Nonstationary stimulated Mandel'shtam-Brillouin scattering at high scattered-light intensities

V. D. Kagan and Yu. V. Pogorel'skiĭ

A. F. Ioffe Physico-technical Institute, Academy of Sciences USSR
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Nonstationary stimulated Mandel'shtam-Brillouin scattering is considered with account of exhaustion of the incident light. It is found that under conditions of strong exhaustion of the incident wave, the process continues to develop in time. Thus the length of the region of effective interaction of light and sound decreases, whereas the sound intensity increases with time.

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In this paper, we consider the development in time of nonstationary stimulated Mandel'shtam-Brillouin scattering (SMBS) with account of the large transfer of energy from the incident light wave to the scattered wave. The case of backscattering will be analyzed, in which the scattered wave of light emerges from the sample in the direction counter to that of the incident wave. It turns out that in the case of strong exhaustion of the incident wave, when the intensity of the scattered wave is close to the intensity of the incident wave, the SMBS process in the sample develops in time—the length of the region of effective interaction decreases, whereas the sound intensity increases with time.

The stimulated scattering is nonstationary in the case in which the laser pulse length T is shorter than the lifetime of the acoustic phonons:[1]

$$\alpha wT \ll 1, \tag{1}$$

where α and w are the damping coefficient and the speed of sound:

$$\Gamma L \ll 1, \tag{2}$$

Γ is the damping coefficient of light, and L is the length of the sample. Conditions (1) and (2) mean that the damping of the light and sound does not depend strongly on the SMBS process. We consider the case

$$wT \ll L \ll cT, \tag{3}$$

where c is the speed of light. This condition means that during the time T the sound wave hardly moves over the sample, and only becomes stronger with time. Light, on the contrary, passes through the sample instantaneously. For Q-switched lasers we have $T \sim 10^{-8}$ sec. The length is then $wT \sim 10^{-2}$ cm and $cT \sim 3$ m.

From Maxwell's equations and from the elasticity-theory equations we can obtain for the amplitudes of the interacting waves the following equations that describe the case (2)–(3):

$$\partial E_0/\partial x = -auE_1, \tag{4}$$

$$\partial E_1/\partial x = -auE_0, \quad a = \varepsilon^{3/2} p\omega q/4c, \tag{5}$$

$$\frac{1}{w}\frac{\partial u}{\partial t} = bE_0E_1, \quad b = \varepsilon^2 p/32\pi\rho w^2. \tag{6}$$

Here $E_0(x,t)$ and $E_1(x,t)$ are the amplitudes of the electric field in the incident and scattered light waves, $u(x,t)$ is the amplitude of the shift in the sound wave. The incident light and sound propagate along the x axis, ε is the permittivity, p is the photoelastic constant, ρ is the density, and q is the sound wave vector. In the coefficients of Eqs. (4) and (5), we do not distinguish between the frequencies of the incident and scattered light, denoting them both as ω (the difference in these frequencies is small in comparison with ω in the ratio w/c.

In comparison with the complete equations for the amplitude, we neglect the damping of the light and sound. Moreover, we discard the derivatives of the amplitude of the field with respect to time and the amplitudes of the shift in the coordinate, using the condition (3).

Let the point $x = 0$ coincide with the input end of the sample. The initial and boundary conditions of the problem are[1)]

$$\begin{aligned} &u(x,0) = u_0, \quad u(L,t) = u_0,\\ &E_1(L,t) = 0, \quad E_0(0,t) = \mathscr{E}. \end{aligned} \tag{7}$$

The scattered light propagates against the direction of the incident; therefore, it is absent on the rear face of the sample. Correspondingly, the sound at the rear face is not amplified.

It is natural for the process of nonstationary SMBS to evolve in time in two stages. Initially, so long as the intensity of the sound and the scattered light is small, we can assume the intensity of the incident light to be given and to be constant along the sample, and we may discard Eq. (4). The solution of the linear theory is:

$$\begin{aligned} u(x,t) &= u_0 I_0(2[dS_0(L-x)t]^{1/2})\\ &\sim u_0[16\pi^2 dS_0 t(L-x)]^{-1/4}\exp 2[dS_0(L-x)t]^{1/2}, \end{aligned} \tag{8}$$

$$\begin{aligned} E_1(x,t) &= au_0\mathscr{E}\left[\frac{L-x}{dS_0 t}\right]^{1/2} I_1(2[dS_0(L-x)t]^{1/2})\\ &\sim au_0\mathscr{E}\frac{(L-x)}{2\pi^{1/2}}[dS_0(L-x)t]^{-3/4}\exp 2[dS_0(L-x)t]^{1/2}, \end{aligned} \tag{9}$$

S_0 is the intensity of the incident light, $d = abw4\pi\varepsilon^{1/2}/c$, I_0 and I_1 are Bessel functions of imaginary argument. The asymptote of the Bessel function is valid if the argument of the exponential is much greater than unity.

According to (8) and (9), the quantity $(dS_0LT)^{1/2}$ determines the increase in the amplitude of the sound and scattered light in the SMBS process. Since the unrenormalized quantity u_0, which can be estimated from the intensity of the

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thermal fluctuations,[1] is very small, the SMBS can be observed experimentally only under the condition

$$dS_0LT \gg 1. \tag{10}$$

If the light intensity is sufficiently great, there is an instant of time $t_0 < T$ at which the scattered light intensity becomes of the order of the incident intensity. This time t_0 can be estimated in order of magnitude by the relation

$$E_1(0,\ t_0) \sim \mathscr{E}, \tag{11}$$

where $E_1(x,t)$ is determined by Eq. (9). (If scans are made, in the experiments on the observation of SMBS, of the pulses of incident and scattered light as functions of time, the time t_0 can be determined directly from these scans.) Beginning with this instant of time, we should take into account the exhaustion of the incident light wave, i.e., we should take Eq. (4) into account. It is convenient then to transform to the function

$$\varphi(x,t) = a\int_0 u(x',t)\,dx', \tag{12}$$

for which we can obtain from (4)–(7) the closed equation

$$\frac{\partial^2\varphi}{\partial x\,\partial t} = dS_0\frac{e^{-2\varphi}(1-e^{-4\varphi_L+4\varphi})}{(1+e^{-2\varphi_L})^2}, \tag{13}$$

where $\varphi_L = \varphi(L,t)$. The functions E_0 and E_1 can be expressed in terms of φ:

$$E_0 = \mathscr{E}\,\mathrm{ch}(\varphi_L-\varphi)/\mathrm{ch}\,\varphi_L, \tag{14}$$

$$E_1 = \mathscr{E}\,\mathrm{sh}(\varphi_L-\varphi)/\mathrm{ch}\,\varphi_L. \tag{15}$$

In the case of weak exhaustion ($\varphi_L \ll 1$) we must linearize the expression in the parentheses in the numerator of (13), while the remaining exponentials in (13) are set equal to unity.

In the case of strong exhaustion ($\varphi_L > 1$), Eq. (13) is simplified:

$$\partial^2\varphi/\partial x\partial t = dS_0e^{-2\varphi}. \tag{16}$$

Equation (16) is not valid in the region of large x; this region can be specified by the condition

$$4\varphi(L,\ t) - 4\varphi(x,\ t) \lesssim 1. \tag{17}$$

The general solution of Eq. (16) is known.[2] Under our boundary and initial conditions, its solution has the form

$$\varphi(x,\ t) = \ln(1+dS_0xt). \tag{18}$$

Thus, at strong exhaustion of the incident light,

$$E_1(x,t) = E_0(x,t) = \frac{\mathscr{E}}{1+dS_0xt}, \tag{19}$$

$$u(x,t) = \frac{dS_0}{a}\frac{t}{1+dS_0xt}. \tag{20}$$

The effective interaction length of the light and sound is

$$x_0 \sim 1/dS_0t. \tag{21}$$

It is seen from the condition $\varphi_L > 1$ and from the solution (18) that $x_0 < L$. It is also seen from (18) that the condition of strong exhaustion $\varphi_L > 1$ is identical with conditions (10), which is necessary for observation of SMBS in any regime—linear or nonlinear. We therefore emphasize that the transition to the regime of strong exhaustion is possible only at $t_0 < T$, where t_0 is defined by the relation (11) and depends on the initial conditions. The solution (18), which comes into play at $t > t_0$, does not depend on the initial conditions, which is reasonable for solution of the nonlinear equation.

The condition (17), which determines the region x in which (16) and (18) are inapplicable, has the form

$$(L-x)/L \lesssim 1/4. \tag{22}$$

Inasmuch as the interaction of sound and light takes place chiefly in the region $x \lesssim x_0 < L$, the region (22) makes a small contribution to the SMBS. Thus, at strong exhaustion of the incident light wave, when the intensity of the scattered light is close to the intensity of the incident, the process continues to develop in time; the effective interaction length x_0 decreases with passage of time, while the amplitude of the shift in the sound wave increases in the region $x < x_0$.

We now discuss in greater detail the conditions under which we can neglect the damping of the light and sound. At $t < t_0$, so long as the exhaustion of the incident light wave is small, these conditions

$$\alpha wT \ll (dS_0LT)^{1/2}, \qquad \Gamma L \ll (dS_0LT)^{1/2} \tag{23}$$

are much more stringent than (1) and (2). (The conditions (23) are easily obtained from a solution of (8), and (9) by equating αu with $w^{-1}\partial u/\partial t$ and E_1 with $\partial E_1/\partial x$).

In the case of strong exhaustion in the region $x < x_0$, the condition under which the damping of the sound has little effect on the SMBS is (1). For neglect of the light damping, it is necessary that

$$\Gamma L < dS_0LT, \tag{24}$$

which is weaker than the condition (2). A more stringent condition is connected with the sound. In the region $x > x_0$ the solution (20) is almost independent of the time; this means that the inclusion of even weak sound absorption leads to a change—a decrease in the amplitude of the sound. Thus, even under the condition (1), the sound will be attenuated in the region $x > x_0$, and the region of applicability of the solution (19) and (20) is $x \lesssim x_0$.

Nonstationary SMBS has been investigated in Ref. 3. In the experiment, scans were made of pulses of incident and scattered light. It is seen from the scans that the forward part of the pulse of incident light passes through the sample without change, while the trailing portion is strongly and almost completely clipped, transforming into the pulse of scattered light. The instant of this clipping can be associated naturally with the instant of time t_0. The quanitity $dSLT$ in this experiment is of the order of 100. This means that $x_0/L \sim 10^{-2}$. We note that in the experiment of Ref. 3, the SMBS was observed in optical fibers of great length $L \sim cT$. Our formulas were obtained at $L \ll cT$ and are not directly applicable to the given experiment. However, the qualitative relation—smallness of x_0 in comparison with L and the decrease in x_0 with increase in time—should probably hold in this case also.

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[1)]In addition to (7) there is also the condition

$$u(0,\ t)=u_0.$$

The complete equation for the amplitude of the shift,

$$\frac{\partial u}{\partial x}+\frac{1}{w}\frac{\partial u}{\partial t}=bE_0E_1$$

describes a short increase in u from u_0 to the values $u(x,t)$ at $x \gg wT$, which are obtained upon solution of (4)–(7). The region $x \ll wT$, where it is necessary to use the complete equation for the shift, is small and makes a small contribution to the interaction of light and sound. Therefore, we shall not consider this region. We emphasize that the solution in the region $x \gg wT$ is determined by the increase in the intensity of the scattered light in the progagation of it from the rear boundary of the sample to the forward boundary and by the increase in the amplitude of the shift with time. The boundary condition for u at $x=0$ has no effect on the solution in the region $x \gg wT$.

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