Randomization of motion of a beam of phased oscillators

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We study the motion of a single oscillator in the field of a given plane electromagnetic wave and of a beam of phased oscillators in a self-consistent electromagnetic field. We show that the motion of a single particle becomes chaotic when the field amplitude satisfies the condition for resonance overlap. The chaotic motion of a single oscillator is the cause for the chaotic behavior of a beam of oscillators in a self-consistent electromagnetic field. We obtain the spectra and correlation functions of the electromagnetic fields excited by the beam. We study the evolution of the distribution function for the beam particle energy. We show that the resulting average level of the oscillations excited by the beam can be estimated by using the resonance overlap condition.

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In different fields of physics a lot of attention has been paid recently to a study of the chaotic behavior of dynamical systems.[1-3] The stochastic motion of such systems is determined by their complex internal dynamics rather than by fluctuating forces. The simplest example of such a system is the non-linear oscillator acted upon by a regular periodic force. Chirikov and Zaslavskiĭ[1] have shown that for sufficiently large values of the acting force the nonlinear resonances start to overlap and a stochastic instability in the motion of the oscillator may occur. It is of value in principle to study the stochastization of the motion using such elementary models, for it makes possible to predict the occurrence of stochasticity in appreciably more complicated systems.

An important example of a nonlinear oscillator is a charged particle moving at an angle to an external magnetic field. It was shown in Refs. 4 and 6 that the motion of such an oscillator in the field of a slow wave ($V_f \ll c$, V_f the phase velocity, c the light velocity in vacuo) is stochastic in nature under conditions for resonance overlap.

The present paper is devoted to a study of the stochastic motion both of a single oscillator in the field of a given plane electromagnetic wave and of a beam of oscillators in a self-consistent electromagnetic field. It is well known[7] that a monoenergetic beam of such oscillators is unstable with respect to the excitation of electromagnetic radiation. If there is no retarding medium the elementary mechanism for such an instability is the normal Doppler effect. In the single-mode regime, at small amplitudes of the wave to be excited (low density beam of oscillators), the exponential growth of the amplitude in the initial stage gives way to amplitude oscillations caused by the phase oscillations of the beam bunches in the field of the wave.[8] Such a picture of the instability is observed under conditions of beam trapping in an isolated cyclotron resonance when the motion of the beam particles is regular. We show below that when the wave amplitude is sufficiently large (dense beam of oscillators) a stochastic regime of the motion of the dynamical system considered is realized. The elementary mechanism for the occurrence of stochasticity is then the overlap of the cyclotron resonances of charged particles.

1. STATEMENT OF THE PROBLEM. BASIC EQUATIONS

We consider the problem of the excitation of electromagnetic oscillations by phased oscillators with a distribution function

$$f_0=\frac{n_p}{P_\perp}\delta(P_\perp-P_{\perp 0})\delta(P_\parallel)\delta\left(\theta-\theta_0+\frac{eH_0}{mc\gamma_0}t\right), \quad (1)$$

where $P_\perp$, $P_\parallel$ are the momentum components perpendicular and parallel to the z-axis, θ the angle between $\mathbf{P}_\perp$ and the x-axis (the z-axis is parallel and the x-axis perpendicular to the external uniform magnetic field $\mathbf{H}_0$), $P_x = P_\perp \cos\theta$, $P_y = P_\perp \sin\theta$, n_p the oscillator density, $\gamma_0 = (1 + P_\perp^2/m^2c^2)^{1/2}$.

The self-consistent set of equations describing the exciting of electromagnetic radiation by the oscillators (1) contains the equation of motion for the particles and the Maxwell equations for the electromagnetic field:

$$\frac{d\mathbf{P}}{dt}=e\mathbf{E}+\frac{e}{mc\gamma}\mathbf{P}\times[\mathbf{H}+\mathbf{H}_0], \qquad \frac{d\mathbf{r}}{dt}=\frac{\mathbf{P}}{mc\gamma}, \quad (2)$$

$$\frac{\partial\mathbf{H}}{\partial t}=-c\,\mathrm{rot}\,\mathbf{E}, \qquad \frac{\partial\mathbf{E}}{\partial t}=c\,\mathrm{rot}\,\mathbf{H}-4\pi\mathbf{j},$$

$$\mathrm{div}\,\mathbf{H}=0, \qquad \mathrm{div}\,\mathbf{E}=4\pi\rho.$$

Here $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field strengths, $\mathbf{P}$ the particle momentum, γ the relativistic factor, $\mathbf{j}$ and ρ the current and charge densities, e and m respectively the charge and rest mass of a particle.

We shall study the evolution of a spatially periodic perturbation of the electromagnetic field with non-vanishing components E_x, E_y, H_z and a wave vector $\mathbf{k} = (k_x,0,0)$ along the x-axis. The expressions for the electric and magnetic field strengths can then be written in the form

$$\mathbf{E}(x,t)=\{\mathrm{Re}\,E_x(t)e^{ikx},\ \mathrm{Re}\,E_y(t)e^{ikx},\ 0\} \quad (3)$$

$$\mathbf{H}(x,t)=\{0,\ 0,\ \mathrm{Re}\,H_z(t)e^{ikx}\}, \quad k\equiv k_x. \quad (4)$$

One should note that the harmonic time-dependence of the fields is not singled out in Eqs. (3) and (4). This approach

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enables us to describe correctly the dynamics of the fields and of the particle motion in the stochastic regime when the excited fields have a broad frequency spectrum.

Using the normal procedure of averaging over a spatial period of the field perturbation and also using Liouville's theorem about the conservation of the phase volume of the particle we easily obtain the following self-consistent set of nonlinear equations for the field and the beam particles:

$$\frac{dp_x}{d\tau}=(\operatorname{Re} he^{i\Psi}+\omega_H)\frac{p_y}{\gamma}+\operatorname{Re}\varepsilon_x e^{i\Psi},$$
$$\frac{dp_y}{d\tau}=-(\operatorname{Re} he^{i\Psi}+\omega_H)\frac{p_x}{\gamma}+\operatorname{Re}\varepsilon_y e^{i\Psi},\qquad \frac{d\Psi}{d\tau}=\frac{p_x}{\gamma},\tag{5a}$$

$$\frac{dh}{d\tau}=-i\varepsilon_y,\qquad \frac{d\varepsilon_y}{d\tau}=-ih-\omega_p^2\frac{1}{\pi}\int_{-\pi}^{\pi}\frac{p_y}{\gamma}e^{-i\Psi}\,d\Psi_0,\tag{5b}$$

$$\frac{d\varepsilon_x}{d\tau}=-\omega_p^2\frac{1}{\pi}\int_{-\pi}^{\pi}\frac{p_x}{\gamma}e^{-i\Psi}\,d\Psi_0.\tag{5c}$$

In Eqs. (5), which are written in dimensionless variables

$$\tau=kct,\quad \Psi=kx,\quad \mathbf{p}=\frac{\mathbf{P}}{mc},\quad \gamma=(1+p_x^2+p_y^2)^{1/2},$$
$$h=\frac{eH_z}{mc}\frac{1}{kc},\qquad \varepsilon_{x,y}=\frac{eE_{x,y}}{mc}\frac{1}{kc},$$

we have used the notation:

$$\omega_H=\frac{eH_0}{mc}\frac{1}{kc};\qquad \omega_p=\frac{1}{kc}\left[\frac{4\pi e^2 n_p}{m}\right]^{1/2}.$$

The integration on the right-hand sides of Eqs. (5b) and (5c) is over the initial values of the particle coordinates.

The set of Eqs. (5) has the integral of motion

$$p_y+\omega_H\Psi+\operatorname{Im} he^{i\Psi}=\text{const}.\tag{6}$$

Equations (5b) are equivalent to the inhomogeneous equation of an oscillator and describe the excitation of a transverse electromagnetic field by particles. In its turn Eq. (5c) takes into account the excitation in the system of a longitudinal electric field which is, in fact, the collective Coulomb field of the charged particles.

2. MOTION OF AN OSCILLATOR IN THE FIELD OF A GIVEN ELECTROMAGNETIC WAVE. RESONANCE OVERLAP CRITERION

We noted in the introduction that when the condition for overlap of nonlinear resonances is satisfied there occurs a stochastic instability in the motion of the oscillator. To obtain a criterion for the resonance overlap we consider the motion of a particle in the field of a transverse monochromatic electromagnetic wave and in a constant external magnetic field. One easily obtains the equations describing the motion of a particle in these fields from the set of Eqs. (5), putting in them

$$\omega_p=0,\ \varepsilon_x=0,\ h=\varepsilon_y=\varepsilon_0 e^{-i\tau},$$

where $\varepsilon_0=$ const is the given wave amplitude. We then get

$$\frac{dp_x}{d\tau}=[\varepsilon_0\cos(\Psi-\tau)+\omega_H]\frac{p_y}{\gamma},$$
$$\frac{dp_y}{d\tau}=-[\varepsilon_0\cos(\Psi-\tau)+\omega_H]\frac{p_x}{\gamma}+\varepsilon_0\cos(\Psi-\tau),\tag{7}$$
$$\frac{d\Psi}{d\tau}=\frac{p_x}{\gamma}.$$

For the analysis of the set (7) it is convenient to use the relations

$$p_x=p_\perp\cos\theta,\ p_y=p_\perp\sin\theta,\ \varphi=\Psi+(p_\perp/\omega_H)\sin\theta$$

to change to the variables $p_\perp$, θ, φ. Using the integral of motion (6) we can write the set of Eqs. (7) in the new variables:

$$\frac{dp_\perp}{d\tau}=\varepsilon_0\sum_{s=-\infty}^{\infty}J_s'(\mu)\sin(s\theta+\tau-\varphi)$$
$$\frac{d\theta}{d\tau}=-\frac{\omega_H}{\gamma}+\varepsilon_0\sum_{s=-\infty}^{\infty}J_s(\mu)\left(\frac{s\omega_H}{p_\perp^2}-\frac{1}{\gamma}\right)\cos(s\theta+\tau-\varphi),\tag{8}$$
$$\varphi+\frac{\varepsilon}{\omega}\sin(\varphi-\mu\sin\theta-\tau)=\text{const}.$$

Here $\mu=p_\perp/\omega_H$, $J_s(\mu)$ is a Bessel function of order s, and the summation is taken over $s=0,\ \pm1,\ \pm2,\dots$.

One sees easily from the set of Eqs. (8) that the particle interacts most effectively with a wave under the resonance conditions:

$$s\omega_H=\gamma.\tag{9}$$

We consider the case of small wave amplitudes when the change in particle energy $\Delta\gamma$ as a result of the interaction with the wave is much less than the distance between neighboring resonances, i.e., $\Delta\gamma\ll\omega_H$. Retaining on the right-hand sides of Eqs. (8) only the resonance term and introducing the resonance phase $\theta_s=s\theta+\tau$ we get a set of equations which describes the motion of an oscillator under conditions of an isolated resonance:

$$\frac{dp_\perp}{d\tau}=\varepsilon_0 J_s'(\mu)\sin\theta_s,\qquad \frac{d\theta_s}{d\tau}=-\frac{s\omega_H}{\gamma}+1.\tag{10}$$

We note that in the second equation of the set (10) we neglected terms of order ε_0.

When the deviations of the particle momentum from the equilibrium value $p_{\perp s}=[1+s^2\omega_H^2]^{1/2}$ are small we can transform Eqs. (10) into the equations of a mathematical pendulum:

$$\frac{d\delta p_\perp}{d\tau}=\varepsilon_0 J_s'(\mu_s)\sin\theta_s,\qquad \frac{d\theta_s}{d\tau}=\frac{p_{\perp s}}{\gamma^2}\delta p_\perp,\tag{10a}$$

where $\delta p_\perp=p_\perp-p_{\perp s}$, $\mu_s=p_{\perp s}/\omega_H$, $\gamma_s=s\omega_H$. The motion of a particle is thus regular when the conditions for an isolated resonance are satisfied. Clearly, the isolated resonance approximation is justified if the maximum width of the nonlinear cyclotron resonance,

$$(\Delta p)_{res}=4[(\gamma_s^2/p_{\perp s})J_s'(\mu_s)\varepsilon_0]^{1/2},\tag{11}$$

which one easily finds from the solution of the set (10a), is appreciably smaller than the distance between neighboring resonances:

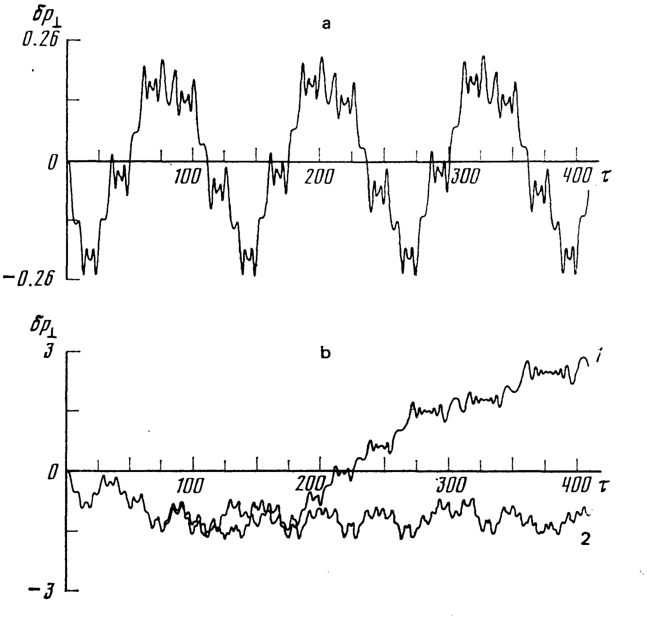


FIG. 1. The transverse momentum $\delta p_\perp$ as function of the time τ for different values of the wave amplitude: a) $\varepsilon_0 = 0.05$; b) $\varepsilon_0 = 0.2$.

$$\omega_H \gg 4[p_{\perp s} J_s'(\mu_s)\varepsilon_0]^{1/2}. \tag{12}$$

If the condition

$$\omega_H \ll 4[p_{\perp s} J_s'(\mu_s)\varepsilon_0]^{1/2} \tag{13}$$

is satisfied the cyclotron resonances overlap. Under those conditions a stochastic instability may occur in the motion of the particle.[1]

To check the criterion for the stochastization of the motion of the oscillator we integrated the set of Eqs. (7) numerically. Numerical calculations were performed for $\omega_H = 0.5$ and an initial energy $\gamma_0 = 2$. The resonance condition is in that case satisfied for $s = 4$. We give the results of the calculations in Fig. 1. In these figures we give the time-dependence of the transverse momentum $\delta p_\perp = p_\perp - p_{\perp s}$ for different values of the dimensionless wave amplitude and for different initial conditions.

For small values of the dimensionless amplitude $\varepsilon_0 = 0.05$ (Fig. 1a) the transverse momentum $\delta p_\perp$ changes periodically with time within the limits from -0.26 to 0.26. As the distance in momentum between neighboring resonances is about 0.5 the phase oscillations of the particle correspond to motion in an isolated resonance. This is in complete accordance with the condition (11) for the applicability of the isolated resonance approximation. As one does not average in the numerical integration over the fast motion there are noticeable small-amplitude fast motions, caused by the effect of the other resonances, against the background of the slow motions in the region of the isolated resonance.

The picture of the particle motion changes qualitatively for sufficiently large wave amplitudes. We show in Fig. 1b the time dependence of the transverse momentum $\delta p_\perp$ for $\varepsilon_0 = 0.2$ when the condition (13) for resonance overlap is satisfied. It is clear from this figure that the particle is not trapped in an isolated resonance but performs a complicated irregular motion. An important feature of the particle motion is that an insignificant change in the initial conditions leads to an appreciable change in the picture of the particle motion. The difference in the initial values of the energy for the curves 1 and 2 of Fig. 1b is only 0.01. Notwithstanding such a small difference the evolution of the particle momentum differs greatly. These results indicate the presence of local instability of the motion in the system.

3. RESULTS OF A NUMERICAL ANALYSIS OF THE COLLECTIVE EXCITATION OF OSCILLATIONS BY A SYSTEM OF OSCILLATORS

In the preceding section we showed that for sufficiently large wave amplitudes, when resonances overlap, the motion of the oscillator becomes stochastic. One may expect that if as the result of the development of a collective instability the amplitude of the excited field in the system of oscillators reaches a value for which resonance overlap starts, the motion of the particles becomes stochastically unstable and the system of oscillators goes over into the regime of generation of stochastic oscillations.

The set of Eqs. (5) describing the excitation of electromagnetic oscillations by an ensemble of phased oscillators was solved numerically for different values of the plasma frequency $\omega_p = 0.1$ and 0.3 and for fixed values of the cyclotron frequency $\omega_H = 0.5$ and of the initial energy $\gamma_0 = 2$. The resonance condition is in that case satisfied for $s = 4$. The initial values for the amplitudes were given as follows: $\mathrm{Re}\,\varepsilon_y = \mathrm{Re}\,h = 5\times10^{-3}$. The numerical calculations were performed for 100 particles. The accuracy of the calculation was monitored against the integral of the motion.

We show in Figs. 2 to 6 the results of a numerical analysis of the set of Eqs. (5). Figures 2 to 5 show the time-dependence of the longitudinal and transverse components E_x and E_y of the electric field at $x = 0$ and also the spectral densities of the power and the correlation functions corresponding to them.

We use for the spectral and correlation analysis of the electric fields the formulae

$$G(\omega) = \frac{1}{\pi}\frac{1}{T}|a_T(\omega)|^2, \quad a_T(\omega) = \int_0^T u(t)\exp(-i\omega t)\,dt,$$
$$R(\tau) = \int_0^\infty G(\omega)e^{i\omega\tau}\,d\omega, \tag{14}$$

where $a_T(\omega)$ is the Fourier transform of the process studied, $u(t)$. Using (14) we found the spectra of the electric fields E_x, E_y in the frequency range $0 < \omega \leqslant 5\pi$ with a resolution $\Delta\omega = 2.5\times10^{-3}\pi$. The spectra and correlation functions shown in Figs. 2 to 5 are normalized to their maximum values.

We give in Fig. 6 the evolution of the particle distribution function for different oscillator densities plotted for 1000 particles per wavelength.

For a low density of charged particles ($\omega_p = 0.1$) the transverse component of the electric field is preferentially excited. In the initial stage of the instability (Fig. 2a) there occurs an exponential growth of the amplitude of the trans-

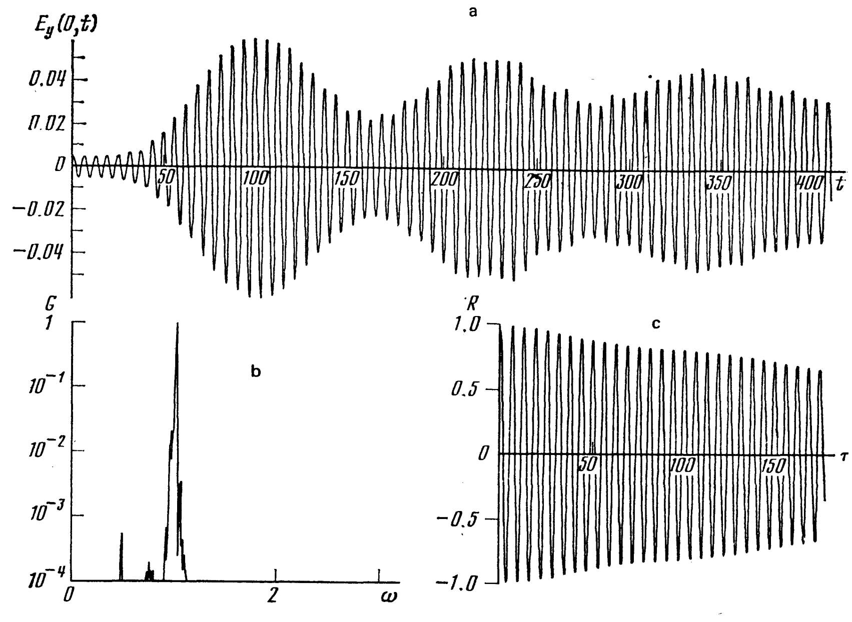


FIG. 2. The amplitude (a), spectral density (b), and correlation function (c) of the transverse field as functions of the time τ at the point $x = 0$ for $\omega_p = 0.1$.

verse electric field. Later its growth is replaced by slow oscillations which are caused by the phase oscillations of the particle bunches trapped by the wave. The spectrum of the transverse component of the electric field has a narrow peak at the fundamental frequency of the oscillations ($\omega = 1$) and two satellites positioned on one side of the peak. The occurrence of the satellites is caused by the modulation of the wave at the frequency of the phase oscillations of the particle bunches in the field of the wave. The correlation function of the transverse electric field is an oscillating function at the fundamental frequency with a slowly decreasing amplitude.

The time-dependence of the longitudinal field (Fig. 3a) has a more complex shape caused by the coincidence of gyrofrequency harmonics. However, here also we observe an exponential growth of the amplitude in the initial stage of the instability, later replaced by amplitude oscillations having the frequency of the phase oscillations of the particles in the field of the transverse wave. The spectrum of the longitudinal electric field (Fig. 3b) has several narrow peaks at the harmonics of the cyclotron frequency of a relativistic particle ($\omega = 0.25$ for $s = 1$; $\omega = 0.5$ for $s = 2$; $\omega = 0.75$ for $s = 3$; $\omega = 1$ for $s = 4$) and the maximum intensity of the longitudinal field occurs at the second harmonic ($s = 2$) whereas the maximum of the transverse field occurs at the fourth harmonic. The correlation function of the longitudinal field is a periodic function of frequency $\omega = 0.5$ (as the spectrum has a maximum at this frequency) with a slowly decreasing amplitude.

A low density oscillator beam thus excites regular oscillations with a spectrum which is discrete in nature. One

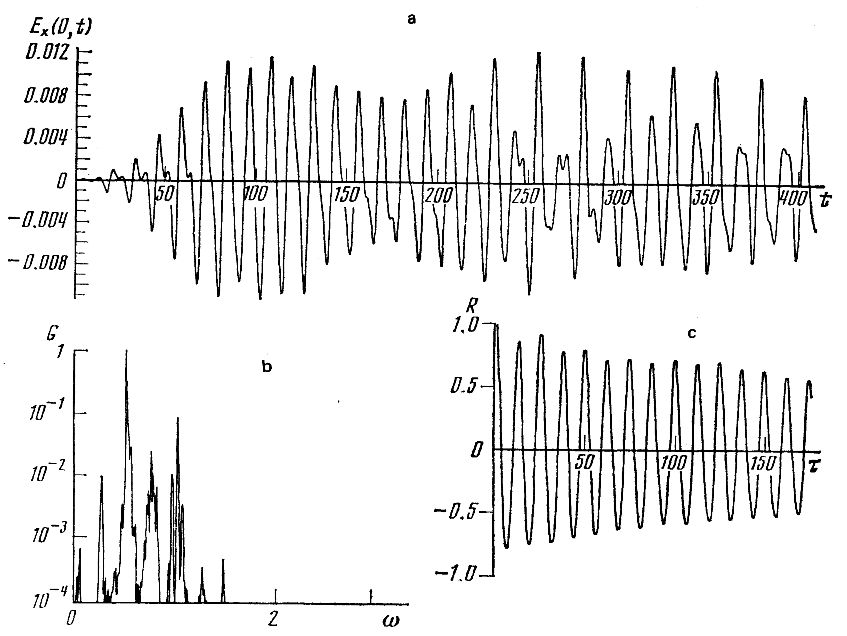


FIG. 3. The amplitude (a), spectral density (b), and correlation function (c) of the longitudinal field as functions of the time τ at the point $x = 0$ for $\omega_p = 0.1$.

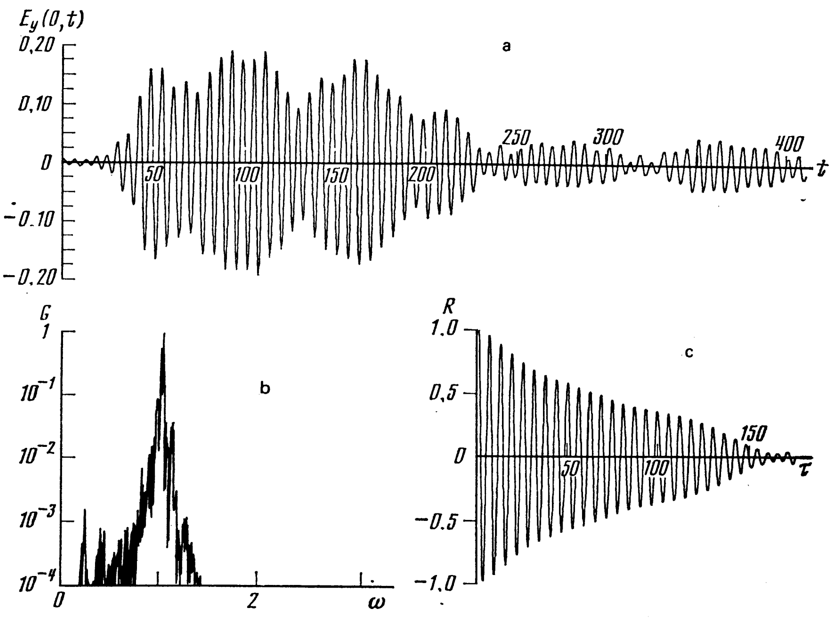


FIG. 4. The amplitude (a), spectral density (b), and correlation function (c) of the transverse field as functions of the time τ at the point $x = 0$ for $\omega_p = 0.3$.

checks easily that the maximum value of the amplitude of the transverse field does not satisfy in this case the condition (13) for resonance overlap, so that the particles are in an isolated resonance with the wave and their motion is regular in nature. A convincing confirmation of this is also the shape of the energy distribution function of the particles at different times. It is clear from Fig. 6 that the excitation of the oscillations is accompanied by a small broadening of the distribution function. As the value of the particle energy lies between the limits from $\gamma = 1.75$ to 2.2 and the energy distance between neighboring resonances is 0.5, all particles are in an isolated ($s = 4$) cyclotron resonance with the wave.

The efficiency coefficient, defined by the relation

$$\eta=(|\mathbf{E}|^2/4\pi)/n_p mc^2(\gamma_0-1),$$

reaches under the conditions for an isolated resonance 37%.

When the density of the charged particles is increased to values at which amplitude of the field excited by the oscillators satisfies the condition for resonance overlap the dynamics of the instability is completely different. Initially, as in the case of an isolated resonance, there is an exponential growth of the transverse field which is limited by the trapping of beam particles by the field of the excited wave (Fig. 4a, $\tau \leqslant 50$). The level of this field is approximately twice the level necessary for resonance overlap so that the motion of the oscillators becomes chaotic. This kind of motion of the particles leads to a chaotic modulation of the amplitude of the transverse field (Fig. 4a, $50 \leqslant \tau \leqslant 200$) and to the occurrence of a chaotic longitudinal field (Fig. 5a). The difference in the behavior of the longitudinal and the transverse fields can be explained as follows. The time-dependence of the longitudinal Coulomb field is, in accordance with Eq. (5c), com-

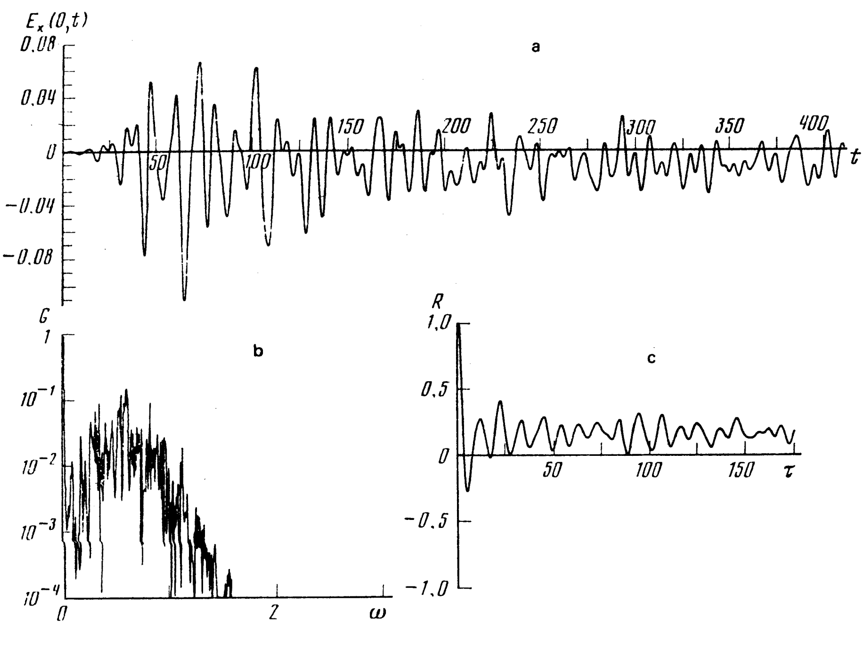
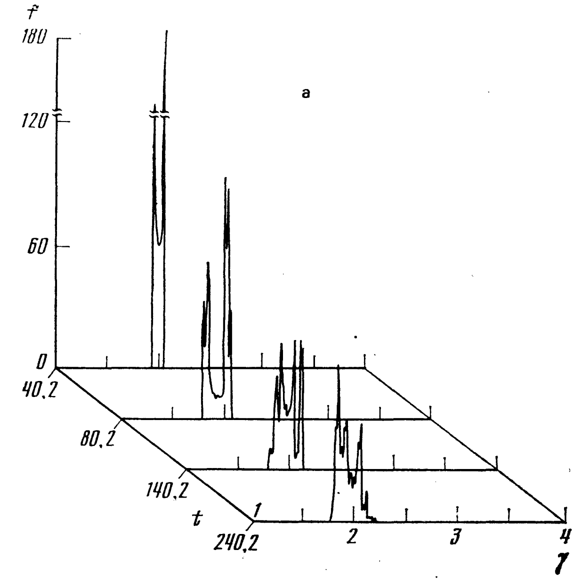


FIG. 5. The amplitude (a), spectral density (b), and correlation function of the longitudinal field as functions of the time τ at the point $x = 0$ for $\omega_p = 0.3$.



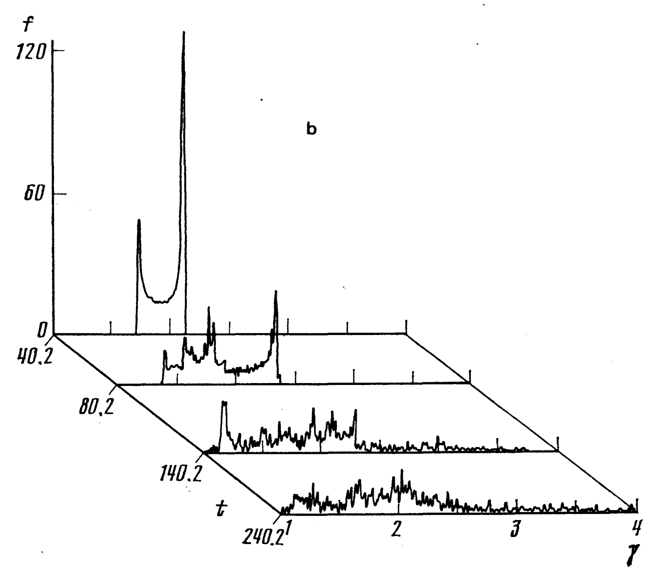


FIG. 6. The evolution of the distribution function of the particle energy γ for (a) $\omega_p = 0.1$ and (b) $\omega_p = 0.3$.

pletely determined by the particle motion so that the chaotization of the charged particle motion causes the chaotization of the self-consistent Coulomb field of those particles. The transverse electromagnetic field is described by the inhomogeneous oscillator Eq. (5b) so that the chaotic beam current which occurs on the right-hand side of this equation can produce only an irregular modulation of the complex amplitude of the transverse field.

In accordance with this picture of the instability we have the shape of the spectrum of the excited oscillations and the shape of the distribution function (Fig. 6b). The spectrum of the transverse field (Fig. 4b) is appreciably broadened even though it has a maximum at the basic frequency $\omega = 1$. The correlation function of this field, in contrast to the case of a low density beam, decreases with time rapidly while oscillating at the main frequency. The spectrum of the longitudinal field (Fig. 5b) is continuous and appreciably broader than the spectrum of the transverse field. The shape of the distribution function (Fig. 6b) shows that up to time $\tau \approx 40$ the instability develops similarly to the case of an isolated resonance. However, already at $\tau \approx 80$ the particle distribution function includes the region of several resonances; apart from the slowed down particles a group of stochastically accelerated particles is clearly visible. When the time increases further the distribution function is much more smeared out and the number of accelerated particles increases even though altogether there are more decelerated than accelerated particles.

The chaotic motion of the oscillators and the smearing out of the distribution function (appearance of accelerated particles) leads to the fact that the level of the self-consistent field that has caused that motion decreases and, starting at time $\tau \approx 225$, the average amplitude of that field corresponds to the value necessary for resonance overlap. This means that the level of field saturation is in final reckoning determined not by the condition for particle capture, but by the condition for resonance overlap.

The motion of a relativistic oscillator in the field of a plane electromagnetic wave with an amplitude which satisfies the condition for overlap of cyclotron resonances is therefore chaotic in nature. Such a motion of a single oscillator is the cause of the chaotic behavior of a more complex system—an oscillator beam moving in self-consistent electromagnetic fields. The chaotization of the motion of an oscillator beam starts at a sufficiently large density of charged particles when the amplitude of the excited electromagnetic field satisfies the resonance overlap criterion. We can then estimate the average resulting level of the excited oscillations from the resonance overlap criterion.

It is necessary to note that in the system considered above, owing to the nonlinearity of the beam, higher spatial harmonics will be excited apart from the fundamental harmonic (governed by the initial conditions). The presence of those harmonics can in the general case lead to a considerable change in the dynamics of the motion of the beam particles. However, a numerical analysis of the model, taking into account the excitation of the spatial harmonics, shows that for sufficiently low beam densities ($\omega_p^2 \ll 1$), particularly for the values considered above, the amplitudes of the higher harmonics are small and they do not change the picture described here of the process of the chaotization of the particles and of the field.

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