Macroscopic vacuum effects in an inhomogeneous and nonstationary electromagnetic field

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Macroscopic effects of vacuum polarization by a strong nonuniform and nonstationary fields, which are kinematically forbidden in the case of a uniform magnetic field, are considered. Calculations are perfomed for the deflection of a light beam in the field of a magnetic dipole, for the production of photon pairs by an inclined rotator, and for doubling and modulation of the frequency in scattering of low-frequency electromagnetic waves by the magnetic field of an inclined rotator.

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1. INTRODUCTION

Interest in the study of phenomena due to vaccum polarization by a macroscopic electromagnetic field has increased of late.[1–3] The possibility of observing vacuum effects in a strong magnetic field in the vicinity of neutron stars is discussed in Refs. 4–8. Dealt with, in particular, is birefringence in a magnetic field, splitting of a photon into two,[4,9–11] vacuum Čerenkov radiation,[5,12] and neutrino generation by photons.[6] It was shown in Refs. 2 and 3 that shock waves can be produced in a magnetized vacuum by nonlinear effects. These processes can be realized in a constant and uniform field of intensity close to the Schwinger critical field[1)] $F_{cr} = m^2/e = 4.41\times10^{13}$ G.

The purpose of the present note is to point out some other macroscopic manifestations of vacuum polarization by a strong electromagnetic field, which can be realized in the case of nonstationary and nonuniform fields. We note that the magnetic field of a neutron star is nonuniform over distances of the order of the radius of the star, and is nonstationary with a characteristic rotation frequency. In a nonuniform magnetic field, the effective refractive index of vacuum becomes coordinate-dependent, and this can lead to the effect of "vacuum" focusing of the light rays. Scattering of a low-frequency electromagnetic wave by the nonstationary field of an inclined rotator can be accompanied by multiplication of the wave frequency and by the appearance of modulation harmonics. The analog of this process in a uniform magnetic field is the coalescence of two photons into one, but this process is suppressed by kinematic forbidenness in the lowest order of perturbation theory. If, however, the field is nonuniform over distances on the order of the wavelength of the electromagnetic (radio) wave, the kinematic forbidenness is lifted. Finally, a nonuniform macroscopic field can itself create single photons and groups of correlated photons. The one-photon "evaporation" of a classical electromagnetic field constitutes a quantum correction to classical radiation. The two-photon effect yields a continuous radiation spectrum that differs qualitatively from the spectrum of classical radiation.

The processes considered can be described with the aid of the Heisenberg-Euler-Schwinger Lagrangian, in which the strong-field contribution is separated and expansion is carried out in powers of the additional weak field. A similar method was used earlier in Refs. 4 and 7 where, however, the perturbing field was assumed to be of special form. The general expansions obtained below allow us to consider a larger group of problems.

2. EXPANSION OF THE EFFECTIVE LAGRANGIAN

If the spatial and temporal scales Δl and Δt, within the limits of which a classical electromagnetic field F can be regarded as uniform, satisfy the conditions

$$\Delta l \gg m^{-1}, \quad \Delta t \gg m^{-1}\max(1,\, eF/m^2), \tag{2.1}$$

where m is the electron mass, the polarization of vacuum by such a field can be described by the effective-Lagrangian method. In a constant and uniform field, the effective Heisenberg-Euler-Schwinger Lagrangian is of the form[13,14]

$$\mathscr{L} = -\frac{e^2}{8\pi^2}\int_0^\infty \frac{ds}{s} e^{-m^2 s}\left(i\frac{g}{2}\frac{\operatorname{Re}\operatorname{ch} esz}{\operatorname{Im}\operatorname{ch} esz} - \frac{1}{e^2s^2} - \frac{f}{3}\right), \tag{2.2}$$

where

$$\begin{gathered} z=(f+ig)^{1/2}, \quad f={}^1/_2F_{\mu\nu}F^{\mu\nu}=\mathbf{B}^2-\mathbf{E}^2, \\ g=-{}^1/_4\varepsilon_{\mu\nu\lambda\tau}F^{\mu\nu}F^{\lambda\tau}=2\mathbf{EB} \quad (\varepsilon_{0123}=-1). \end{gathered} \tag{2.3}$$

The expansion of the Lagrangian (2.2) in powers of the tensor $F_{\mu\nu}$ starts with terms of fourth order. In the case of a coordinate-dependent field, account must be taken of the contribution of the second-order diagram (Fig. 1) that yields under the condition (2.1) an increment[11,14,15]

$$\mathscr{L}' = -\frac{e^2}{120\pi^2m^2}\frac{\partial F^{\mu\nu}}{\partial x^\mu}\frac{\partial F^\lambda{}_\nu}{\partial x^\lambda}. \tag{2.4}$$

We estimate now the ratio of (2.4) and (2.2). Assuming $\partial F^{\mu\nu}/\partial\chi^\nu \sim l^{-1}F^{\mu\nu}$, we find that at field intensities $F \ll F_{cr}$

$$\frac{\mathscr{L}'}{\mathscr{L}} \sim \left(\frac{F_{cr}}{F}\right)^2\left(\frac{\lambda_c}{l}\right)^2, \tag{2.5}$$

where λ_c is the Compton wavelength. In the actual cases considered below $F \lesssim F_{cr}$ and $l \gg \lambda_c$, so that the contribution (2.4) of the second-order diagram is small.

In the problem considered, the field $F_{\mu\nu}$ takes the form

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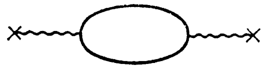


FIG. 1. Second-order contribution to the effective Lagrangian of a nonuniform field.

of the sum

$$F_{\mu\nu}=\mathscr{F}_{\mu\nu}+f_{\mu\nu}, \tag{2.6}$$

where $\mathscr{F}_{\mu\nu}$ is the tensor of the external field of arbitrary intensity and $f_{\mu\nu}$ is the weak-field tensor. Expanding the Lagrangian (2.2) in powers of $f_{\mu\nu}$, we can carry out next a second quantization in the field $f_{\mu\nu}$, so that the results can be formulated in terms of photon creation and absorption (in such a theory, of course, we must confine ourselves to Feynman diagrams of the "tree" type). We construct the sought expansion under the additional condition that the pseudoscalar invariant of the field $\mathscr{F}$ be zero ($\varepsilon_{\mu\nu\lambda\tau}\mathscr{F}^{\mu\nu}\mathscr{F}^{\lambda\tau}=0$). Introducing the dimensionless parameters

$$\begin{gathered}\beta=ef_0^{1/2}/m^2, \quad y=esf_0^{1/2},\\ f_1=\mathscr{F}_{\mu\nu}f^{\mu\nu}/f_0, \quad f_2={}^1/_2 f_{\mu\nu}f^{\mu\nu}/f_0,\\ g_1=-{}^1/_2\varepsilon_{\mu\nu\lambda\tau}\mathscr{F}^{\mu\nu}f^{\lambda\tau}/f_0, \quad g_2=-{}^1/_4\varepsilon_{\mu\nu\lambda\tau}f^{\mu\nu}f^{\lambda\tau}/f_0,\end{gathered} \tag{2.7}$$

where $f_0=\frac{1}{2}\mathscr{F}_{\mu\nu}\mathscr{F}^{\mu\nu}$, we represent (2.2) in the form

$$\mathscr{L}=-\frac{f_0e^2}{8\pi^2}\int_0^\infty\frac{dy}{y^2}e^{-y/\beta}\sum_{i=0}^\infty\Phi_i. \tag{2.8}$$

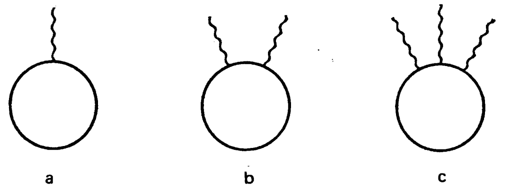
The function

$$\Phi_0=\operatorname{cth}y-\frac{y^2}{3}-\frac{1}{y} \tag{2.9}$$

corresponds here to the initial Heisenberg-Euler Lagrangian for the field $\mathscr{F}_{\mu\nu}$; the function

$$\Phi_1=\frac{f_1}{2}\left(\operatorname{cth}y-\frac{y}{\operatorname{sh}^2y}-\frac{2}{3}y\right) \tag{2.10}$$

describes the processes of creation and absorption of a photon by the vacuum (Fig. 2a); the succeeding terms of the expansion correspond to two-photon (Fig. 2b), three-photon (Fig. 2c) processes etc. We note that when condition (2.1) is satisfied the contribution of the multiphoton diagrams is small, since it contains the factor $(\omega/m)^N$, where N is the number of photons. We present the terms of second and third order of $f_{\mu\nu}$:



a b c

FIG. 2. Expansion of the Heisenberg-Euler-Schwinger Lagrangian. The thick line corresponds to the Green's function of an electron in the field $\mathscr{F}_{\mu\nu}$.

$$\begin{aligned}\Phi_2=&\frac{f_2}{2}\left(\operatorname{cth}y-\frac{y}{\operatorname{sh}^2y}-\frac{2}{3}y\right)\\ &-\frac{f_1^2}{8}\left(\operatorname{cth}y+\frac{y}{\operatorname{sh}^2y}-2y^2\frac{\operatorname{cth}y}{\operatorname{sh}^2y}\right)\\ &+\frac{g_1^2}{8}\left(\operatorname{cth}y-\frac{y}{\operatorname{sh}^2y}-\frac{2}{3}y^2\operatorname{cth}y\right),\end{aligned} \tag{2.11}$$

$$\begin{aligned}\Phi_3=&\frac{f_1^3}{16}\left[\operatorname{cth}y+\left(1-\frac{4}{3}y^2\right)\frac{y}{\operatorname{sh}^2y}-2\frac{y^3}{\operatorname{sh}^4y}\right]\\ &-\frac{3}{16}f_1g_1^2\left[\left(1-\frac{2}{9}y^2\right)\operatorname{cth}y\right.\\ &\left.-\frac{1}{3}\left(1+\frac{2}{3}y^2\right)\frac{y}{\operatorname{sh}^2y}-\frac{2}{3}y^2\frac{\operatorname{cth}y}{\operatorname{sh}^2y}\right]\\ &-\frac{f_1f_2}{4}\left(\operatorname{cth}y+\frac{y}{\operatorname{sh}^2y}-2y^2\frac{\operatorname{cth}y}{\operatorname{sh}^2y}\right)\\ &+\frac{g_1g_2}{4}\left(\operatorname{cth}y-\frac{y}{\operatorname{sh}^2y}-\frac{2}{3}y^2\operatorname{cth}y\right).\end{aligned} \tag{2.12}$$

We note that expansions (2.11) and (2.12) contain no combinations of the type $f_1g_1, f_1^2g_1, g_1, g_2$, etc. by virtue of the P-invariance of the electrodynamics. The terms proportional to f_1^3 and $f_1g_1^2$ were previously calculated in Ref. 4. In the limiting case of a weak field, $\beta\ll1$, it suffices to retain in (2.11) and (2.12) the principal terms of the power-series expansions

$$\Phi_0=-\frac{y^3}{45}\left(1-\frac{2}{21}y^2\right), \tag{2.13}$$

$$\Phi_1=-\frac{2}{45}y^3\left(1-\frac{y^2}{7}\right)f_1, \tag{2.14}$$

$$\begin{aligned}\Phi_2=-\frac{y^3}{45}\left[\left(1-\frac{2}{7}y^2\right)f_1^2\right.\\ \left.+\frac{7}{4}\left(1-\frac{13}{147}y^2\right)g_1^2+2\left(1-\frac{y^2}{7}\right)f_2\right].\end{aligned} \tag{2.15}$$

$$\begin{aligned}\Phi_3=-\frac{2}{45}y^3\left[\left(1-\frac{2}{7}y^2\right)f_1f_2\right.\\ \left.+\frac{7}{4}\left(1-\frac{13}{147}y^2\right)g_1g_2-\frac{y^2}{21}\left(f_1^3+\frac{13}{8}f_1g_1^2\right)\right].\end{aligned} \tag{2.16}$$

In the opposite strong-field limit, $\beta\gg1$, the main contribution to the integral (2.8) is made by large y and we can confine ourselves to the asymptotic representation for Φ_i:

$$\begin{gathered}\Phi_0=-\frac{y}{3}, \quad \Phi_1=-\frac{y}{3}f_1,\\ \Phi_2=-\frac{1}{8}\left(f_1^2+\frac{8}{3}yf_2+\frac{2}{3}y^2g_1^2\right),\\ \Phi_3=\frac{1}{16}\left[(f_1^3-4f_1f_2)+\frac{2}{3}y^2(f_1g_1^2-4g_1g_2)\right].\end{gathered} \tag{2.17}$$

3. DEFLECTION OF LIGHT RAY IN A NONUNIFORM MAGNETIC FIELD

It is known that allowance for the second-order terms (2.11) in Maxwell's equations for $f_{\mu\nu}$ leads to the appearance of an effective refractive index n_λ of the vacuum ($\lambda = 1, 2$ is the polarization index), which depends on the external-field intensity $\mathscr{F}$. In the case of a uniform magnetic field the two stable modes $\lambda = 1, 2$ have definite CP parity (the corresponding functions $n_\lambda(\beta)$ are given in Refs. 4 and 7). Within the limits of the restriction (2.1), these expressions can be used for a nonuniform field (but the polarization is no longer necessarily conserved along the ray). Since the refractive index depends on the coordinates, the ray trajectory is no longer a straight line. Let us estimate the deflection of the ray in the magnetic field of the dipole that simulates the magnetic field of a neutron star:

$$\mathbf{B}=\begin{cases} 2\boldsymbol{\mu}/R^3, & r<R, \\ (3\mathbf{n}(\boldsymbol{\mu}\mathbf{n})-\boldsymbol{\mu})/r^3, & r>R, \end{cases} \tag{3.1}$$

where R is the radius of the star, $\boldsymbol{\mu}$ is the magnetic moment, and $\mathbf{n} = \mathbf{r}/r$.

For rays propagating in a plane perpendicular to the magnetic moments, the polarization states with magnetic vectors perpendicular ($\lambda = 1$) and parallel ($\lambda = 2$) to the vector $\boldsymbol{\mu}$ are stable. The corresponding refractive indices are, subject to the condition (2.1),

$$n_\lambda=1+\frac{e^2}{90\pi}\left(\frac{\mu}{F_{cr}}\right)^2\frac{Q_\lambda(\beta)}{r^6}, \tag{3.2}$$

$$Q_1(\beta)=7\varphi_1(\beta)-3\varphi_2(\beta), \quad Q_2(\beta)=7\varphi_2(\beta)$$

(the functions φ_1 and φ_2 are given in Ref. 7). By virtue of the symmetry, trajectories of rays initially located in the equatorial plane do not leave this plane. To find the ray trajectory we write down the eikonal equation

$$(\mathrm{grad}\,\psi_\lambda)^2-n_\lambda^2(\partial\psi_\lambda/\partial t)^2=0. \tag{3.3}$$

Since n_λ depends only on the absolute value of the radius vector, we obtain the solution of (3.3), in polar coordinates, in the form

$$\psi=-\omega t+l\varphi+\psi_r, \quad \psi_r=\int dr\left(n^2\omega^2-\frac{l^2}{r^2}\right)^{1/2}, \tag{3.4}$$

where ω and l are constants. The ray trajectory is defined by the equation $\partial\psi/\partial l = \text{const}$, whence

$$\varphi=\int\frac{dr}{r^2}\left(\frac{n^2}{\rho^2}-\frac{1}{r^2}\right)^{1/2}, \tag{3.5}$$

where $\rho = l/\omega$ is the impact parameter.

Using (3.5) we easily obtain the deviations from linearity of the ray trajectory. The asymptotic expansions (2.15) and (2.17) of the function Φ_2 in the case of a weak field, $\mu/(\rho^3F_{cr})\ll 1$, we obtain the deflection angle

$$\Delta\varphi_\lambda=q_\lambda(B_s/F_{cr})^2(R/\rho)^6, \tag{3.6}$$

where $q_1 = \frac{7}{48}e^2$, $q_2 = e^2/12$; $B_s = \mu/R^3$ is the magnetic field intensity on the surface of the neutron star. In the limiting case of a superstrong magnetic field, $\mu/(\rho^3F_{cr})\gg 1$, we have

$$\Delta\varphi_2\gg\Delta\varphi_1, \quad \Delta\varphi_2\approx\frac{e^2}{3\pi}\frac{B_s}{F_{cr}}\left(\frac{R}{\rho}\right)^3. \tag{3.7}$$

Assuming by way of estimate $B_s \sim F_{cr}$ and $\rho \sim R$, we obtain $\Delta\varphi_2 \sim 2''$. The described "attraction" of the ray to the neutron star is a small correction to the gravitational deflection of the ray, but can be discerned in principle because of its stronger dependence on the impact parameter.

4. FREQUENCY DOUBLING IN SCATTERING OF AN ELECTROMAGNETIC WAVE BY A ROTATING DIPOLE

If the length of the electromagnetic wave is comparable with the parameter R that characterizes the inhomogeneity of the magnetic field, photon coalescence $\gamma_1 + \gamma_1 \to \gamma_2$ becomes possible in lowest-order perturbation theory (this process is kinematically forbidden in the lowest order of perturbation theory in the case of a uniform magnetic field[4]). To calculate the probability of this process we substitute for $f_{\mu\nu}$ in (2.12) the sum of the field $\varphi_{\mu\nu}$ of the initial field and the second-quantized field described by a vector potential A_μ, and replace $\mathscr{F}$ by a nonuniform nonstationary magnetic field $\mathbf{B}$. Retaining terms linear in A_μ, we reduce the action term proportional to Φ_3 to the form

$$\int\mathscr{L}_3d^4x=-\int J^\mu_{eff}(x)A_\mu(x)\,d^4x. \tag{4.1}$$

The process considered can then be described as "emission" of electromagnetic field by the effective vacuum polarization current $J_{\text{eff}}{}^\mu$. Expressing the initial-wave field tensor in the form

$$\varphi_{\mu\nu}=(a_\mu k_\nu-a_\nu k_\mu)\sin\alpha, \quad \alpha=kx=\omega t-\mathbf{kr}, \tag{4.2}$$

we obtain the effective current in the case of a weak magnetic field $\beta\ll 1$:

$$J^\nu_{eff}(x)=\frac{1}{45}\left(\frac{e}{\pi F_{cr}}\right)^2\frac{\partial}{\partial x^\mu} \times\left[\mathbf{kaB}(a^\mu k^\nu-a^\nu k^\mu)-\frac{7}{4}\,\omega\mathbf{aB}\varepsilon^{\mu\nu\lambda\tau}a_\lambda k_\tau\right]\sin^2\alpha. \tag{4.3}$$

The effective cross section of the process is determined by the Fourier transform of the current (4.3). Substituting in (4.3) the expression for the magnetic field of a rotating dipole (3.1), where we must put

$$\boldsymbol{\mu}(t)=\boldsymbol{\Omega}^0\mu_\parallel+\frac{\mu_\perp}{\sqrt{2}}(\mathbf{e}_+e^{i\Omega t}+\mathbf{e}_-e^{-i\Omega t}), \quad \boldsymbol{\Omega}^0=\frac{\boldsymbol{\Omega}}{\Omega}, \tag{4.4}$$

($\boldsymbol{\Omega}$ is the vector of the angular velocity of the dipole; $\mathbf{e}_\pm\times\boldsymbol{\Omega} = 0$, $|\mathbf{e}_\pm| = 1$, $\mathbf{e}^2_\pm = 0$; and $\mu_\parallel$ and $\mu_\perp$ are constants), we obtain after simple calculations[2)]

$$J^\nu_{eff}(q)=J_\parallel{}^\nu(q)\delta(q_0-2\omega) +J_+{}^\nu(q)\delta(q_0-2\omega+\Omega)+J_-{}^\nu(q)\delta(q_0-2\omega-\Omega), \tag{4.5}$$

where

$$J_\parallel{}^\nu(q)=\frac{8}{135}\left(\frac{e}{F_{cr}}\right)^2 \times\mu_\parallel\left[\mathbf{ka}\boldsymbol{\Omega}^0(a^\mu k^\nu-a^\nu k^\mu)q_\mu-\frac{7}{4}\,\omega\mathbf{a}\boldsymbol{\Omega}^0\varepsilon^{\mu\nu\lambda\tau}q_\mu a_\lambda k_\tau\right],$$

$$J_\pm{}^\nu(q)=\frac{4}{135}\left(\frac{e}{F_{cr}}\right)^2\mu_\perp \times\left[\mathbf{ka}\mathbf{e}_\pm(a^\mu k^\nu-a^\nu k^\mu)q_\mu-\frac{7}{4}\,\omega\mathbf{a}\mathbf{e}_\pm\varepsilon^{\mu\nu\lambda\tau}q_\mu a_\lambda k_\tau\right].$$

The first term in (4.5) corresponds to scattering with doubling of the frequency. This process takes place in the particular case of an axisymmetric rotator ($\mu_\perp = 0$). The two other terms describe Raman scattering that occurs only in the nonstationary field of an inclined rotator. The total cross section for the process is the sum of the cross sections for frequency doubling and for frequency doubling with modulation:

$$d\sigma = d\sigma_\parallel + d\sigma_+ + d\sigma_-. \tag{4.6}$$

Integrating over the directions of the scattered wave, we obtain the total cross section for doubling

$$\sigma_\parallel = 5.1\cdot 10^{-6} R^6 \omega^4 \left(\frac{B_\parallel}{F_{cr}}\right)^2 \left(\frac{E}{F_{cr}}\right)^2 \left(1 + \frac{33}{16}\cos^2\alpha\right)\sin^2\vartheta, \tag{4.7}$$

where $B_\parallel = \mu_\parallel / R^3$, $E = a\omega$; ϑ is the angle between the vectors $\mathbf{k}$ and $\mathbf{\Omega}$; α is the angle of inclination of the polarization of the initial vector to the plane plassing through $\mathbf{k}$ and $\mathbf{\Omega}$. At $B_\parallel \sim F_{cr}$ we have

$$\sigma_\parallel \sim 2.4\cdot 10^{-2} \frac{R^6}{\lambda^4}\left(\frac{E}{F_{cr}}\right)^2, \tag{4.8}$$

where λ is the length of the incident wave. Putting in (4.7) $B_\parallel$, $E \sim 0.1\ F_{cr}$; $\omega \sim 10^4$ sec^{-1}; $R \sim 10^6$ cm, we obtain the estimate $\sigma_\parallel \sim 0.2\times 10^{-2}$ cm^2.

The partial cross sections for frequency doubling with modulation are found to be

$$\sigma_\pm = 1.3\cdot 10^{-6} R^6 \left(\omega \mp \frac{\Omega}{2}\right)^4 \left(\frac{B_\perp}{F_{cr}}\right)^2 \left(\frac{E}{F_{cr}}\right)^2 \left[\sin^2\vartheta + \frac{49}{16} \times (1 - \cos^2\alpha \sin^2\vartheta)\right], \tag{4.9}$$

where $B_\perp = \mu_\perp / R^3$.

5. PHOTON PRODUCTION BY AN INCLINED ROTATOR

Photon production by the rotating magnetic field of an inclined rotator is described by diagrams of the type shown in Fig. 2, where the wavy lines must be taken to mean the lines of outgoing photons. The process represented by diagram 2a can be regarded as the radiation generated by the effective current defined by the function (2.10). The intensity of the rotator "vacuum" radiation is, at $B_s \ll F_{cr}$,

$$P_{vac} = 1.5\cdot 10^{-8} \mu_\perp^2 \Omega^4 (B_s/F_{cr})^4, \tag{5.1}$$

and at $B_s \gg F_{cr}$

$$P_{vac} \sim 10^{-6} \mu_\perp^2 \Omega^4 \ln^2 (B_s/F_{cr}). \tag{5.2}$$

Comparing these expressions with the power $P_{cl} = (2/3)\mu_\perp^2 \Omega^4$ of the classical radiation of an inclined rotator we see that the ratio P_{vac}/P_{cl} is small in both limiting cases. The absolute value of P_{vac}, however, can be quite large and at $B_s \sim F_{cr}$, $\Omega \sim 10^4$ sec^{-1}, and $R \sim 10^6$ cm we obtain $P_{vac} \sim 10^{35}$ erg/sec. Nonetheless, this process is only a small correction to the classical radiation, from which it is practically indistinguishable.

In contrast to one-photon production, the production of photon pairs (diagram in Fig. 2b) by a nonstationary field yields a continuous spectrum of photons with frequency in the interval $0 < \omega < 2\Omega$. Using for Φ the expansion (2.15), we obtain the amplitude of the process in the form

$$S = 2\pi [S_\parallel \delta(\Omega - \omega_1 - \omega_2) + S_\perp \delta(2\Omega - \omega_1 - \omega_2)], \tag{5.3}$$

where $S_\parallel$ is proportional to $\mu_\parallel$ and is different from zero in the axisymmetric case, whereas $S_\perp$ vanishes in this case. We note the two terms in (5.3), which correspond to different kinematic conditions, do not interfere with each other. Averaging over the polarizations of the photons and integrating over their emission angles we obtain the following spectral distribution of the probability of the process in the frequency of one of the photons of the pair:

$$\frac{dw}{d\omega_1} = 1.2\cdot 10^{-8} R^6 \omega_1^3 \left(\frac{B_\perp}{F_{cr}}\right)^4 \left[(2\Omega - \omega_1)^3 \theta(2\Omega - \omega_1) + 4\left(\frac{B_\parallel}{B_\perp}\right)^2 (\Omega - \omega_1)^3 \theta(\Omega - \omega_1)\right], \tag{5.4}$$

where $\theta(x)$ is the Heaviside function. (The spectral curve for two-photon production at $B_\parallel / B_\perp = 3$ is shown in Fig. 3).

The total production probability is obtained by integrating (5.4) with respect to frequency, and is equal to

$$w = 1.1\cdot 10^{-8} R^6 \Omega^7 \left(\frac{B_\perp}{F_{cr}}\right)^4 \left[1 + \frac{1}{32}\left(\frac{B_\parallel}{B_\perp}\right)^2\right]. \tag{5.5}$$

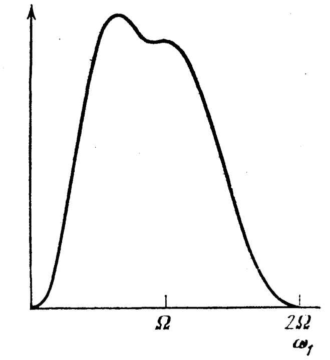


FIG. 3. Spectrum of two-photon production by an inclined rotator.

[1)]We use a system of units with $\hbar = c = 1$, $e^2 = 1/137$, and a scalar product $a_\mu b^\mu = a_0 b_0 - \mathbf{a}\times\mathbf{b}$.

[2)]We assume that $2\omega > \Omega$. At $2\omega < \Omega$, the harmonic $\Omega - 2\omega$ is generated with the same probability as the harmonic $2\omega - \Omega$ is generated in the considered case $2\omega > \Omega$.

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