Perturbation transformations on a fast shock wave in a longitudinal magnetic field

S. F. Pimenov

Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences
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The transformation coefficients of low-amplitude magnetohydrodynamic and entropy waves in a rapid shock wave moving in a longitudinal magnetic field are investigated. Cases of appreciable wave amplification are considered. It is shown that in a weak homogeneous magnetic field it is possible to enhance the field's perturbations considerably.

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Perturbations can change their spatial and temporal scales considerably and also be significantly amplified in interactions with shock waves. Transformations of a perturbation by a shock wave were first considered by Blokhintsev,[1] and also by Kontorovich in ordinary hydrodynamics and magnetohydrodynamics.[2–4] A numerical study of the transformation coefficients in magnetohydrodynamics was carried out by J. F. McKenzie and K. O. Westphal.[5–7]

Compact expressions are derived in the present paper for the transformation coefficients of magnetohydrodynamic waves in the case of oblique incidence on a fast shock wave in a longitudinal magnetic field. The dependence of the transformation coefficients on the angle of incidence is investigated analytically. The possibility of significant amplification of the perturbations of the magnetic field in supernova fragments via excitation of vortex motions in a shock wave is discussed.

1. STATEMENT OF THE PROBLEM

We shall consider a shock wave in a system of coordinates in which the unperturbated discontinuity is at rest and coincides with the *yz* plane. We shall assume the plasma on both sides of the discontinuity to be an ideal conducting liquid, which we shall describe by the equations of ideal magnetohydrodynamics.[8] In the case of small perturbations that is of interest to us, we shall limit ourselves to a linear approximation; we set $\partial/\partial z = 0$, and take the dependence of the perturbations on the time t and the coordinate y in the form $\exp\{-i(\omega t - qy)\}$. On each side of the discontinuity, the perturbations can be represented in the form of the superposition of families of waves: entropy, two slow magnetosonic, two fast magnetosonic, and two Alfvén waves, which, in correspondence with the direction of the group velocity,[4] divide into waves incident on the discontinuity and into waves going out from it. According to the evolutionarity condition[9] there are only incident waves in front of the fast shock wave ($x < 0$) and only a single fast magnetosonic wave behind the shock wave. This latter is an incident wave, the others are outgoing waves.

Perturbation of the velocity and the magnetic field in the *xy* plane, and also perturbations of the entropy and pressure, are due only to the entropy and magnetosonic waves. We express these perturbations in terms of the amplitude of the waves δA_{il} and the polarization vector $\mathbf{y}_{il}$:

$$\left(\frac{\delta p_{il}}{\rho_i v_i^2}\quad \frac{\delta v_{xil}}{v_i},\ \frac{\delta v_{yil}}{v_i},\ \frac{\delta B_{xil}}{(4\pi\rho_i)^{1/2} v_i},\ \frac{T_i \delta\sigma_{il}}{v_i^2}\right) \equiv \delta A_{il}\mathbf{y}_{il}. \qquad (1)$$

The index i here is equal to unity in the region in front of the shock wave, and is absent in back of the shock; l denotes the type of wave; ρ_i, v_i and T_i are the unperturbed density, flow velocity, and temperature, respectively; δp_{il}, $\delta \mathbf{v}_{il}$, δB_{xil}, $\delta\sigma_{il}$ are the perturbations, created by the l-th wave, of the pressure, velocity, magnetic field and entropy.

The polarization vector $\mathbf{y}_{il}$ is determined from the linearized equations of magnetohydrodynamics:

$$\hat{A}_i \mathbf{y}_{il} = k_{xil} \hat{B}_i \mathbf{y}_{il}, \qquad (2)$$

where k_{xil} is the derivative of the component of the wave vector, and the matrices $\hat{A}_i$, $\hat{B}_i$ have the form

$$\hat{A}_i \equiv \begin{pmatrix} M_i^2/v_i & 0 & -Q_i/v_i & 0 & r_i v_i/\rho_i T_i \\ -Q_i & 0 & 1 & -Q_i/M_{ai} & 0 \\ M_i^2 & 1 & -Q_i(1-M_{ai}^{-2}) & -M_{ai}^{-1} & r_i v_i^2/\rho_i T_i \\ 0 & 0 & -Q_i/v_i^{1/2} M_{ai} & v_i^{-1/2} & 0 \\ 0 & v_i & 0 & 0 & v_i \end{pmatrix}, \qquad (3)$$

$$\hat{B}_i \equiv \begin{pmatrix} M_i^2 & 1 & 0 & 0 & r_i v_i^2/\rho_i T_i \\ 0 & 0 & v_i(1-M_{ai}^{-2}) & v_i/Q_i M_{ai} & 0 \\ v_i(M_i^2+1) & 2v_i & 0 & -v_i/M_{ai} & r_i v_i^3/\rho_i T_i \\ 0 & 0 & 0 & v_i^{1/2} & 0 \\ v_i^2 & v_i^2 & 0 & 0 & v_i^2 \end{pmatrix}. \qquad (4)$$

Here $M_i = v_i/s_i$ is the Mach number, $M_{ai} = v_i/v_{axi}$ is the magnetic Mach number, $v_{axi} = B/(4\pi\rho_i)^{1/2}$ is the Alfvén velocity,

$$s_i^2 = \left(\frac{\partial p_i}{\partial \rho_i}\right)_{\sigma_i}, \quad Q_i = \frac{q v_i}{\omega}, \quad r_i = \left(\frac{\partial \rho_i}{\partial \sigma_i}\right)_{p_i}.$$

Equation (2) determines the polarization vectors $\mathbf{y}_{il}$ to within a multiplicative factor. We represent its solutions for the entropy waves in the form

$$\mathbf{y}_{i\sigma} = (0,\ 0,\ 0,\ 0,\ 1), \qquad (5)$$

and for the magnetosonic waves,

$$\mathbf{y}_{il} = \left(1,\ \frac{1-\nu}{\nu},\ \frac{M_i^2\nu^2-(1-\nu)^2}{\nu Q_i},\ \frac{M_i^2\nu^2-(1-\nu)^2}{M_{ai}\nu^2},\ 0\right)\Bigg|_{\nu=\nu_{il}}, \qquad (6)$$

where $\nu_{il} \equiv (\omega - k_{xil} v_i)/\omega$ is the normalized frequency of the waves in the system of coordinates of the plasma at rest.

Perturbations of the projections of the velocity and the magnetic field intensity on the x axis are due exclusively to

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the Alfvén waves, which we shall describe with the help of the amplitudes δB_{zi+} and δB_{zi-} in correspondence with the dispersion law $\omega = k_{xia} v_i [1 \pm 1/M_{ai}]$.

Waves departing from the discontinuity appear obviously as the result of the interaction of the incident waves with the discontinuity. This interaction takes place with conservation of the projections of the mass and energy fluxes and momentum-flux tensor normal to the discontinuity f. The normal component of the magnetic field intensity and the tangential component of the electric field intensity are also continuous.[10] This means that the surface of the discontinuity itself undergoes oscillations near the zy plane under the action of the perturbations. These oscillations have the form

$$R(y,t)=\eta\exp[-i(\omega t-qy)], \tag{7}$$

where $R(y,t)$ is the deviation of the surface element of the discontinuity from the zy plane along the x axis, and η is the amplitude of the oscillations. From the conservation conditions at the discontinuity,[10] we arrive at the following set of equations which connect the amplitudes of the incident and departing (entropy and the magnetosonic) waves:

$$\sum_l \hat{B}\delta A_l \mathbf{y}_l-\boldsymbol{\alpha}\eta=-\hat{B}\delta A_{\rm inc}\mathbf{y}_{\rm inc}+\sum_m \hat{B}_1\delta A_{1m}\mathbf{y}_{1m}. \tag{8}$$

Here $\hat{B}$ and $\hat{B}_1$ are defined in (4), the sum over l contains only the departing waves, $\delta A_{\rm inc}$ and $\mathbf{y}_{\rm inc}$ are the amplitude and polarization vector of the incident wave in the region $x>0$, and the vector $\boldsymbol{\alpha}$ is equal to

$$\boldsymbol{\alpha}\equiv i\omega\{v\}\left(\frac{1}{vv_1},\ -\frac{Q}{v},\ 0,\ 0,\ -1\right),\quad \{v\}=v-v_1. \tag{9}$$

2. TRANSFORMATION COEFFICIENTS. GENERAL PROPERTIES

The equations for the amplitudes of the Alfvén waves are separable; the perturbations of the magnetic fields in the transmitted and incident Alfvén waves are connected in the following way:

$$\delta B_{z+}={}^1/_2(M_a+1)^{-1}[\delta B_{z1+}(\sqrt{\varkappa}+1)(M_a\sqrt{\varkappa}+1)+\delta B_{z1-}(\sqrt{\varkappa}-1)(M_a\sqrt{\varkappa}-1)],$$

$$\delta B_{z-}={}^1/_2(M_a-1)^{-1}[\delta B_{z1+}(\sqrt{\varkappa}-1)(M_a\sqrt{\varkappa}+1)+\delta B_{z1-}(\sqrt{\varkappa}+1)(M_a\sqrt{\varkappa}-1)], \tag{10}$$

where $\varkappa$ is the compressibility in the shock wave.

From Eq. (8) we obtain for the amplitudes δA_i of the waves departing from the discontinuity

$$\delta A_l=\sum_m \Pi_{lm}\delta A_{1m}+O_l\delta A_{\rm inc}. \tag{11}$$

In the solution of (8) by Cramer's rule, the transformation coefficients Π_{lm} and O_l turn out to be functions[6] of the wave vectors k_{xj-} k_{xs+}, k_{sx-} of all three magnetosonic waves departing from the discontinuity (the angular dependence.) This complicates the analysis of the transformation coefficients, since the longitudinal wave vectors of the magnetosonic waves are the roots of the dispersion equation

$$\det\|\hat{A}-k_{xl}\hat{B}\|=0$$

which is of the fourth degree, and has the following form in the variables Q,ν:

$$Q^2=\frac{(M^2\nu^2-(1-\nu)^2)(M_a^2\nu^2-(1-\nu)^2)}{(M^2+M_a^2)\nu^2-(1-\nu)^2}. \tag{12}$$

The function $Q^2(\nu)$ is shown in Fig. 1, where we have used the notation

$$\nu_{g1}=-(M_a-1)^{-1},\quad \nu_{g2}=-[(M^2+M_a^2)^{1/2}-1]^{-1},$$

$$\nu_{g3}=[(M^2+M_a^2)^{1/2}+1]^{-1},$$

$$\nu_{g4}=(M_a+1)^{-1},\quad \nu_{g5}=(1+M)^{-1},\quad \nu_{g6}=(1-M)^{-1},$$

and ν_{k_1}, ν_{k_2}, ν_{k_3},ν', ν'' are multiple roots. However, Π_{lm} and O_l can be represented as functions of only two roots of the dispersion equation (12), ν_l and $\nu_{\rm inc}$. Actually, let the basis vectors $\mathbf{z}_l$, $\mathbf{z}_{\rm inc}$ be mutually orthogonal to the basis vectors $\mathbf{y}_l$, $\mathbf{y}_{\rm inc}$.[11] Multiplying (8) by the vector $\mathbf{t}_{\rm inc}\equiv\hat{B}^{-1*}\mathbf{z}_{\rm inc}$, we find the amplitude of the oscillations of the surface of discontinuity η:

$$\eta=(\mathbf{t}_{\rm inc}\boldsymbol{\beta})/(\mathbf{t}_{\rm inc}\boldsymbol{\alpha}),\quad \boldsymbol{\beta}\equiv\sum_m \hat{B}_1\delta A_{1m}\mathbf{y}_{1m}-\hat{B}\delta A_{\rm inc}\mathbf{y}_{\rm inc}. \tag{13}$$

Multiplying (8) by $\mathbf{t}_l\equiv B^{-1*}\mathbf{z}_l$, we find the amplitude of the outgoing wave δA_l:

$$\delta A_l=\frac{(\mathbf{t}_l\boldsymbol{\beta})(\mathbf{t}_{\rm inc}\boldsymbol{\alpha})-(\mathbf{t}_l\boldsymbol{\alpha})(\mathbf{t}_{\rm inc}\boldsymbol{\beta})}{(\mathbf{t}_{\rm inc}\boldsymbol{\alpha})(\mathbf{t}_l\hat{B}\mathbf{y}_l)}. \tag{14}$$

The biorthogonal-basis vectors $\mathbf{z}_l$, $\mathbf{z}_{\rm inc}$ are determined with the help of the usual procedures;[12] then the equation for the vectors $\mathbf{t}_l$, $\mathbf{t}_{\rm inc}$ takes the form

$$\hat{A}^*\mathbf{t}_l=k_{xl}^*\hat{B}^*\mathbf{t}_l. \tag{15}$$

Thus, the equations (2), (14), and (15) determine the transformation coefficients Π_{lm} and O_l as functions of $k_{xl}(k_{x\,\rm inc}),k_{x\,\rm inc}$ or $\nu_l(\nu_{\rm inc}),\nu_{\rm inc}$:

$$O_l=\frac{(\mathbf{t}_l\boldsymbol{\alpha})(\mathbf{t}_{\rm inc},\hat{B}\mathbf{y}_{\rm inc})}{(\mathbf{t}_{\rm inc}\boldsymbol{\alpha})(\mathbf{t}_l,\hat{B}\mathbf{y}_l)},$$

in

$$\Pi_{lm}=\frac{(\mathbf{t}_{\rm inc}\boldsymbol{\alpha})(\mathbf{t}_l,\hat{B}_1\mathbf{y}_{1m})-(\mathbf{t}_l\boldsymbol{\alpha})(\mathbf{t}_{\rm inc},\hat{B}_1\mathbf{y}_{1m})}{(\mathbf{t}_{\rm inc}\boldsymbol{\alpha})(\mathbf{t}_l,\hat{B}\mathbf{y}_l)}. \tag{17}$$

The multiplier $(\mathbf{t}_l,\ \hat{B}\mathbf{y}_l)=(\mathbf{z}_l\mathbf{y}_l)$ in the denominators of (16) and (17) does not vanish in the general case. The dispersion equation (12) can have multiple roots (Fig. 1) $\nu_l=\nu_j$, and then $(\mathbf{t}_l,\hat{B}\mathbf{y}_l)\to-(\mathbf{t}_j,\hat{B}\mathbf{y}_j)\to 0$. If both the l and j waves are outgoing (the points ν' and ν'', $\operatorname{Im}k_{xl}=\operatorname{Im}k_{xj}>0$ in the case $\operatorname{Im}\omega>0$), then $O_l\to-O_j\to\infty$ and $\Pi_{lm}\to-\Pi_{jm}\to\infty$; however, the infinite terms cancel one another in the summation over the modes; for example,

$$O_l\exp(ik_{xl}x)+O_j\exp(ik_{xj}x)\neq\infty,$$

i.e., the perturbations of the physical quantities are bounded. In the case in which one of the waves l is outgoing, and the other is an incident wave ($\nu_l=\nu_{\rm inc}$) the coefficients O_l and Π_{lm} are themselves bounded by virtue of the vanishing of the numerators of (16) and (17).

The transformation coefficents become infinite at $(\mathbf{t}_{\rm inc}\,\boldsymbol{\alpha})=0$, which then gives the following condition:

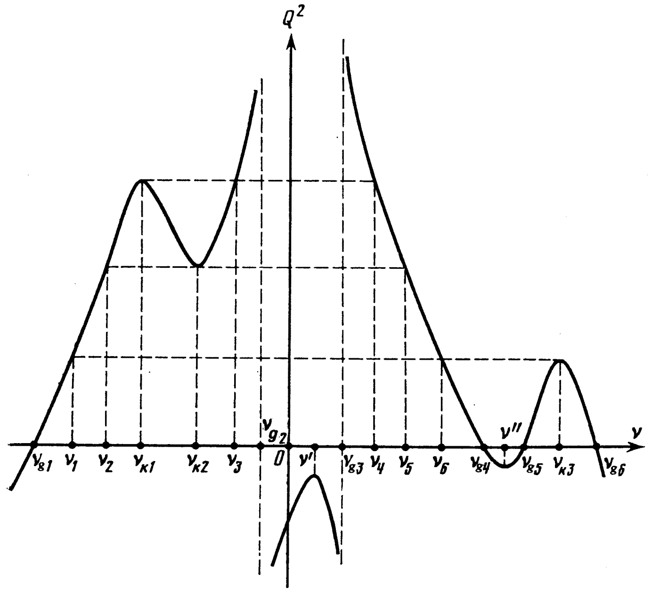

FIG. 1. Dispersion curve (12).

$$\left[J^2\left(\frac{\partial V}{\partial p}\right)_H+1\right]\{1+\varkappa[M^2\nu_{\rm inc}^2-(1-\nu_{\rm inc})^2]\}$$
$$-2\nu_{\rm inc}(1-M^2)=0. \tag{18}$$

Here J is the mass flow across the discontinuity, $(\partial V/\partial p)_H$ is the derivative of the specific volume $V=1/\rho$ with respect to the pressure p along the shock adiabat. Satisfaction of (18) at $\operatorname{Im}\omega=0$ corresponds to spontaneous radiation of waves by the discontinuity[13,14]; at $\operatorname{Im}\omega>0$ it corresponds to absolute instability.[15] In an ideal gas with constant heat capacity, the spontaneous radiation of waves does not take place in ordinary hydrodynamics; however, a sufficiently strong magnetic field leads to such a possibility.[16]

We note here another region of spontaneous radiation of the waves by the discontinuity (not noted in Ref. 16). It arises in the case that the roots of Eq. (18) fall in the interval $\nu_{kl}<\nu_{\rm inc}<\nu_{k2}$ (Fig. 1). Then $\operatorname{Im}\omega=\operatorname{Im}(qv/Q(\nu_{\rm inc}))=0$ while the projection of the group velocity $v_{{\rm gr}\,x}=d\omega/dk_{x\,{\rm inc}}$ is negative, which corresponds to the spontaneous radiation of waves by the discontinuity.[16] From (18) we obtain

$$-1+\frac{2\nu_{k1}(1-M^2)}{1+\varkappa[M^2\nu_{k1}^2-(1-\nu_{k1})^2]}$$
$$<J^2\left(\frac{\partial V}{\partial p}\right)_H<-1$$
$$+\frac{2\nu_{k2}(1-M^2)}{1+\varkappa[M^2\nu_{k2}^2-(1-\nu_{k2})^2]}. \tag{19}$$

The left limit of (19) is negative but greater than -1, while the right does not exceed $-M^2$. In the general case, the region (19) borders on regions of absolute instability of the shock wave. In extremely intense magnetic fields $(M_a\to 1)$, the left limit coincides with the boundary of absolute instability and is equal to -1. In an ideal gas with constant heat capacity, the inequality (19) is not satisifed.

In a stable shock wave, the quantity $(\mathbf{t}_{\rm inc}\,\alpha)$ does not vanish in the region $\operatorname{Im}\omega>0$, $\operatorname{Im}k_{x\,{\rm inc}}\leqslant 0$ (Fig. 2), but can be sufficiently small if one of the roots of Eq. (18) $\nu_\pm$ is located near the boundary of this region, in particular, in the case $\nu_\pm\to\nu_{k3}(\nu_{k2},\nu_{k1}$. In the latter case, the reflection coefficients O_l are as before bounded by virtue of the multiplier $(\mathbf{t}_{\rm inc},\hat{B}\mathbf{y}_{\rm inc})$ which vanishes in the case $\nu_{\rm inc}=\nu_{k3}(\nu_{k1},\nu_{k2})$.

The transformation coefficients Π_{lm} can be large at $\nu_{\rm inc}\to\nu_{k3}(\nu_{k1},\nu_{k2})$ while at sufficiently large distance from $\mathbf{t}_{\rm inc}$ (17) simplifies to

$$\Pi_{lm}\approx-\frac{(\mathbf{t}_l\alpha)(\mathbf{t}_{\rm inc},\hat{B}_l\mathbf{y}_{lm})}{(\mathbf{t}_{\rm inc}\alpha)(\mathbf{t}_l,\hat{B}\mathbf{y}_l)}\approx\frac{\rm const}{\nu_{\rm inc}-\nu_\pm}. \tag{20}$$

Moreover, the reflection and refraction coefficients become proportional to one another:

$$O_l/O_j\approx\Pi_{lm}/\Pi_{jm}. \tag{21}$$

In an ideal gas with constant heat capacity Π_{lm} is large in two cases. First, in a strong shock wave (as $M_1\to\infty$) and in a bounded magnetic field B when the larger root of (18), ν_+ and the boundary of the real $\nu_{\rm inc}$ tend to the same limit $\nu=(1-M_{\rm min}^2)^{-1}$ from different sides (a similar situation occurs in ordinary hydrodynamics). Second, in a sufficiently strong magnetic field in the case of bounded Mach numbers $M_1\to M_{1\,{\rm cr}}(B_{\rm cr})$. Here $M_{1\,{\rm cr}}\sim B_{\rm cr}^2/4\pi n_1T_1$ is the smallest of the Mach numbers at which the shock wave is absolutely stable (that is, spontaneous radiation of waves does not occur[16]).

3. REFLECTION OF A FAST MAGNETOSONIC WAVE

For the medium we take the equation of state of an ideal gas with constant heat capacity and give the expression for the transformation coefficients O_l.

The departing waves in the region $x>0$ can be traveling and surface waves, depending on the angles of incidence of the waves at the discontinuity. The traveling waves $(\operatorname{Im}\omega=0,\ \operatorname{Im}_{x\,l}=0)$ correspond to portions of the real axis (Fig. 1). In the general case there exist two regions of real values of $\nu_{\rm inc}$. The region $\nu_{k3}<\nu_{\rm inc}<\nu_{g6}$ arises in the case of

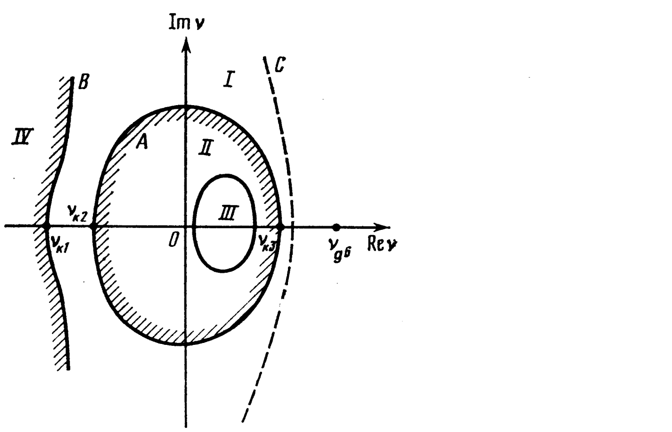

FIG. 2. Regions of values of the roots of the dispersion equation in the complex plane: I—incident waves ($\operatorname{Im}k_x<0$, $\operatorname{Im}\omega>0$); II-IV—outgoing waves; I and II—fast magnetosonic waves (ν_f, $\nu_{\rm inc}$); III and IV—slow magnetosonic waves ($\nu_{s\pm}$). In a weak magnetic field, I is bounded by the curve C, III does not change qualitatively, II and IV are separated by a cut.

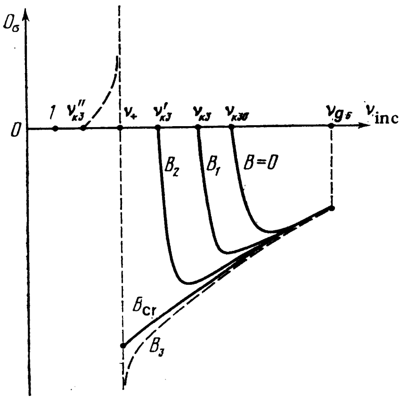

FIG. 3. Dependence of the transformation coefficient O_σ on ν_{inc}.

arbitrary values of the magnetic field ($M_a > 1$). The roots of (12), corresponding to the departing waves, lie in the intervals $\nu_{r5} < \nu_f < \nu_{k3}$ (fast), $\nu_6 < \nu_{s+} < \nu_{r4}$ and $\nu_{ri} < \nu_{s-} < \nu_1$ (slow). The region $\nu_{k1} < \nu_{inc} < \nu_{k2}$ is possible only in sufficiently strong magnetic fields; the roots corresponding to the departing waves lie in the intervals $\nu_{k2} < \nu_f < \nu_3$, $\nu_2 < \nu_{s-} < \nu_{k1}$, $\nu_4 < \nu_{s+} < \nu_5$. The boundaries of the regions I–IV in the complex ν plane correspond to surface waves ($\text{Im}\,\omega = 0$, $\text{Im}\,k_{xl} = 0$) correspond to the boundaries of the regions I–IV in the complex plane ν (Fig. 2).

A fast magnetosonic wave incident on the discontinuity from the region $x > 0$ excites upon reflection an entropy, a fast, and two slow magnetosonic waves. The entropy wave is characterized by the transformation coefficient O_σ:

$$O_\sigma = 2(\varkappa_m - 1)\{(M^2\nu + 1 - \nu)[M_a^2\nu^2 - (1-\nu)^2] \times [(M_a^2 + M^2)\nu^2 - (1-\nu)^2] + M^2\nu(1-\nu)[M^2\nu^2 - (1-\nu)^2]\}\{[(\varkappa_m - 1 + M^2)(\nu-1)^2 - M^2] \times M_a^2\nu^2[M_a^2\nu^2 - (1-\nu)^2]\}^{-1}|_{\nu=\nu_{inc}}. \tag{22}$$

Here $\varkappa_m = (\gamma+1)/(\gamma-1)$ is the maximum compressibility in the shock wave, γ is the adiabatic coefficient. For traveling waves, the modulus of O increases monotonically with the magnetic field but remains bounded in the region of stability. Qualitatively, the properties of O are shown in Fig. 3 for the region $\nu_{k3} < \nu_{inc} < \nu_{g6}$. At normal incidence ($\nu_{inc} = \nu_{g6}$) the coefficient O_σ does not depend on the magnetic field. At arbitrary ν_{inc} the modulus of O_σ does not exceed

$$(\nu_{k3} - \nu_+)^{-1}[(\nu - \nu_+)O_\sigma(\nu)]_{\nu=\nu_+}.$$

The fields in the drawing haves values $B_1 < B_2 < B_{cr} < B_3$, where B_{cr} corresponds to the boundary of the instability.

The magnetosonic waves are described by the transformation coefficients O_l which can be expressed in terms of the O_σ in the following way:

$$O_l = O_\sigma(\nu_{inc})/O_\sigma(\nu_l). \tag{23}$$

We note that at $\nu_{inc} \to \nu_{kj}$ ($j = 1,2,3$) only a single outgoing wave is formed, l, for which $\nu_l \to \nu_{kl}$ (Fig. 1). Here $O_\sigma \times (\nu_{inc} \approx -O_\sigma(\nu_l) \to 0$, since the expression in the first curly brackets in (22) is proportional to the derivative $dQ^2/d\nu$. Then $Q_l = -1$ while $O_p = 0$ at $p \neq l$.

In a weak magnetic field ($M_n \to \infty$) the ratio of the transformation coefficient O_l to the corresponding coefficient in ordinary hydrodynamics O_{j0} is given in the case of equal ν_{inc} by

$$\frac{O_f}{O_{f0}} = 1 + \frac{Q^2}{M_a^2}\left(\frac{1}{\nu_{inc}^2} - \frac{1}{\nu_f^2}\right), \tag{24}$$

which is less than unity because $\nu_{inc} > \nu_j$.

For slow magnetosonic waves in the case $M_a \to \infty$ we get

$$O_{s\pm} = \nu_{s\pm}\frac{(\varkappa_m - 1)(M^2\nu_{inc} + 1 - \nu_{inc})Q^2}{[(\varkappa_m - 1 + M^2)(\nu_{inc} - 1)^2 - M^2](Q^2 + 1)}, \tag{25}$$

$$\nu_{s\pm} \to \pm\frac{1}{M_a},$$

i.e., the perturbation of the pressure in the reflected slow magnetosonic waves is much less than in the incident wave. As is seen from (6), the perturbations of the magnetic fields are the following:

$$\frac{\delta B_{xs\pm}}{\delta B_{x\,inc}} = -\frac{(\varkappa_m - 1)(M^2\nu_{inc} + 1 - \nu_{inc})\nu_{inc}^2}{\nu_{s\pm}[(\varkappa_m - 1 + M^2)(\nu_{inc} - 1)^2 - M^2](Q^2 + 1)}. \tag{26}$$

However, near the shock wave ($x = 0$) the perturbations δB_{xs+} and δB_{xs-} cancel one another, since they are opposed in phase: $\delta B_{xs+} + \delta B_{xs-} = -\delta B_{x\,inc}$. Since the waves $s\pm$ have identical frequency ω and differ little in the wave vectors $k_{xs\pm} \approx (\omega/v_x(1 \pm M_a^{-1})$, a significant phase lag takes place at the distance $L \sim (\pi v_x/2\omega)M_a$. The components $\delta B_{xs\pm}(L)$ will have identical sign and the perturbation of the field becomes large. Behind the shock wave a picture of spatial beats of the perturbations of the magnetic field develops (a component of the perturbation of the velocity $\delta\mathbf{v}$ beats in antiphase). In this sense we can speak of a significant amplification of the perturbations of the magnetic field of the shock in a weak uniform magnetic field.

4. PASSAGE OF THE WAVES ACROSS THE DISCONTINUITY

The transformation coefficients Π_{lm} (17) are in the general case more complicated functions of the wave vectors than O_l. Here we give the relation between the perturbation $(\delta T_1/T_1)_e$ of the temperature in an incident entropy wave and the perturbation $\delta B_{xs\pm}$ of a magnetic field by the magnetosonic waves excited by it:

$$\frac{\delta B_{xs\pm}}{B_x(\delta T_1/T_1)} = \mp M_a\frac{(\varkappa - 1)Q^2}{2(1+Q^2)} \times \left\{1 + \frac{1 - \varkappa + (\nu_{inc} - 1)\varkappa(\varkappa_m - 1)}{\varkappa\varkappa_m(\nu_{inc} - 1)^2 - 1}\right\}. \tag{27}$$

This transformation coefficient is a monotonically decreased function (in amplitude) of ν_{inc} in the interval $\nu_{k3} < \nu_{inc} < \nu_{g6}$. The physical picture behind the shock in this case is the same as in the case of reflection. i.e., $(\delta B/B)/(\delta T_1/T_1) \sim M_a$. Significant amplification of the magnetic field develops upon incidence of fast magnetosonic waves on the discontinuity from the region $x < 0$. We note also that similar phenomena should be observed at any orientation of the magnetic field.

We now consider the possibility of this mechanism of excitation and amplification of the perturbations of the magnetic field under astrophysical conditions. According to Ref. 17, the galactic magnetic field and the mean concentration of particles in interstellar space amount to $B \sim (2\text{–}3)10^{-6}$ Oe and $n_1 \sim 1\ \text{cm}^{-3}$. From the condition $M_a \gg 1$ we obtain $v_1 \gg B/(\pi\rho_1)^{1/2} \approx 1.1\times10^6$ cm/s, which is generally achievable in supernova fragments. The direct application to them of the results set forth above is connected with two conditions: 1) The emission of energy from the shock wave should be negligible, a condition satisfied in the adiabatic phase of expansion; 2) the width of the shock front δ_f should be significantly less that the radius of the relic R_S.

We shall use a model of a supernova with ejection of mass $\approx 4M_\odot$ and initial velocity 5×10^8 cm/s with typical density and temperature of interstellar space $n_1 \sim 1\ \text{cm}^{-3}$, $T_1 \sim 10^2\text{–}10^4$ K.[18] The adiabatic phase in this case begins at a velocity $v_{1b} \approx 5\times10^8$ cm/s and ends at $v_{1e} \approx 1.7\times10^7$ cm/s (when the temperature behind the shock front drops to 10^6 K). Condition 2) begins to be fully satisfied at expansion velocities $v_1 \leqslant 7\times10^7$ cm/s. At this stage, the radius of the relic $R_S \approx 12$ pc, the temperature behind the front $T_S = (3/2)mv_1^2/k_B \approx 1.3\times10^{-9}v_1^2 \approx 6.4\times10^6$ K; consequently, the free path length is $\Lambda_i = 2\times10^4 T_S^2/n \approx 2\times10^{17}$ cm $\approx 7\times10^{-2}$ pc, $\delta_\phi \sim \Lambda_i \ll R_S$. The relic slows its motion from a speed of $v_1 \approx 7\times10^7$ cm/s to $v_{1e} \approx 2.7\times10^7$ cm/s in a time of 2.4×10^4 yrs, which is about 80% of the adiabatic stage.

The magnetic Mach number at a speed of expansion $v_1 \sim 5\times10^7$ cm/s in a magntic field $B = 3\times10^{-6}$ Oe amounts to $M_a = v_1(\pi\rho_1)^{1/2}/B \approx 38$ which assures a significant gain in amplification (excitation) of the perturbations ofthe magnetic field. At a temperature of interstellar space $T_1 \sim 10^4\text{–}10^2$ K, we get from (27)

$$\left[\frac{\delta B_{xs}}{B_x(\delta T_1/T_1)}\right]_{\max} \approx 1.2M_a \approx 0.45\cdot10^2,$$

i.e., relative perturbations of the temperature of the order of 10^{-2} can lead to a significant ($\delta B/B \sim 1$) perturbation of the magnetic field.

An observational test of the given mechanism of formation of inhomogeneities of the magnetic field can be provided by a dual structure of radio details. Actually, when a shock front passes through a hot (or cold) region, vertical perturbations of the velocity are formed behind it, compressed by a factor of 4 along the x axis. This perturbation in a weak magnetic field is the superposition of two wave packets of the slow magnetosonic waves, moving with different group velocities $v_{gr} = v_x \pm v_{ax}$. Consequently, the packets move apart within a certain time (by 1 pc within 5×10^4 yr) and form two details with amplified perturbation of the magnetic field $\pm\delta B$.

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