S=T_\tau\exp\left(-\int_0^\beta \hat{\mathscr{H}}_{int}(\tau)d\tau\right),\quad \beta=1/T,\quad x=(\mathbf{r},\tau).$$

Here T_τ is the τ-time-ordering operator, $\langle...\rangle$ are Gibbs mean values, T is the temperature, and ψ^M are Matusbara operators.

It must be borne in mind that in the principal approximation in the interaction, both G_{ij} and F_{ij} are diagonal in i and j:

$$G^{(0)}_{ij}(x,x')=\delta_{ij}G^{(0)}_j(x,x'),\quad F^{(1)}_{ij}(x,x')=\delta_{ij}F^{(1)}_j(x,x'). \qquad (3)$$

(i and j are not tensor indices, and there is no summation over repeated indices.)

From (1)–(3) we obtain in the usual manner, near the critical point, a linearized system of Gor'kov equations for such a two-band model[15]:

$$F_i^+(\mathbf{r})=\sum_{j=1}^{2}\Lambda_{ij}\int d^3\mathbf{r}'K_i(\mathbf{r},\mathbf{r}')F_j^+(\mathbf{r}'),$$

$$K_i(\mathbf{r},\mathbf{r}')=T\sum_\omega G^{(0)}_{-\omega,i}(\mathbf{r}',\mathbf{r})G^{(0)}_{\omega,i}(\mathbf{r},\mathbf{r}'), \qquad (4)$$

$$\omega_n=\pi T(2n+1),\qquad n=0,\pm1,\pm2.$$

Here $G^0_{\omega,1}(\mathbf{r},\mathbf{r}')$ and $G^0_{\omega,2}(\mathbf{r},\mathbf{r}')$ are respectively the Green's functions of the volume and localized electrons,

$$G^{(0)}_{\omega,2}(\rho-\rho',z,z')=\sum_s\frac{\varphi_s(\mathbf{r})\varphi_s(\mathbf{r}')}{i\omega_n-\mu+\varepsilon_s(p_{\|})}. \qquad (5)$$

The wave functions φ_s depend on the type of inhomogeneity. In this paper we consider planar inhomogeneity. We confine ourselves for simplicity to the case of a single localized level near the inhomogeneity.

The Kernel $K_1(R)$ of the integral equation (4) is of the usual form[16]:

$$K_1(R)=(N_3T_0/2v_0)\,[R^2\,\mathrm{sh}\,(2\pi TR/v_0)]^{-1}, \qquad (6)$$

where N_3 is the three-dimensional density of states on the Fermi surface and v_0 is the Fermi velocity.

The kernel $K_2(\rho-\rho',z,z')$ is of the form

$$K_{\omega,2}(\rho-\rho'\,z,z')$$
$$=\int\frac{\varphi^2(z/D)\exp[i(\rho-\rho')\mathbf{Q}]}{(i\omega+\mu-\varepsilon_p)(-i\omega+\mu-\varepsilon_{Q-p})}\varphi^2\left(\frac{z'}{D}\right)\frac{d^2\mathbf{p}}{(2\pi)^2}\frac{d^2\mathbf{Q}}{(2\pi)^2};$$

$$\rho=(x,y),\qquad \mathbf{Q}=(Q_x,Q_y),\qquad \mathbf{p}=(p_x,p_y). \qquad (7)$$

In a magnetic field there appears in the kernels K_i a phase factor[17]

$$K_i(\mathbf{r},\mathbf{r}',H)\equiv K_i(\mathbf{r},\mathbf{r}')\exp\left(i\frac{2e}{c}\int_{\mathbf{r}}^{\mathbf{r}'}\mathbf{A}(\mathbf{s})\,d\mathbf{s}\right). \qquad (8)$$

(The spacing of the magnetic levels is $\varepsilon_H\ll T$.)

3. TRANSITION TEMPERATURE IN THE ABSENCE OF A MAGNETIC FIELD

To determine the temperature of the superconducting transition it is necessary to solve Eq. (4) with account taken of the forms (5) and (6) of K_i. As a result we have for the functions $F_1^+(z)$ and F_{20}^+,

$$F_{20}=\int_{-\infty}^{\infty}F_2(z)\,dz,\qquad F_2(z)=F_{20}\varphi^2(z/D),$$

the system of equations

$$-\xi_{T_0}{}^2\nabla_z{}^2F_1{}^+(z)+\varepsilon_0F_1{}^+(z)-\frac{Dg_{12}}{g_1}\Phi(z)F_{20}{}^+=0; \qquad (9)$$

$$F_{20}{}^+=\frac{g_{21}q}{g_2}F_1{}^+(0),\qquad g_{ik}=N_i\Lambda_{ik},$$

$$\xi_{T_0}=\frac{0.18v_{0i}}{T_0},\qquad q=\ln\frac{1{,}14\omega_D}{T}\Big/\ln\frac{T_0}{T_2},$$

$$\varepsilon_0=\ln(T_0/T_1),\qquad T_i=1.14\omega_D\exp(-1/g_i), \qquad (10)$$

$$\Phi(z)=(T_0/2v_{0i})\int_{R>r_0}[R^2\,\mathrm{sh}(2\pi TR/v_{0i})]^{-1}d^2\rho,$$

$$R=(\rho^2+z^2)^{1/2},\qquad r_0=0.3v_{0i}/\omega_D,\qquad \Lambda_{ik}\equiv\Lambda_{ikik}.$$

Here N_i is the density of states on the Fermi surface and ω_D is the Debye frequency.

In the derivation of the system (9) and (10) we used the fact that the function $F_2^+(z)\sim\varphi^2(z)$ decreases rapidly over distances $z\sim D$ compared with the other functions, while $F_1^+(z)$ decreases over a distance greatly exceeding ξ_{T_0} and could therefore be expanded in a series.

From (9) we obtain for the Fourier component $F_1^+(p)$ of $F_1^+(z)$:

$$F_1{}^+(p)=\frac{Dg_{12}}{g_1}\frac{\Phi_pF_{20}{}^+}{\xi^2p^2+\varepsilon_0}. \qquad (11)$$

Taking the inverse Fourier transform and recognizing that the characteristic length ($\sim 1/\xi_T$) of the variation of Φ_p is much less than $1/p$ at the pole of the denominator of (11), we obtain

$$F_1{}^+(z)=F_1{}^+(0)\exp(-|z|/L),\qquad L=\xi_1/\sqrt{\varepsilon_0}\,. \qquad (12)$$

The dispersion equation for the linear homogeneous system (9), (10) is of the form ($g_{11}\neq0$, $g_{22}\neq0$)

$$\ln\frac{T_0}{T_1}\ln^2\frac{T_0}{T_2}=\frac{D^2}{4\xi_1{}^2}\varkappa^2\ln^4\frac{1{,}14\omega_D}{T_0},\qquad \varkappa=\frac{g_{12}g_{21}}{g_1g_2}. \qquad (13)$$

This equation determines the temperature T_0 at which the superconducting sets in. The temperatures T_2 and T_1 are the transition temperatures of the localized and free electrons neglecting the interactions between them, and the parameter $\varkappa$ characterized the intensity of the interaction.

We consider some limiting cases. We introduce the parameters γ and t that characterize respectively the proximity of the transition temperatures and the value of the interaction:

$$\gamma=\left|\ln\frac{T_1}{T_2}\right|=\left|\frac{1}{g_1}-\frac{1}{g_2}\right|,\quad t=\frac{\varkappa}{2}\frac{1}{g_1^2}\frac{D}{\xi_1}\frac{1}{\gamma^{1/2}}. \tag{14}$$

1) At $t\ll 1$ and $T_1>T_2$ we obtain

$$\frac{T_0-T_1}{T_1}=\frac{\varkappa^2}{4}\frac{1}{g_1^4}\frac{D^2}{\xi_1^2}\frac{1}{\gamma^2},\quad L=\xi_1\left(\frac{T_1}{T_0-T_1}\right)^{1/2}. \tag{15}$$

In this case the main cause of the transition is the interaction between the volume electrons, and the localized electrons in fact merely redefine the interaction. This situation corresponds to the effective-proximity situation considered by us earlier. The effective interaction constant g takes then in the vicinity of the lcoalization plane the form

$$g^{-1}=g_{11}^{-1}-G^{-1},\quad G^{-1}=\varkappa/2g_1^2\gamma. \tag{16}$$

2) At $t\ll 1$ and $T_2>T_1$ we obtain

$$\frac{T_0-T_2}{T_2}=\frac{\varkappa}{2\gamma^{1/2}g_2^2}\frac{D}{\xi_1},\quad L=\frac{\xi_1}{\gamma^{1/2}}. \tag{17}$$

In this case the main cause of the transition is the interaction of the localized electrons that induce, via interaction with the volume electrons, superconductivity in the volume at a distance L from the plane. With respect to both the character of the dependence on D and of the size of the localization region, Eqs. (17) differ substantially from the equations for the proximity effects.

3) At $t\gg 1$ the interaction between the subsystem is strong compared with the difference between their critical temperatures. In this case

$$\frac{T_0-T_c}{T_c}=\left(\frac{\varkappa}{2g^2}\frac{D}{\xi_1}\right)^{2/3},\quad L=\xi_1\left(\frac{T_c}{T_0-T_c}\right)^{1/2}, \tag{18}$$

$$T_c\approx T_1\approx T_2.$$

A special case occurs when there is no pairing in one of the subsystem.

4) The values of T_0 and L at $g_{22}=0$ ar obtained by a transition to the limit from Eq. (15). At $g_{11}=0$, separating the small terms in (9), we obtain

$$F_1^+(z)\sim F_1^+(0)\Phi(z), \tag{19}$$

$$\frac{T_0-T_2}{T_2}=\frac{2g_{12}g_{21}}{g_{22}^2}\frac{D}{r_0},\quad L^2\sim\frac{v_{01}^2}{T_2^2}.$$

4. DIAGRAM OF COEXISTENCE OF SUPERCONDUCTING AND NORMAL PHASES IN A MAGNETIC FIELD

We determine now the coexistence curve $H_0(T)$ of the superconducting and normal phases for different magnetic-field directions. We confine ourselves for simplicity to the superconducting-transition point, where the Ginzburg-Landau equations are valid. In this case we obtain in place of the system (9)–(10)

$$\left[\ln\frac{T}{T_1}+\xi_1^2\left(i\nabla+\frac{2e}{c}\mathbf{A}\right)^2\right]F_1^+(\mathbf{r})$$
$$=\frac{\Lambda_{12}D}{g_1}\int K_1(\rho-\rho',z)F_{20}^+(\rho')d^2\rho', \tag{20}$$

$$\left[\ln\frac{T}{T_2}+\xi_2^2\left(i\nabla+\frac{2e}{c}\mathbf{A}\right)^2\right]F_{20}^+(\rho)+\frac{7\zeta(3)}{4\pi^2}\mu^2\frac{H^2}{T^2}F_{20}^+(\rho)$$
$$=\frac{g_{21}}{g_2}\ln\frac{1.14\omega}{T}F_1^+(0,\rho),\quad \mathbf{A}=(0,H_zx-H_xz,0).$$

Here ρ is a vector in the localization plane and μ is the bohr magneton.

The magnetic-field components $H_z=H\sin\theta$ and $H_x=H\cos\theta$ (θ is the angle between the magnetic-field direction and the localization plane) act differently on the supercondcuting state. The component H_z leads to localization in the plane of the plate and cannot be accounted for by perturbation theory. The field component H_x can be calculated by perturbation theory in weak fields such that $aH\gg L$, where aH is the magnetic radius:

$$a_H=(c/2eH)^{1/2}. \tag{21}$$

In strong fields $a_H\ll L$ use can be made of the smallness of a_H/L.

We consider first weak fields. In this case the solution takes in the principal approximation the form

$$F_1(\mathbf{r})=\exp(-|z|/L)\exp(-x^2/2a_{H_z}^2)F_1(0),$$
$$F_{20}(x)=\exp(-x^2/2a_{H_z}^2)F_{20},\quad a_{H_z}^2=c/2eH\sin\theta. \tag{22}$$

The dispersion equation that determines the $H_0(T)$ dependence is

$$\left(\ln\frac{T}{T_1}+\frac{2e}{c}\xi_1^2H_0\sin\theta+\frac{2e^2}{c^2}H_0^2\cos^2\theta\, L^2\xi_1^2\right)$$
$$\times\left(\ln\frac{T}{T_2}+\frac{2e}{c}\xi_2^2H_0\sin\theta+\frac{7\zeta(3)}{4\pi^2}\mu^2\frac{H_0^2}{T_2^2}\right)^2$$
$$=\frac{D^2}{4\xi_1^2}\varkappa^2\ln^4\frac{1.14\omega_D}{T}. \tag{23}$$

We present asymptotically exact solutions of this equation. We note first that the terms quadratic in H_0 are significant only at very small angles. For the expressions in the first and second parentheses in (23) these angles are bounded:

$$\theta_1\sim(L/a_H)^2,\quad \theta_2\sim(a/a_H)^2,\quad \theta_1\gg\theta_2. \tag{24}$$

1) At $t\ll 1$ and $T_1>T_2$ we can neglect in the second parentheses the terms with the magnetic field and set $T\approx T_1$. We obtain for $H_0(T,\theta)$ angular-dependence equations that agree with Tinkham's result for thin films[18]:

$$\frac{H_0}{H_\perp}\sin\theta+\left(\frac{H_0}{H_\parallel}\cos\theta\right)^2=1,\quad \tau=\frac{T_0-T}{T_0},$$
$$H_\perp=\frac{c}{2e\xi_1^2}\tau,\quad H_\parallel\sim\frac{c}{e}\frac{1}{L\xi_1}\tau^{1/2}. \tag{25}$$

2) In the case $t\ll 1$ and $T_2>T_1$ we obtain at $\theta\gg(\tau/\gamma)(\xi_1/\xi_2)^2$:

$$H_0=H_\perp/\sin\theta,\quad H_\perp=c\tau/2e\xi_2^2. \tag{26}$$

At $\theta<\tau\xi_1^2/\gamma\xi_2^2$ we land in the strong-field region, which will be considered below.

3) At $t\gg 1$ and $T_1\approx T_2\approx T_c$ formula (25) is valid with ξ_1^2

replaced by ξ_1^2 replaced by $2\xi_2^2/3 + \xi_1^2/3$.

In strong fields ($a_H \ll L$) the interaction is weak compared with the terms with the magnetic field. In the principal approximation the critical field simply coincides with the larger of the critical fields of the volume of localized electrons. These critical fields are given by

$$\tau_1 + \frac{2e}{c} H_0 \xi_1^2 = 0, \tag{27}$$

$$\tau_2 + \frac{2e}{c} \xi_2^2 H_0 \sin\theta + \frac{7\zeta(3)}{4\pi^2} \frac{\mu^2}{T^2} H_0^2 = 0, \tag{28}$$

where $\tau_i = (T - T_i)/T_i$.

Depending on the parameters τ_i, ξ_i and θ several situations are possible. We denote the solution of (27) by H_{01}, and the solution of (18) by H_{02}. We obtain

a) at $|\tau_2| > |\tau_1|$ and $\xi_2 < \xi_1$, at all $\theta > \theta_2$ (Fig. 1a)

$$H_0 = H_{02}; \tag{29}$$

b) at $|\tau_2| < |\tau_1|$ and $\xi_2 < \xi_1$ at all angles we have $H_0 = H_{02}$ in the temperature interval $\tau > \tau_0$ and $H_0 = H_{01}$ at $\tau < \tau_0$ (Fig. 1b)

$$\tau_0 = \xi_1^2 \Delta\tau / (\xi_2^2 \sin\theta - \xi_1^2), \qquad \Delta\tau = \tau_2 - \tau_1; \tag{30}$$

c) at $\tau_2 < \tau_1$ and $\xi_2 > \xi_1$, $H_0 = H_{01}$ always in the angle interval $\theta > \theta_0$; $H_0 = H_{01}$ in the angle interval $\theta < \theta_0$ at $\tau > \tau_0$; $H_0 = H_{02}$ at $\tau < \tau_0$ (Fig. 1c),

$$\sin\theta_0 = \xi_1^2 / \xi_2^2; \tag{31}$$

d) at $\tau_2 > \tau_1$ and $\xi_1 < \xi_2$ we have $H_0 = H_{02}$ in the angle inteval $\theta > \theta_0$ at $\tau > \tau_0$; if $\tau < \tau_0$, then $H_0 = H_{01}$; $H_0 = H_{02}$ at $\theta < \theta_0$ for all τ.

The conditions under which the magnetic field is strong can be written in the form $\tau \gg \gamma$ in the case of weak coupling between the subsystems (cases 1 and 2) and $\tau \gg t^{1/3}\tau^{1/2}$ for strong coupling (case 3).

It should be noted that in situations (c) and (d) the kink on the phase diagram lands in the region of applicability of the theory at angles θ close to θ_0.

In the region far from the kink on the phase diagram ($\tau = \tau_0$) the corrections to the critical field on account of the interaction of the subsystems are small compared with the field itself. To find the corrections in the system (20) it is necessary when $H_0 = H_{01} + h_1$, to seek F_1 and F_2 in the form

$$F_1(z,x) = \varphi_{01}(x) F_1(z), \qquad F_{20}(x) = \sum c_n \varphi_{2n}(x). \tag{32}$$

Here φ_{1n} and φ_{2n} are the normalized eigenfunctions of the operators $(i\nabla + (2e/c)\mathbf{A})^2$ and $(i\nabla + (2e/c)\mathbf{A})_1^2$, respectively, at $H_0 = H_{02} + h_2$

$$F_{20}(x) = \varphi_{20}(x), \qquad F_1(z,x) = \sum_n c_n F_1(z) \varphi_{1n}(x). \tag{33}$$

In the equations obtained by substituting (32) and (33) in (20) it is necessary to transform to Fourier compoennets, so that from the obtained dispersion equation we obtain expressions for the increments to the critical field

$$h_1 = \frac{c\varkappa^2 D^2}{8e\xi_1^2\xi_2^2} \ln^4 \frac{1.14\omega_D}{T} \frac{\sin\theta}{1+\sin\theta} \sum_n \frac{f_n}{1+s_1+2n}; \tag{34}$$

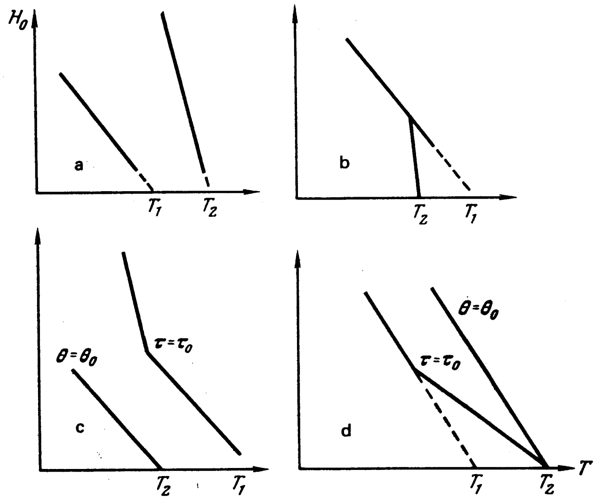

FIG. 1

$$s_1 = \frac{\tau_2}{\tau_1} \frac{\xi_1^2}{\xi_2^2} \sin\theta, \qquad H_{01} = \frac{c}{2e\xi_1^2} |\tau_1|,$$

$$h_2 = \frac{cD \ln^2(1.14\omega_D/T)}{4e\xi_1^2\xi_2\tau_2^{1/2}} \frac{1}{1+\sin\theta} \sum_n \frac{f_n}{(1+s_2+2n)^{1/2}},$$

$$s_2 = \frac{\tau_1}{\tau_2} \frac{\xi_2^2}{\xi_1^2} \sin\theta, \quad H_{02} = \frac{c}{2e\xi_2^2} |\tau_2|, \quad f_n = \frac{2(2n)!}{4^n (n!)^2} \left(\frac{1-\sin\theta}{1+\sin\theta} \right)^n. \tag{35}$$

Near the kink on the phase diagram, the corrections h_i are no longer small and to find the critical magnetic field in this region it is necessary to solve the secular equation. Expanding the functions $F_1(\mathbf{r})$ and $F_2(\mathbf{r})$ in terms of the eigenfunctions φ_{1n} and φ_{2n} we obtain the dispersion equation in this case

$$1 = \frac{\varkappa D}{\xi_1} \ln^2 \left(\frac{1{,}14\omega_D}{T} \right) I \bigg/ \left(\ln \frac{T}{T_1} + \varepsilon_{01} \right)^{1/2} \left(\ln \frac{T}{T_2} + \varepsilon_{02} \right),$$

$$\varepsilon_{01} = \frac{2e}{c} \xi_1^2 H, \qquad \varepsilon_{02} = \frac{2e}{c} \xi_2^2 H \sin\theta + \frac{7\zeta(3)}{4\pi^2} \frac{\mu^2 H^2}{T^2},$$

$$I = \begin{cases} 2\pi^{-1}[\sin\theta(1+\sin\theta)]^{-1/2}, & \theta \gg \Delta h / H_{c2}(\tau_0) \\ 4\pi^{-1}(\sin\theta)^{1/2}/(a_H p_0)^2, & \theta \ll \Delta h / H_{c2}(\tau_0) \end{cases}, \tag{36}$$

$$(a_H p_0)^2 = \Delta h / H_{c2}(\tau_0).$$

For the increment Δh to the critical field in the kink region we obtain from (36)

$$\left(\frac{\Delta h}{H_0} + \frac{\delta\tau}{\tau_0} \right) \left(\frac{\Delta h}{H_0} + \frac{\delta\tau}{\tilde{\tau}} \right)^2 = \frac{1}{\tilde{\tau}^2 \tau_0} \frac{\varkappa^2 D^2}{\xi_1^2} \ln^4 \left(\frac{1.14\omega_D}{T} \right) I^2,$$

$$\tilde{\tau} = \frac{T_0 - T_2}{T_2}, \qquad \delta\tau = \frac{T_0 - T}{T_0}. \tag{37}$$

The critical magnetic field at the kink point differs from H_0 by the amount

$$\Delta h = \frac{cI^{2/3}}{2e\xi_1^2} \left(\frac{\varkappa D}{\xi_1} \right)^{2/3} \ln^{4/3} \frac{1.14\omega_D}{T}. \tag{38}$$

The general form of the diagram of the coexistence of the superconducting and normal phase is shown in Fig. 2 (solid curve).

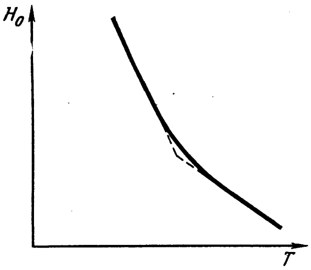

FIG. 2

5. CONCLUSION

The expounded theory of interacting localized and free electrons describes the superconducting transition in systems containing localized electrons. The effects indicated above can be observed in a single-crystal semiconducting superconductors, near a twinning plane, or near the surface of the sample, where localized electronic states are usually present, as well as in systems containing thin films on a semiconducting or metallic surface.

To observe the effects indicated in the paper, the following experiments can be performed.

When the magentic moment of such a system is varied, the derivative $\partial M/\partial T$ undergoes a jump at $\tau = t_0$.

The temperature dependence of the resistance of systems, in the case when the current flows parallel to the surface, effects analogous to those of Kosterlitz and Thouless (KT) for superconductors[19] can be observed. Indeed, from the electrodynamic viewpoint the near-surface superconducting states are analogous to a pair of superconducting films on a dielectric surface, with thicknesses D and L and with respective order parameters $\Lambda_{22}F_2$ and $\Lambda_{11}F_1$. Near the KT transition point at $T_{KT} < T_0$ te resistance decreases to zero in a region whose width is determined by the relation

$$\Delta T_{KT}=T_0-T_{KT}, \qquad \Delta T_{KT}=\delta T\sim T_0 a^2/\xi_2 d; \tag{39}$$

here d is the film thickness and a is the interatomic distance.

To estimate the contribution made to the free energy by the localized electrons and of the superconducting "tails" that penetrate deep into the superconductor, we write down the mean value of the interacting part of the Hamiltonian (1) in the form

$$\langle \mathcal{H}_{int}\rangle=\Lambda_{11}F_1{}^2L+2\Lambda_{12}F_1F_{20}D+\Lambda_{22}F_2{}^2D. \tag{40}$$

Using the connection between F_{20} and $F_1(0)$ we find that in case 1 the condition $\Lambda_{11}F_1^2L \gg \Lambda_{22}F_2^2D$, is satisfied, i.e., the main contribution to the energy is made by the superconducting "tails" in the interior of the superconductor. To estimate the values of δT it is necessary in this case to substitute $d\sim L$ in (39). This leads to the estimate

$$\Delta T_{KT}\sim T_0(a/\xi_2)^3, \qquad T_0-T_1\sim T_0(a/\xi_1)^2. \tag{41}$$

In this case T_{KT} differs only insignificantly from the formal transition temperature T_0. In case 2 we have

$$F_2{}^2D\gg F_1{}^2L, \qquad d=D, \qquad \Delta T_{KT}\sim T_0(a^2/D\xi_2). \tag{42}$$

The same estimates holds for case 3.

The estimates obtained here can be used for comparison with the experiments of Khaĭkin and Khlyustikov.[20] In these experiments the electric resistance of single-crystal tin containing a twin plane decreased with a characteristic variation interval $\delta T = 0.6$ K. Recognizing that for tin $T_0 \approx 3.7$ K$\xi \approx$ 3500 Å, and $a = 5$ Å, we find that one of two cases, 2 or 3, in which $\delta T \approx 10^{-2}$ K, is possibly realized in the experiments. Another possible explanation is that case 1 is realized, but a number of parallel twin planes are present, and the size of the temperature region increases rapidly with increasing number of twinning planes.

In addition, since the twin planes in the experiments of Ref. 20 is not parallel to the current flowing through the sample, a resistance is produced not due to the Kostelitz-Thouless vortices, but to the penetration of the electric field and to the resultant inhomogeneous superconducting layer.

In our theory the localized electrons have only one bound energy level. When a large number of such levels is taken into account, the temperatures T_1 and T_2 will no longer be independent, and T_1 tends to T_2.

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