Dependence of the sharpness of an interference pattern on the quantum state of the electromagnetic field

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It is demonstrated theoretically for a number of cases that the sharpness of an interference pattern of intersecting light beams with identical space-time structures may vary with the quantum state of the electromagnetic field. The interference pattern becomes smeared if the two interfering beams belong to orthogonal quantum states, i.e., if they consist of "unlike" photons. The smearing considered is not due to the randomness of the initial phases as in the case of incoherent sources.

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1. Communications that describe the results of optical-interference experiments wherein the quantum properties of the light might manifest themselves are published from time to time. Many papers were devoted to the investigation of the dependence of the sharpness of an interference pattern on the intensity of the light.[1–6] In particular, the interference pattern was investigated under the condition that one photon is present in the region where the components of the split beam overlap.[3,4] The results of Ref. 3 did not agree with those of Ref. 4. In Refs. 5 and 6 measurements of the correlation of the photocounts were used to investigate the interference pattern of two laser beams. No dependence of the sharpness of the interference pattern on the light intensity was observed there.

In the present paper, a standard quantum-mechanical formalism is used to calculate the probabilities of photodetection at various points of the region of overlap of two light beams propagating at an angle to each other. In this case one and the same space-time structure of the beam can correspond to different quantum states of the field. In the experiment these can be, for example, the following light sources: 1) a laser whose beam is split into two components that intersect at a certain angle; 2) two lasers whose beams follow the same directions as in the first case; 3) independent light sources of different nature having the same space-time structure.

We consider the following pure states of an electromagnetic field (making thereby no statements whatever with respect to the light source to which the particular state corresponds):

I) coherent state of a field whose spatial spectrum consists of two components;

II) sum of two coherent states, each with its own beam-propagation direction;

III) two- and four-photon states.

The calculation leads to the following results.

I. In the case of a coherent state with a beam "split" into two components one observes a sharp (corresponding to the classical) interference pattern. The same holds for arbitrary n-photon states that make up the given coherent state (each photon is split here into two components.

II. For a sum of two coherent states, each of which forms one of the beams (each identical in its space-time structure to the corresponding component of the first example), the interference pattern is smeared more strongly the larger the number the interference fringes that must be contained in the beam-overlap region (the number of fringes is in this case a measure of the orthogonality of the considered two coherent states).

III. For a two-photon state in which each of the photons is contained in a corresponding beam, the interference vanishes under the same conditions as in case II. An interference term, however, remains here if the probability of the coincidences is considered. If, on the other hand, not one but two photons are contained in each of the beams, partial smearing of the interference pattern for the coincidence probability takes place, too.

Smearing of the interference pattern occurs for pure quantum states and therefore does not reduce to averaging over the random phases, as in the case of incoherent sources. As can be seen from the second example, this smearing is not necessarily connected also with the large quantum indeterminacy of the phase, which takes place at a fixed number of photons[1)] (such an indeterminacy exists in third order but is absent in the second, where the average number of photons can be arbitrarily large). From the formal point of view, the interference vanishes in those cases when two interfering beams belong to orthogonal states (i.e., consist of "unlike" photons).

2. When calculating the photorecording probabilities we start from the theory of photodetection, expounded, e.g., in Refs. 7 and 8. Let $\hat{\mathbf{E}}(\mathbf{r},t)$ be the Heisenberg operator of the transverse part of the electric field. It can be represented in the form

$$\hat{\mathbf{E}}(\mathbf{r},\ t)=\hat{\mathbf{E}}_{(+)}(\mathbf{r},\ t)+\hat{\mathbf{E}}_{(-)}(\mathbf{r},\ t), \tag{1}$$

where $\hat{\mathbf{E}}_{(+)}$ and $\hat{\mathbf{E}}_{(-)}$ are the positive- and negative-frequency parts of the operator $\hat{\mathbf{E}}$:

$$\hat{\mathbf{E}}_{(+)}(\mathbf{r},t)=\frac{i(\hbar c)^{1/2}}{2\pi}\int d^3k k^{1/2}\mathbf{e}_\lambda(\mathbf{k})\exp[i(\mathbf{kr}-kct)]\hat{c}_\lambda(\mathbf{k}), \tag{2}$$

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$$\hat{\mathbf{E}}_{(-)}(\mathbf{r},t) = -\frac{i(\hbar c)^{1/2}}{2\pi}\int d^3k k^{1/2}\mathbf{e}_\lambda(\mathbf{k})\exp[-i(\mathbf{kr}-kct)]\hat{c}_\lambda^+(\mathbf{k})$$
$$=(\hat{\mathbf{E}}_{(+)}(\mathbf{r},t))^+. \quad (3)$$

Here $k=|\mathbf{k}|, \mathbf{e}_\lambda(\mathbf{k})(\lambda=1,2)$ are real polarization unit vectors, for which $\mathbf{k}\cdot\mathbf{e}_\lambda(\mathbf{k})=0$, $\mathbf{e}_\lambda(\mathbf{k})\cdot\mathbf{e}_\mu(\mathbf{k})=\delta_{\lambda\mu}$, and $e_\lambda{}^i(\mathbf{k})e_\lambda{}^l(\mathbf{k})=\delta_{il}-(k_ik_l/k^2)$ (summation is carried out over repeated polarization indices). The operators $\hat{c}_\lambda$ and $\hat{c}_\lambda{}^+$ satisfy the canonical commutation relations

$$[\hat{c}_\lambda(\mathbf{k}),\ \hat{c}_{\lambda'}{}^+(\mathbf{k}')]=\delta_{\lambda\lambda'}\delta(\mathbf{k}-\mathbf{k}'). \quad (4)$$

It is known[7,8] that the probability $dp(\mathbf{r},t)$ of the recording a photon at a point $\mathbf{r}$ in the time interval $(t,t+dt)$ is equal to $dp(\mathbf{r},t)=\nu_1(\mathbf{r},t)dt$, where

$$\nu_1(\mathbf{r},\ t)=\eta\langle\psi|\hat{E}_{(-)}(\mathbf{r},\ t)\hat{E}_{(+)}(\mathbf{r},\ t)|\psi\rangle. \quad (5)$$

Here $|\psi\rangle$ is the state vector of the electromagnetic field, $\hat{E}_{(\pm)}(\mathbf{r},t)=\mathbf{n}\hat{\mathbf{E}}_{(\pm)}(\mathbf{r},t)$, $\mathbf{n}$ is a unit vector along the direction of the dipole moment of the detected atom. The coefficient η is connected with the effective photodetection cross section σ by the formula $\eta=c\sigma/4\pi\hbar\omega$.

The probability of joint registration of two photons at the points $\mathbf{r}_1$ and $\mathbf{r}_2$ in the time intervals (t_1,t_1+dt_1) and (t_2,t_2+dt_2) is equal to

$$dp(\mathbf{r}_1,\ t_1;\ \mathbf{r}_2,\ t_2)=\nu_2(\mathbf{r}_1,\ t_1;\ \mathbf{r}_2,\ t_2)dt_1dt_2,$$

where

$$\nu_2(\mathbf{r}_1,\ t_1;\ \mathbf{r}_2,\ t_2)$$
$$=\eta^2\langle\psi|\hat{E}_{(-)}(\mathbf{r}_1,\ t_1)\hat{E}_{(-)}(\mathbf{r}_2,\ t_2)\hat{E}_{(+)}(\mathbf{r}_1,\ t_1)\hat{E}_{(+)}(\mathbf{r}_2,\ t_2)|\psi\rangle. \quad (6)$$

3. We consider hereafter the following states of the electromagnetic field.

I. Coherent state $|z\rangle$, for which

$$\hat{c}_\lambda(\mathbf{k})|z\rangle=z_\lambda(\mathbf{k})|z\rangle. \quad (7)$$

The coherent state of the electromagnetic field can be expressed in the form

$$|z\rangle=\exp\left\{-{}^1/_2\int d^3k|z_\lambda(\mathbf{k})|^2\right\}\exp\left\{\int d^3kz_\lambda(\mathbf{k})\hat{c}_\lambda^+(\mathbf{k})\right\}|0\rangle, \quad (8)$$

where $|0\rangle$ is the vacuum state $(\hat{c}_\lambda(\mathbf{k})|0\rangle$, $\langle 0|0\rangle=1$. To describe the coherent state of the electromagnetic field it is convenient to introduce the operators

$$\hat{c}^+=\gamma\int d^3kz_\lambda(\mathbf{k})\hat{c}_\lambda^+(\mathbf{k}),\qquad \hat{c}=\gamma\int d^3kz_\lambda^*(\mathbf{k})\hat{c}_\lambda(\mathbf{k}) \quad (9)$$

and choose the coefficient γ to satisfy the condition

$$[\hat{c},\ \hat{c}^+]=1. \quad (10)$$

It is easy to verify that

$$\gamma=\left[\int d^3k|z_\lambda(\mathbf{k})|^2\right]^{-1/2}=1/N^{1/2}, \quad (11)$$

where

$$N=\langle\psi|\int d^3k\hat{c}_\lambda^+(\mathbf{k})\hat{c}_\lambda(\mathbf{k})|\psi\rangle=\int d^3k|z_\lambda(\mathbf{k})|^2$$

is the average number of photons in the state $|\psi\rangle=|z\rangle$. We can then rewrite (8) with the aid of the formula

$$|z\rangle=\exp(-N/2)\exp(\gamma\bar{N}\hat{c}^+)|0\rangle,\quad \langle z|z\rangle=1, \quad (12)$$

which is analogous to the formula for the coherent state of an oscillator. The possibility of expressing the coherent state of a field with the aid of the operator $\hat{c}^+$ in the form (12) has a definite physical meaning: In the coherent state of the field we encounter photons that always have one and the same structure, and these photons are generated by the operator $\hat{c}^+$.

We shall assume in the present paper that the function $z_\lambda(\mathbf{k})$, which determines the spectral makeup of the field, is of the form

$$z_\lambda(\mathbf{k})=\alpha_\lambda(\mathbf{k})+\beta_\lambda(\mathbf{k}), \quad (13)$$

where $\alpha_\lambda(\mathbf{k})$ is concentrated in a small vicinity of the point $\mathbf{k}_1=k_0\mathbf{n}_1$, while $\beta_\lambda(\mathbf{k})$ is located close to the point $\mathbf{k}_2=k_0\mathbf{n}_2$. The unit vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ specify the propagation directions of the two components of the beam. The dimensions $2\pi/l$ and $2\pi/b$ of the regions in which the functions $\alpha_\lambda(\mathbf{k})$ and $\beta_\lambda(\mathbf{k})$ are concentrated (see Fig. 1) determine the dimensions of the wave packets. We shall put $k_0l\gg 1, k_0b\gg 1$, corresponding to quasimonochromatic and quasiplane wave packets.

We shall find it convenient to introduce the operators

$$\hat{a}^+=N_1^{-1/2}\int d^3k\alpha_\lambda(\mathbf{k})\hat{c}_\lambda^+(\mathbf{k}),\qquad \hat{a}=N_1^{-1/2}\int d^3k\alpha_\lambda^*(\mathbf{k})\hat{c}_\lambda(\mathbf{k}),$$
$$\hat{b}^+=N_2^{-1/2}\int d^3k\beta_\lambda(\mathbf{k})\hat{c}_\lambda^+(\mathbf{k}),\qquad \hat{b}=N_2^{-1/2}\int d^3k\beta_\lambda^*(\mathbf{k})c_\lambda(\mathbf{k}), \quad (14)$$

where

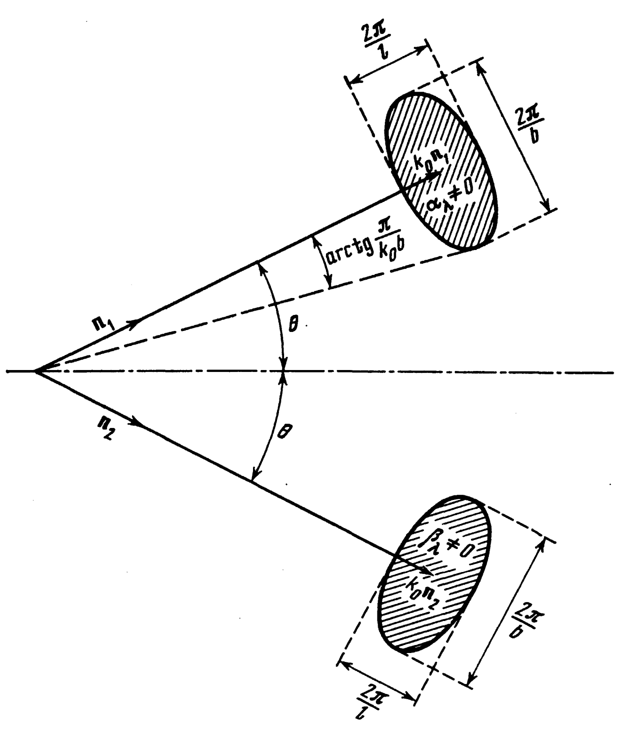

FIG. 1. Regions in wave-number space where the functions $\alpha_\lambda(\mathbf{k})$ and $\beta_\lambda(\mathbf{k})$ are concentrated: l—longitudinal and b—transverse scales of the wave packets; $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors along their propagation direction.

$$N_1=\int d^3k|\alpha_\lambda(\mathbf{k})|^2,\quad N_2=\int d^3k|\beta_\lambda(\mathbf{k})|^2.$$

We then have according to (9) and (13)

$$\hat{c}^+=(N_1/N)^{1/2}\hat{a}^+ +(N_2/N)^{1/2}\hat{b}^+,\quad \hat{c}=(N_1/N)^{1/2}\hat{a}+(N_2/N)^{1/2}\hat{b}. \tag{15}$$

The operators $\hat{a}$ and $\hat{b}$ satisfy the commutation relations

$$[\hat{a},\hat{a}^+]=[\hat{b},\hat{b}^+]=1,\quad [\hat{a},\hat{b}^+]=R,\quad [\hat{b},\hat{a}^+]=R^*, \tag{16}$$

where the quantity

$$R=(N_1N_2)^{-1/2}\int d^3k\alpha_\lambda^*(\mathbf{k})\beta_\lambda(\mathbf{k}) \tag{17}$$

determines the degree of overlap of the states $|\alpha\rangle$ and $|\beta\rangle$ [see (23)]. According to the Cauchy-Schwarz inequality we have

$$|R|\leqslant 1. \tag{18}$$

Returning to the state (12), in which $z_\lambda(\mathbf{k})$ takes the form (13), we note that this model can be set in correspondence with coherent radiation passing from one source through an optical system that splits the beam into two components that propagate at an angle to each other.

II. The second state we shall consider is a sum of two coherent states:

$$|\psi\rangle=M[|\alpha\rangle+|\beta\rangle]. \tag{19}$$

Here

$$|\alpha\rangle=\exp(-N_1/2)\exp(N_1^{1/2}\hat{a}^+)|0\rangle,\quad \langle\alpha|\alpha\rangle=1,$$
$$|\beta\rangle=\exp(-N_2/2)\exp(N_2^{1/2}\hat{b}^+)|0\rangle,\quad \langle\beta|\beta\rangle=1 \tag{20}$$

are coherent states that are generated by the operators (14) and satisfy the relations

$$\hat{c}_\lambda(\mathbf{k})|\alpha\rangle=\alpha_\lambda(\mathbf{k})|\alpha\rangle,\quad \hat{c}_\lambda(\mathbf{k})|\beta\rangle=\beta_\lambda(\mathbf{k})|\beta\rangle. \tag{21}$$

The normalization coefficient M is determined from the condition $\langle\psi|\psi\rangle=1$ and is connected with $\langle\alpha|\beta\rangle$ by the relation

$$2M^2[1+\operatorname{Re}\langle\alpha|\beta\rangle]=1. \tag{22}$$

On the basis of (20) we have

$$\langle\alpha|\beta\rangle=\exp[-(N_1+N_2)/2]\langle 0|\exp(N_1^{1/2}\hat{a})\exp(N_2^{1/2}\hat{b}^+)|0\rangle.$$

Using the known Glauber formula

$$\exp(\hat{A}+\hat{B})=\exp(\hat{A})\exp(\hat{B})\exp(-{}^1/_2[\hat{A},\hat{B}])$$
$$=\exp(\hat{B})\exp(\hat{A})\exp({}^1/_2[\hat{A},\hat{B}]),$$

we easily reduce the averaged equation to a normally ordered form:

$$\exp(N_1^{1/2}\hat{a})\exp(N_2^{1/2}\hat{b}^+)=\exp(N_2^{1/2}\hat{b}^+)\exp(N_1^{1/2}\hat{a})\exp[R(N_1N_2)^{1/2}].$$

Taking into account the equality

$$\langle 0|\exp(N_2^{1/2}\hat{b}^+)\exp(N_1^{1/2}\hat{a})|0\rangle=1$$

we then obtain

$$\langle\alpha|\beta\rangle=\exp\{-{}^1/_2[N_1+N_2-2R(N_1N_2)^{1/2}]\}. \tag{23}$$

In particular, if $N_1=N_2=N_0$, i.e, the mean values of the photons in both wave packets are equal, then

$$\langle\alpha|\beta\rangle=\exp\{-N_0(1-R)\}. \tag{24}$$

From this we see that if

$$N_0(1-\operatorname{Re}R)\gg 1, \tag{25}$$

then $|\langle\alpha|\beta\rangle|\ll 1$, i.e., the states $|\alpha\rangle$ and $|\beta\rangle$ are approximately orthogonal.

We note that the considered state (19) outside the region of intersection of the wave packets leads to the same space-time picture of the electromagnetic field as the "split" coherent state (12). This will be seen directly from the formulas for the quantity $\nu_1(\mathbf{r},t)$ calculated for these states.

III. In the next example we consider states of the form

$$|\Psi_{2n}\rangle=M_{2n}(\hat{a}^+\hat{b}^+)^n|0\rangle \tag{26}$$

at $n=1$ and 2. Here $\hat{a}^+$ and $\hat{b}^+$ are defined as before by Eqs. (14), but the quantities $N_{1,2}$ no longer have the previous physical meaning, but are simply normalization factors. The coefficient M_{2n} is determined from the condition $\langle\Psi_{2n}|\Psi_{2n}\rangle=1$, and its values for $n=1$ and 2 are

$$M_2=(1+|R|^2)^{-1/2},\quad M_4=[4(1+4|R|^2+|R|^4)]^{-1/2}. \tag{27}$$

Each state (26) contains n photons that propagate in the directions $\mathbf{n}_1$ and $\mathbf{n}_2$.

4. We now obtain the values of ν_1 and ν_2 for the considered states.

I. For the coherent state (12), using (7), we obtain

$$\hat{E}_{(+)}(\mathbf{r},t)|z\rangle=|z\rangle[E_1(\mathbf{r},t)+E_2(\mathbf{r},t)], \tag{28}$$

where

$$E_{1,2}(\mathbf{r},t)=\frac{i(\hbar c)^{1/2}}{2\pi}\int d^3k\,k^{1/2}(\mathbf{n}\mathbf{e}_\lambda(\mathbf{k}))\exp[i(\mathbf{kr}-kct)]\begin{Bmatrix}\alpha_\lambda(\mathbf{k})\\ \beta_\lambda(\mathbf{k})\end{Bmatrix} \tag{29}$$

are analytic signals corresponding to the coherent states $|\alpha\rangle$ and $|\beta\rangle$. Equation (5) leads next to the expression

$$\nu_1(\mathbf{r},t)=\eta|E_1(\mathbf{r},t)+E_2(\mathbf{r},t)|^2$$
$$=\eta\{|E_1(\mathbf{r},t)|^2+|E_2(\mathbf{r},t)|^2+2\operatorname{Re}E_1(\mathbf{r},t)E_2^*(\mathbf{r},t)\}. \tag{30}$$

The last term is here the interference term. To express it in explicit form, we obtain an approximate expression for $E_{1,2}$. To this end we introduce in the integral for E_1 a new integration variable, putting $\mathbf{k}=k_0\mathbf{n}_1+\boldsymbol{\varkappa}$. Since $\varkappa\ll k_0$ in the region essential for the integration, we can use the expansion

$$k=(k_0^2+2k_0\boldsymbol{\varkappa}\mathbf{n}_1+\varkappa^2)^{1/2}=k_0+\boldsymbol{\varkappa}\mathbf{n}_1+\ldots$$

and retain two terms of this series in the exponential, and one term in the pre-exponential factor. We then obtain

$$E_1(\mathbf{r},t)=A_1(\mathbf{r}-\mathbf{n}_1ct)\exp[ik_0(\mathbf{n}_1\mathbf{r}-ct)], \tag{31}$$

where

$$A_1(\mathbf{r}-\mathbf{n}_1ct)=|A_1|\exp(i\varphi_1)=\frac{i(\hbar ck_0)^{1/2}}{2\pi}$$
$$\times(\mathbf{n}\mathbf{e}_\lambda(k_0\mathbf{n}_1))\int d^3\varkappa\,\alpha_\lambda(k_0\mathbf{n}_1+\boldsymbol{\varkappa})\exp[i\boldsymbol{\varkappa}(\mathbf{r}-\mathbf{n}_1ct)] \tag{32}$$

is the wave-packet envelope, which is a smooth function of the coordinates in the wavelength scale. We obtain exactly the same expression for $E_2(\mathbf{r},t)$. Substituting (31) in (30), we have

$$\nu_1(\mathbf{r},t)=\eta\{|A_1(\mathbf{r}-\mathbf{n}_1ct)|^2+|A_2(\mathbf{r}-\mathbf{n}_2ct)|^2 +2|A_1A_2|\cos[k_0(\mathbf{n}_1-\mathbf{n}_2)\mathbf{r}+\varphi_1-\varphi_2]\}. \qquad (33)$$

It can be seen directly from (33) that in this case the interference pattern is sharpest (the modulation depth equals unity).

It follows from (28) that the coincidence counting rate ν_2 is given in this case by the known formula

$$\nu_2(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2)=\nu_1(\mathbf{r}_1,t_1)\nu_1(\mathbf{r}_2,t_2). \qquad (34)$$

It can also be shown that an equation that differs from (33) only by a common numerical coefficient holds also for states of the type

$$|n\rangle=(n!)^{-1/2}(\hat{c}^+)^n|0\rangle. \qquad (35)$$

This state can be treated as an n-photon state in which each of the photons is split into two components that propagate along the directions $\mathbf{n}_1$ and $\mathbf{n}_2$.

II. We consider now the state (19). Using (21), we obtain

$$\hat{E}_{(+)}(\mathbf{r},t)|\psi\rangle=M[|\alpha\rangle E_1(\mathbf{r},t)+|\beta\rangle E_2(\mathbf{r},t)],$$
$$\langle\psi|\hat{E}_{(-)}(\mathbf{r},t)=M[E_1^*(\mathbf{r},t)\langle\alpha|+E_2^*(\mathbf{r},t)\langle\beta|]. \qquad (36)$$

Then

$$\nu_1(\mathbf{r},t)=\eta M^2\{|E_1(\mathbf{r},t)|^2+|E_2(\mathbf{r},t)|^2 +2\,\mathrm{Re}[\langle\alpha|\beta\rangle E_1^*(\mathbf{r},t)E_2(\mathbf{r},t)]\}. \qquad (37)$$

Equation (37) differs from the corresponding (30) in the presence of an additional factor $\langle\alpha|\beta\rangle$ in the last term. This factor can according to (23) be very small in absolute value if the condition (25) is satisfied. In this case, as follows from (37), the interference pattern becomes smeared.

We consider also the quantity ν_2. Applying to (36) the operator $\hat{E}_{(+)}(\mathbf{r}',t')$ and again using (21), we obtain

$$\hat{E}_{(+)}(\mathbf{r}',t')\hat{E}_{(+)}(\mathbf{r},t)|\psi\rangle=M[|\alpha\rangle E_1(\mathbf{r}',t')E_1(\mathbf{r},t) +|\beta\rangle E_2(\mathbf{r}',t')E_2(\mathbf{r},t)],$$
$$\langle\psi|\hat{E}_{(-)}(\mathbf{r},t)\hat{E}_{(-)}(\mathbf{r}',t')=M[E_1^*(\mathbf{r}',t')E_1^*(\mathbf{r},t)\langle\alpha| +E_2^*(\mathbf{r}',t')E_2^*(\mathbf{r},t)\langle\beta|].$$

We then have for the coincidence counting rate

$$\nu_2(\mathbf{r},t;\mathbf{r}',t')=\eta^2M^2\{|E_1(\mathbf{r},t)E_1(\mathbf{r}',t')|^2+|E_2(\mathbf{r},t)E_2(\mathbf{r}',t')|^2 +2\,\mathrm{Re}\,[\langle\alpha|\beta\rangle E_1^*(\mathbf{r},t)\cdot E_1^*(\mathbf{r}',t')E_2(\mathbf{r},t)E_2(\mathbf{r}',t')]\}. \qquad (38)$$

the last term which is responsible for the interference, also vanishes in this formula if $|\langle\alpha|\beta\rangle|\ll 1$. Thus, in this case the interference patterns become smeared both for ν_1 and for ν_2.

III. We consider now the states $|\Psi_{2n}\rangle$ (26). We note that the method of exciting states with a finite number of photons was proposed in Ref. 9 and realized in Ref. 10. First, using the definition (14), we can find the formulas

$$[\hat{c}_\lambda(\mathbf{k}),\hat{a}^+]=\alpha_\lambda(\mathbf{k})/N_1^{1/2}, \quad [\hat{c}_\lambda(\mathbf{k}),\hat{b}^+]=\beta_\lambda(\mathbf{k})/N_2^{1/2},$$

after which it is easy to derive the commutation relations

$$[\hat{E}_{(+)}(\mathbf{r},t),\hat{a}^+]=E_1(\mathbf{r},t)/N_1^{1/2}, \quad [\hat{E}_{(+)}(\mathbf{r},t),\hat{b}^+]=E_2(\mathbf{r},t)/N_2^{1/2}. \qquad (39)$$

Using them we obtain in the case $n=1$

$$\hat{E}_{(+)}(\mathbf{r},t)|\Psi_2\rangle=M_2\{\hat{b}^+|0\rangle[E_1(\mathbf{r},t)/N_1^{1/2}]+\hat{a}^+|0\rangle[E_2(\mathbf{r},t)/N_2^{1/2}]\},$$
$$\langle\Psi_2|\hat{E}_{(-)}(\mathbf{r},t)=M_2\{[E_1^*(\mathbf{r},t)/N_1^{1/2}]\langle0|\hat{b}+[E_2^*(\mathbf{r},t)/N_2^{1/2}]\langle0|\hat{a}\}. \qquad (40)$$

We have then for the counting rate $\nu_1(\mathbf{r},t)$

$$\nu_1(\mathbf{r},t)=\eta M_2^2\{\langle0|\hat{b}\hat{b}^+|0\rangle|E_1(\mathbf{r},t)|^2/N_1 +\langle0|\hat{a}\hat{a}^+|0\rangle|E_2(\mathbf{r},t)|^2/N_2+[\langle0|\hat{a}\hat{b}^+|0\rangle E_1(\mathbf{r},t)E_2^*(\mathbf{r},t) +\langle0|\hat{b}\hat{a}^+|0\rangle E_1^*(\mathbf{r},t)E_2(\mathbf{r},t)]/(N_1N_2)^{1/2}\}.$$

But according to (16)

$$\langle0|\hat{a}\hat{a}^+|0\rangle=\langle0|\hat{b}\hat{b}^+|0\rangle=1,\quad \langle0|\hat{a}\hat{b}^+|0\rangle=R,\quad \langle0|\hat{b}\hat{a}^+|0\rangle=R^*,$$

whence

$$\nu_1(\mathbf{r},t)=\frac{\eta}{1+|R|^2}\left\{\frac{|E_1(\mathbf{r},t)|^2}{N_1}+\frac{|E_2(\mathbf{r},t)|^2}{N_2} +\frac{2}{(N_1N_2)^{1/2}}\mathrm{Re}[RE_1(\mathbf{r},t)E_2^*(\mathbf{r},t)]\right\}. \qquad (41)$$

It can be seen from this formula that as $|R|\to0$ the interference term vanishes, just as in the preceding example.

We now obtain ν_2. Acting on the first equation of (40) by the operator $\hat{E}_{(+)}(\mathbf{r}',t')$ and using (39), we obtain [designating for brevity $E_{1,2}=E_{1,2}(\mathbf{r},t),E'_{1,2}=E_{1,2}(\mathbf{r}'t')$]

$$\hat{E}_{(+)}(\mathbf{r}',t')\hat{E}_{(+)}(\mathbf{r},t)|\Psi_2\rangle=|0\rangle[M_2/(N_1N_2)^{1/2}][E_1E_2'+E_1'E_2],$$
$$\langle\Psi_2|\hat{E}_{(-)}(\mathbf{r}',t')\hat{E}_{(-)}(\mathbf{r},t)=[M_2/(N_1N_2)^{1/2}][E_1E_2'+E_1'E_2]^*\langle0|.$$

Therefore

$$\nu_2(\mathbf{r},t;\mathbf{r}',t')=\{\eta^2/[N_1N_2(1+|R|^2)]\} \times\{|E_1E_2'|^2+|E_1'E_2|^2+2\,\mathrm{Re}\,[(E_1E_2')(E_1'E_2)^*]\}. \qquad (42)$$

We see that as $|R|\to0$ only a common numerical coefficient changes in this equation, but the interference term does not vanish. Thus, as $|R|\to0$ the usual interference pattern for the state $|\Psi_2\rangle$ becomes smeared but the interference pattern for the coincidences remains sharp.

The situation is similar in the case of the state $|\Psi_4\rangle$. Since the calculations here are more unwieldy, we present only the final equations:

$$\nu_1=\frac{2\eta[1+2|R|^2]}{1+4|R|^2+|R|^4}\left\{\frac{|E_1|^2}{N_1}+\frac{|E_2|^2}{N_2} +\frac{2[2+|R|^2]}{[1+2|R|^2](N_1N_2)^{1/2}}\mathrm{Re}[RE_1E_2^*]\right\}, \qquad (43)$$

$$\nu_2=\frac{2\eta^2}{1+4|R|^2+|R|^4}\left\{\frac{|E_1E_1'|^2}{N_1^2}+\frac{|E_2E_2'|^2}{N_2^2}+\frac{2}{N_1N_2} \times|E_1E_2'+E_1'E_2|^2+\mathrm{Re}\frac{4R}{(N_1N_2)^{1/2}}\left[\frac{(E_1E_2'+E_1'E_2)(E_2E_2')^*}{N_2} +\frac{(E_1E_2'+E_1'E_2)^*(E_1E_1')}{N_1}\right]+\frac{2}{N_1N_2} \times[|R|^2|E_1E_2'+E_1'E_2|^2+\mathrm{Re}((E_1E_1')(E_2E_2')^*R^2)]\right\}. \qquad (44)$$

Just as in the preceding case, as $|R|\to 0$ the interference term vanishes in the formula for ν_1 but remains in the formula for ν_2. However, the depth of modulation of the interference pattern for ν_2 in the state $|\Psi_4\rangle$ is less than in the state $|\Psi_2\rangle$. To verify this we substitute in (42) and (44), taken at $R=0$, the representation (31) for $E_{1,2}$. For simplicity we assume that $N_1=N_2=N_0$ and $|A_1|=|A_2|=|A'_1|=|A'_2|$ (i.e., we consider the interference pattern in the central part of intersecting identical wave packets). Then

$$\nu_2 \approx \frac{2\eta^2|A|^4}{N_0^2}\{1+\cos[k_0(\mathbf{n}_1-\mathbf{n}_2)(\mathbf{r}-\mathbf{r}')]\} \quad \text{for} \quad |\Psi_2\rangle,$$
$$\nu_2 \approx \frac{12\eta^2|A|^4}{N_0^2}\left\{1+\frac{2}{3}\cos[k_0(\mathbf{n}_1-\mathbf{n}_2)(\mathbf{r}-\mathbf{r}')]\right\} \quad \text{for} \quad |\Psi_4\rangle. \tag{45}$$

It can be suggested that at $R=0$ and when n is increased the depth of modulation of the interference pattern in the expression for ν_2 will decrease.

Let us cast light on the physical meaning of the parameter R, which determines the sharpness of the interference pattern. According to (17), it determines the fraction of the "common" photons in the two interfering beams. The condition under which $|R|\ll 1$ is that the regions in which the functions $\alpha_\lambda(\mathbf{k})$ and $\beta_\lambda(\mathbf{k})$ are concentrated must not overlap. It is seen directly from Fig. 1 that this takes place in the case $b\ll l$ if $\arctan(\pi/k_0 b)\ll\theta$ or

$$K=k_0 b\,\mathrm{tg}\,\theta/\pi\gg 1. \tag{46}$$

On the other hand, K is the number of classical-interference-pattern fringes that are contained in the cross section of the beams intersecting at the angle θ. Indeed, according to (33) the period Λ of the interference pattern is determined by condition $2k_0\Lambda\sin\theta=2\pi$ and is equal to $\Lambda=\pi/k_0\sin\theta$. The transverse dimension of the region of intersection of two beams of diameter b is equal to $b/\cos\theta$. Therefore the number of interference fringes is

$$K=b/\Lambda\cos\theta=k_0 b\,\mathrm{tg}\,\theta/\pi.$$

Thus, if $K\gg 1$, i.e., if the classical interference pattern should subtend over a large number of fringes, then $|R|\ll 1$, and in example II and III the interference pattern should be smeared out. Conversely, if $K\ll 1$, the regions where the functions $\alpha_\lambda(\mathbf{k})$ and $\beta_\lambda(\mathbf{k})$ are concentrated overlap almost completely, and in this case $|R|\approx 1$.

By way of example we consider the case of coherent states with Gaussian spectra

$$\left.\begin{matrix}\alpha_\lambda(\mathbf{k})\\ \beta_\lambda(\mathbf{k})\end{matrix}\right\}=A\delta_{\lambda 1}\exp\left\{-\frac{(\mathbf{k}\mathbf{n}_{1,2}-k_0)^2 l^2}{2}-\frac{[k^2-(\mathbf{k}\mathbf{n}_{1,2})^2]b^2}{2}\right\}. \tag{47}$$

According to (31) and (32), they correspond to electromagnetic wave packets of the form

$$\left.\begin{matrix}\langle\alpha|\hat{E}_{(+)}(\mathbf{r},t)|\alpha\rangle\\ \langle\beta|\hat{E}_{(+)}(\mathbf{r},t)|\beta\rangle\end{matrix}\right\}=E_{1,2}(\mathbf{r},t)=\frac{iA(2\pi\hbar\omega_0)^{1/2}}{lb^2}(\mathbf{n}\mathbf{e}_1(k_0\mathbf{n}_{1,2}))$$
$$\times\exp\left\{ik_0(\mathbf{n}_{1,2}\mathbf{r}-ct)-\frac{(\mathbf{n}_{1,2}\mathbf{r}-ct)^2}{2l^2}-\frac{\mathbf{r}_\perp^2}{2b^2}\right\},\quad \mathbf{r}_\perp=\mathbf{r}-\mathbf{n}_{1,2}(\mathbf{r}\mathbf{n}_{1,2}). \tag{48}$$

It is easy to find that in this case

$$R=lb[(l^2\cos^2\theta+b^2\sin^2\theta)(l^2\sin^2\theta+b^2\cos^2\theta)]^{-1/2}$$
$$\times\exp\{-k_0^2l^2b^2\sin^2\theta/(l^2\cos^2\theta+b^2\sin^2\theta)\}, \tag{49}$$

and if $(l/b)\gg\tan\theta$, then $R\propto\exp(-K^2)$.

5. Let us summarize the main points. From the viewpoint of quantum theory a situation is possible wherein the interference pattern of intersecting light beams is partly or completely smeared out. The size of this smearing depends on the quantum state of the electromagnetic field and is a consequence of the orthogonality of the states to which the interfering beams belong. In other words, if the quantum states are orthogonal, the photons that enter in the different beams are "unlike" and do not interfere. By the same token we make somewhat more specific Dirac's known statement[11] that a photon interferes only with itself. Here is a manifestation of the difference between classical and quantum electrodynamics, namely that the former is linear in the fields and the latter is linear in the state vectors. It follows also that orthogonal coherent states do not interfere.

In connection with the arguments advanced above, when experiments on interference of light are performed and interpreted attempts should be made to identify to some degree the quantum state of the field; this is of course a difficult task. Without this identification, however, it would be difficult to point out the cause of the discrepancies between the results of different experiments.

Although it follows from the results of the present paper that the character of the interference pattern should depend substantially on the quantum state of the electromagnetic field, it is difficult to indicate *a priori* which quantum state is generated by one or other light source or by their aggregate. If it can be assumed, for example, that the model of the coherent state $|z\rangle$ with the function $z_\lambda(\mathbf{k})=\alpha_\lambda(\mathbf{k})+\beta_\lambda(\mathbf{k})$ is applicable for laser emission passing through an optical system that splits the beam into two intersecting components, for a field produced by two lasers it would be possible, generally speaking, to propose different models of quantum states. If the operators $\hat{a}^+$ and $\hat{b}^+$ introduced above are used, all the quantum states of the form

$$|\psi\rangle=\sum_{n,m=0}^{\infty}A_{nm}(\hat{a}^+)^n(\hat{b}^+)^m|0\rangle \tag{50}$$

lead to one and the same space-time structure of the electromagnetic field. All the field-state examples considered above were particular cases of (50). Yet we have verified that the calculation results depend essentially on the choice of the coefficients A_{nm} of this expansion. It is not clear beforehand to which particular case of the general expression (50) corresponds, for example, radiation produced by two lasers.

Of course, it is difficult to create in experiment conditions under which there would be no smearing of the interference pattern on account of random losses of phase coherence. However, as can be seen with Refs. 5 and 6 as examples, such experiments can be organized. It is in the interpretation of these experiments that the question arises of the quantum state of the field, and without answering this question it is impossible to conclude whether the theoretical conclusions agree with experiment or not.

At any rate, however, it must be borne in mind that smearing of an interference pattern for pure quantum states of an electromagnetic field does not contradict, generally speaking, quantum theory.

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[1)]B. Ya. Zel'dovich called the author's attention to the possibility of such an interpretation.

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Note added in proof (20 December 1982). After the article went to press, the author became acquainted with a paper by V. P. Bykov and G. V. Shepelev [Kvant. Elektron. (Moscow) **9**, 1844 (1982), J. Quant. Electronics **12**, 1188 (1982)] in which it is shown that a lasing regime is possible with smearing of the interference pattern, but the mechanism of this smearing differs from the one considered in the present article.

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