Scattering of light by electrons of metals and semimetals with complex Fermi surfaces

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The scattering of light by conduction electrons of metals with anisotropic Fermi surfaces is considered over a broad range of scattering frequencies $\omega = \omega_i - \omega_s \gtrsim v_F/\delta$, restricted only by the condition $\omega \gg \nu$, where ν is the electron relaxation frequency and δ is the penetration depth of light into the metal. It is shown that detailed analysis of the frequency dependence of the light-scattering cross section can give Fermi-surface-structure information that supplements data obtained by other methods.

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The scattering of light by electrons in metals (in the normal and superconducting states) has been studied theoretically[1,2] in the range of low scattering frequencies $\omega = \omega_i - \omega_s \gtrsim v_F/\delta$. The frequencies of the incident ω_i and scattered ω_s radiations belong to the optical range: $\delta \approx c/\omega_p$ is the light penetration depth, $\omega_p^2 = 4\pi ne^2/m_0$ is the square of the plasma frequency. In this case, it follows from the conservation laws that only electrons from a strip of the Fermi surface (FS) take part in the scattering:

$$\mathbf{nv}=0, \quad \varepsilon(\mathbf{p})=\varepsilon_F, \quad \mathbf{v}=\partial\varepsilon(\mathbf{p})/\partial\mathbf{p}, \tag{1}$$

where $\mathbf{n}$ is the direction of the normal to the surface of the sample. This leads to anisotropy of the scattering cross section: upon change of orientation of the surface of the sample, i.e., of $\mathbf{n}$, the strip moves over the FS. In this case the dependence of the cross section on the polarization of the incident and scattered light is due to the anisotropy of the effective mass of the electrons of the strip (1) on the FS. However, the magnitude of the cross section in the low-frequency region is small, which makes experimental study of the effect difficult.

In the present work we consider the scattering of light by the conduction electrons of a metal with an anisotropic FS over a wide range of scattering frequencies $\omega \lesssim v_F/\delta$, restricted only by the condition $\omega \gg \nu$, where ν is the relaxation frequency of the conduction electrons. The scattering cross section is maximal at frequencies $\omega \approx v_F/\delta$. In this region of frequencies, it is determined by the contribution of all the electrons of the FS and can exceed in order of magnitude the cross section of electron scattering in semiconductors (in which, in contrast to metals, the phenomenon has been well studied[3]). A detailed analysis of the frequency dependence of the scattering cross section gives information on the structure of the FS, supplementing data obtained by other methods.

1. The expression for the differential cross section for light scattering by conduction electrons in a metal $d^2\sigma/d\omega d\Omega$ has been obtained in Refs. 1 and 2 for the case of specular reflection of electrons from the surface of a metal and under the condition $\nu \to 0$. A more general derivation of the cross section, taking into account, in particular, the volume and surface scattering of the electrons, is given in the Appendix. The cross section can be written down in the form

$$\frac{d^2\sigma}{d\omega d\Omega} = \frac{1}{2\pi}\frac{\omega_i}{\omega_s} r_0^2 J_\omega, \tag{2}$$

where $d\Omega$ is the element of solid angle, $r_0 = e^2/m_0c^2$ is the classical radius of the electron, m_0 is the mass of the free electron, and

$$J_\omega = S\frac{2\hbar}{1-\exp(-\hbar\omega/T)} R_{\alpha\beta}R_{\gamma\delta}^* \int_{-\infty}^{+\infty}\frac{dk}{2\pi} W_{\alpha\beta\gamma\delta}(k,\omega)\,U^2(k). \tag{3}$$

Here S is the area of the metal surface; the function $U(k)$ is determined by the field distribution of the incident and scattered waves in the metal. In the case of a simple exponential dependence of the fields on the z coordinate (chosen along the normal to the surface into the metal) $U(k)$ has the form

$$U(k) = 4\delta[4+(k\delta)^2]^{-1}. \tag{4}$$

The factors $R_{\alpha,\beta}$ are expressed in terms of the projections of the unit polarization vectors of the incident e_α^i and scattered e_β^s waves and the coefficients $C(\alpha,\beta)$, which are determined with the help of the Fresnel formulas. Thus, for example,

$$R_{xx} = e_x^i (e_x^s)^* C(x,x),$$
$$C(x,x) = \left(\frac{2\cos\varphi}{[\varepsilon-\sin^2\varphi]^{1/2}+\cos\varphi}\right)\left(\frac{2\cos\varphi'}{[\varepsilon-\sin^2\varphi']^{1/2}+\cos\varphi'}\right)^*. \tag{5}$$

Here φ and φ' are angles of incidence and reflection. In what follows, we limit ourselves to the case of almost normal incidence and scattering, $\varphi', \varphi'' \ll 1$, for which all the coefficients $C(\alpha,\beta)$ are identical:

$$C(\alpha,\beta) = \frac{4}{|\sqrt{\varepsilon}+1|^2} = 4[(n+1)^2+\varkappa^2]^{-1},$$

where ε is permittivity and $\sqrt{\varepsilon} = n + i\varkappa$.

The quantity $W_{\alpha\beta\gamma\delta}(k,\omega)$ in (3) represents the generalized susceptibility of the electrons relative to an external field with frequency $\omega = \omega_i - \omega_s$ and wavelength $2\pi/k$; at $\hbar\omega \ll \varepsilon_F$ and $\hbar k \ll p_F$ it can be written down in the form of an integral over the FS:

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$$W_{\alpha\beta\gamma\delta}(k,\omega)=\frac{2}{(2\pi\hbar)^3}\oint\frac{dS_{\mathbf{p}}}{v}\tilde{\mu}_{\alpha\beta}(\mathbf{p},k,\omega)\tilde{\mu}_{\gamma\delta}^{*}(\mathbf{p},k,\omega)\times\operatorname{Im}\frac{k^2v_z^2}{k^2v_z^2-\omega^2-i2\omega\nu},\tag{6}$$

$$\tilde{\mu}_{\alpha\beta}(\mathbf{p},k,\omega)=\mu_{\alpha\beta}(\mathbf{p})-\oint\frac{dS_{\mathbf{p}}}{v}\mu_{\alpha\beta}(\mathbf{p})\frac{kv_z}{kv_z-\omega-i\nu}\left(\oint\frac{dS_{\mathbf{p}}}{v}\frac{kv_z}{kv_z-\omega-i\nu}\right)^{-1}$$

$$\mu_{\alpha\beta}(\mathbf{p})=m_0\frac{\partial^2\varepsilon(\mathbf{p})}{\partial p_\alpha\partial p_\beta},\tag{7}$$

where $v_z=\mathbf{n}\cdot\mathbf{v}_{\mathbf{p}}$ and $\mu_{\alpha\beta}(\mathbf{p})$ is the dimensionless tensor of the reciprocal effective electron mass. Screening of the external force acting on the electrons is taken into account in the quantity $\mu_{\alpha\beta}(\mathbf{p},k,\omega)$. In view of the relatively low frequency of the external field and its strong inhomogeneity ($k\sim1/\delta$), the susceptibility (6) has strong spatial dispersion.

2. Proceeding to the analysis of the frequency dependence of the scattering cross section, we first note that, thanks to the condition $\omega\gg\nu$, we can write

$$\operatorname{Im}\,[k^2v_z^2-\omega^2-2i\omega\nu]^{-1}=\pi\delta(k^2v_z^2-\omega^2).$$

Here the function $W_{\alpha\beta\gamma\delta}$ depends only on the ratio $\omega/k\equiv u$; this allows us to establish in general form the frequency dependence of the scattering cross section in the region of small ($\omega\ll v_F/\delta$) and large ($\omega\gg v_F/\delta$) scattering frequencies. Actually, transforming in (3) to integration over u we obtain

$$J_\omega=S\frac{16F(\omega)}{\pi[(n+1)^2+\varkappa^2]^2}\int_0^\infty du\frac{16\delta^2u^2}{[4u^2+\omega^2\delta^2]^2}W(u),\tag{8}$$

where

$$F(\omega)=\hbar\omega[1-\exp(-\hbar\omega/T)^{-1}$$

and

$$W(u)=\frac{2\pi u}{(2\pi\hbar)^3}\oint\frac{dS_{\mathbf{p}}}{v}|\tilde{\mu}^{(i,s)}|^2\delta(u-v_z),\tag{9}$$

$$\tilde{\mu}^{(i,s)}=e_\alpha^{\,i}\tilde{\mu}_{\alpha\beta}(\mathbf{p},k,\omega)(e_\beta^{\,s})^{*}.\tag{10}$$

It follows from (9) that $W(u)\propto u$ as $u\to0$; therefore $J_\omega\propto F(\omega)\ln(v_F/\omega\delta)$ at small ω. On the other hand, $W(u)=0$ at $u>v_{z\max}$; therefore at $\omega\gg v_{z\max}/\delta$ we have $J_\omega\propto F(\omega)/\omega^4$.

The analytic properties of the susceptibility (6) allow us to obtain for the scattering cross section integral expressions of sum-rule type.[4] In the case of low temperatures $T\ll\hbar v_F/\delta$ we have from the Kramers-Kronig relation.

$$\int\frac{d^2\sigma}{d\omega d\Omega}\frac{d\omega}{\omega}=\frac{16\hbar}{[(n+1)^2+\varkappa^2]^2}r_0^2\langle|\mu^{(i,s)}-\langle\mu^{(i,s)}\rangle|^2\rangle S\delta\nu(\varepsilon_F).\tag{11}$$

At high temperatures $T\gg\hbar v_F/\delta$, when $T\gg\hbar\omega$ for all important frequencies, the similar relation takes the form

$$\int\frac{d^2\sigma}{d\omega d\Omega}d\omega=\frac{16}{[(n+1)^2+\varkappa^2]^2}r_0^2\langle|\mu^{(i,s)}-\langle\mu^{(i,s)}\rangle|^2\rangle S\delta\nu(\varepsilon_F)T,$$

$$\mu^{(i,s)}=e_\alpha^{\,i}\mu_{\alpha\beta}(e_\beta^{\,s})^{*}.\tag{12}$$

Here $\nu(\varepsilon_F)$ is the density of states on the Fermi level, and averaging over the FS is denoted by the angle brackets:

$$\langle\mu^{(i,s)}\rangle=\frac{2}{(2\pi\hbar)^3\nu(\varepsilon_F)}\oint\frac{dS_{\mathbf{p}}}{v}\mu^{(i,s)}(\mathbf{p}).$$

The relation (12) has an explicit physical meaning: in the classical limit, the integrated cross section is determined by the mean square of the fluctuations of the effective mass tensor of the electrons located in the volume $S\delta$ and having energy in the range $\sim T$ near the FS. It should be noted that, just as in the initial formula (6), these relations do not take into account transitions between energy bands, i.e., the considered frequencies are given by $\hbar\omega\ll\varepsilon_g$, where ε_g is the minimum separation of the bands near the FS. Therefore the integration in (11) and (12) is actually limited to these frequencies (see Refs. 5 and 6).

3. The frequency dependence of the scattering cross section in the interval $\omega\gtrsim v_F/\delta$ is determined by the behavior of the function $W(u)$, which in turn depends on the structure of the FS as follows from its definition (9).

The relation (8) can be regarded as an integral equation for finding the function $W(u)$ from the experimentally measured differential scattering cross section. The reconstruction of the function $W(u)$ requires solution of Eq. (8) in general form. Its solution is similar to the solution of the Stieltjes equation, which is considered, for example, in Refs. 7 and 8. In place of u we introduce the dimensionless variable $u'=u/v$ (v is the characteristic value of the velocity, and we shall omit the prime in the following). The substitution

$$u=e^{\tau},\quad W(e^{\tau})e^{\tau}=\Psi(\tau),$$

$$\frac{\omega\delta}{2v}=e^{\eta},\quad\left\{\frac{\pi}{64}[(n+1)^2+\varkappa^2]^2\frac{J_\omega\omega^2v}{SF(\omega)}\right\}_{\omega=2(v/\delta)e^{\eta}}=\Sigma(\eta)\tag{13}$$

converts Eq. (8) to an equation with a difference kernel:

$$\Sigma(\eta)=\int_{-\infty}^{+\infty}\frac{d\tau}{4\operatorname{ch}^2(\eta-\tau)}\Psi(\tau),\tag{14}$$

which reduces by the Fourier transformation

$$\tilde{\Sigma}(p)=(2\pi)^{-1/2}\int_{-\infty}^{+\infty}\Sigma(\eta)e^{ip\eta}d\eta$$

to an algebraic one:

$$\tilde{\Sigma}(p)=(\pi p/\operatorname{sh}\pi p)\tilde{\Psi}(p).\tag{15}$$

Finally, the solution of Eq. (8) can be written down in the form

$$W(u)=\int_{-\infty}^{+\infty}\tilde{\Sigma}(p)\frac{\operatorname{sh}\pi p}{\pi p}u^{-ip-1}\frac{dp}{(2\pi)^{1/2}}.\tag{16}$$

It should be noted that the poor convergence of the integral in (16) does not permit us to carry out the integration in general form. For this reason, it is more convenient to have the solution of Eq. (14) in the form of a series. To find it, we use the formula

$$\frac{\operatorname{sh}\pi p}{\pi p}e^{-ip\eta}=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\left(\pi\frac{d}{d\eta}\right)^{2n}e^{-ip\eta}\equiv\hat{R}e^{-ip\eta}. \quad (17)$$

We operate on the right and left sides of Eq. (14) with the operator $\hat{R}$. Using the expansion for $\Psi(\tau)$ in a Fourier series, it is not difficult to establish the fact that the operator inverts the integral operator in the right side of (14). Carrying out the substitution inverse to (13), we obtain

$$W(u)=\frac{1}{u}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\left(\pi u\frac{d}{du}\right)^{2n}$$

$$\times\left\{\frac{\pi}{64}[(n+1)^2+\varkappa^2]^2\frac{J_\omega\omega^2 v}{SF(\omega)}\right\}_{\omega=2uv/\delta}. \quad (18)$$

Use of the formal solution of (16) or (18) for finding $W(u)$ from the experimental frequency dependence of the scattering cross section presupposes a sufficient accuracy of the measurement of the cross section. Reliability in the establishment of the function $W(u)$ can be achieved by the inclusion of the corresponding mathematical reduction of the experimental data in the measurement process itself.[9]

4. The function $W(u)$ satisfies definite integral relations with respect to u that follow, just as do (11) and (12), from the analytical properties of the generalized susceptibility (as usual, the variable has the dimension of velocity). In particular,

$$\int_0^\infty W(u)\frac{du}{u}=\pi\nu(\varepsilon_F)\langle|\mu^{(i,s)}-\langle\mu^{(i,s)}\rangle|^2\rangle. \quad (19)$$

Actually, the interval of integral in (19) should be limited by the condition $\hbar u/\delta<\varepsilon_g$ which separates the range of frequencies in which there are no interband transitions.

Analysis of the singularities of the behavior of the function $W(u)$ is similar to the analysis of the singularities of the sound absorption coefficient in Refs. 10 and 11.

According to (9), electrons of the strip $v_z=u$ make the contribution to $W(u)$ at a specific value of u. With increase in u, the strip contracts to the limiting point on the FS where $v_z=v_{z\max}$; at $u>v_{z\max}$ there is no strip and $W=0$. Singularity of $W(u)$ corresponds to disappearance (generation) of the strip. For an everywhere-convex FS, such a singularity (and, furthermore, only one) exists at arbitrary orientation of the surface of the metal. However, in real metals, the FS are irregular and have hollows, necks, and open directions, which complicate the dependence $W=W(u)$. Further, the character of the function $W(u)$ depends significantly on the direction of the normal $\mathbf{n}$ to the metal surface. Strips on FS of a dumbbell shape are shown in Fig. 1 in the case of various directions of the vector $\mathbf{n}$. Each change in the topology of the strip (at $u=u_c$) leads to a singularity in the function. As shown in Ref. 10, topological changes in the strips can be of two forms: the appearance (disappearance) of a loop of (or of the entire) strip (accompanied by an O-type singularity), break of the strip (singularity of the X-type). In the calculation of singular part of $W(u)$ it is impossible to make direct use of the results of Refs. 10 and 11, since, as a consequence of screening, the integrand in (9) vanishes at the critical point

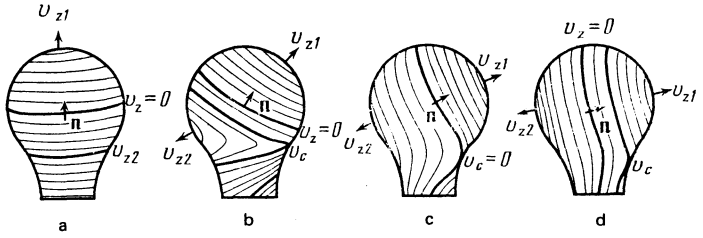

FIG. 1.

(at the point of generation of the strip or at the point of its self-intersection). Actually, the intergal

$$\langle R\rangle\equiv\frac{2}{(2\pi\hbar)^3\nu(\varepsilon_F)}\oint\frac{dS_p}{v}\frac{1}{v_z-u-i0}$$

irrespective of the type of singularity, diverges logarithmically as $u\to u_c$: if the singularity is O-type,

$$\operatorname{Re}\langle R\rangle\propto\ln|v_c/\Delta u|,$$

if it is X-type

$$\operatorname{Im}\langle R\rangle\propto\ln|v_c/\Delta u|, \quad (\Delta u=u-u_c).$$

From (7) and (10), omitting the indices, we have

$$\tilde{\mu}=\mu-(\langle\mu\rangle+u\langle\mu R\rangle)/(1+u\langle R\rangle). \quad (7')$$

The integral $\langle\mu R\rangle\to\mu_c\langle R\rangle$ as $u\to u_c$, where μ_c is the value of μ at the critical point. It is seen that $\mu\to0$ as $u\to u_c$. With the help of (7') and (9), we can calculate the singularity of $W(u)$ as $u\to u_c$.

Upon the approach of u to $v_{z\max}$ we have

$$W_{\text{sing}}(u)\approx\beta_c\nu(\varepsilon_F)\frac{[\langle\mu\rangle-\mu_c+u_c\langle(\mu-\mu_c)R\rangle]^2}{\ln^2|v_c/\Delta u|},$$
$$u_c=v_{z\max},\quad \beta_c=\left(\oint dS_p/v\right)\Big/u_cm_c^2, \quad (20)$$

here m_c is a parameter of the dimensions of mass, determined by the curvature of the FS at the critical point, near which we can introduce the set of coordinates such that $v_z=u_c\pm v_c(\xi^2\pm\eta^2)$ (in the case of a singularity of the O-type the sign is of η^2+; in the case of a singularity of the X-type, the sign is $-$). The parameter m_c is determined by the equation $dS/v_c^2=m_c^2d\xi d\eta$, in order of magnitude it is equal to the mass of the electron and can be calculated in terms of the derivatives of the components of the effective mass tensor with respect to the quasimomentum. In the general case, $\beta_c\sim1$.

If the strip has a point of self-intersection at $u=u_c$, then

$$W_{\text{sing}}(u)-W_c\approx\beta_c\nu(\varepsilon_F)\frac{[\langle\mu\rangle-\mu_c+u_c\langle(\mu-\mu_c)R\rangle]^2}{\ln|v_c/\Delta u|},$$
$$W_c=\pi\nu(\varepsilon_F)u_c\langle(\mu-\mu_c)^2\delta(v_z-u_c)\rangle. \quad (21)$$

It is seen that the derivatives of $W(u)$ to the left and right at the point $u=u_c$ are equal to $\mp\infty$. Under certain conditions (and with some "rough" approximation) the vicinity of

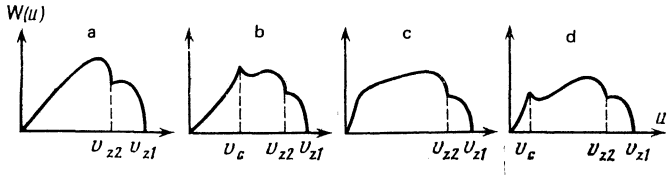

FIG. 2.

the point $u = u_c$ will be perceived as the vicinity of a sharp maximum.

We note: W_c and the coefficients of $\ln^{-2}|v_c/\Delta u|$ and $\ln^{-1}|v_c/\Delta u|$ contain integrals over the entire FS, and are not expressed in terms of the value of the electronic characteristics at the critical point.

The qualitative behavior of the function $W(u)$ for a surface of the "dumbbell" type at the orientations shown in Fig. 1, a–d of the normal to the surface relative to the axes of the dumbbell, are given here in Figs. 2, a–d.

As is seen from the foregoing, the character of the singularity of $W(u)$ depends essentially on the local geometry of the FS. For example, the existence of local flattenings on the FS should enhance the singularity. The vanishing of μ at the critical point (which led to a significant roughening of the singularity) was connected with the assumption that to each value of u_c on the FS there corresponds a single critical point. If there are several, and they are nonequivalent, then $W(u)$ at $u = u_c$ has a jump or a logarithmic singularity:

$$W_{\text{sing}}(u) \approx \pi\nu(\varepsilon_F) \sum_{n=1}^{N} \frac{|\mu_n - \bar{\mu}|^2}{\beta_n} \begin{cases} \pi & (O\text{-type}) \\ \ln\left|\dfrac{v_c}{|\Delta u|}\right| & (X\text{-type}) \end{cases}. \tag{22}$$

Here

$$\beta_n = \left(\oint dS_p/v\right) \Big/ u_c m_n^2, \quad \bar{\mu} = \left(\sum_{n=1}^{N} \mu_n m_n^2\right) \Big/ \sum_{n=1}^{N} m_n^2,$$

μ_n and m_n are the values of μ and m_c at the n-th point.

5. The function $W(u)$ can be calculated in explicit form in the nearly free electron model, in which the deviation of the FS from spherical is significant only near the Bragg-reflection planes. Near one of the edges of the Brillouin zone, the energy of the electrons is described by the equation[12]

$$\varepsilon(\mathbf{p}) = \frac{1}{2}(\varepsilon_{\mathbf{p}+\mathbf{G}/2} + \varepsilon_{\mathbf{p}-\mathbf{G}/2}) \mp \frac{1}{2}[(\varepsilon_{\mathbf{p}+\mathbf{G}/2} - \varepsilon_{\mathbf{p}-\mathbf{G}/2})^2 + 4|V_G|^2]^{1/2}. \tag{23}$$

Here V_G is the Fourier component of the pseudopotential of the lattice corresponding to the reciprocal lattice vector $\mathbf{G}/\hbar$. Equation (23) describes both "necks" (here and below these correspond to the upper sign) and "lenses" (the lower sign). The reciprocal effective-mass tensor entering into the cross section (7) is as a whole determined by the anisotropic contribution to the spectrum. Here

$$\mu_{\alpha\beta} = \delta_{\alpha\beta} \mp \frac{G_\alpha G_\beta |V_G|^2}{4m_0[(\mathbf{Gp}/2m_0)^2 + |V_G|^2]^{1/2}}. \tag{24}$$

It should be noted that Eq. (24) and what follows are valid for the description of the scattering processes under the condition $V_G \gg \hbar\omega$. In the opposite limiting case the Bragg reflection of the electrons can be taken into account by perturbation theory. The value of the cross section in this region is proportional to $(V_G/\varepsilon_F)^2$.

For a FS with a single neck (lens) it is not difficult to calculate the quantity which, according to (12) and (19), determines the scattering cross section:

$$\langle|\mu^{(i,s)} - \langle\mu^{(i,s)}\rangle|^2\rangle = \frac{|\mathbf{G}^{(i)}|^2|\mathbf{G}^{(s)}|^2}{4G^2(\mathbf{p}\pm\mathbf{G}/2)^2}\left[\frac{3\pi}{16}\frac{\mathbf{G}(\mathbf{p}\pm\mathbf{G})}{2m_0 V_G} - 1\right]. \tag{25}$$
$$G = |\mathbf{G}|, \quad \mathbf{G}^{(i)} = G\mathbf{e}^i, \quad \mathbf{G}^{(s)} = G\mathbf{e}^s.$$

It is seen from (25) that the presence on the FS of a portion of large curvature leads to an increase in the integrated cross section by a factor $\sim\varepsilon_F/V_G$, while in the approximation of lowest order in V_G/ε_F the scattering cross section is determined by the immediate vicinity of this portion. In this approximation, if the FS has several necks (lenses) their contributions to the cross section are additive.

In the calculation of the function $W(u)$, we take it into account that the quantity $\mu_{\alpha\beta}(\mathbf{p})$ is anomalously large in a narrow region of small $\mathbf{p}\cdot\mathbf{G}$. Just this region determines (apart from corrections $\sim V_G/\varepsilon_F$) the value of $W(u)$ for all strips which pass through it. Thus, for example, for a lens at $\mathbf{n}\perp\mathbf{G}$, all the strips pass through the region of small $\mathbf{p}\cdot\mathbf{G}$. Correspondingly, by using (24) and carrying out integration in (9) over the vicinity of the region of small $\mathbf{p}\cdot\mathbf{G}$, we obtain

$$W(u) = \frac{3\pi}{128}\frac{\mathbf{G}^{(i)}\mathbf{G}^{(s)}}{G(\mathbf{p}-\mathbf{G}/2)}\frac{\nu(\varepsilon_F)u}{V_G[2m_0\varepsilon_F - (G/2)^2 - (m_0u)^2]^{1/2}}. \tag{26}$$

Equation (26) is valid up to

$$u \approx v_0 = [2m_0^{-1}(\varepsilon_F - V_G) - (G/2m_0)^2]^{1/2}.$$

In the narrow interval

$$v_0 < u < v_{max} = [(2\varepsilon_F/m_0) - (G/2m_0)^2]^{1/2}$$

the function $W(u)$ falls off, as a result of screening, from its maximum value $\sim(\varepsilon_F/V_G)$ to zero.

At arbitrary orientation of the vector $\mathbf{n}$ relative to the axis of the lens, the $W(u)$ dependence has two characteristic portions: 1) the portion $u < v_1 = v_{z\max}\sin\alpha$ [where $\cos\alpha = (\mathbf{n}\cdot\mathbf{G})/G$] in which the strips $\mathbf{n}\cdot\mathbf{v}_\mathbf{p} = u$ intersects the region of large $\mu(\mathbf{p})$; here $W(u) \propto (v_1 - u)^{-1/2}$; 2) the region $v_1 < u < v_{z\max}$, in which the quantity $W(u)$ is determined by the vicinity of the O-type point. The function $W(u)$ for the lens is shown in Fig. 3.

In similar fashion, we can determine $W(u)$ for a sphere with several necks. Additional information on the anisotropy of the electron spectrum and the shape of the FS is contained in the dependence of $W(u)$ on the polarization of the incident and scattered radiations. Thus, at the polarization-vectors orientations $\mathbf{e}^i \perp \mathbf{G}$ and $\mathbf{e}^s \perp \mathbf{G}$, where $\mathbf{G}$ is the reciprocal lattice vector parallel to the symmetry axis of the region of large curvature, the contribution of this region to $W(u)$ is sharply reduced, in accord with (25).

6. In conclusion, we discuss the effect of volume and

surface collisions of electrons on the spectrum of the light scattered by the electrons.

The singularities in the dependence of $W(u)$ appear upon satisfaction of the condition $\omega \gg \nu$, at which the contribution to the distribution function of the electrons, determined from the kinetic equation (A7), has a delta-function character. For this reason, as in the problem of the anomalous skin effect,[12] we can take the scattering of the electrons into account by the introduction of a finite relaxation time $\tau[\varepsilon(\mathbf{p})]$, which has the meaning of the time of "departure" of the electron from the strip $\mathbf{n}\cdot\mathbf{v}_\mathbf{p} = u$. As is well known, the relaxation time decreases rapidly when the excitation energy rises above the Fermi level, so that the quantity $1/\tau[\varepsilon(\mathbf{p})]$ becomes equal to the energy of excitation at $[\varepsilon(\mathbf{p}) - \varepsilon_F]/\hbar \approx (3\text{–}6)\cdot 10^{14}\ \mathrm{s}^{-1}$ (see Ref. 12). Correspondingly, in the reconstruction of the function $W(u)$ from the frequency dependence of the light-scattering cross section, the singularities in the region $u \lesssim \delta/\tau(\varepsilon_F + \hbar u\delta^{-1})$ will be smeared out. In view of the strong dependence of τ on the energy, better conditions for the determination of the shape of the Fermi surface should obviously be achieved in semimetals and alloys with sufficiently large penetration depth of the radiation, in which the inequality $u \ll \delta/\tau[\varepsilon(\mathbf{p})]$ can be satisfied with sufficient margin.

Scattering of electrons by surface defects is taken into account in the boundary conditions (A8) to the kinetic equation (A7), as in the problem of the anomalous skin effect.[5,12] However, in the problem of light scattering (in contrast to Refs. 5 and 12), the response of the system is determined at a specific distribution of the external field (see the Appendix), which materially simplifies the problem. Use for δn_p of the integral boundary condition (A8), which describes the scattering from surface roughnesses, leads to the appearance of an additional term J_ω' in the expression for the function J_ω that determines the scattering cross section (2):

$$J_\omega' = S\frac{32F(\omega)}{[(n+1)^2+\varkappa^2]^2} \times \int_0^\infty du \int_0^\infty du' W(u,u') \frac{16\delta^2 u^2}{[4u^2+\omega^2\delta^2][4u'^2+\omega^2\delta^2]}, \quad (27)$$

$$W(u,u') = \frac{2}{(2\pi\hbar)^3}\oint \frac{dS_\mathbf{p}}{v}\frac{\tilde{\mu}^{(i,s)}(\mathbf{p},u)v_z}{u^2-v_z^2}\int \frac{d^2p_\perp}{(2\pi)^3\hbar^4} p_z p_z' V(\mathbf{p}_\perp-\mathbf{p}_\perp') \times\left[\frac{\tilde{\mu}^{(i,s)}(\mathbf{p}',u')v_z'^2}{v_z'^2-u'^2} - \frac{\tilde{\mu}^{(i,s)}(\mathbf{p},u')v_z^2}{v_z^2-u'^2}\right]. \quad (28)$$

This term does not change the asymptotic values of the scattering cross section and of its integrated intensity, and has the smallness of $(a/d)^2$ in the case of a sufficiently smooth surface [see (A8)].

APPENDIX

The effective Hamiltonian of the interaction of the electron with the field of the incident and scattered electromagnetic waves has the form[1,2]

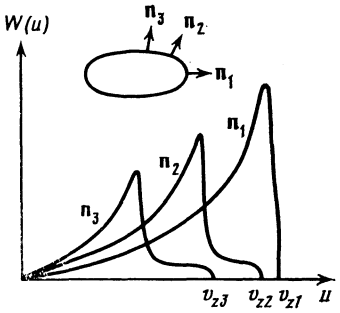

FIG. 3

$$H = \frac{e^2}{m_0c^2}\int d^3r A_\alpha^i(\mathbf{r},t)A_\beta^s(\mathbf{r},t)\sum_\mathbf{p}\mu_{\alpha\beta}(\mathbf{p})n_\mathbf{p}(\mathbf{r},t). \quad (A1)$$

Here $A_\alpha^i(\mathbf{r},t)$ and $A_\beta^s(\mathbf{r},t)$ are the vector potentials of the fields of the incident and scattered waves. As was shown in Refs. 1 and 13, at $\omega_i - \omega_s \ll \omega_i$ the terms in the interaction Hamiltonian that are linear in the field of the electromagnetic wave give a small ($\sim v/c$) contribution to the scattering cross section in comparison with the quadratic term (A1). The vector potentials entering into (A1) correspond to the field within the metal and are expressed in terms of the field outside the metal with the help of Maxwell's equations. The Hamiltonian (A1) corresponds to the point potential energy of interaction of a photon with an electron. This allows us to express the scattering cross section of the photon, by a standard method,[3] in terms of the correlator of fluctuations of the effective mass tensor of a unit volume, namely the function J_ω in (2) is equal to

$$J_\omega = \frac{1}{|A_0^i|^2|A_0^s|^2}\int d^3r \int d^3r' A_\alpha^i(\mathbf{r})A_\beta^{*s}(\mathbf{r})A_\gamma^{*i}(\mathbf{r}')A_\delta^s(\mathbf{r}') \times\langle\delta\mu_{\alpha\beta}(\mathbf{r},t)\delta\mu_{\gamma\delta}(\mathbf{r}',0)\rangle_\omega, \quad (A2)$$

$$\delta\mu_{\alpha\beta}(\mathbf{r},t) = \sum_\mathbf{p}\mu_{\alpha\beta}(\mathbf{p})\delta n_\mathbf{p}(\mathbf{r},t). \quad (A3)$$

Here A_0^i and A_0^s are the amplitudes of the electromagnetic waves outside the metal, and $\delta n_p = n_p - n_0$. The correlator in (A2) can be expressed in terms of the generalized susceptibiity of the electrons relative to the weak external field

$$U_{\alpha\beta} = \frac{e^2}{m_0c^2}A_\alpha^i(\mathbf{r},t)A_\beta^{*s}(\mathbf{r},t). \quad (A4)$$

We represent the response of the system to the external field (A4) in the form

$$\langle\delta\mu_{\alpha\beta}\rangle_\omega = -\int d^3\mathbf{r}'\alpha_{\alpha\beta\gamma\delta}(\mathbf{r},\mathbf{r}',\omega)U_{\gamma\delta}(\mathbf{r}',\omega). \quad (A5)$$

Then, according to the fluctuation-dissipation theorem, the correlator from (A2) is expressed in terms of the generalized susceptibility $\alpha_{\alpha\beta\gamma\delta}$ by the equation

$$\langle\delta\mu_{\alpha\beta}(\mathbf{r},t)\delta\mu_{\gamma\delta}(\mathbf{r}',0)\rangle_\omega = \frac{2\hbar}{1-\exp(-\hbar\omega/T)}\operatorname{Im}\alpha_{\alpha\beta\gamma\delta}(\mathbf{r},\mathbf{r}',\omega). \quad (A6)$$

The generalized susceptibility α can be found from a solution of the kinetic equation for $\delta n_p(\mathbf{r})$

$$-i\omega\delta n_{\mathbf{p}}+v_z\frac{\partial}{\partial z}\delta n_{\mathbf{p}}+\frac{1}{\tau_{\mathbf{p}}}\delta n_{\mathbf{p}}$$

$$=\left[v_z\mu_{\alpha\beta}(\mathbf{p})\frac{\partial}{\partial z}U_{\gamma\delta}(z,\omega)-e\mathbf{E}\mathbf{v}\right]\frac{\partial n_0}{\partial\varepsilon} \qquad \text{(A7)}$$

with the boundary condition determined by the character of the reflection of the electrons from the boundary of the metal. Under sufficiently general assumptions, this condition can be written in the form[5,14]

$$\delta n_p^{>}(z=0)=\delta n_p^{<}(z=0)+p_z\int\frac{d^2p_\perp}{(2\pi\hbar)^2}p_z'V(\mathbf{p}_\perp-\mathbf{p}_\perp')$$
$$\times[\delta n_{\mathbf{p}'}^{<}(z=0)-\delta n_{\mathbf{p}}^{<}(z=0)]. \qquad \text{(A8)}$$

Here $V(\mathbf{p}_\perp-\mathbf{p}_\perp')$ is the Fourier component of the binary correlator of the surface roughnesses. It differs from zero in the region $|\mathbf{p}_\perp-\mathbf{p}_\perp'|\lesssim\hbar/d$, where its value is $V(0)\sim a^2d^2$; a is the mean amplitude, d is the correlation length of the roughnesses. The electric field $\mathbf{E}$ is found from the condition of neutrality.

The sign $\gtrless$ at δn_p denotes: $v_z\gtrless 0$.

Calculation of the response (A5) with the boundary condition (A8) is similar to finding the current in the problem of the anomalous skin effect.[5,12] The final result for the quantity J_ω, which determines the scattering cross section, is given in the case of specular reflection of the electrons from the surface, when $V(\mathbf{p}_\perp-\mathbf{p}_\perp')=0$, by the equations (3)–(7), and allowance for the surface scattering leads to the appearance of the additional term (27).

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