Thermodynamic properties of disordered conductors

B. L. Al'tshuler, A. G. Aronov, and A. Yu. Zyuzin

A. F. Ioffe Physicotechnical Institute, USSR Academy of Sciences
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We investigate the effect of interaction between electrons in disordered conductors of arbitrary dimensionality on the temperature and field dependences of the magnetic susceptibility and of the specific heat. We show that in the presence of repulsion between the electrons the temperature and field dependences of the magnetic susceptibility of the conduction electrons imitate the Curie-Weiss law. The specific heat contains, besides the term linear in temperature, contributions that are not analytic in the temperature and depend on the effective dimensionality of the sample. We study the effect of spin scattering of the conduction electrons on the thermodynamics of disordered conduction. The entire analysis is valid in the temperature $T\tau/\hbar < 1$ and in classically weak magnetic fields $\omega_c\tau < 1$, where τ is the momentum relaxation time and ω_c is the cyclotron frequency.

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1. INTRODUCTION

It was shown in Refs. 1 and 2 that interaction between electrons in a disordered metallic system leads to anomalies in the state density, and this leads in turn to anomalous temperature dependences of thermodynamic quantities such as the specific heat and the magnetic susceptibility. This pertains to systems of any dimensionality, three-dimensional[1] as well as two-dimensional.[2,3] In Ref. 3 account was taken of the effect of interelectron interaction on the magnetic susceptibility of the conduction electrons in the two-dimensional case, and it was shown that the magnetic susceptibility has a logarithmic temperature dependence. In the localized state, the Hubbard repulsion increases the magnetic susceptibility with decreasing temperature. This phenomenon manifests itself also in the delocalized state. In Ref. 4 were investigated both the temperature and the field dependences of the paramagnetic susceptibility in the two- and three-dimensional cases and it was shown that the susceptibility increases with decreasing temperature.

The present paper is devoted to the study of the influence of the interaction between the electrons in disordered conductors of arbitrary dimensionality on the temperature and field dependences of the magnetic susceptibility and on the specific heat. It is assumed throughout that $p_F l \gg \hbar$, where p_F is the Fermi momentum and l is the mean free path for scattering by impurities or defects. The entire analysis is valid at temperatures $T\tau \ll \hbar$ and in classically weak magnetic fields $\omega_c\tau \ll 1$, where these corrections describe the field and temperature dependences of the observed quantities ($\omega_c = eH/m^*c$ is the cyclotron frequency, m^* is the effective mass, and τ is the momentum relaxation time.

In Sec. 2 we derive the corrections to the thermodynamic potential for the interaction of the electrons in disordered conductors in a magnetic field. Account of the interaction is taken in both the diffusion and the Cooper channels. The main contribution to the orbital susceptibility comes from the electron interaction with the opposite spins in the Cooper channels.

The influence of superconducting fluctuations on the temperature dependence of the orbital magnetic susceptibility was investigated in Refs. 5 and 6 both near and far from the superconducting-transition point, in pure as well as in dirty metals. It was observed[6] that substantial corrections are needed not only in substances with superconducting transition, but also in nonsuperconducting materials.

The orbital magnetic susceptibility can be represented in the form

$$\delta\chi = \frac{1}{3}\beta_D^2 \int_{-\infty}^{\infty} \Delta\nu(T,\varepsilon)\frac{\partial n_0}{\partial\varepsilon}d\varepsilon, \tag{1}$$

where $\beta_D = 2eD/c$ is the effective magneton for a particle with charge $2e$ and mass $\hbar/2D$, while $\Delta\nu(T,\varepsilon)$ is the correction to the state density, necessitated by the interaction in the Cooper channel[7]; D is the electron diffusion coefficient. The effective magneton β_D is much larger than $\beta^* = \hbar e/2m^*c$ relative to the parameter $p_F l/\hbar \gg 1$, and therefore, despite the smallness of the correction to the state density, $\delta\chi$ turns out to be large enough:

$$\frac{\delta\chi(T)}{\chi_0} = -2\left(\frac{\pi}{6}\right)^{1/2}\zeta\left(\frac{1}{2}\right)\left(\frac{T\tau}{\hbar}\right)^{1/2}\left(\ln\frac{T_c}{T}\right)^{-1}, \quad d=3;$$
$$\frac{\delta\chi(T)}{\chi_0} = \frac{4}{3}\frac{l}{a}\ln\left[\frac{\ln(T_c/T)}{\ln(T_c\tau/\hbar)}\right], \quad d=2; \tag{2}$$

where a is the film thickness, $\chi_0 = -(\frac{2}{3})\beta^{*2}\nu_0$ is the diamagnetic susceptibility of the electrons, d is the effective dimensionality of the sample,[2] ν_0 is the single-spin state density, $\zeta(x)$ is the Riemann zeta function,

$$T_c = \frac{2\gamma}{\pi}\omega_D \exp\frac{1}{\lambda_c},$$

ω_D is the Debye frequency of the phonons, $\ln\gamma = c = 0.577$, and λ_c is the effective electron-electron interaction constant[7] determined both by the virtual-phonon exchange, λ_{ph}, and by the Coulomb repulsion, λ_{coul} (Ref. 7)

$$\lambda_c = \lambda_{ph} + \frac{\lambda_{coul}}{1+\lambda_{coul}\ln(\varepsilon_F/\omega_D)} \tag{3}$$

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(ε_F is the Fermi energy). For free electrons, when the reciprocal Debye radius $\varkappa$ is small compared with the Fermi momentum ($\varkappa \ll p_F$) we have

$$\lambda_{coul}=\frac{\varkappa^2}{8p_F^2}\ln\left(1+\frac{4p_F^2}{\varkappa^2}\right). \tag{4}$$

It can be seen from (2) that the total orbital magnetic susceptibility of the electrons decreases with increasing temperature in the case of electron repulsion ($\lambda_c > 0$), thus imitating the Curie-Weiss paramagnetism.[6] We note that at $d = 2$ we have $\delta\chi(T) \gg \chi_0$ if $l \sim a$, and at $\lambda_c > 0$ the orbital susceptibility is paramagnetic.

In a magnetic field $1/\tau \gg \Omega_H \gg T/\hbar$ we have

$$\frac{\delta\chi(H)}{\chi_0}=-\frac{1}{\ln(T_c/\hbar\Omega_H)}\sqrt{\frac{3}{2}}\frac{(1-2^{3/2})\cdot 15\zeta(5/2)}{(2\pi)^2}(\tau\Omega_H)^{1/2},\ d=3;$$
$$\frac{\delta\chi(H)}{\chi_0}=\frac{4}{3}\frac{l}{a}\ln\left[\frac{\ln(T_c/\hbar\Omega_H)}{\ln(T_c\tau/\hbar)}\right],\quad d=2; \tag{5}$$

where

$$\Omega_H=\frac{4D_eH}{\hbar c}=\frac{2\beta_D H}{\hbar}.$$

In the derivation of (5) it was assumed that

$$\hbar\Omega_H \gg g\mu_B H=\hbar\omega_s$$

(g is the Landé factor of the conduction electron and $\mu_B = \hbar e/2m_0c$ is the Bohr magneton).

We note that according to (5) and in a magnetic field the correction to the orbital susceptibility imitates at $\lambda_c > 0$ the free-spin susceptibility.

The relatively large orbital susceptibility is due to the fact that the probability that particles located at given point will interact depends strongly on the magnetic field. This sensitivity compensates to a considerable degree the allotted-phase-space smallness due to the small fraction of the trajectories on which the particles acquire the equal phase differences and which lead to an effective increase of the contribution of the interaction energy to the thermodynamic potential.

A distinguishing feature of the orbital susceptibility is its strong anisotropy: for a field parallel to the film plane the magnetic susceptibility is much smaller than for a perpendicular field:

$$\frac{\delta\chi_\parallel}{\chi_0}=\frac{2\pi Ta}{v_F\ln(T_c/T)}\ll\frac{\delta\chi_\perp}{\chi_0}, \tag{6}$$

where v_F is the Fermi velocity.

We note that when a becomes of the order of $L_T = (D/T)^{1/2}$ expressions (5) and (6) in two-dimensional space become of the same order and agree in order of magnitude with the magnetic susceptibility in the three-dimensional case [expression (2)].

The orbital magnetic susceptibility of thin wires depends on the orientation of the magnetic field relative to the wire axis and on the shape of the cross section:

$$\frac{\delta\chi_1}{\chi_0}=-\alpha\cdot 24\left(\frac{2}{3}\right)^{1/2}\pi^{3/2}\zeta\left(\frac{1}{2}\right)\frac{(T\tau/\hbar)^{1/2}}{\ln(T_c/T)}, \tag{7}$$

where $\alpha = 1/8\pi$ for a round wire at H parallel to the wire axis, $\alpha = a/12B$ for a rectangular wire with $H \parallel b$, a and b are the transverse dimensions of the wire.

When b becomes of the order of L_T expression (7) coincides in order of magnitude with (6), and at $a \sim L_T$ the temperature dependence of $\delta\chi_1$ drops out and Eq. (7) agrees with (5) apart from the logarithm.

Besides the orbital susceptibility, there are quantum corrections to the spin susceptibility.[4] In a weak magnetic field we have at different dimensionalities

$$\delta\chi_s^{(d)}=\delta\chi_s^{(d)C}+\delta\chi_s^{(d)D}$$
$$=\frac{3\xi(3/2)(g\mu_B)^2}{16\sqrt{2}\pi^{3/2}(DT\hbar)^{1/2}}\left(\frac{2}{\ln(T_c/T)}-\lambda^{(j=1)}\right),\quad d=1;$$
$$\delta\chi_s^{(d)}=\delta\chi_s^{(d)C}+\delta\chi_s^{(d)D}=-\frac{(g\mu_B)^2}{8\pi^2D\hbar}$$
$$\times\left(2\ln\left[\frac{\ln(T_c\tau/\hbar)}{\ln(T_c/T)}\right]-\lambda^{(j=1)}\ln\frac{T\tau}{\hbar}\right),\quad d=2;$$
$$\delta\chi_s^{(d)}=\delta\chi_s^{(d)C}+\delta\chi_s^{(d)D}$$
$$=\frac{\zeta(1/2)(g\mu_B)^2T^{1/2}}{16\sqrt{2}\pi^{1/2}(D\hbar)^{3/2}}\left(\frac{2}{\ln(T_c/T)}-\lambda^{(j=1)}\right),\quad d=3. \tag{8}$$

Here $\delta\chi^{(2)}$ and $\delta\chi^{(1)}$ are the susceptibilites per unit area and per unit length of the film, respectively. Contributions to (8) are made both by the interaction of the electrons in the Cooper channel, $\delta\chi^{d(C)}$ (with small total momentum) and by the interaction in the diffuse channel $\chi_s^{d(C)}$ (with small difference between the momenta of the interacting electron and hole). In the latter case a substantial role is played by electron-hole interaction with a summary spin j equal to 1; $\lambda^{(j=1)}$ is an effective dimensionless constant [$\lambda^{(j=1)} < 0$ for electron repulsion], which can be expressed in the general case in terms of the Fermi-liquid amplitude of the interaction. Expression (8) goes over into the expressions given in Ref. 4 for the susceptibility at a small interaction constant in the Cooper channel, when $\lambda^{(j=1)} = -2\lambda_c = -F$.

Unlike the orbital, the spin susceptibility for low-dimensionality samples is isotropic in weak magnetic fields.

In the case of electron repulsion the spin suceptibility also increases with decreasing temperature and decreases in the case of attraction.

In addition, the spin susceptibility of interacting conduction electrons depends significantly on the magnetic field in the region of relatively weak fields:

1) The diffusion contribution to the susceptibility begins to depend on the magnetic field when $\hbar\omega_s = g\mu_B H \gtrsim T$. In this case $\delta\chi_s^{(d)D}$ becomes independent of temperature, and the dependence on the magnetic field takes at $\hbar\omega_s \gg T$ the form

$$\delta\chi_s^{(d)D}=-(g\mu_B)^2\lambda^{(j=1)}/4\pi(2D\hbar\omega_s)^{1/2},\quad d=1;$$
$$\delta\chi_s^{(d)D}=(g\mu_B)^2\lambda^{(j=1)}\frac{1}{8\pi^2D}\ln\tau\omega_s,\quad d=2; \tag{9}$$
$$\delta\chi_s^{(d)D}=(g\mu_B)^2\lambda^{(j=1)}\omega_s^{1/2}/8\sqrt{2}\pi^2D^{3/2},\quad d=3.$$

2) The Cooper contribution to the susceptibility in the three-dimensional case, and in the two-dimensional if the magnetic field is perpendicular to the plane of the film, de-

pends strongly on H if $\hbar\Omega_H \gtrsim T$. It is impossible to separate exactly the spin and and orbital susceptibilities in this field region. If, as is usually the case, $\Omega_H/\omega_s \sim Dm_0/g\hbar \gg 1$, the orbital part of the susceptibility (5) is much larger than the contribution made to the susceptibility by the interaction in the Cooper channel and due to the spin. In this case the total correction to the magnetic susceptibility is equal to the sum of the contributions (5) and (8) or (9), depending on the ratio of $\hbar\omega_s$ and T.

3) In the one-dimensional case and at $d = 2$ if the magnetic field is parallel to the plane of the film, the dependence of $\delta\chi_s^{(d)C}$ on H is determined by the relation between ω_s and τ_H^{-1} (at $d = 1$ we have $\tau_H^{-1} = 4\alpha De^2SH^2/c^2\hbar^2$, where S is the wire cross-section area, and in the two dimensional case $\tau_H^{-1} = De^2H^2a^2/3c^2\hbar^2$, Ref. 8). If $\tau_H^{-1} \gg \omega_s T$, the Cooper contribution to the magnetic susceptibility is determined by the approximations (6) and (7). In the region $\omega_s \gg T, \tau_H^{-1}$ the most substantial is the Cooper contribution to the spin susceptibility

$$\delta\chi_s^{(d)C} = \frac{2^{1/2}(g\mu_B)^2}{4\pi^{1/2}\ln(T_c/\hbar\omega_s)(D\omega_s\hbar)^{1/2}} \quad d=1;$$
$$\delta\chi_s^{(d)C} = -\frac{(g\mu_B)^2}{4\pi^2 D\hbar}\ln\left[\frac{\ln(T_c\tau/\hbar)}{\ln(T_c/\hbar\omega_s)}\right], \quad d=2. \tag{10}$$

Besides the usual (linear in temperature) electron contribution to the specific heat, there exist also corrections due to the diffusion as well as the Cooper channels:

$$\delta c_V = \delta c_V^{(D)} + \delta c_V^{(C)}.$$

The correction connected with the interaction in the diffusion channel is of the form

$$\delta c_V^{(D)} = \left[\frac{2}{d} + \frac{3}{2}\lambda^{(j=1)}\right]\frac{45}{(8\pi)^2}\left(\frac{\pi}{2}\right)^{1/2}\zeta\left(\frac{5}{2}\right)\left(\frac{T}{\hbar D}\right)^{3/2}, d=3;$$
$$\delta c_V^{(D)} = \left[\frac{2}{d} + \frac{3}{2}\lambda^{(j=1)}\right]\frac{T}{12D}\ln\frac{T\tau}{\hbar}, \quad d=2; \tag{11}$$
$$\delta c_V^{(D)} = -\left[\frac{2}{d} + \frac{3}{2}\lambda^{(j=1)}\right]\frac{3\zeta(3/2)}{16\sqrt{2\pi}}\left(\frac{T}{\hbar D}\right)^{1/2}, \quad d=1.$$

Here $\delta c_V^{(D)}$ is the specific heat per unit film area at $d = 2$, and the specific heat per unit length of the wire at $d = 1$.

The first term in the square brackets in (11) describes the contribution from the interaction of the electron and the hole with zero total spin. We call attention to the fact that this contribution is universal—independent of all interaction constants. This is due to the Coulomb character of the electron interaction at large distances.

The correction due to the interaction in the Cooper channel takes at $d = 3$ the form

$$\delta c_V^{(C)} = -\frac{1}{\ln(T_c/T)}\frac{45\zeta(3/2)}{32\sqrt{2}\pi^{1/2}}\left(\frac{T}{\hbar D}\right)^{3/2} \approx -\frac{0{,}24}{\ln(T_c/T)}\left(\frac{T}{\hbar D}\right)^{3/2}, \tag{12}$$

and in the two-dimensional case

$$\delta c_V^{(C)} = \frac{T}{6\hbar D}\ln\left[\frac{\ln(T_c/T)}{\ln(T_c\tau/\hbar)}\right]. \tag{13}$$

For a wire

$$\delta c_V^{(C)} = \frac{3\zeta(3/2)}{8(2\pi)^{1/2}}\frac{1}{\ln(T_c/T)}\left(\frac{T}{\hbar D}\right)^{1/2}. \tag{14}$$

When a magnetic field such that $\hbar\Omega_H > T$ is applied, the Cooper part of the correction to the specific heat becomes a linear function of the temperature, but depends also substantially on the magnetic field:

$$\delta c_V^{(C)}(H) = \frac{(1-1/\sqrt{2})\zeta(1/2)T\Omega_H^{1/2}}{6\sqrt{2}\ln(T_c/\hbar\Omega_H)D^{3/2}} \quad d=3;$$
$$\delta c_V^{(C)}(H) = \frac{T}{6D\hbar}\ln\left[\frac{\ln(T_c/\hbar\Omega_H)}{\ln(T_c\tau/\hbar)}\right], \quad d=2, \quad \mathbf{H}\|\mathbf{n} \tag{15}$$

($\mathbf{n}$ is the normal to the plane of the film). If $\mathbf{H}\perp\mathbf{n}$ at $d = 2$ and in the one-dimensional case the dependence of the specific heat on the magnetic field is determined by the ratio of ω_s and τ_H^{-1}:

$$\delta c_V^{(C)}(H) = \frac{T}{6D\hbar}\ln\left[\frac{\ln[2\pi T_c\min\{\tau_H, \omega_s^{-1}\}]}{\ln(T_c\tau/\hbar)}\right], d=2, \mathbf{H}\perp\mathbf{n};$$
$$\delta c_V^{(C)}(H) = \frac{2^{1/2}\pi}{6}\frac{T}{\hbar(\omega_s D)^{1/2}}\frac{1}{\ln(T_c/\hbar\omega_s)}, \quad d=1, \quad \omega_s\tau_H \gg 1; \tag{16}$$
$$\delta c_V^{(C)}(H) = \frac{\pi}{3}\frac{T}{\hbar(D/\tau_H)^{1/2}}\frac{1}{\ln(T_c\tau_H/\hbar)}, \quad d=1, \quad \omega_s\tau_H \ll 1.$$

As for the diffusion contribution to the specific heat, it becomes strongly dependent on the magnetic field if $\hbar\omega_s \gtrsim T$. The magnetic-field-dependent part of the diffusion correction to the specific heat is due to interaction of electrons from different spin subbands. At $\hbar\omega_s \gg T$ it takes the form

$$\delta c_V^{(D)}(H) = \lambda^{(j=1)}\frac{T(\hbar\omega_s)^{1/2}}{24\sqrt{2}(\hbar D)^{3/2}}, \quad d=3;$$
$$\delta c_V^{(D)}(H) = \lambda^{(j=1)}\frac{T}{24\hbar D}\ln\tau\omega_s, \quad d=2; \tag{17}$$
$$\delta c_V^{(D)}(H) = -\lambda^{(j=1)}\frac{\pi}{12\sqrt{2}}\frac{T}{(D\omega_s)^{1/2}}, \quad d=1.$$

Measurement of the thermodynamic quantities for samples with low effective dimensionality is extremely difficult. There exist, however, artificial systems (see, e.g., Ref. 9) comprising a large number of practically independent metallic filaments, with diameter on the order of several times ten angstrom.

All the foregoing corrections to the thermodynamic quantitites change in the presence of spin scattering of the electrons:

1) The diffusion correction to the susceptibility does not depend on T and H at $\hbar/t_s \gg T, \hbar\omega_s$, where t_s is the total time of the electron spin relaxation:

$$t_s^{-1} = {}^4/_3(\tau_s^{-1} + \tau_{so}^{-1}), \tag{18}$$

τ_s is the time of relaxation on the magnetic impurities, and τ_{so} is the time of spin relaxation due to the spin-orbit interaction of the electron with an ordinary impurity. The magnetic-field-dependent part of the diffusion correction to the specific heat also vanishes in this region. All that remains is the part $c_V^{(D)}$ connected with the interaction of electrons with zero total momentum.[10]

2) All the corrections to the thermodynamic quantities on account of the electron interaction in the Cooper channel remain unchanged in the presence of a finite τ_{so}. They are,

however, independent of T and H in the presence of paramagnetic impurities if $\hbar/t_s \gg T, \hbar\Omega_H, \hbar/\tau_H, \hbar\omega_s$.

All the foregoing results can be obtained from the general expressions, derived in the remaining part of the article, for the thermodynamic potential.

2. THERMODYNAMIC POTENTIAL OF DISORDERED CONDUCTORS

Non trivial corrections to the thermodynamic potential in disordered conductors are necessitated only by the interaction between the electrons. These corrections can be of two types.

1. Due to the interaction in the diffusion channel, i.e., due to interaction in a channel with small momentum transfer q and low energy ω_m. This effect is due to the enhancement of the interaction between particles because of the diffuse character of their motion.

The equation for the diffusion is shown in Fig. 1. Besides the difference momentum q and the energy m, the diffusion depends on four spin variables: the two-particle Green's function $D_{\alpha\beta\gamma\delta}(q\omega_m)$ averaged over the random potential of the electron (with initial and final spin projections α and β) and of the hole (with initial and final spin projections δ and γ). It will be convenient to expand this Green's function over states with specified combined spin j of the electron and hole and its projection M in the form:

$$D_{\alpha\beta\gamma\delta} = \sum_j C_{\alpha,\delta}^{j,\alpha+\delta} C_{\beta,\gamma}^{j,\beta+\gamma} \delta(\alpha+\delta-\beta-\gamma) D^{(j,\alpha+\beta)}, \tag{19}$$

where $C_{M_1M_2}^{(j,M)}$ are Clebsch-Gordan coefficients. We took it into account that both the combined spin and its projection are conserved in the averaged two-particle propagator. Moreover, j and M are conserved in the two-particle Green's function even if no account is taken of the interaction between the electrons. This means in turn that the total correction to the thermodynamic potential can be represented as a sum of contributions that describe the interaction between particles with total combined spin and its projection.

The equation represented by Fig. 1 can be written in the presence of a magnetic field in the form

$$(|\omega_m| - D\nabla^2 - iM\omega_s \operatorname{sign}\omega_m) D^{(j,M)}(\mathbf{r}, \mathbf{r}', \omega_m, \varepsilon_n) = \frac{\delta(\mathbf{r}-\mathbf{r}')}{2\pi\nu_0\tau^2}\theta(-\varepsilon_n(\varepsilon_n+\omega_m)). \tag{20}$$

Here $\omega_s = g\mu_B H$ is the Zeeman splitting and

$$\theta(x) = \begin{cases} 1, & x>0 \\ 0, & x<0 \end{cases}.$$

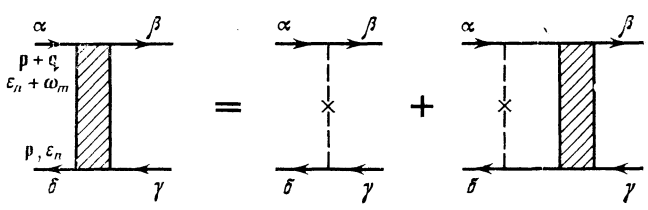

FIG. 1

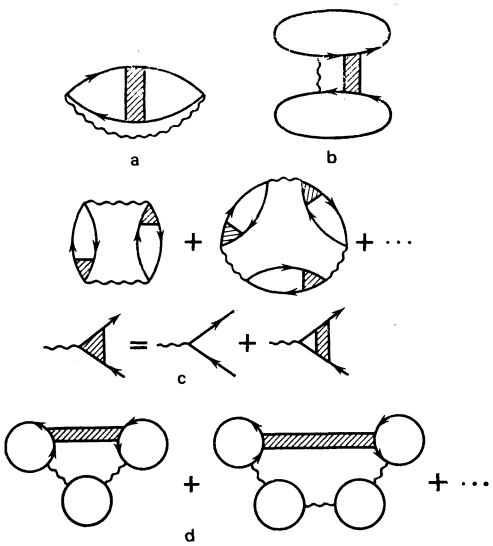

FIG. 2

The diffusion corrections to the thermodynamic potential are represented in first order in the interaction by diagrams a and b of Fig. 2. These diagrams describe the diffusion contributions to the exchange energy and to the direct-interaction energy, respectively. Owing to the long-range character of the Coulomb interaction, first-order perturbation theory is insufficient. We consider first the diffusion contribution to the exchange energy. To this end it is necessary to take into account besides diagram a also the sum of the diagrams c. As a result the contribution of the diagrams a and c to the thermodynamic potential can be written in the form

$$\frac{\delta\Omega_{ex}^{(D)}}{V_d} = \frac{T}{2} \sum_{|\tau\omega_m|<1} \int (dq) \ln\{1 + V_d^{(0)}(q)\Pi(q, \omega_m)\}. \tag{21}$$

Here $(dq) = d^d q/(2\pi)^d$, V_3 is the volume of the sample, V_2 is the area of the sample, $\Pi(q,\omega_m)$ is a polarization operator that takes at $\omega_m\tau < 1, Dq^2\tau < 1$ the form[1,2]

$$\Pi(q, \omega_m) = 2\nu_0 Dq^2/(|\omega_m| + Dq^2). \tag{22}$$

Next, $V_d(0)(q)$ is the unscreened Coulomb potential:

$$V_3^{(0)}(q) = 4\pi e^2/q^2, \quad V_2^{(0)}(q) = 2\pi e^2/|q|. \tag{23}$$

Changing in (21) from summation over the imaginary frequencies to integration over the real ones and discarding the inessential constant that is independent of temperature as well as of the magnetic field, we obtain

$$\frac{\delta\Omega_{ex}^{(D)}}{V_d} = \frac{1}{\pi}\int_0^\infty d\omega N(\omega) \int (dq) \operatorname{arctg}\frac{\omega}{Dq^2} = \frac{2}{\pi d}\int_0^\infty d\omega\omega N(\omega) \int (dq) \frac{Dq^2}{(Dq^2)^2+\omega^2}. \tag{24}$$

Here $N(\omega) = 1/(\exp(\omega/T) - 1)$.

We note that expression (24) does not contain the electron charge. Moreover, (24) describes fully the diffusion contribution to the energy, due to the interaction having zero total spin $\delta\Omega_{ex}^{(D)} = \delta\Omega_{(j=0)}^{(D)}$.

We consider now the diffusion contribution to the energy of an interaction have $j=1$. In the lowest order of perturbation theory it is determined by diagram b of Fig. 2. To take the Coulomb screening into account it is necessary to add to this diagram the sum of the series d. The combinatorial coefficient corresponding to each of these diagrams is equal to unity, so that it is sufficient to consider[1)] the contribution of diagram b, taking the wavy line to mean the screened Coulomb interaction. The same result is obtained by using the expression that connects the thermodynamic potential with the two-particle Green's function.[11] Therefore

$$\frac{\delta\Omega_{(j=1)}^{(D)}}{V_d}=\frac{\pi\nu_0\tau^2}{2}FT\sum_{|\omega_m\tau|<s}|\omega_m|\sum_{M=0,\pm1}\int(dq)D^{(1,M)}(q,\omega_m), \tag{25}$$

where

$$F=\frac{1}{2\pi^2\nu_0\tau^2}\int(dp)(dp')V_d(\mathbf{p}-\mathbf{p}')|G(\mathbf{p}',0)|^2|G(\mathbf{p},0)|^2$$

is the characteristic constant of the interaction in the diffusion channel. In the purely two-dimensional case[2]

$$F=\int_0^{2\pi}\frac{d\varphi}{2\pi}\left[1+\frac{2p_F}{\varkappa}\sin\left(\frac{\varphi}{2}\right)\right]^{-1}.$$

In the remaining cases

$$F=\frac{\varkappa^2}{4p_F^2}\ln\left(1+\frac{4p_F^2}{\varkappa^2}\right).$$

Expression (25) describes the contribution of the interaction with $j=1$ to the thermodynamic potential only at small F. In the general case it suffices to replace F by $-\lambda^{(j=1)}$, an effective constant that can be expressed in terms of the Fermi-liquid amplitude of the interaction.

Changing again from summation over the imaginary frequencies to integration with respect to the real ones and discarding the inessential constant, we obtain

$$\frac{\delta\Omega_{(j=1)}^{(D)}}{V_d}=\frac{\lambda^{(j=1)}}{2\pi}\int_0^\infty d\omega\sum_{M=0,\pm1}(\omega+M\omega_s)N(\omega+M\omega_s)\times\int(dq)\frac{Dq^2}{(Dq^2)^2+\omega^2}. \tag{26}$$

The sum of expressions (24) and (26) yields to total thermodynamic-potential correction connected with the interaction in the diffusion channel

$$\frac{\delta\Omega^{(D)}}{V_d}=\frac{1}{\pi}\int_0^\infty d\omega\left\{\frac{2}{d}\omega N(\omega)+\frac{\lambda^{(j=1)}}{2}\sum_{M=0,\pm1}(\omega+M\omega_s)\times N(\omega+M\omega_s)\right\}\int(dq)\frac{Dq^2}{(Dq^2)^2+\omega^2}. \tag{27}$$

(In the three-dimensional case the integral with respect to (dq) in (27) diverges at the upper limit. The upper limit must be taken to be l^{-1}, inasmuch as at $ql>1$ the diffusion approximation does not hold. We shall not take into account the resultant contribution to $\delta\Omega^{(D)}$, which is quadratic in T and does not depend on H, since it leads only to an insignficiant change of the constant in the usual linear specific-heat law.)

We note that the entire dependence of the diffusion correction to the thermodynamic potential from the magnetic field enters only via the Zeeman splitting ω_s. Therefore the diffusion correction contributes to the spin susceptibility but not to the orbital susceptibility of the electron gas.

We consider now the effect of the electron spin scattering on the thermodynamic potential. The spin-dependent part of the electron scattering by the impurity consists of two parts:

$$f_s(\mathbf{p},\mathbf{p}')=u_s\boldsymbol{\sigma}\mathbf{S}+iu_{so}\boldsymbol{\sigma}[\mathbf{p}\times\mathbf{p}'], \tag{28}$$

where $\mathbf{p}$ and $\mathbf{p}'$ are the momenta of the electrons before and after the scattering and $\mathbf{S}$ is the impurity spin. The spin-relaxation times are

$$1/\tau_s=2\pi\nu_0N_s|u_s|^2S(S+1),$$
$$1/\tau_{so}=2\pi\nu_0N_i|u_{so}|^2[\overline{\mathbf{p}\times\mathbf{p}'}]^2, \tag{29}$$

where N_s is the concentration of the paramagnetic impurities and N_i is the total impurity concentration.

Direct summation of the ladder diagrams lead to the following expression for the diffusion pole:

$$D^{(j,M)}(\omega_m,q)=[2\pi\nu_0\tau^2(|\omega_m|+Dq^2+j/t_s-iM\omega_s\operatorname{sign}\omega_m)]^{-1}, \tag{30}$$

where $t_s^{-1}=\frac{4}{3}(\tau_s^{-1}+\tau_{s0}^{-1})$.

The quantity $D^{(j=0)}=D_{\alpha\beta\beta\alpha}$ describes the fluctuations of the electron density and, as can be seen from (30), does not depend on the spin relaxation. On the other hand, the quantities $D^{(j=1,M)}$ describe the fluctuations of the off diagonal elements of the density matrix, and therefore are damped in the presence of spin relaxation.[10] As a result, the spin scattering of the electrons does not affect that part of the thermodynamic potential which is connected with the interaction at $j=0$. At the same time, at $\hbar/t_s>T,\hbar\omega_s$ the contribution made to $\delta\Omega^{(D)}$ by the interaction with $j=1$ becomes inessential.

The complete expression for the diffusion correction to the thermodynamic potential is of the form

$$\frac{\delta\Omega^{(D)}}{V_d}=\sum_{j,M}\frac{\delta\Omega_{(j,M)}^{(D)}}{V_d}=\frac{\delta\Omega_{j=0}^{(D)}(T)}{V_d}+\sum_{M=0,\pm1}\frac{\delta\Omega_{(j=1,M)}^{(D)}(T,t_s,M\omega_s)}{V_d}, \tag{31}$$

where

$$\frac{\delta\Omega_{(j=0)}^{(D)}}{V_d}=\frac{2}{\pi d}\int_0^\infty d\omega\omega N(\omega)\int\frac{(dq)Dq^2}{(Dq^2)^2+\omega^2}, \tag{32}$$

$$\frac{\delta\Omega_{(j=1,M}^{(D)}}{V_d}=\frac{\lambda^{(j=1)}}{2\pi}\int_0^\infty d\omega(\omega+M\omega_s)N(\omega+M\omega_s)\int\frac{(dq)(Dq^2+1/t_s)}{\omega^2+(Dq^2+1/t_s)^2}. \tag{33}$$

Expression (32) can be written in the form

$$\frac{\delta\Omega_{(j=0)}^{(D)}}{V_d}=\begin{cases}c_d\Gamma\left(\frac{d+2}{2}\right)\zeta\left(\frac{d+2}{2}\right)\frac{2}{d}T^{\frac{d+2}{2}}\frac{1}{(\hbar D)^{d/2}}, & d=1,3;\\ -\frac{1}{12d\hbar D}T^2\ln\frac{T\tau}{\hbar}, & d=2;\end{cases} \tag{34}$$

$$c_1=\sqrt{2}/4\pi,\quad c_3=-\sqrt{2}/8\pi^2.$$

In the case of strong spin scattering of the electrons,

$\hbar/t_s \gg T, \hbar\omega_s$ only $\delta\Omega^{(D)}_{(j=0)}$ leads to nontrivial corrections to the thermodynamic quantities, while $\delta\Omega^{(D)}_{(j=1)}$ can be disregarded.

In the absence of spin scattering, if $\hbar/t_s \ll \hbar\omega_s \ll T$, then

$$\frac{\delta\Omega^{(D)}(T,\omega_s)}{V_d}=\frac{\delta\Omega^{(D)}_{(j=0)}}{V_d}\left\{1+\frac{3d}{4}\lambda^{(j=1)}+\frac{d\lambda^{(j=1)}}{16}\left(\frac{\hbar\omega_s}{T}\right)^2 \times\frac{(4-d)(2-d)\Gamma(d/2-1)\zeta(d/2-1)}{\Gamma(d/2+1)\zeta(d/2+1)}\right\}. \tag{35a}$$

If, however, $\hbar\omega_s \gg T \gg \hbar/t_s$, we have

$$\frac{\delta\Omega^{(D)}}{V_d}-\left(1+\frac{d}{4}\lambda^{(j=1)}\right)\frac{\delta\Omega^{(D)}_{(j=0)}}{V_d}=\sum_{M=\pm1}\frac{\delta\Omega^{(D)}_{(j=1,M)}(T,\omega_s)}{V_d}$$

$$=\lambda^{(j=1)}\frac{c_d}{2}\frac{(\hbar\omega_s)^{(d+2)/2}}{(\hbar D)^{d/2}}\left\{\frac{4}{d(d+2)}+\frac{\pi^2}{6}\left(\frac{T}{\hbar\omega_s}\right)^2\right\},\quad d=1,3; \tag{35b}$$

$$\frac{\delta\Omega^{(D)}}{V_d}-\left(1+\frac{d}{4}\lambda^{(j=1)}\right)\frac{\delta\Omega^{(D)}_{(j=0)}}{V_d}=\sum_{M=\pm1}\frac{\delta\Omega^{(D)}_{(j=1,M)}(T,\omega_s)}{V_d}$$

$$=-\frac{\lambda^{(j=1)}}{8\pi^2\hbar D}\ln\omega_s\tau\left\{\frac{(\hbar\omega_s)^2}{2}+\frac{\pi^2}{6}T^2\right\},\quad d=2.$$

2) We consider now the interaction in the Cooper channel. In first-order perturbation theory, the corrections to the thermodynamic potential are represented by diagrams a and b of Fig. 3. We note right away that the Cooper contributions to the exchange energy (diagram a) and to the direct-interaction energy (diagram b) cancel each other for particles with identical spins. Allowance for higher orders in the Cooper channel make it necessary to take diagrams c and d into account. As a result we have 6

$$\frac{\delta\Omega_d^{(C)}}{V_d}=T\sum_{\omega_m}\frac{eH}{\pi c}\sum_{n\geq0}\int\frac{dq_z}{2\pi}\ln(1-\lambda_c\Pi_c), \tag{36}$$

where q and ω_m are respectively the combined momentum and the combined energy of the interacting aprticles, q_z is the component of q in the H direction, the constant λ_c is given by Eq. (3), and

$$\Pi_c=\ln\frac{T}{\omega_D}-\psi\left(\frac{1}{2}\right)+\mathrm{Re}\,\psi\left(\frac{|\omega_m|+Dq_z^2+\Omega_H(n+1/2)+i\omega_s+1/\tau_s}{4\pi T}+\frac{1}{2}\right)\quad d=3;$$

$$\Pi_c=\ln\frac{T}{\omega_D}-\psi\left(\frac{1}{2}\right)+\mathrm{Re}\,\psi\left(\frac{|\omega_m|+\Omega_H(n+1/2)+i\omega_s+1/\tau_s}{4\pi T}+\frac{1}{2}\right),\quad d=2. \tag{37a}$$

In the one-dimensional case and in the two-dimensional if $\mathbf{H}\perp\mathbf{n}$

$$\frac{\delta\Omega_d^{(C)}}{V_d}=T\sum_{\omega_m}\int(dq)\ln(1-\lambda_c\Pi_c), \tag{37b}$$

$$\Pi_c=\ln\frac{T}{\omega_D}+\mathrm{Re}\,\psi\left(\frac{|\omega_m|+Dq^2+i\omega_s+1/\tau_s+1/\tau_H}{4\pi T}+\frac{1}{2}\right)-\psi\left(\frac{1}{2}\right),$$

where $\psi(x)$ is the logarithmic derivative of the gamma function.

Thus, the spin-orbit scattering does not influence the interaction between the electrons in the Cooper channel, while the scattering by the paramagnetic centers, as will be shown below, suppresses the corrections to the thermodynamic potential that are necessitated by this interaction (for the same reasons for which it suppresses the superconductivity).

From (36) and (37) we obtain

$$\frac{\delta\Omega_d^{(C)}}{V_d}=\frac{T^{(d+2)/2}}{(4D\hbar)^{d/2}\ln(T_c/T^x)}\times\begin{cases} f_{-1/2}\left(\frac{\hbar\Omega_H}{2\pi T},\frac{\hbar\omega_s}{\pi T},\frac{\hbar}{\pi T\tau_s}\right), & d=3;\\ f_{1/2}\left(0,\frac{\hbar\omega_s}{\pi T},\frac{\hbar}{\pi T}\left(\frac{1}{\tau_s}+\frac{1}{\tau_H}\right)\right), & d=1.\end{cases} \tag{38}$$

Here

$$T^x=\max\left\{T,\frac{\hbar\Omega_H}{2\pi},\frac{\hbar\omega_s}{\pi},\frac{\hbar}{\pi\tau_s}+\frac{\hbar}{\pi\tau_H}\right\}, \tag{39}$$

and the function f_γ is defined by the relation

$$f_\gamma(x_1,x_2,x_3)=x_1\int_0^\infty\frac{dt\,t^\gamma}{\mathrm{sh}^2 t}\frac{\cos x_2 t}{\mathrm{sh}\,x_1 t}e^{-x_3 t}. \tag{40}$$

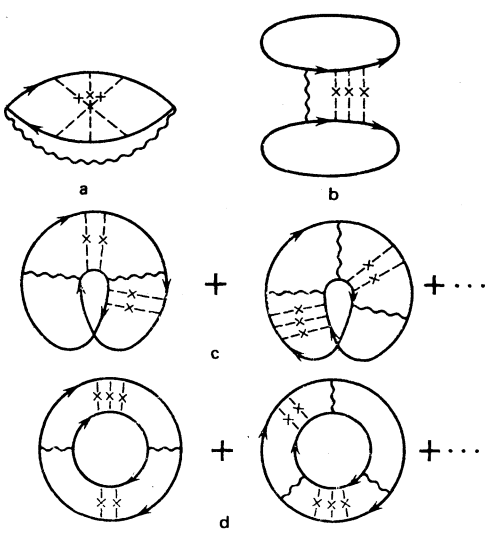

FIG. 3

TABLE I. Values of the coefficients $a_i(\gamma)$.

	$\gamma=-1/2$	$\gamma=1/2$
$i=0$	0.51	−1.04
$i=1$	0.22	−0.61
$i=2$	$4\sqrt{2\pi}/15$	$-2\sqrt{2\pi}/3$
$i=3$	$-8\sqrt{\pi}/15$	$4\sqrt{\pi}/3$

This integral converges at the lower limit if $\gamma > 2$. The values of these functions at $\gamma = -\frac{1}{2}$ and $\frac{1}{2}$ will be understood in the sense of analytic continuation in the parameter γ, which corresponds to discarding the inessential constant contribution that is linear in T, and at $d = 3$ also the contribution quadratic in the temperature, to the thermodynamic potential.

If one of the quantitites $x_0 = 1, x_1, x_2, x_3$ is much larger than the remaining ones (we denote this quantity by x_i) it is convenient to write the expression for f_γ in the form

$$f_\gamma(x_1, x_2, x_3) = x_i^{2-\gamma}\,[a_i(\gamma) + f_\gamma^{(i)}(x_1 x_2 x_3)]. \tag{41}$$

Expressions for $f_\gamma^{(i)}$ and for the coefficients a_i are given in the Appendix.

In the two-dimensional case

$$\frac{\delta\Omega_2^{(C)}}{V_2} = \frac{T^2}{4D\hbar}\int\limits_{\frac{T\tau}{\hbar}}^{\infty}\frac{dt}{\ln(T_c t/\pi T)}\left\{\frac{\cos(\hbar\omega_s t/\pi T)}{\mathrm{sh}^2 t}\,\frac{\hbar\Omega_{H_\perp}}{2\pi T}\right.$$
$$\left.\times\exp\left[-\frac{\hbar t}{\pi T}\left(\frac{1}{\tau_s}+\frac{1}{\tau_{H_\parallel}}\right)\right]\mathrm{sh}^{-1}\left(\frac{\hbar\Omega_{H_\perp} t}{2\pi T}\right)-\frac{1}{t^3}+\frac{\hbar}{\pi T t^2}(\tau_s^{-1}+\tau_{H_\parallel}^{-1})\right\}. \tag{42}$$

Here $H_\parallel$ and $H_\perp$ are the projections of the magnetic field respectively on the plane of the film and on the normal to it.

Those contributions to $\delta\Omega_2^{(C)}$ which are of no interest were subtracted from (42) explicitly. Expression (42) at $T^* \ll \hbar/\tau$ can be rewritten in the form

$$\frac{\delta\Omega_2^{(C)}}{V_2} = -\frac{T^2}{24D\hbar}\left\{\ln\left[\frac{\ln(T_c/T^*)}{\ln(T_c\tau/\hbar)}\right]\left[2+\left(\frac{\Omega_{H_\perp}\hbar}{2\pi T}\right)^2+3\left(\frac{\hbar\omega_s}{\pi T}\right)^2-\frac{3\hbar^2(\tau_s^{-1}+\tau_H^{-1})}{(\pi T)^2}\right]+\frac{6a_i}{\ln(T_c/T^*)}\frac{\hbar(\tau_s^{-1}+\tau_H^{-1})}{\pi T^2}T^*\right\}. \tag{43}$$

Here T^* is defined by (39). The values $i = 0, 1, 2, 3$ are determined by which of the quantities in the right side of (39) coincides with T^*. The coefficients a_i are equal to

$$a_0 = -1, \quad a_1 = -\ln 2, \quad a_2 = -\pi/2, \quad a_3 = 0.$$

Using expressions (35) and (38)–(43) we can obtain by simple differentiation all the corrections to the thermodynamic quantities written out above

APPENDIX

The functions $f^{(i)}$ can be expanded in powers of $x/x_i (j \neq i)$ and are of the form

$$f_\gamma^{(0)} = -x_3\frac{\Gamma(\gamma+1)\zeta(\gamma)}{2^{\gamma-1}} - \left(\frac{x_1^2}{3}+x_2^2-x_3^2\right)\frac{\Gamma(\gamma+2)\zeta(\gamma+1)}{2^{\gamma+1}},$$
$$x_1, x_2, x_3 \ll 1, \tag{A1}$$

$$f_\gamma^{(1)} = -\frac{2x_3}{x_1}\Gamma(\gamma)(1-2^{-\gamma})\zeta(\gamma) - \frac{2+3x_2^2-3x_3^2}{3x_1^2}$$
$$\times\Gamma(\gamma+1)\zeta(\gamma+1)(1-2^{-\gamma-1}), \quad x_1 \gg x_2, x_3, 1, \tag{A2}$$

$$f_\gamma^{(2)} = -\frac{x_3}{x_2}\Gamma(\gamma-1)\cos\frac{\pi(\gamma-1)}{2} - \frac{2+x_1^2-3x_3^2}{6x_2^2}\Gamma(\gamma)$$
$$\times\cos\frac{\pi\gamma}{2}, \quad x_2 \gg 1, x_3, x_1, \tag{A3}$$

$$f_\gamma^{(3)} = -\frac{\Gamma(\gamma)}{6x_3^2}(2+x_1^2+3x_2^2), \quad x_3 \gg 1, x_1, x_2. \tag{A4}$$

The coefficients $a_i(\gamma)$ can also be calculated. Their numerical values are listed in the table.

[1)] Corresponding to a diagram of order n is a factor $1/n$, but in this case it is necessary to fix one of the interaction lines.[11] Since this can be done in an nth order diagram (Fig. 2d) in n different ways, a factor equal to n appears and cancels in fact the usual combinatorial factor.

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