Theory of electromagnetic excitation of two-dimensional plasma waves in multilayer superlattices

M. V. Krasheninnikov

Institute of Semiconductor Physics, Siberian Division, USSR Academy of Sciences

A. V. Chaplik

Siberian State Research Institute for Metrology

(Submitted 24 June 1982)
Zh. Eksp. Teor. Fiz. **84**, 1075–1081 (March 1982)

We solve the problem of interaction of an electromagnetic field with a semi-infinite system of equidistant electron planes immersed in a dielectric medium (multilayer superlattice). The calculation is carried out for the same experimental setup which is used for the observation of two-dimensional plasmons in a single plasma layer. The shape of the absorption band is found. At relatively weak electron scattering, the latter influences the absorption substantially only at frequencies close to the edges of the plasma bond, whereas in the inner part of the band the absorption is due mainly to excitation of plasma waves. We calculate the amplitude of the electric field and the evolution of the time-limited electromagnetic pulse that is transformed into plasma waves.

PACS numbers: 71.45.Gm

1. INTRODUCTION

Artificial periodic structures—superlattices—in which a two-dimensional electron gas is realized are recently attracting much attention. The number of layers in the experimentally obtained superlattices reaches $10^2 = 10^3$ (Ref. 1) at insulating-gap thicknesses $\sim$200–300 Å. The thicknesses of the regions occupied by electrons are of the same order or lower. The particle surface density is $\sim 10^{11}$–10^{12} cm^{-2}. Under these conditions, the electrons can populate only the first transverse-quantization level, and the tunneling between the layers is negligibly small (the Visscher-Falicov model[2]).

Plasma waves in multilayer superlattices have been investigated theoretically in sufficient detail,[2–7] but to our knowledge there are no reports of their experimental observation. We discuss in this paper, by way of one of the possibilities of observing plasma oscillations in multilayer sublattices, their interaction with infrared electromagnetic radiation. We obtain the shape of the absorption band, the frequency dependence of the amplitude of the electric field in a plasma wave passing through a superlattice, and the broadening of a time-limited electromagnetic pulse of given shape, which is transformed into plasma waves.

If any of the semiconductors making up the superlattice is piezoelectric, coupled plasma-acoustic waves are excited in the system. In this case, as we shall show, the initial electromagnetic pulse splits into two, three, or four pulses, one of which propagates with a velocity typical of plasma waves, and the remainder at sound-wave velocities.

2. RESONANT EXCITATION OF PLASMA WAVES

In the case of a single-layer system with a two-dimensional electron gas, resonant excitation of plasmons is effected by a diffraction grating produced on the sample surface.[8] The period of this grating sets the two-dimensional plasmon momentum **k**, and the absorption resonance takes place at $\omega = \omega_p(k)$, where ω is the frequency of the incident electromagnetic wave and ω_p is the two-dimensional plasma frequency.

We consider now a semi-infinite multilayer superlattice occupying the region $z > 0$, on the surface of which is produced an analogous periodic structure. The electric field of the wave passing through the diffraction grating is represented by a Fourier series in $\cos k_s x$, where $k_s = 2\pi s/L$, L is the period of the diffraction grating, and $s = 0, 1, 2,\ldots$ The zeroth harmonic does not excite plasma oscillations and produces in the absorption the usual Drude background (see the experiments, Ref. 8). The remaining harmonics are not waves that propagate (along the z axis), and the electric field in them attenuates exponentially with increasing distance from the diffraction-grating plane. Indeed, under typical experimental conditions the wavelength of the infrared radiation is much larger than the wavelength of a plasmon having the same frequency ($\omega \sim 10^{12}$–10^{13} Hz), i.e., a quasistatic limit is realized. Therefore the sth harmonic of the electromagnetic wave passing through the diffraction grating should be proportional to $\exp(ik_s x - k_s z)$. Thus, for the considered excitation method, the model of a semi-infinite superlattice is fully justified if the structure thickness is much larger than $1/k$.

Resonantly interacting with the superlattice plasma, the harmonics with $s > 0$ are transformed into plasma oscillations. The latter are already traveling waves and can carry energy into the interior of the superlattice at a group velocity determined by the dispersion law of the plasmons in the multilayer structure. We note that the intensity of the first harmonic can reach approximately 10% of the intensity of the incident radiation (see, e.g., Ref. 8).

The problem consists of solving the equations for the electrostatic potential φ and of the nonequilibrium increments to the surface density $\widetilde{N}_n$ of the charge, where n is the number of the superlattice layer:

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$$\frac{\partial^2\varphi}{\partial z^2}-k^2\varphi=\frac{4\pi e}{\varepsilon}\sum_{n=0}^{\infty}\tilde{N}_n\delta(z-n\Delta). \quad (1)$$

Here ε is the dielectric constant, Δ is the superlattice period, and e is the electron charge.

The quantities $\tilde{N}_n$ can be obtained from the kinetic equation (see, e.g., Ref. 9). At a given extraneous field $E_{ex}=E_0\exp(-kz+ikx)$ in the approximation $\omega\gg kv$, where v is the characteristic electron velocity, we obtain

$$\tilde{N}_n=\frac{eN_sk}{\omega(\omega+i\nu)m}[k\varphi(z=n\Delta)+iE_{ext}(z=n\Delta)]. \quad (2)$$

N_s in (2) denotes the equilibrium surface density of the electrons in the layers; m and ν are respectively the effective mass and the carrier collision frequency.

We seak the solution in the form

$$\varphi(x,z)=\sum_{m=0}^{\infty}A_m\exp\{-k|z-m\Delta|+ikx\}. \quad (3)$$

Substituting (2) and (3) in (1), we obtain a system of equations for the coefficients A_m:

$$A_{n\geqslant 0}=\frac{\omega_p^2}{\omega(\omega+i\nu)}\left[\sum_{m=0}^{\infty}A_me^{-k|m-n|\Delta}+i\frac{E_0}{k}e^{-k\Delta n}\right], \quad (4)$$

$$\omega_p^2=2\pi e^2N_sk/m\varepsilon.$$

The system (4) is a discrete analog of the inhomogeneous Wiener-Hopf integral equation. Its solution is based on methods expounded, e.g., in Refs. 10 and 11, and is given in the Appendix. The results take the form

$$A_{n\geqslant 0}=2i\frac{\omega_p^2}{\omega(\omega+i\nu)}\frac{E_0}{k}\frac{\operatorname{sh}k\Delta}{e^{iq\Delta}-e^{-k\Delta}}e^{-iq\Delta n}. \quad (5)$$

The plasmon quasimomentum q at given ω and k is determined from the dispersion equation

$$\cos q\Delta=\operatorname{ch}k\Delta-\frac{\omega_p^2}{\omega(\omega+i\nu)}\operatorname{sh}k\Delta. \quad (6)$$

For the x-component of the total electric field $E_{tot}(n)$, which acts on the particles in the nth layer, we obtain the expression

$$E_{tot}(n)=E_0e^{-k\Delta n}-ik\varphi(z=n\Delta)=\frac{2E_0\operatorname{sh}k\Delta}{e^{iq\Delta}-e^{-k\Delta}}e^{-iq\Delta n+ikx}. \quad (7)$$

In the collisionless approximation ($\nu=0$), $E_{tot}(n)$ attenuates exponentially with increasing n if the frequency lies outside the plasma-oscillation band defined by the dispersion relation (6). In the experimentally most realistic limiting case $k\Delta\ll 1$ the limits of the band are defined by

$$\omega_{max}=\omega_p(2/k\Delta)^{1/2},\quad \omega_{min}=\omega_p(k\Delta/2)^{1/2}.$$

Near the upper threshold $\omega\gtrsim\omega_{max}$ the spatial damping of the wave (7) equals

$$q''=-k[2(\omega-\omega_{max})/\omega_{max}]^{1/2}, \quad (8a)$$

and at $\omega\lesssim\omega_{min}$

$$q''=-2\Delta^{-1}[2(\omega_{min}-\omega)/\omega_{min}]^{1/2}. \quad (8b)$$

Inside the band, neglecting the collisions, the damping q'' is of course zero. In this sense one can speak of resonant excitation of plasma waves in superlattices; in the frequency band $\omega_{min}<\omega<\omega_{max}$ the total electric field in the nth layer does not vanish as $n\to\infty$, although the exciting field attenuates like $\exp(-k\Delta n)$. The frequency dependence of the transmitted wave is given by

$$|E_{tot}|^2=2E_0^2\operatorname{sh}k\Delta e^{k\Delta}(\omega^2/\omega_p^2),\quad \omega_{min}<\omega<\omega_{max}. \quad (9)$$

Allowance for the electron scattering gives rise to collision damping. Its spatial decrement, say for $\omega=\omega_{max}/2$ (in which case $q'\Delta\ll 1$) is equal to $q''=-k\nu/2^{1/2}\omega_{max}$. We shall estimate this quantity for the GaAs–GaAlAs structure at the following values of the characteristic parameters: $\Delta=2\times10^{-6}$ cm, $\nu=3\times10^{11}$ sec^{-1}, $N_s=10^{12}$ cm^{-2}, $m=6\times10^{-29}$ g, $\varepsilon=12.5$. If the period of the diffraction grating is 3 μm, then $k\Delta=4\times10^{-2}$, $\omega_{max}=4\times10^{13}$ sec^{-1}, so that $|q''|=0.5\times10^{-2}k$. Thus, when the number of layers exceeds 60 the external field decreases by more than an order of magnitude, i.e., the passage of the "diffracted" field through the system is due mainly to its transformation into plasma waves.

3. SHAPE OF THE ABSORPTION BAND

The total (i.e., summed over all layers) work performed by the external field on the system per unit time is given by the formulas

$$Q=\frac{1}{2}\operatorname{Re}\sum_{n=0}^{\infty}E_0e^{-k\Delta n}j_n;$$

$$j_n=E_{tot}(n)\sigma_{xx},\quad \sigma_{xx}=\frac{iN_se^2}{m\omega(\omega+i\nu)}.$$

Using (7) and summing, we get

$$Q=\frac{E_0^2N_se^2}{m\omega}\frac{\operatorname{sh}k\Delta}{1+\nu^2/\omega^2}$$
$$\times\frac{e^{q''\Delta}[e^{q''\Delta}\sin q'\Delta(1-e^{-2k\Delta})+(\nu/\omega)\operatorname{sh}k\Delta(1-e^{-2k\Delta+2q''\Delta})]}{(1-2\cos q'\Delta e^{-k\Delta+q''\Delta}+e^{-2k\Delta+2q''\Delta})^2}$$
$$\times\frac{\cos q'\Delta}{A}, \quad (10)$$

where $q'\equiv\operatorname{Re}q>0$, $q''\equiv\operatorname{Im}q<0$. The frequency dependence of $Q(\omega)$ is contained also in q' and q'', which are defined by Eq. (6):

$$\sin^2q'\Delta=C+(C^2+B^2)^{1/2},\quad e^{q''\Delta}=A/\cos q'\Delta-B/\sin q'\Delta,$$

$$A=\operatorname{ch}k\Delta-\frac{\omega_p^2}{\omega^2+\nu^2}\operatorname{sh}k\Delta,\quad B=\frac{\nu}{\omega}\frac{\omega_p^2}{\omega^2+\nu^2}\operatorname{sh}k\Delta, \quad (11)$$

$$C=\frac{1}{2}(1-A^2-B^2).$$

Near the end points of the band, the behavior of $Q(\omega)$ is characterized by the following expressions ($k\Delta\ll1$, $|q''|\ll k$): in the vicinity of the upper threshold

$$Q=Q_0\frac{\nu}{[2(\omega-\omega_{max})\omega_{max}]^{1/2}},\qquad \omega>\omega_{max},\quad \omega-\omega_{max}\gg\nu; \tag{12a}$$

$$Q=Q_0(\nu/2\omega_{max})^{1/2},\qquad |\omega-\omega_{max}|\ll\nu; \tag{12b}$$

$$Q=Q_0[2(\omega_{max}-\omega)/\omega_{max}]^{1/2},\qquad \omega<\omega_{max},\quad \omega_{max}-\omega\gg\nu \tag{12c}$$

(here $Q_0 = E_0^2 N_s e^2 k\Delta /2m\omega_{max}$); in the vicinity of the lower threshold the $Q(\omega)$ frequency dependence is described by the same formulas, with ω_{max} replaced by ω_{min}, $\omega_{max}-\omega$ by $\omega-\omega_{min}$, and Q_0 by $E_0^2 N_s e^2 (k\Delta)^2/m\omega_{min}$.

Inside the band, at a frequency distance larger than ν from the edges, the electron scattering becomes insignificant. The work performed by the external field goes into plasma-wave excitation. The absorption is described in this case by the formula

$$Q(\omega)=\frac{E_0^2N_se^2}{2m\omega}\,\mathrm{sh}\,k\Delta e^{k\Delta}\frac{\omega^2}{\omega_p^2}\left[2\frac{\omega^2}{\omega_p^2}\,\mathrm{cth}\,k\Delta-1-\frac{\omega^4}{\omega_p^4}\right]^{1/2}. \tag{13}$$

A plot of $Q(\omega)$ [see Eq. (10)] for the characteristic system parameters $k\Delta = 4\times10^{-2}$, $\nu/\omega_p = 0.05$ is shown in the figure.

To conclude this section, we estimate the parameters of a multilayer superlattice as a decelerating system. Let the amplitude of the exciting field have a slow Gaussian variation

$$E_0(t)=E_0\exp(-t^2/T^2),\qquad \omega T\gg1.$$

It is known that the transmitted pulse at $z = n\Delta$ will also have a Gaussian shape of width

$$T^{*}=T\left[1+4\left(\frac{\partial^2 q}{\partial\omega^2}\right)^2\frac{(n\Delta)^2}{T^4}\right]^{1/2}.$$

We consider the frequency region in which $q\Delta, k\Delta \ll 1$, but $q \ll k$. From (6) we obtain then the approximate relation $\omega\approx\omega_{max}\,k/q$. At the numerical values of the parameters used in Sec. 2 and at $k/q\sim0.1$ we obtain $\omega\approx4\times10^{12}$ sec^{-1} and $\omega/\nu\approx12$; the group velocity is $v_{gr} = 2\times10^7$ cm/sec. The relative broadening of the pulse for $n = 10^3$ layers is $(10^{-11}/t)^4$, where T is in seconds.

The foregoing estimate pertains to the frequency interval in which $\omega\sim(\omega_{max}\omega_{min})^{1/2}$. It is precisely this interval which is optimal for obtaining small group velocities. Near the upper edge of the band it is necessary to take into account the electron scattering; in the region $\omega_{max}-\omega\ll\nu$ we obtain $v_{gr} = (2\omega_{max}\nu)^{1/2}/k\approx2.5\times10^8$ cm/sec at the same ω_{max}, ν, and k as above. The lower edge of the band is hardly of interest, since $\omega_{min}/\nu\sim1$ for the presently available superlattices.

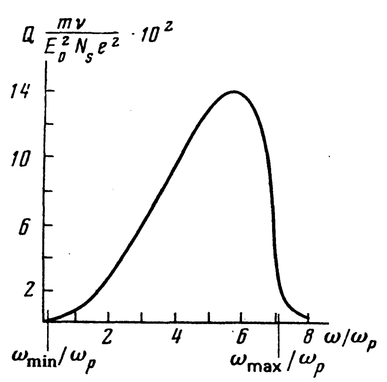

FIG. 1.

4. PIEZOELECTRIC COUPLING OF PLASMONS WITH ELASTIC WAVES

Gallium arsenide is known to be piezoelectric, and this should lead to interaction of plasma waves with acoustic ones in multilayers GaAs-GaAlAs superlattices. This problem does not differ in principle from the one considered above, although it does entail rather cumbersome calculations. We do not present here a complete solution, and confine ourselves to a qualitative discussion of the results. In the general case, at arbitrary orientation of the superlattice layers relative to the crystallographic axes, the electric-wave potential has four components. Given ω and k, the dispersion equation has four roots $q_j(k,\omega)$, $j = 1,2,3,4$. One of them corresponds to a group velocity of the order of the plasmon velocity, and the three others to velocities of the order of acoustic. The function $G(w)$ introduced in the Appendix has respectively four pairs of complex-conjugate zeros $w_j = \exp(\pm q_j\Delta)$. The electric field in the nth layer will thus consist of four waves traveling with different velocities. Since the electromechanical-coupling coefficient γ is usually small compared with unity ($\gamma\sim10^{-2}$ for GaAs), the main fraction of the energy is contained in the plasma-type wave.

In the absence of electron scattering, the broad plasmon band breaks up into a set of plasma-acoustic bands of width $\sim2\pi c/\Delta$, where c is the speed of sound (see Ref. 7). The width of the forbidden band is less than $2\pi c/\Delta$ in terms of the parameter γ. A similar structure should manifest itself in the shape of the absorption band of an electromagnetic wave. However, the electron collision frequencies actually attainable at the present time do not permit resolution of this structure. Therefore the shape of the absorption band remains practically the same as obtained in Sec. 3, but an interesting experimental manifestation of the many-part character of the plasma acoustic waves is the splitting of a time-limited pulse in accordance with the different values of the group velocities.

By way of illustration we present the results for the simplest model: the electron planes are imbedded in a piezocrystal of symmetry c_{6v}, with the C_6 axis parallel to the layers of the structure (in this case we have a two-part potential wave). The electric field in the nth layer is

$$E_{tot}(n)=e^{-iq_1\Delta n}\frac{2E_0\,\mathrm{sh}\,k\Delta}{e^{iq_1\Delta}-e^{-k\Delta}}+[1-e^{i(q_2-\varkappa)\Delta}]e^{iq_2\Delta n}\frac{2E_0\,\mathrm{sh}\,k\Delta}{1-e^{-k\Delta-iq_1\Delta}}, \tag{14}$$

where $\varkappa^2 = \omega^2/c^2 - k^2$, $q_{1,2}$ are the roots of the dispersion equation,[7] and

$$\omega^2=\omega_p^2\left(\frac{\mathrm{sh}\,k\Delta}{\mathrm{ch}\,k\Delta-\cos q\Delta}-\frac{\gamma k}{\varkappa}\frac{\sin\varkappa\Delta}{\cos\varkappa\Delta-\cos q\Delta}\right). \tag{15}$$

Equation (14) was obtained under the assumption $\gamma \ll 1$ and $\omega \gg ck$. The latter means that the plasma and acoustic waves are weakly coupled; therefore q_1 can be regarded as determined from the dispersion equation (6), and q_2 differs from $\varkappa$ by a small amount of the order of $\gamma ck/\omega$.

Let us discuss in conclusion the question of the local modes. Although the plasma layers occupy in the considered problem a half-space, we do not obtain a solution of the surface-plasmon type ($\omega = \omega_p/\sqrt{2}$). The point is that motion normal to the layers is completely excluded for two-dimensional electrons, whereas in a surface plasmon the particle displacement must contain two components, parallel and perpendicular to the surface. When account is taken of the acoustic degrees of freedom, a local mode of the type of a Rayleigh or a Bluestine-Gulyaev wave appears. At the k values of interest to us the frequency of these waves is smaller by two or three orders than $\omega_{\min}$, so that the local modes do not play an important role in the phenomena considered.

APPENDIX

We solve the system (4) in full accord with Refs. 10 and 11. Such a system can be reduced to the Hilberg boundary-value problem. We redefine the coefficients A_n to include $n<0$

$$A_{n<0} = \frac{\omega_p^2}{\omega(\omega+i\nu)} \sum_{m=0}^{\infty} A_m e^{-k|m-n|\Delta}.$$

We introduce a function of the complex variable w, defined by the series

$$G(w) = \frac{\omega_p^2}{\omega(\omega+i\nu)} \times \sum_{n=-\infty}^{\infty} e^{-k\Delta|n|} w^n - 1 = -\frac{(w-e^{iq\Delta})(w-e^{-iq\Delta})}{(w-e^{k\Delta})(w-e^{-k\Delta})}, \quad \text{(A1)}$$

$$A^+(w) = \sum_{n=0}^{\infty} A_n w^n; \quad \text{(A2)}$$

$$A^-(w) = \sum_{n=-1}^{-\infty} A_n w^n, \quad A^-(\infty) = 0. \quad \text{(A3)}$$

We have introduced here the notation $\exp(\pm iq\Delta)$, where q is determined from the relation

$$\cos q\Delta = \operatorname{ch} k\Delta - \frac{\omega_p^2}{\omega(\omega+i\nu)} \operatorname{sh} k\Delta. \quad \text{(A4)}$$

It is obvious from (A1) that the function $G(w)$ is regular in the ring $e^{-k\Delta} < |w| < e^{k\Delta}$. Assume that there exist r and R such that $A^+(w)$ is regular at $|w| < R$ and $A^-(w)$ is regular at $|w| > r$. We multiply (4) by w^n and sum over n from $-\infty$ to $+\infty$. By varying the order of summation we obtain a functional equation that relates $A^+(w)$ with $A^-(w)$:

$$A^-(w) = G(w) A^+(w) - \frac{S}{w-e^{k\Delta}}, \qquad S = i \frac{\omega_p^2 e^{k\Delta}}{\omega(\omega+i\nu)} \frac{E_0}{k}. \quad \text{(A5)}$$

The functions $A^{\pm}(w)$ can be constructed in the region of the ring $r < |w| < R$ by first solving the an auxiliary boundary-value problem. We choose a contour L ($|w| = \rho$) such that it contains no zeros of $G(w)$. We arrive at the Hilbert boundary-value problem[12]: find $A^+(w)$ which is regular at $|w| < \rho$, $A^-(w)$ that is regular at $|w| > \rho$, which are connected on the contour L by relation (A5). Since the function $G(w)$ is the ratio the polynomials (A1), the factorization is elementary:

$$G(w) = G^+(w) G^-(w),$$

$$G^+(w) = -\frac{(w-e^{iq\Delta})(w-e^{-iq\Delta})}{w-e^{k\Delta}}, \qquad G^-(w) = \frac{1}{w-e^{-k\Delta}},$$

where $G^+(w)$ and $G^-(w)$ are regular inside and outside the contour L, respectively. The solution of the boundary value problem is then[12]:

$$A^+(w) = \frac{P_m(w)}{G^+(w)} + S \frac{w-e^{-k\Delta}}{G^+(w)(w-e^{k\Delta})}, \qquad |w|<\rho; \quad \text{(A6a)}$$

$$A^-(w) = P_m(w) G^-(w), \qquad |w|>\rho. \quad \text{(A6b)}$$

Here $P_m(w)$ is a polynomial of degree m. The condition $A^-(\infty) = 0$ yields $P_m = C$, where C is an arbitrary constant.

Obviously, (A5) realizes an analytic continuation of $A^+(w)$ defined in (A6a) into the region $|w| > \rho$, and it is important to note that $A^+(w)$ can have poles where $G(w) = 0$.

The coefficients A_n are obtained from (A2):

$$A_{n\geq 0} = \frac{1}{2\pi i} \oint_L \frac{A^+(w)\,dw}{w^{n+1}}. \quad \text{(A7)}$$

Deforming the integration contour in (A7) in the region $|w| > \rho$ we obtain the solution of the system (4):

$$A_{n\geq 0} = \frac{2i}{\sin q\Delta} \{ e^{-iq\Delta(n+1)} [C(e^{iq\Delta} - e^{k\Delta}) + S(e^{iq\Delta} - e^{-k\Delta})] + e^{iq\Delta(n+1)} [C(e^{-iq\Delta} - e^{k\Delta}) + S(e^{-iq\Delta} - e^{-k\Delta})] \}. \quad \text{(A8)}$$

Since it follows from (A.4) that at $q' \equiv \operatorname{Re} q > 0$ we have $q'' \equiv \operatorname{Im} q < 0$, the coefficient of $\exp(iq\Delta n)$ in (A8) must be set equal to zero. We thus determine the constant C:

$$C = -S \frac{e^{iq\Delta} - e^{-k\Delta}}{e^{-iq\Delta} - e^{k\Delta}};$$

the solution of the system (4) takes then the form (5).

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Translated by J. G. Adashko