Nonstationary Josephson effect and nonequilibrium properties of SNS junctions

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A theory of the nonstationary Josephson effect in SNS junctions with low impurity density is constructed on the basis of the microscopic equations of superconductivity theory. The general expressions obtained for the current are used to determine the linear response of the system to a small alternating voltage, the stimulation of the critical contact by microwave irradiation is investigated, and the nonstationary Josephson effect is analyzed at low and high voltages. The decisive role in the nonstationary behavior of SNS junctions is played by the disequilibrium of the quasiparticle distribution in the system; this disequilibrium leads, in particular, to an essentially nonsinusoidal $I(\varphi)$ dependence at all T and d.

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1. INTRODUCTION

Superconductor–normal metal–superconductor (SNS) junctions are among the most important (both theoretically and for practical applications) types of weak superconducting links (see, e.g. Ref. 1). The microscopic theory of the stationary Josephson effect in SNS junctions has in fact already been completely developed, but the features of their nonstationary and nonequilibrium behavior have so far been little investigated. Yet the deviation of the distribution function of quasiparticles in SNS from equilibrium, which takes place when a potential difference is present in the system, influences strongly the nonstationary properties of such junctions. Correct allowance for the disequilibrium of the distribution function is in fact the main difficulty encountered in the development of a theory for the nonstationary Josephson effect in weakly bound superconducting systems.

It is known that this difficulty can be relatively easily avoided in the case of tunnel (SIS) junctions, whose nonstationary properties are described by perturbation theory in term of the transparency of the dielectric barrier, wherein the superconductor is regarded as in equilibrium in the zeroth approximation. The corresponding theory was developed in the papers of Werthamer[2] and Larkin and Ovchinnikov.[3] In systems with direct (nontunnel) conduction this approach is no longer applicable, and the problem calls for an exact solution.

Alamazov and Larkin[4] and Likharev and Yakobson[5] investigated the Josephson effect in superconducting bridges near T_c, using the nonstationary Ginzburg-Landau equations, which are valid for zero-gap superconductors and do not take into account the nonequilibrium effects. In the case of superconductors with a gap, to take into account the disequilibrium of the distribution function it is necessary to introduce into these equations also additional terms, as was done indeed by Alamazov and Larkin.[6] Even within the framework of such an approach it was shown that superconducting bridges have a number of interesting properties (the excess current,[5] the critical-current stimulation due to oscillation of the gap in the junction[6]), which are absent in tunnel junctions.[2,3]

The next step in the study of nonstationary phenomena in superconducting weak links was made by Artemenko, Volkov, and Zaĭtsev[7] and by Zaĭtsev,[8] who constructed a theory of the nonstationary Josephson effect in short superconducting bridges (SCS) on the basis of consistent microscopic equations for quasiclassical Green's functions integrated with respect to energy.[9,10] It was shown in particular that the nonstationary behavior of SCS junctions becomes much more complicated (compared with the simple resistive model[4]) principally because of the presence of nonequilibrium electrons.

The nonstationary Josephson effect in SNS systems containing a dielectric layer (SINS and SNISNS) was investigated[1)] by Zharkov and myself.[11,12] To construct the theory, use was made also of the microscopic equations,[9,10] while the presence of a dielectric barrier was taken into account with the aid of the boundary conditions obtained in Ref. 11, which are valid for arbitrary transparency of such a barrier. In Refs. 11 and 12 was considered the case of low transparency (tunnel conduction) and it was shown that the nonstationary Josephson effect in SNINS junctions has a number of interesting features. Thus, the anomalous part of the Josephson component of the current has, even at high temperatures, no factor that depends exponentially on the N-layer thickness. In pure SNINS junctions at definite voltages there exist logarithmic peaks in the expression for this current component, and are due to resonant tunneling of the electron between the systems of Andreev levels. These effects can lead to a "double" stimulation of the critical current of an SNINS system in a microwave field.

The present paper deals with nonstationary and nonequilibrium properties of SNS systems that have direct conductivity (unity transparency coefficient). The deviation of the distribution function from equilibrium plays a decisive role in the behavior of such junctions. In Sec. 2, a general expression for the current is obtained and describes the nonstationary Josephson effect in the considered system. This expression is investigated for different limiting cases in Secs. 3 and 4. The linear response to a low alternating voltage is considered, and the singularities of stimulation of the critical current in a microwave field as well as the nonstationary Josephson effect at low and high voltages are considered. The results are discussed in the concluding section.

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2. FUNDAMENTAL EQUATIONS. GENERAL EXPRESSIONS FOR A CURRENT IN AN SNS JUNCTION

The analysis is based on the microscopic equations, integrated with respect to $\xi_p = v(p - p_F)$, for the Green's functions[9,10]

$$\mathbf{v}_F \frac{\partial \check{G}}{\partial \mathbf{R}} + \hat{\tau}_3 \frac{\partial \check{G}}{\partial t} + \frac{\partial \check{G}}{\partial t'} \hat{\tau}_3 + [\check{H} + \check{\Sigma}, \check{G}] = 0, \qquad (1)$$

where

$$\check{G} = \check{G}(v_F, R; t, t') = \begin{pmatrix} \hat{G}^R & \hat{G} \\ \hat{0} & \hat{G}^A \end{pmatrix},$$

$$\check{\Sigma} = \frac{v_F}{2l} \langle \check{G} \rangle + \check{\Sigma}_{ph} = \begin{pmatrix} \hat{\Sigma}^R & \hat{\Sigma} \\ \hat{0} & \hat{\Sigma}^A \end{pmatrix},$$

$$\check{H} = \{- ie\mathbf{v}_F \mathbf{A}(t) \hat{\tau}_3 + ie\Phi(t) - i\hat{\Delta}(t)\} \check{1} \delta(t - t'), \qquad (2)$$

$$\hat{\Delta} = \begin{pmatrix} 0 & \Delta \\ -\Delta^* & 0 \end{pmatrix}$$

$v_F = p_F/m$ is the Fermi velocity, $\mathbf{A}$ and Φ are respectively the vector and scalar potentials, l is the mean free path, Δ is the order parameter, and $\hat{\tau}_i$ ($i = 1, 2, 3$) are Pauli matrices. The square brackets in (1) denote a commutator, and the angle brackets denote averaging over the directions of the vector $\mathbf{v}_F$. The product of matrices denotes everywhere convolution with respect to the internal variable. The current in the system is given by

$$\mathbf{j}(t) = (ep_F/4\pi) \langle \mathrm{Sp}\, \hat{\tau}_3 \mathbf{p}_F \hat{G}(t, t) \rangle. \qquad (3)$$

There is an additional normalization condition for the matrices $\check{G}$ (Ref. 10)

$$\check{G}^2 = \check{1}\delta(t - t'). \qquad (4)$$

The model for the SNS junction will be the known model of a variable-thickness bridge: two bulky superconducting banks are connected by a thin normal-metal filament of length d and of cross-section area S, with $a \sim \sqrt{S} \ll l, \xi_0$($\xi_0$ is the the coherence length of the superconductor). In this case the situation is in fact quasi-one-dimensional. The order parameter of the system is of the form $\Delta(x) = \Delta [\Theta(x - d/2) + \Theta(-x - d/2)]$, where $\Theta(x)$ is the Heaviside function.

We shall assume that the mean free path is large compared with all the characteristic parameters of the problem (the pure limit). In this case the Green's functions of the SNS junction can be obtained analytically for all energies. We note that in experiment it is quite difficult to realize the situation of a pure SNS junction. However, many properties of dirty ($l \ll \xi_0$) and pure SNS junctions are qualitatively similar, so that the investigation of the problem posed is not only of theoretical but also of definite experimental interest. We shall return to this question later.

Assume that there is in the system a potential difference $V(t)$ such that

$$\Phi(t) = \pm {}^1/{}_2 V(t), \quad x \gtrless \pm d/2, \quad |x| - d/2 \gg a. \qquad (5)$$

We neglect the vector potential. Equations (1) are solved in the superconducting and in the normal regions, and the obtained solutions are matched on the transit trajectories, it being assumed that reflection from the N-filament boundaries is specular. Since the problem is quasi-one-dimensional the Green's function depends only on the coordinate x and on the velocity component v_x. Solution of (1) in the superconducting regions on the transit trajectories is of the form[8,11]

$$\check{G}_j(v_x, x) = \check{A}_j + \exp\{-\check{K}_j(x - (-1)^j d/2)\} \check{B}_j(v_x) \times \exp\{\check{K}_j(x - (-1)^j d/2)\}. \qquad (6)$$

Here and elsewhere the subscript 1 pertains to the left-hand superconductor ($x < -d/2$) and 2 to the right ($x > d/2$). The expressions for the matrices $\check{A}_j$ are well known[7]:

$$\check{A}_j = \begin{pmatrix} \hat{A}_j^R & \hat{A}_j \\ \hat{0} & \hat{A}_j^A \end{pmatrix}, \quad \hat{A}_j^{R(A)} = \hat{S}_j(t) \int \hat{g}^{R(A)}(\varepsilon) e^{-i\varepsilon(t-t')} \frac{d\varepsilon}{2\pi} \hat{S}_j^+(t'),$$

$$\hat{g}^{R(A)} = g^{R(A)}(\varepsilon)\hat{\tau}_3 + f^{R(A)}(\varepsilon) i\hat{\tau}_2 = \frac{(\varepsilon + i\gamma_1^{R(A)})\hat{\tau}_3 + (\Delta + i\gamma_2^{R(A)}) i\hat{\tau}_2}{[(\varepsilon + i\gamma_1^{R(A)})^2 - (\Delta + i\gamma_2^{R(A)})^2]^{1/2}}$$

$$\hat{A}_j = \hat{A}_j^R \hat{n}_j - \hat{n}_j \hat{A}_j^A, \qquad (7)$$

$$\hat{n}_j(t, t') = \hat{S}_j(t) \int \mathrm{th} \frac{\varepsilon}{2T} \exp\{-i\varepsilon(t - t')\} \frac{d\varepsilon}{2\pi} \hat{S}_j^+(t'),$$

$$\hat{S}_j(t) = \hat{1} \cos\frac{\varphi(t)}{4} + i\hat{\tau}_3(-1)^j \sin\frac{\varphi(t)}{4}, \quad \varphi(t) = \varphi_0 + 2e\int V(t_1) dt_1.$$

The expressions for the parameters $\gamma_{1,2}^{R(A)}$, which describe the electron-phonon relaxation in the superconductors, are given in Ref. 7. The matrices $\check{A}$ and $\check{B}$ satisfy the conditions[8]

$$\check{B}_j(v_x)\check{A}_j = -\check{A}_j \check{B}_j(v_x) = (-1)^j \check{B}_j(v_x) \operatorname{sign} v_x. \qquad (8)$$

The solution of Eqs. (1) in the N region are represented in the form

$$\check{G}(v_x, x; t, t') = \hat{W}(v_x, t) \check{G}^{(0)}(v_x, x; t, t') \hat{W}^*(v_x, t'),$$

$$\hat{W} = \begin{pmatrix} \exp\{i\chi(v_x, t)\} & 0 \\ 0 & \exp\{i\chi(-v_x, t)\} \end{pmatrix},$$

$$\chi = -e \int^t dt_1 \Phi(x - v_x(t - t_1), t_1),$$

where the matrix $\check{G}^{(0)}$ is the solution of (1) under the condition $\Phi(x,t) = 0$, $|x| \leqslant d/2$. It should be noted first of all that as $t' \to 1$ the matrix $\check{G}$ is identical with $\check{G}^{(0)}$. Since the current in the system is expressed in terms of the Green's functions at equal times, for calculation at any point of the N layer we need know only $\check{G}^{(0)}$ (but not $\check{G}$), i.e., the concrete form of the potential Φ at $|x| \leqslant d/2$ has no effect whatever on the expression for the current in the N layer. We note also that by virtue of the symmetry of the problem we have $\Phi(0,t) = 0$, consequently at the point $x = 0$ the matrices $\check{G}$ and $\check{G}^{(0)}$ coincide at all t and t' (and not only at $t = t'$). We shall find it convenient to find an expression for the function $\check{G}(v_x, 0) \equiv \check{G}_0(v_x)$. To this end we introduce the matrix

$$\hat{C}(\varepsilon,\varepsilon',v_x)=\left(\hat{1}\cos\frac{\varepsilon d}{2v_x}+i\hat{\tau}_3\sin\frac{\varepsilon d}{2v_x}\right)\delta(\varepsilon-\varepsilon'). \quad (9)$$

We then have (we leave out the energy variables for the sake of brevity)

$$\check{A}_1+\check{B}_1(v_x)=\check{C}(v_x)\check{G}_0(v_x)\hat{C}^+(v_x),$$
$$\check{A}_2+\check{B}_2(v_x)=\check{C}^+(v_x)\check{G}_0(v_x)\hat{C}(v_x). \quad (10)$$

Using (8) and (10) we obtain after simple transformations

$$\check{G}_0(v_x)=(\check{1}+\check{Q}_-(v_x)\operatorname{sign}v_x)\check{Q}_+^{-1}(v_x),$$
$$\check{Q}_\pm(v_x)={}^1/_2\{\hat{C}^+(v_x)\check{A}_1\hat{C}(v_x)\pm\hat{C}(v_x)\check{A}_2\hat{C}^+(v_x)\}, \quad (11)$$

whence follows directly an expression for the matrix $\hat{G}_0(v_x)$:

$$\hat{G}_0(v_x)=\hat{G}_0{}^r(v_x)+\hat{G}_0{}^a(v_x), \quad (12)$$

$$\hat{G}_0{}^r=\hat{P}^R\hat{n}_+-\hat{n}_+\hat{P}^A,\qquad \hat{P}^{R(A)}=(\hat{1}+\hat{Q}_-^{R(A)}\operatorname{sign}v_x)(\hat{Q}_+^{R(A)})^{-1}$$
$$\hat{G}_0{}^a=-(\hat{Q}_+{}^R)^{-1}(\hat{Q}_-{}^R\hat{n}_--\hat{n}_-\hat{Q}_-{}^A)(\hat{Q}_+{}^A)^{-1}$$
$$+\operatorname{sign}v_x[(\hat{Q}_+{}^R)^{-1}\hat{n}_-(\hat{Q}_+{}^A)^{-1}+\hat{Q}_-{}^R(\hat{Q}_+{}^R)^{-1}\hat{n}_-\hat{Q}_-{}^A(\hat{Q}_+{}^A)^{-1}-\hat{n}_-],$$
$$\hat{n}_\pm(v_x)={}^1/_2(\hat{C}\hat{n}_1\hat{C}^+\pm C^+\hat{n}_2\hat{C}).$$

The current at the point $x=0$ is connected with the function $\hat{G}_0$ by the relation

$$I=I^r+I^a=\frac{1}{4eR}\operatorname{Sp}\hat{\tau}_3\int_{-1}^{1}\alpha\,d\alpha\hat{G}_0(v_F\alpha;t,t),\qquad R=\frac{\pi^2}{p_F{}^2Se^2}, \quad (13)$$

where R is the resistance of the junction in the normal state.[13] Of course, the current can be calculated at any other point of the N metal, and by virtue of the continuity equation the result should not depend on the choice of this point.[2]

Thus, relations (7), (9) and (11)–(13) yield the complete solution of determining $I(t)$ in pure SNS bridges at arbitrary $V(t)$. As $d\to 0$ our expressions for the system Green's functions go over, as expected, into the corresponding equations[8] obtained for pure SCS junctions.

We note also that the matrix $\hat{G}_0^a$ can be represented in the form

$$\hat{G}_0{}^a(v_x)=\hat{P}^R\hat{F}^a-\hat{F}^a\hat{P}^A. \quad (14)$$

The matrix $\hat{F}^a$ describes in fact the deviation of the distribution function from the equilibrium function. It can be shown that $\hat{F}^a$ satisfies the equations

$$\hat{Q}_\pm{}^R\hat{F}^a-\hat{F}^a\hat{Q}_\pm{}^A=\hat{Q}_\mp{}^A\hat{n}_--\hat{n}_-\hat{Q}_\mp{}^A, \quad (15)$$

which are a direct generalization of the corresponding equations in Refs. 7 and 8 ($d\to 0$).

We proceed to investigate the obtained general expressions.

3. CASE OF LOW ALTERNATING VOLTAGE

a) Linear response

Let a constant current I_0 flow through the junction. The current is parametrized by a time-independent phase φ_0. When a low alternating voltage appears on the junction, the phase difference takes the form $\varphi(t)=\varphi_0+\varphi_1(t),\varphi_1\ll 1$. The presence of the alternate component $\varphi_1(t)$ means the appearance of an additional current $I_1(t)$ through the junction. We obtain an expression for such a current in terms in an approximation linear in φ_1, using for this purpose Eqs. (12). It must be noted that the terms linear in φ_1 will be contained in the expansions of the matrices $\hat{P}^{R,A}$ and $\hat{n}_-$. Using this, we have after simple calculations

$$I_{1\omega}=\frac{i\varphi_{1\omega}}{8R}\int d\varepsilon\int_{-1}^{1}\alpha\,d\alpha\cos\frac{\omega d}{2v_F\alpha}\left\{\operatorname{th}\frac{\varepsilon_+}{2T}X^{AA}Y^{AA}(\omega)\right.$$
$$\left.-\operatorname{th}\frac{\varepsilon_-}{2T}X^{RR}Y^{RR}(\omega)+\left(\operatorname{th}\frac{\varepsilon_-}{2T}-\operatorname{th}\frac{\varepsilon_+}{2T}\right)X^{RA}Y^{RA}(\omega)\right\}$$
$$X^{ij}=\operatorname{sign}\alpha\{g^i(\varepsilon_+)g^j(\varepsilon_-)+f^i(\varepsilon_+)f^j(\varepsilon_-)\cos\chi_+\cos\chi_-\} \quad (16)$$
$$\times\{1-f^i(\varepsilon_+)f^j(\varepsilon_-)\sin\chi_+\sin\chi_-\}-i\{f^i(\varepsilon_+)\sin\chi_+$$
$$+f^j(\varepsilon_-)\sin\chi_-\}\{g^i(\varepsilon_+)f^j(\varepsilon_-)\cos\chi_-+f^i(\varepsilon_+)g^j(\varepsilon_-)\cos\chi_+\},$$
$$Y^{ij}(\omega)=\{[1+(f^i(\varepsilon_+)\sin\chi_+)^2][1+(f^j(\varepsilon_-)\sin\chi_-)^2]\}^{-1},$$
$$\chi_\pm=\varphi_0/2+\varepsilon_\pm d/v_F\alpha,\qquad \varepsilon_\pm=\varepsilon\pm\omega/2,\ i,j=R,A.$$

Here $I_{1\omega}$ and $\varphi_{1\omega}$ are the Fourier transforms of $I_1(t)$ and $\varphi_1(t)$. Expression (16) becomes noticeably simpler under the conditions $T\sim T_c$, $\omega\ll\Delta(T)<v_F/d$ and go over into

$$I_{1\omega}=\frac{i\omega\varphi_{1\omega}}{2R}+\varphi_{1\omega}\frac{\pi\Delta^2\sin^2(\varphi_0/2)}{4eRT}\left[1-\frac{1}{i\omega\tau_\varepsilon}\right]^{-1}+I_c\varphi_{1\omega}\cos\varphi_0. \quad (17)$$

Here I_c is the critical current of the bridge, $\tau_\varepsilon=\frac{1}{2}\gamma_1\propto T^{2)}$ (at $T\sim T_c$) is the time of the electron-phonon relaxation on the banks. As $d\to 0$ Eq. (17) goes over into the Zaĭtsev result.[8] In the case $d\gg\xi_0$ we have $I_0\propto\exp\{-d/\xi_0\}$ so that the last term in (17) can be neglected. At low frequencies $\omega\ll 1/\tau_\varepsilon$ we then obtain

$$I_1=\frac{V}{\tilde{R}(\varphi_0)},\qquad V(t)=\frac{1}{2e}\frac{\partial\varphi_1}{\partial t},$$

$$\tilde{R}(\varphi_0)=R\left[1-\frac{\pi\Delta^2\tau_\varepsilon}{2T}\sin^2\frac{\varphi_0}{2}\right]^{-1}. \quad (18)$$

Thus in SNS bridges (just as in other types of weak links with direct conduction) the conductivity is increased because of the deviation of the distribution function from equilibrium. We call attention also to the fact that at $\omega<v_F/d$ the expression for the current (17) is in fact independent of d.

At low temperatures the behavior of the bridge is also quite interesting. At $T\ll\Delta\ll v_F/d$ we have $I_{1\omega}=0$, $\omega<\Delta\cos(\varphi_0/2)$;

$$I_{1\omega}=\frac{\pi\varphi_{1\omega}}{4eR\omega}\Delta\sin\frac{\varphi_0}{2}\left\{\Delta\sin\frac{\varphi_0}{2}+\left[\Delta^2-\left(\omega-\Delta\cos\frac{\varphi_0}{2}\right)^2\right]^{1/2}\right\}$$
$$\times\frac{2\Delta\cos(\varphi_0/2)-\omega}{2\Delta\cos(\varphi_0/2)+\omega},\qquad \Delta\cos\frac{\varphi_0}{2}<\omega<\Delta. \quad (19)$$

The absence of a linear response at $\omega<\Delta\cos(\varphi_0/2)$ is actually due to the fact that the Green's function pole corresponding to the presence of a discrete level in the system makes no contribution to the current. Obviously this result is valid only if one neglects electron-scattering processes that lead to broadening of the discrete level, i.e., at $\omega\gg\gamma_N$, where γ_N is the characteristic width of the level. At large $d \gg \xi_0$ and $T \ll v_F/d$, Eq. (16) yields

$$I_{1\omega} = \frac{2v_F}{eRd} \varphi_{1\omega} \left[\ln \left(2 \operatorname{tg} \frac{\omega d}{4v_F} \right) + \cos \frac{\omega d}{2v_F} \right], \quad \omega \gg \frac{v_F}{d}. \quad (20)$$

The logarithmic singularities in the expression for the linear response (20) at $\omega = 2\pi v_F n/d$ are similar to the analogous singularities on the CVC of SNINS junctions. They are due to electron hopping between Andreev levels under the influence of the external alternating field. Thus, even at low voltages the behavior of SNS junctions is quite nontrivial and, as can be seen, it differs substantially at high and low temperatures.

b) Stimulation of critical current in a microwave field

One of the important properties of SNS junctions is the appreciable increase of the critical current by external radiation. This effect was observed in experiment.[14] It was shown in Refs. 15 and 12 that stimulation of the critical current in SNS junctions (as well as in other superconducting structures) is due to the disequilibrium of the distribution function, and a distinguishing feature of these junctions is the magnitude of the effect: the critical current in a microwave field can increase by several orders of magnitude compared with the equilibrium situation. To investigate the stimulation of the superconducting current of pure SNS junctions we use the general formulas (12). Assume that an alternating voltage $V(t) = V_1 \cos\omega_1 t$ is applied to the junction. Then

$$\varphi_{1\omega} = (eV_1/\omega_1)[\delta(\omega+\omega_1) - \delta(\omega-\omega_1)]. \quad (21)$$

We calculate the critical current of the SNS junction under the assumptions $eV_1 \ll \omega_1$ and $d \gg \xi_0$. To this end, just as in the case of an SNINS junction,[12] it is necessary to expand the general expression for the current up to terms quadratic in $\varphi_{1\omega}$. Carrying out the corresponding calculations and averaging over the time, we arrive at the final expression for the current which, however, is quite complicated and is therefore given in the Appendix [Eqs. (A1) and (A2)]. We note also that Eqs. (A1) and (A2) give a general expression for the "nonequilibrium" part of the current through the junction. Here, however, we are interested in the equation for I^r, inasmuch as at $t \gg v_F/d$ (i.e., at temperatures at which the stimulation effect manifests itself) $I^r \propto \exp\{-d/\xi_T\}$, where $\xi_T = v_F/2\pi T$. We shall not investigate the obtained general expression in various limiting cases. We note only that this expression becomes much simpler at $T \sim T_c$ and $\omega_1 \ll \Delta(T) < v_F/d$. Under these conditions it goes over into

$$I_c = \frac{\pi}{8} \frac{\Delta^2}{RT} \frac{eV_1^2}{\omega_1^2 + 1/\tau_\varepsilon^2}, \quad \varphi_0 = \frac{\pi}{2}, \quad (22)$$

i.e., the expression for the critical current of SNS bridges in a microwave field does not depend at all, under definite conditions ($\omega_1 \ll v_F/d$) on the N-layer thickness. It can be seen that at $\omega_1 \ll 1/\tau_s$ we have $I_c \propto \tau_\varepsilon^2$, and at $\omega_1 \gg 1/\tau_\varepsilon$ the critical current can become of order I_c for short bridges under equilibrium condition. In other words, the disequilibrium of the distribution function leads, as it were, to a shortening of the bridge. Of course, at $\omega_1 \gtrsim v_F/d$ the characteristic energies at which the distribution function is essentially not in equilibrium is of the order of (or larger then) the reciprocal transit time of the electrons through the junction, so that in this case I_c no longer depends on d. Such a dependence (under certain conditions) follows a power law,[12,15] rather than an exponential law as in the equilibrium case. Then the "nonequilibrium" critical current of the bridge can exceed by several orders its equilibrium value, as is indeed observed in experiment.

We note also that the results are valid if the electron-phonon relaxation in the N layer is neglected. The relaxation of the distribution function to equilibrium takes place in our case in the superconducting banks. The same condition was used in Ref. 12. A different physical situation was considered in Ref. 15, whose authors proposed that the energy relaxation of the nonequilibrium quasiparticles trapped in the N layer is due to electron-phonon interaction in this layer. The nonequilibrium correction to the distribution function was calculated by perturbation theory directly from the kinetic equation for the N metal with the usual "equilibrium" boundary conditions for the Green's function on the NS boundary. It can be seen here, too, that even a small deviation from equilibrium can lead to a considerable increase of the critical current of the junction.

4. NONSTATIONARY JOSEPHSON EFFECT. LOW AND HIGH VOLTAGES

As already noted, the general expressions obtained for the Green's functions make it possible in principle to determine the current through the contact at an arbitrary voltage. In a number of cases (e.g., at $eV \sim \Delta$), however, to reach numerical results and by the same token determine the CVC of a junction is quite difficult. We confine ourselves therefore to an investigation of the nonstationary Josephson effect at low ($eV \ll \Delta$) and high ($eV \gg \Delta$) voltages.

A. Low voltages

We consider first the temperature region $T \sim T_c$. In this case $\Delta(T)$ is small, so that in a really attainable experimental situation the condition $d < v_F/\Delta(T)$ will be satisfied. The analysis is based then on the use of relations (14) and (15) and does not differ in fact from that in Refs. 7 and 8 for short bridges. We have ultimately

$$I = \frac{1}{2eR} \frac{\partial\varphi}{\partial t} + \frac{\pi\Delta^2}{4eRT} P\{\varphi\} + I_c \sin\varphi. \quad (23)$$

Thus, the current-phase relation for SNS bridges in the resistive regime near T_c is the same as in the case of short SCS junctions[7,8] (the form of the functional $P\{\varphi\}$ was obtained in Ref. 8). However, whereas in short junctions the coefficients of $P\{\varphi\}$ and $\sin\varphi$ are equal and coincide with the critical current of the junction, in SNS junctions they differ substantially. Thus, at $d \gg \xi_0$ the current I_c is exponentially small and the first two terms in (23) can be seen to be entirely independent of d, i.e., the relative contribution of the "nonequilibrium" terms in the case of sufficiently long junctions is larger than for short ones. The functional $P\{\varphi\}$ was investigated in Ref. 8, so that there is no need here to write down the expressions for the current in different limiting cases. We note only that at low voltages $eV \ll 1/\tau$ the CVC of the junction takes the form

$$V = I\tilde{R}, \quad \tilde{R} = R[1-(\pi\Delta^2/4T)\tau_\varepsilon]^{-1}, \tag{24}$$

i.e., just as in the case of a low alternating voltage, the deviation from equilibrium leads to an effective increase of the conductivity of the junction. With decreasing temperature, the situation changes substantially. Let us consider the case $T \ll \Delta$ and $eV \ll \min\{\Delta, v_F/d\}$. In addition, we assume V to be constant or slowly varying in time. Under these conditions the matrices $\hat{Q}^{R,A}_{\pm}$ and $(\hat{Q}^{R}_{+})^{-1}$ are obtained for the energy region of interest to us, in the adiabatic approximation, by replacing φ_0 with $\varphi(t)$ in the corresponding equilibrium expressions. As a result we get

$$I(\varphi) = I^r(\varphi) + I^a(\varphi), \tag{25}$$

where $I^r(\varphi)$ is actually a generalization of the result of Refs. 16 and 17 to include the nonstationary case ($T \to 0$):

$$I^r(\varphi) = \begin{cases} (v_F/6eRd)\varphi, & -\pi<\varphi<\pi, \quad d \gg \xi_0. \\ (\Delta/eR)\sin(\varphi/2), & d \ll \xi_0 \end{cases} \tag{26}$$

The expressions for $I^a(\varphi)$ differ noticeably, depending on the region of the variation of the problem parameters. Thus, under the condition

$$\cos\frac{\varphi(t)}{2} \gg \max\left\{\frac{eV}{\Delta}, \frac{eVd}{v_F}, \frac{T}{\Delta}, \frac{Td}{v_F}\right\}$$

$I^a(\varphi)$ does not depend on d:

$$I^a = \frac{V}{R\cos^2(\varphi/2)}. \tag{27}$$

At arbitrary $\varphi(t)$, but under the condition $T \ll eV \ll v_F/d < \Delta$, we have

$$I^a = \frac{V}{R\cos^2(\varphi/2)} \times\left[1 + \frac{b^2}{2}\left(1+\sin^2\frac{\varphi}{2}\right)\mathrm{tg}^2\frac{\varphi}{2}\ln\left|1 - \frac{1}{b^2}\mathrm{ctg}^2\frac{\varphi}{2}\right|\right],$$

$$b = \frac{eVd}{2v_F} \ll 1. \tag{28}$$

In the case $\cos(\varphi/2) \gg eVd/v_F$ the expression (28) coincides with (27). The resonant singularities in (28), just as in other cases, are due to the presence of a discrete structure of the Andreev levels in the junction. As $\varphi(t) \to \pm\pi + 2\pi n$ we obtain $I^a = V/2R$.

Thus, at low temperatures $I(\varphi)$ deviates from the equilibrium relation (26) (and all the more from sinusoidal) even at low voltages. This difference is directly connected with the appearance in (25) of the term $I^a(\varphi)$, which describes the deviation of the distribution function from equilibrium.

Averaging (28) over the period $\varphi(t)$ and assuming at the same time that V changes little during this period, we obtain

$$\bar{I} = (e^2V^3d/Rv_F^2)\ln(2v_F/eVd), \tag{29}$$

i.e., the CVC of an SNS junction, under the considered conditions, deviates greatly from Ohm's law. This deviation is due to the fact that as $T \to 0$ there are no quasiparticle excitations in the system, and the current is due to condensate flow.

B. High voltages

In the case $eV \gg \Delta$ the normal current exceeds considerably the Josephson and the interference components. In this case $\varphi = \varphi_0 + 2Vt$, and we have for $(\hat{Q}^{R,A}_{+})^{-1}$ in analogy with Ref. 8

$$[\hat{Q}^{(R)A}_{+}(\varepsilon,\varepsilon')]^{-1} = 2[g^{R(A)}(\varepsilon+{}^1/_2eV)g^{R(A)}(\varepsilon-{}^1/_2eV)+1]^{-1}\hat{Q}^{R(A)}_{+}(\varepsilon,\varepsilon'). \tag{30}$$

The current through the junction has in the first-order approximation the form

$$I = I_0 + I_1\sin\varphi + I_2\cos\varphi. \tag{31}$$

Calculating I_0 with the aid of (30) we obtain

$$I_0 = \frac{V}{R} + I_{ex}, \quad I_{ex} = \frac{8}{3}\frac{\Delta}{eR}\mathrm{th}\frac{eV}{2T}. \tag{32}$$

The quantity I_{ex} in (32) defines the excess current in the junction, the expression for which can be seen to be independent of the N-layer thickness and to be exactly equal to Zaĭtsev's result[8] for short SCS junctions. We emphasize also that I_{ex} depends on d only in pure SNS junctions.

For sufficiently large d the ratio I_{ex}/I_c is much larger than unity even at $T=0$ (and all the more at $T \gg v_F/d$). Thus, at $d \gg \xi_0$ and $T=0$ we have

$$I_{ex}/I_c = (16/\pi)(\Delta d/v_F) \gg 1. \tag{33}$$

We present expressions for I_1 and I_2 or the case of short SNS junctions with $d \ll v_F/eV$. These expressions are quite cumbersome even in the first-order approximation in Δ/eV, and are therefore relegated to the Appendix. At $T \ll \Delta$ Eqs. (A3)–(A5) become much simpler. In this case we have[3)]

$$I_1 = \frac{\pi\Delta^2}{2e^2RV}, \quad I_2 = \frac{\Delta^2}{2e^2RV}\left(\frac{3}{4} - \ln\frac{4eV}{\Delta}\right). \tag{34}$$

For comparison we present the expressions for I_1 and I_2 in SIS junctions[2,3] at $T \ll \Delta \ll eV$:

$$I_1 = \frac{\pi\Delta^2}{e^2RV}, \quad I_2 = -\frac{2\Delta^2}{e^2RV}\ln\frac{2eV}{\Delta}. \tag{35}$$

It can be seen that I_1 and $|I_2|$ turn out to be less in the case considered by us than the corresponding expressions (35). The quantity $\gamma(V) = I_1/I_c$ takes the form

$$\gamma(V) = \Delta/2eV, \quad eV \gg \Delta \gg T.$$

The analogous quantity for SIS junctions in the considered temperature and voltage range is equal to $\gamma_{SIS} = 2\Delta/eV$. The decrease of $\gamma(V)$ in point junctions with direct conductivity, compared with the case of tunnel junctions, was observed in experiment.[19] It is quite difficult, however, to compare quantitatively our results with those of the cited experiments, inasmuch as in Ref. 19 were used superconductors with large amounts of impurities, as well as not too high voltages $eV \lesssim 4\Delta$.

In the case of broad SNS junctions, the currents I_1 and I_2 contain an additional smallness [compared with (34)] in terms of the parameter v_F/eVd (see Ref. 11).

5. DISCUSSION OF RESULTS

Our analysis shows thus that many nonstationary properties of SNS junctions differ noticeably from the previously investigated properties of tunnel junctions[2,3] and of short superconducting junctions.[7,8] At high temperatures $T \sim T_c$ the decisive role is played by the disequilibrium of the distribution function, since the Josephson current is exponentially small at these T and at $d \gg \xi_0$, while the "nonequilibrium" terms do not contain such a small quantity (at sufficiently low voltages and frequencies, they do not depend on d at all). This is in fact the cause of the stimulation of the critical current of SNS junctions in a microwave field. We recall that in short junctions the superconductivity is suppressed by low-intensity microwave radiation,[7] since the increase of the pair current I^r prevails under these conditions over the anomalous component I^a (which is absent in the equilibrium case). In sufficiently long SNS junctions, I^a can exceed I^r by many orders of magnitude, and this leads, so to speak, to shortening of the junction under the influence of the irradiation.

At low temperatures $T \ll \min\{\Delta, v_F/d\}$ the quasiparticle excitations in the system are in fact absent. In the case of low voltages the current through the junction differs from zero only because of the condensate flow. We note we have here a manifestation of one of the essential differences between systems with tunnel and direct conduction. It is known that in tunnel junctions the quasiparticle and interference components of the current are zero at $T=0$ (in first order in the transparency) at $eV < 2\Delta$ (for SIS, See Refs. 2 and 3) or $eV < \Delta$ (for SINS and SNISNS, see Ref. 11). In junctions with direct conduction the current I^a differs from zero even at $T=0$ and at arbitrarily low voltages, and the current flow is due in this case to the Andreev reflection of the excitations with $E < \Delta$ from the NS boundaries (for SNS junctions) or the direct transition of the electrons from condensate to condensate (for superconducting junctions). In tunnel junctions these phenomena contribute to the current only in second order in the transparency, and therefore play no significant role in the current transport. In systems with direct conduction the current I^a due to the passage of the electrons between superconducting condensates of two metals should obviously depend strongly on $\varphi(t)$ [see (27) and (28)], and the CVC of the junction deviates in this case from Ohm's law.

One more significant difference from junctions with direct conduction is the presence, on the CVC at $eV \gg \Delta$, of an excess current that is actually absent in tunnel junctions. The reason is that in tunnel systems the quasiparticles with $E < \Delta$ make no contribution to the current in first order in the transparency, whereas in SCS and SNS junctions all the states contribute to the current. Thus, the peculiarity of the nonstationary behavior of SNS junctions is connected in many respects with the conductivity of the quasiparticles having $E < \Delta$ and trapped in the N layer. The presence of characteristic logarithmic singularities on the CVC in a number of cases is due to the discrete character of the spectrum of such quasiparticles.[20] Of course, these logarithmic peaks on the CVC become smeared out when account is taken of the Andreev-level width, which is made finite by the scattering (elastic and inelastic) of the quasiparticles.

The presence of a discrete quasiparticle spectrum at $E < \Delta$ is a feature of pure SNS systems, so that the singularities indicated can be realized only in such systems and do not occur in SNS junctions with large amounts of impurities. In all other respects the physical situation in pure and dirty SNS junctions is qualitatively similar. The role of the transit time $\tau_c \sim d/vF$ of a quasiparticle in the N layer of a contaminated junction is played by the characteristic diffusion time $\tau_d \sim d^2/D$ (where $D = v_F l/3$). The reason why it is difficult to calculate nonstationary phenomena in dirty SNS junctions is that it is impossible to find an analytic expression for the Green's functions of the junctions at energies $\varepsilon \sim D/d^2$ (in contrast to the case of pure SNS systems, whose Green's functions are known for all energies). Nonetheless, in a number of limiting cases the current in dirty SNS junctions can likewise be calculated accurately. We illustrate this using by way of example the calculation of the critical current of a dirty SNS junction in a microwave field.

We consider the case $T \gg D/d^2$ and assume an applied alternating voltage $V(t) = V_1 \cos\omega_1 t$, with $eV_1 \ll \omega_1 \ll D/d^2$. To calculate the current we need to know the Green's function only at low energies $\varepsilon \ll D/d^2$, and the expressions for these functions are known.[7] Consequently, we can use right away the result of Ref. 7 [Eq. (54)]. Neglecting the exponentially small Josephson current and averaging over the time, we obtain

$$I_c = \frac{\pi}{4}\left(1 - \frac{\pi}{4}\right)\frac{\Delta^2}{RT}\frac{eV_1^2}{\omega_1^2 + 1/\tau_\varepsilon^2}, \tag{36}$$

i.e., the critical current is stimulated in the microwave field: the current does not contain the exponential small factor $\sim \exp\{-d/\xi_N\}$ with $\xi_N = (D/2\pi T)^{1/2}$ (the junction is so to speak shortened).

We note also that at low voltages $V \ll \Delta(T) < D/d^2$ and $T \sim T_c$ the expression for the current takes the form (23), with the functional $P\{\varphi\}$ calculated in Ref. 7, and the expression for I_c of dirty SNS calculated in a number of papers (see, e.g., Ref. 21).

As already noted, relations (9) and (11)–(13) make it possible to determine the current at the point $x = 0$, i.e., at the midpoint of the junction. Yet the matrix $\check{G}^{(0)}$ at other points of the N layer differs generally speaking from $\check{G}^{(0)}$, namely

$$\check{G}^{(0)}(v_x, x) = \hat{C}(v_x, x)\check{G}_0(v_x)\hat{C}(v_x, x), \quad |x| < d/2, \tag{37}$$

where the matrix $\hat{C}(v_x, x)$ is defined by Eq. (9) in which $d/2$ must be replaced by x. In the case of low voltages $eVd/v_F \ll 1$ the matrix $\check{G}$, in the principal approximation in terms of this parameter, is independent of x. At high voltages, however, this dependence becomes significant. For example, in the case $eV \gg \Delta \gg v_F/d$ we have

$$I=I_0+I_++I_-,\ I_\pm=\tfrac{1}{2}[I_1 \sin \varphi_\pm(x,t)+I_2 \cos \varphi_\pm(x,t)].\quad \varphi_\pm(x,t)=\varphi_0+2eV(t\pm x/v_F). \tag{38}$$

The electric field in the N layer, not too close to the NS boundaries, is of the form

$$E=E_++E_-,\ E_\pm(x)=(\pi/VS)[I_1 \cos \varphi_\pm(x,t)-I_2 \cos \varphi_\pm(x,t)]. \tag{39}$$

Thus, in the situation considered there are present in the N layer unusual waves of the current and of the electric field. We emphasize that we have in mind the field inside the N filament. Near NS boundaries in a superconductor, however, the electric field changes quite abruptly, so that the potential difference between the superconducting banks turns out to be equal to V. The penetration depth of the electric field into the superconductor depends on a number of factors (temperature, system geometry, and others) and, as is well known, is greatest at $T \sim T_c$. In the case considered, since a is small, the electric field in the semiconductor can be appreciable only at distances of the order $a \ll \xi_0$ from the NS boundaries.

One more final remark. We have dealt throughout with an SNS structure of bridge type, and the weakness of the binding was due in fact to the smallness of the cross section of the N filament. All the results, however, turn out to be valid (just as in the stationary case[22]) also for an SNS structure of the sandwich type provided that $v_{FN} \ll v_{FS}$. In this case the weakness of the bond is due already to the small Fermi velocity of the N layer, and the formula takes the same form: $R = \pi^2/P_{FN}{}^2Se^2$.

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APPENDIX

We present for reference several complicated expressions. The general expression for the current I^a in the presence of a small alternating phase difference (21) is

$$I^a=\frac{eV_1{}^2}{32\omega_1{}^2R}\int d\varepsilon \int_{-1}^{1} \alpha d\alpha \cos\frac{\omega_1 d}{2v_F\alpha}\sum_{k=1}^{2} T_k Y^{RA}\{\operatorname{sign}\alpha[iL_1{}^R N_+{}^{RA} - iL_1{}^A N_+^{AR}+L_2{}^{AR}N_-{}^{RA}-L_2{}^{RA}N_-{}^{AR}] - [N_1+L_3{}^{RA}N_2{}^{RA}-L_3{}^{AR}N_2{}^{AR}]\}, \tag{A1}$$

where

$$L_1{}^j=[f^j(\varepsilon)\sin\chi(\varepsilon)+f^j(\varepsilon+\omega_k)\sin\chi(\varepsilon+\omega_k)] \times[1+(f^j(\varepsilon+\omega_k)\sin\chi(\varepsilon+\omega_k))^2]^{-1},$$

$$L_2{}^{ij}=if^i(\varepsilon)\sin\chi(\varepsilon)[1-f^j(\varepsilon)f^j(\varepsilon+\omega_k)\sin\chi(\varepsilon)\sin\chi(\varepsilon+\omega_k)] \times[1+(f^j(\varepsilon+\omega_k)\sin\chi(\varepsilon+\omega_k))^2]^{-1},$$

$$L_3{}^{ij}=L_1{}^i[f^j(\varepsilon)\sin\chi(\varepsilon)+f^j(\varepsilon+\omega_k)\sin\chi(\varepsilon+\omega_k)],$$

$$Y^{RA}=[1+(f^R(\varepsilon)\sin\chi(\varepsilon))^2]^{-1}[1+(f^A(\varepsilon)\sin\chi(\varepsilon))^2]^{-1},$$

$$N_\pm{}^{ij}=[g^i(\varepsilon)f^i(\varepsilon+\omega_k)\cos\chi(\varepsilon+\omega_k) +g^i(\varepsilon+\omega_k)f^i(\varepsilon)\cos\chi(\varepsilon)]g^j(\varepsilon) \pm f^j(\varepsilon)\cos\chi(\varepsilon)[g^i(\varepsilon)g^i(\varepsilon+\omega_k) +f^i(\varepsilon)f^i(\varepsilon+\omega_k)\cos\chi(\varepsilon)\cos\chi(\varepsilon+\omega_k)], \tag{A2}$$

$$N_1=[g^R(\varepsilon)g^A(\varepsilon)+f^R(\varepsilon)f^A(\varepsilon)\cos^2\chi(\varepsilon)]\{g^R(\varepsilon)-g^R(\varepsilon+\omega_k) -g^A(\varepsilon)+g^A(\varepsilon+\omega_k)-[g^R(\varepsilon)f^A(\varepsilon)+g^A(\varepsilon)f^R(\varepsilon)]\}\cos\chi(\varepsilon) \times\{[f^R(\varepsilon)-f^A(\varepsilon)]\cos\chi(\varepsilon) +[f^R(\varepsilon+\omega_k)-f^A(\varepsilon+\omega_k)]\cos\chi(\varepsilon+\omega_k)\},$$

$$T_k=\operatorname{th}\frac{\varepsilon+\omega_k}{2T}-\operatorname{th}\frac{\varepsilon}{2T},\quad \omega_k=(-1)^k\omega_1,\quad i,j=R,A.$$

The expression for N_2^{ij} is obtained from the expression for N_+^{ij} by interchanging the quantities $g^j(\varepsilon)$ and $f^j(\varepsilon)\cos\chi(\varepsilon)$. The Josephson and interference components of the current take at $eV \gg \Delta$ the form

$$I_1=\frac{i}{4eR}\int d\varepsilon\left\{\frac{1}{2}\left[T_+\left(\frac{3V}{2}\right)+T_+\left(\frac{V}{2}\right)\right]\times\left[f^R\left(\varepsilon+\frac{eV}{2}\right)f^R\left(\varepsilon-\frac{eV}{2}\right)Z^R(\varepsilon)-f^A\left(\varepsilon+\frac{eV}{2}\right)\times f^A\left(\varepsilon-\frac{eV}{2}\right)Z^A(\varepsilon)\right]+T_-\left(\frac{V}{2}\right)\times\left[f^R\left(\varepsilon+\frac{eV}{2}\right)f^A\left(\varepsilon-\frac{eV}{2}\right)M_+ +f^R\left(\varepsilon-\frac{eV}{2}\right)f^A\left(\varepsilon+\frac{eV}{2}\right)M_-\right]\right\}, \tag{A3}$$

$$I_2=\frac{1}{4eR}\int d\varepsilon\left\{\frac{1}{2}\left[T_-\left(\frac{3V}{2}\right)+T_-\left(\frac{V}{2}\right)\right]\times\left[f^R\left(\varepsilon+\frac{eV}{2}\right)f^R\left(\varepsilon-\frac{eV}{2}\right)Z^R(\varepsilon) +f^A\left(\varepsilon+\frac{eV}{2}\right)\times f^A\left(\varepsilon-\frac{eV}{2}\right)Z^A(\varepsilon)\right] -T_-\left(\frac{V}{2}\right)\left[f^R\left(\varepsilon+\frac{eV}{2}\right)f^A\left(\varepsilon-\frac{eV}{2}\right)M_+ +f^R\left(\varepsilon-\frac{eV}{2}\right)f^A\left(\varepsilon+\frac{eV}{2}\right)M_-\right]\right\}, \tag{A4}$$

$$M_\pm=Z^R(\varepsilon\pm eV)Z^A(\varepsilon)+\frac{1}{4}\left\{\left[\pm g^R\left(\varepsilon\mp\frac{3eV}{2}\right)\mp g^R\left(\varepsilon\mp\frac{eV}{2}\right)\right]\times Z^R(\varepsilon\mp eV)\pm\left[g^R\left(\varepsilon+\frac{eV}{2}\right)+g^R\left(\varepsilon-\frac{eV}{2}\right)\right]Z^R(\varepsilon)\right\} \times\left\{\left[g^A\left(\varepsilon-\frac{eV}{2}\right)-g^A\left(\varepsilon+\frac{eV}{2}\right)\right]Z^A(\varepsilon) \pm\left[g^A\left(\varepsilon\pm\frac{eV}{2}\right)+g^A\left(\varepsilon\pm\frac{3eV}{2}\right)\right]Z^A(\varepsilon\pm eV)\right\}, \tag{A5}$$

$$Z^{R(A)}(\varepsilon)=\left[1+g^{R(A)}\left(\varepsilon+\frac{eV}{2}\right)g^{R(A)}\left(\varepsilon-\frac{eV}{2}\right)\right]^{-1}$$

$$T_{\pm}(x)=\text{th}\,\frac{\varepsilon+ex}{2T}-\text{th}\,\frac{\varepsilon-ex}{2T}.$$

[1)]I take the opportunity to point out an error in Ref. 1. The currents j_0, j_1^a, and j_2 actually do not depend on T under the conditions $v_F/d \ll eV \ll \Delta$ and $T \ll \Delta$. Accordingly Eqs. (4.29) and (4.31) of Ref. 11 take the form

$$j_1^a(T \gg v_F/d) = j_1^a(T=0), \qquad (4.29)$$

$$j_0 = V/R, \qquad (4.31)$$

and the coefficient η in Ref. 12 [Eqs. (5) and (7)] is $\eta = 2$ and is independent of temperature.

[2)]Strictly speaking, this statement pertains only to the time-independent current component. We shall deal further with this question below.

[3)]The currents I_1 and I_2 for bridge structures at $eV \gg \Delta$ were investigated independently in a very recent paper.[18] The expression given there for I_2 differs little from (34).

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